FINM 37601

HW2

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1. $$E(I) = \rho_\ell \left(b - \frac{a+b}{2}\right) + \rho_m \left(\frac{a+b}{2} - \frac{a+b}{2}\right) + \rho_u \left(a - \frac{a+b}{2}\right)$$

$$= \rho_\ell \frac{b-a}{2} + \rho_u \frac{a-b}{2} + \rho_m 6$$

$$= \frac{1}{2} (\rho_\ell - \rho_u)(b-a)$$

2. $$Var(I) = \sum_i \left[p(i) \left[I(i) - E(I) \right]^2 \right]$$

$$= \rho_\ell \left(\frac{b-a}{2} - E(I)\right)^2 + \rho_u \left[\frac{a-b}{2} - E(I)\right]^2 + \rho_M \left(0 - E(I)\right)^2$$

$$= \frac{1}{4} \left(\rho_\ell - \rho_u\right)^2 (b-a)^2$$

3. $u = \frac{1}{2}(\rho_e - \rho_u)(b-a) + \frac{\lambda}{4}(\rho_e - \rho_u)^2(b-a)^2$

$= \frac{1}{2}(\rho_e - \rho_u)(b-a)\left[1 + 2\lambda(\rho_e - \rho_u)(b-a)\right]$

Due to the term $(\rho_e - \rho_u)^2$ we observe a strong effect of large spread.

So this shows risk averse and a big impact of λ.

— ①

4. using ① and $\rho_e \approx \rho_u$ we get $u = 0$

This is independent of fm and the spread

There will be a cancelling out effect in aversion to var and expected market impact when the probability is balanced

5. Holding risk = var of impact over the horizon of execution.

$$Var(I) = \rho_l \, Var(I)_b \, T_m + \rho_u \, var(I)_a \, T_m + f_m \, n_{threshold} \, var\left[I \frac{(a+b)}{2}\right] (T_s - T_m)$$

So we have $Var(I)_a = Var(I)_b = \frac{T}{3}$

and $Var(I_{\frac{a+b}{2}}) = 0$

So $Var(I) = \frac{\rho_l \, T \, T_m}{3} + \frac{\rho_u \, T \, T_m}{3}$ —— 3

For $f_m = 0$, the time is T_m

$Var(I) = (\rho_l + \rho_u) \frac{T}{3} T_s \, f_m \, threshold$

Using the given information we get,

$$Var(I) = \frac{2 T T_s}{3} f_m \, threshold \, \rho_l$$

$$= \frac{T T_s}{3} f_m \, threshold \, (1 - f_m \, threshold)$$

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6. Using ② and f_m threshold $\sim 1-e^{-xT_m}$,

$$Var(I) = \frac{T T_s e^{-xT_m}}{3}\left(1-e^{-xT_m}\right)$$

For minimizing this, T_m^* should be $1-e^{-xT_m}=0$ or $e^{-xT_m}=0$ as f_m threshold lies between 0 & 1.

As $T_m = \infty$, $e^{-xT_m}=1$, $-xT_m=0$
Then $T_m=0$ which does not hold

So we need max T_m and equal to T_s

7. From (3), we use

$$Var(I) = \frac{\rho_e T T_m}{3} + \frac{\rho_u T T_m}{3}$$

But $T_m = \frac{(\rho_e + \rho_u) T_s}{r} f_{m\ threshold}$

Then $Var(I) = \frac{\rho_e T T_s}{3} \left(\frac{\rho_e + \rho_u}{r} \right) f_{m\ threshold}$

$$+ \frac{\rho_u T T_s}{3} \left(\frac{\rho_e + \rho_u}{r} \right) f_{m\ threshold}$$

$$= \frac{T T_s}{3} (\rho_e + \rho_u) f_{m\ threshold}$$

$$= \frac{T T_s}{3} (1 - f_{m\ threshold}) f_{n\ threshold}$$

Using results in problem 6 to derive minimum Var(I), we have

$e^{-\alpha T_M} = 0$ or $1 - e^{-\alpha T_M} = 0$

$T_m = \infty$ which implies $e^{-\alpha T_m} = 1$
and $\alpha T_m = 0$
$T_m = 0$ which does not hold

So $T_m = \infty$ should be maximum possible or equal to T_s