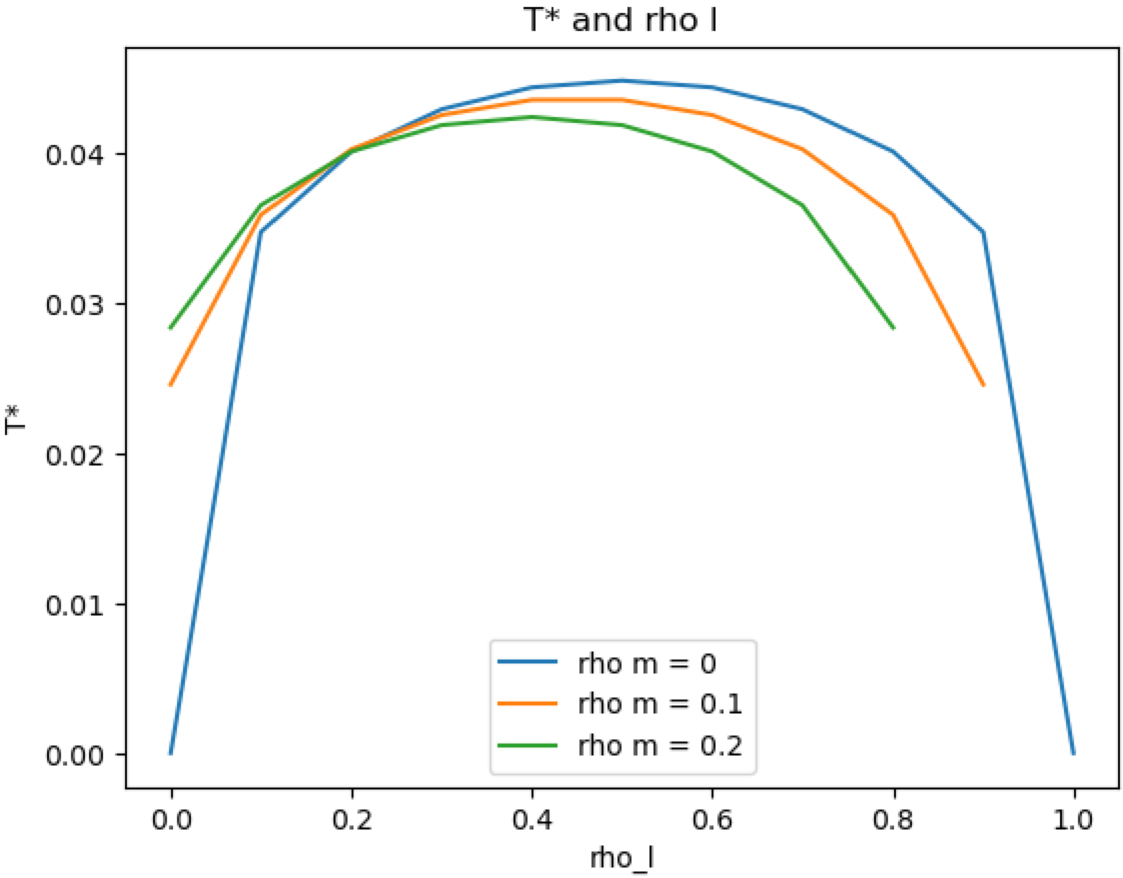
Problem 5

In [2]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

In [3]:
```
#given values assigned and sigma annualised
t_star_values = []
b_value = 62
a_value = 62.05
# 20% annualized
sigma_value = 0.2
p0_value = 62.025
```

In [4]:
```
for rho_m_value in [0, 0.1, 0.2]:
    rho_l_values = np.arange(0, 1.1 - rho_m_value, 0.1)
    temp_values = []

    for rho_l_value in rho_l_values:
        rho_u_value = 1 - rho_l_value - rho_m_value
        expected_p = rho_l_value * b_value + rho_u_value * a_value + rho_m_v
        variance_p = rho_l_value * ((b_value - expected_p) ** 2) + rho_u_va
        temp_values.append(variance_p ** 0.25 / ((p0_value * sigma_value) **

    t_star_series = pd.Series(temp_values, index=rho_l_values)
    t_star_values.append(t_star_series)
```

In [8]:
```
#plot
fig, ax = plt.subplots()

for index, rho_m_value in enumerate([0, 0.1, 0.2]):
    ax.plot(t_star_values[index], label=f"rho m = {rho_m_value}")

plt.legend()
plt.xlabel("rho_l")
plt.ylabel("T*")
plt.title("T* and rho l")
plt.show()
```



T* and rho l
T*
rho_l
0.00
0.01
0.02
0.03
0.04
0.0
0.2
0.4
0.6
0.8
1.0
rho m = 0
rho m = 0.1
rho m = 0.2

FINM 37601
HW1
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1. mean of transaction price

$$E[p] = \rho_e a + \rho_u b + \frac{\rho_m}{2}(a+b) \quad \text{—— ①}$$

variance of transaction price

$$Var[p] = \rho_e (a - E(p))^2 + \rho_u (E(p) - b)^2 + \rho_m \left(E(p) - \frac{a+b}{2}\right)^2 \quad \text{—— ②}$$

2. Since $\rho_e \approx \rho_u + \delta$ where δ is very small, we can approximate $\rho_e = \rho_u$

Then in ①,

$$E[p] = (b+a)\left[\frac{\partial \rho_u + \rho_m}{2}\right] = \frac{b+a}{2} + \partial b \quad \text{—— ③}$$

and in ②

$$Var[p] = \rho_e \left(\frac{b-a}{2}\right)^2 + \rho_u \left(\frac{a-b}{2}\right)^2 + \rho_m (0)$$

$$= \left(\frac{a-b}{2}\right)^2 (\rho_e + \rho_u) = \left(\frac{a-b}{2}\right)^2 \rho_u$$

3. since $\rho_m << \rho_\ell, \rho_m$

Then in ①, $E[p] \approx \rho_\ell a + \rho_u b$ ⑤

and in ②, $Var[p] \approx \rho_\ell (b - E(p))^2 + \rho_u (a - E(p))^2 + \rho_m \left(\frac{a+b}{2} - E(p)\right)^2$

$= (\rho_\ell^3 + \rho_u^3)(b-a)^2$ ⑥

4. Given $\gamma = \frac{1}{2}$ and $\sqrt{Var[p]}$ scales with $\sigma T_*^{1/2}$ as $\sqrt{Var[p]} \sim p_0 \sigma T_*^{1/2}$

Assuming $|\rho_\ell - \rho_u| = \sigma << 1$ from Q2, we can substitute in ④ as

$p_0 \sigma T_*^{1/2} \sim \sqrt{\rho_u (a-p)^2 + \rho_\ell (p-b)^2}$

Assuming $\rho_m << 1$ from Q3, we have ⑥ as follows.

$\sqrt{Var[p]} = T \sim \sqrt{\sqrt{\rho_\ell^3 + \rho_u^3}\ \frac{|b-a|}{p_0 \sigma}}$

5. Check at the end

6. The impact of dark pools on market quality can be complex. Some generalizations that we can draw from problems above are:

(a) Dark pools can reduce price volatility because block trades do not immediately hit market price. Probability of hitting mid quote ρ_m becomes low which means lower ρ_m values means lower variance = lower volatility in short run.

(b) How dark pools can fragment the market cos trading happens off public exchange. This can be a concern if we look at it from fairness of the price discovery process for market participants

(c) Regulators actively monitor dark pool in reality. So the true impact of dark pools is not direct and is controlled by regulators.

(d) It is difficult to measure direct impact because of many factors being involved.