EN 2040 Random Signals and Processes

Simulation Assignment

Rectangular pulses of $\pm A$ carry binary equiprobable data over a communication channel. Binary data $D \in \{0, 1\}$ is mapped to the amplitude of the rectangular pulses S as follows:

$$S = \begin{cases} +A & \text{if} \quad D = 1 \\ -A & \text{if} \quad D = 0 \end{cases}.$$

The channel is corrupted by additive white Gaussian noise (AWGN) of zero mean and variance σ^2. At the receiver,the received signal is sampled and compared with a threshold $\tau = 0$. A received signal sample is given by

$$R = S + N,$$

where N represents the random effect of noise. Decoding is done by considering the amplitude of R. To this end, the decision is taken as follows:

$$Y = \begin{cases} +A & \text{if} \quad R > \tau \\ -A & \text{if} \quad R \leq \tau \end{cases}.$$

1. Generate a binary sequence of length $L = 1000$, considering $D \in \{0, 1\}\, and \Pr(D = 0) = \Pr(D = 1) = 1/2$. Use the binary sequence to generate a stream of rectangular pulses of amplitude S, where $S \in \{-A, +A\}$ with $A = 1$.
2. Generate an AWGN sequence, also of length L = 1000, considering $\sigma^2 = 1$.
3. Plot the sequence of R and observe the impact of the variance of noise on R by varying σ^2.
4. Sketch and compare the sequence of Y with the transmitted signal.
5. Repeat the above steps for a sequence of length $L = 100,000$. Write a code to generate and plot the histogram of the received sequence taking the no of bins as 10. Compare your result with the one generated from the built-in function **hist**() of MATLAB.
   (a) Change the no of bins from 10 to 100 and observe the impact.
   (b) By selecting a suitable value for the no of bins, sketch the conditional PDFs $f_{R|S}(r|S = A)$ and $f_{R|S}(r|S = -A)$ with the use of the normalized histograms. Change the value of A and observe the impact.
   (c) Find $E[R|S = A], E[R|S = -A]$ and $E[R]$.

(d) Similarly, sketch the PDF $f_R(r)$.

6. Now, consider that there is interference I from other transmitters in addition to noise. Thus, the received sample can then be written as

$$R = S + N + I.$$

Assuming that I is also Gaussian distributed with zero mean and variance $\sigma_i^2 = 1$, repeat step $5 - (b)$ to (d) and discuss the impact of the addition of interference.

7. Finally, consider that the received signal is amplified by a factor of α, such that

$$R = \alpha S + N.$$

Repeat step $5 - (b)$ to (d) and discuss the impact of scaling.