University of Moratuwa

Department of Electronic and Telecommunication Engineering

EN2040

Random Signals and Processes

Simulation Assignment

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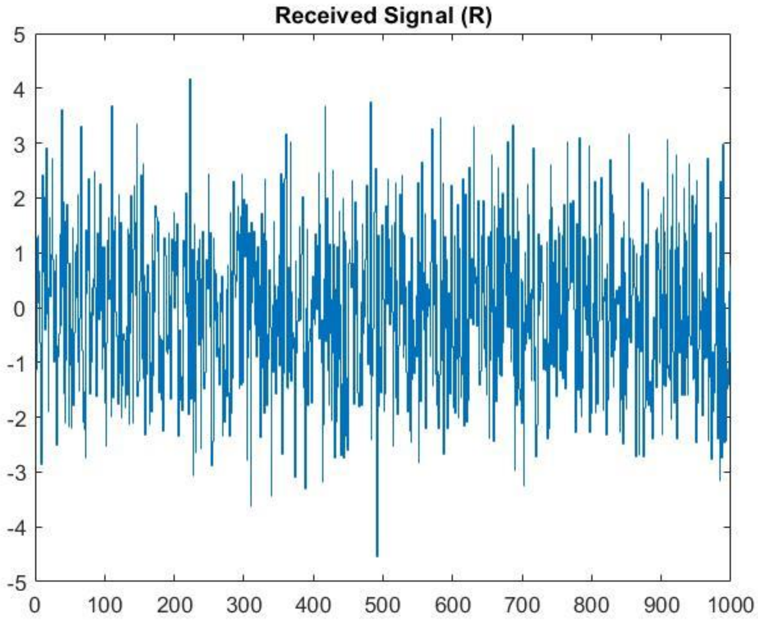
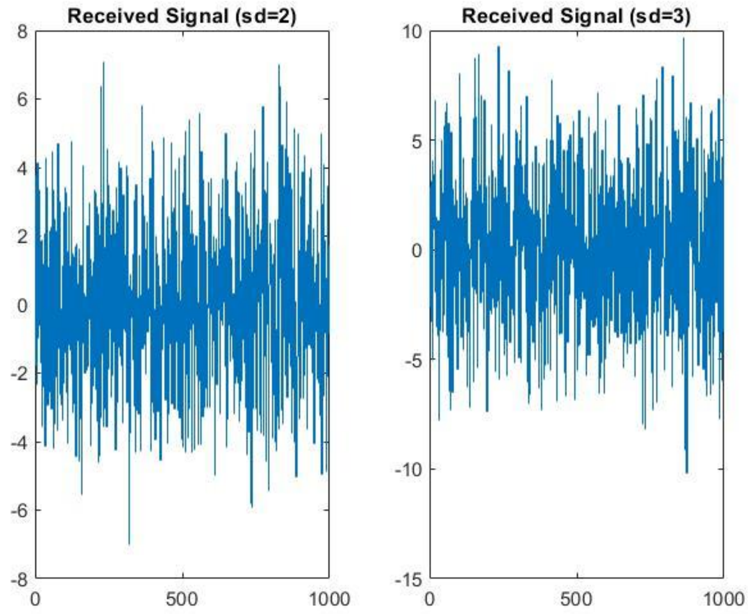
Question-3

First, we plot the received signal(R).



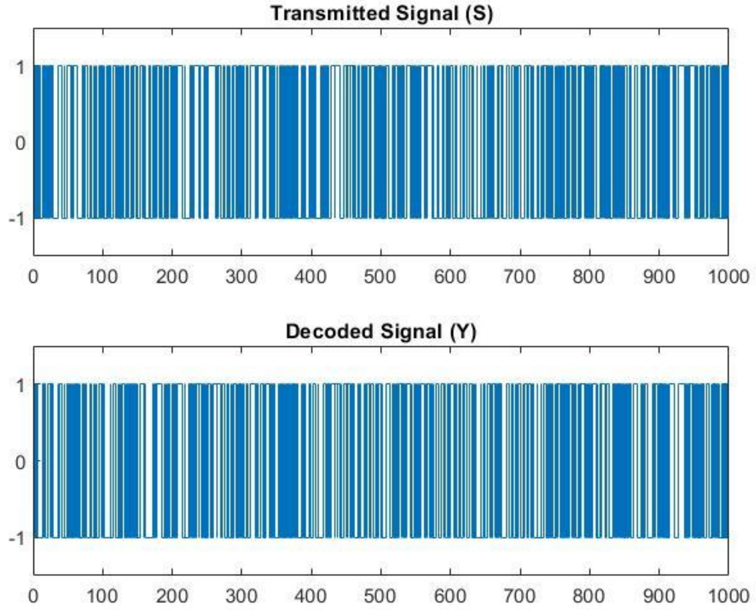
Next, we generate AWGN of different variances for observing the impact of the variance on AWGN on R (mean remains constant). Let's take variances; 4 and 9. Then we plot the received signals.



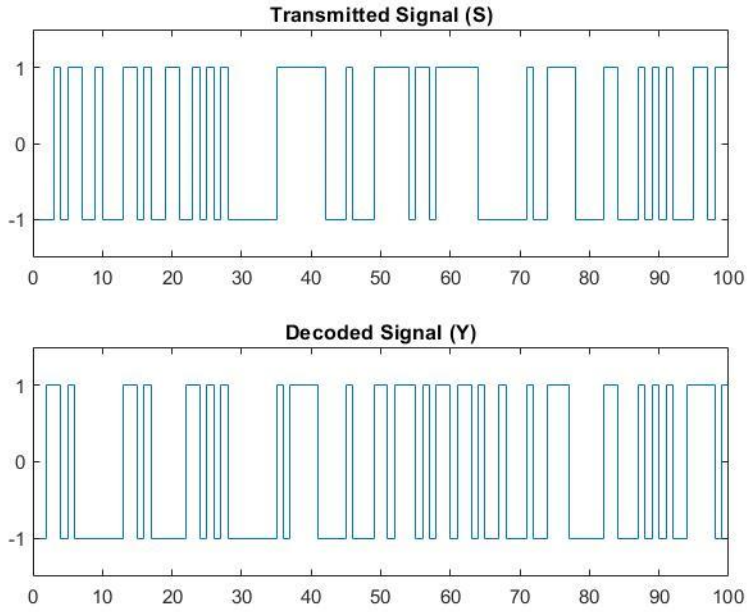
Answer: *From the received signals we can observe, the amplitude of the received signal increases with the increment of the variance of AWGN.*

Question-4

First, we generate the decoded signal; Y, and next, we plot the transmitted signal and the decoded signal.



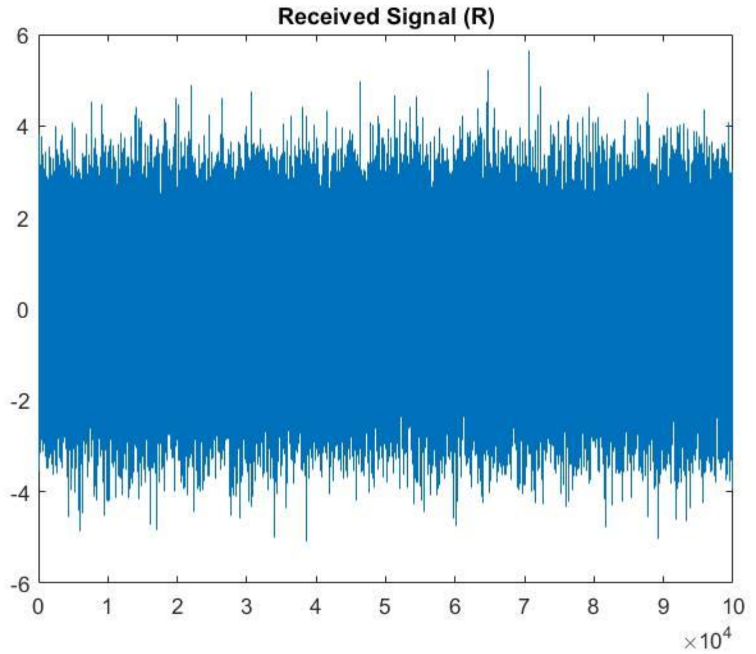
For comparing, we can zoom in S & Y.

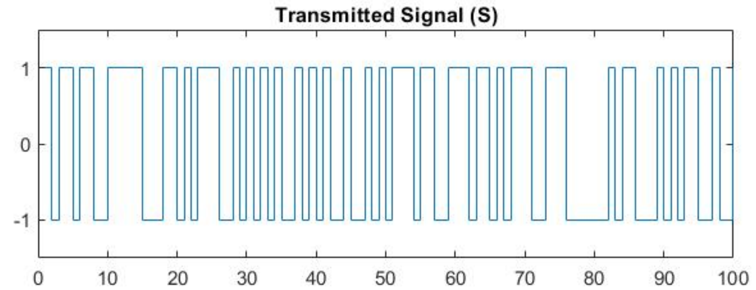


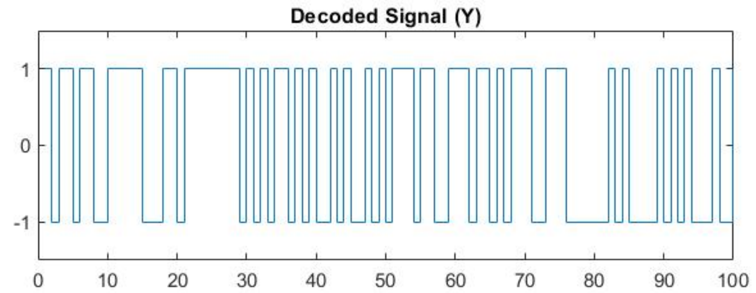
Answer: *From the above plots, we can see there are some errors in the received signal. The probability of an error occurring is significant.*

Question-5

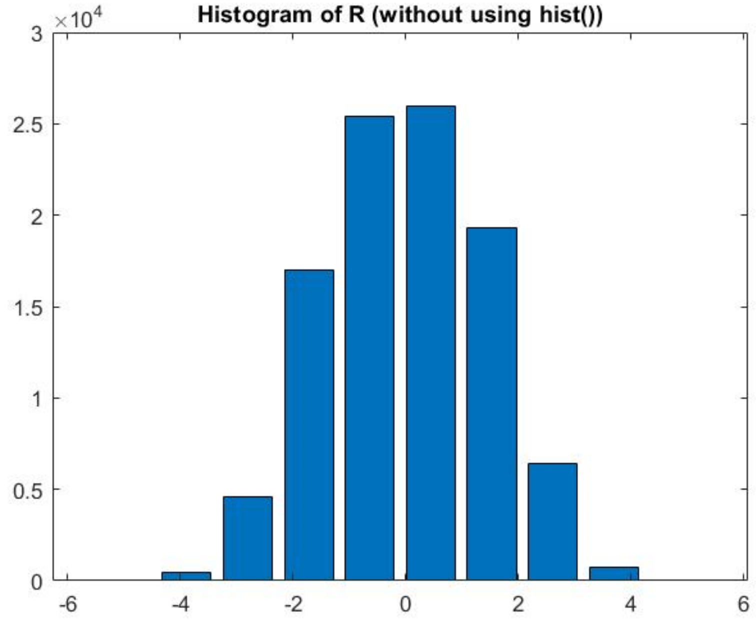
Now we change variable L to 100,000 and run the code.



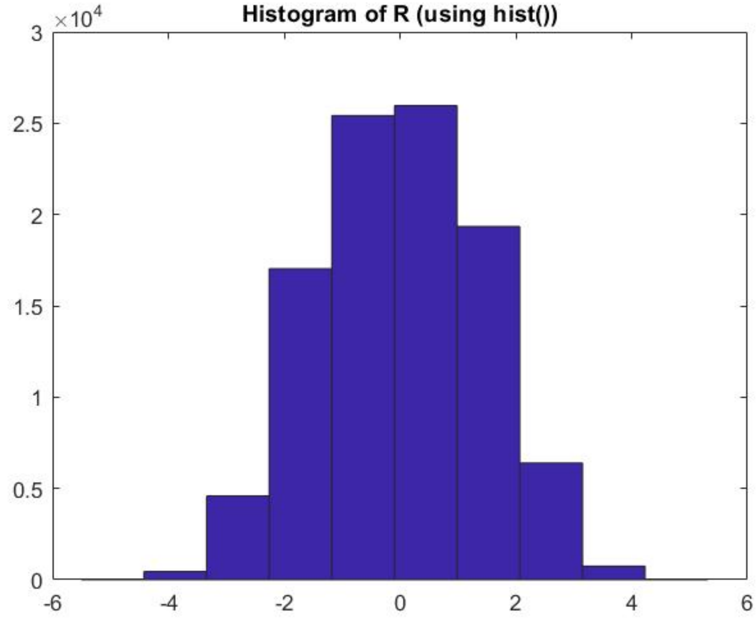




Next, we generate the histogram of R without using the built-in function; hist().



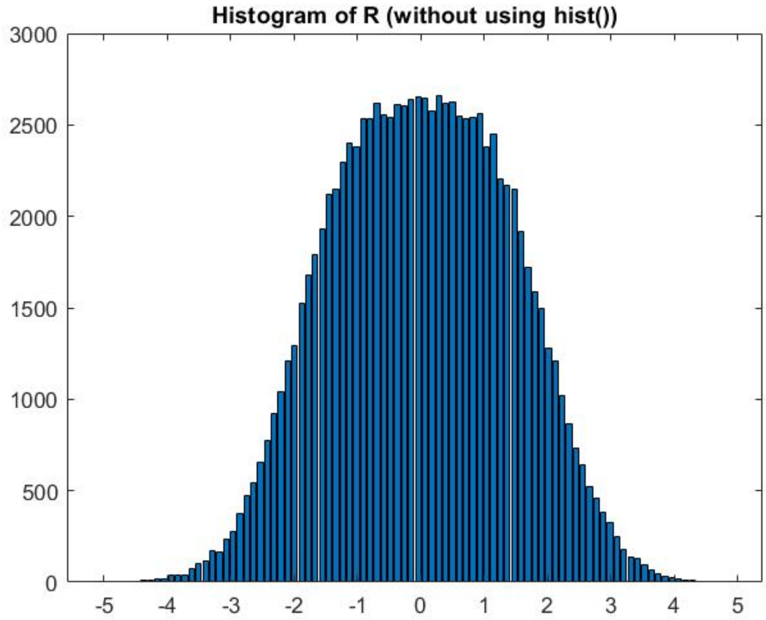
Next, we generate the histogram of R using the built-in function; hist().

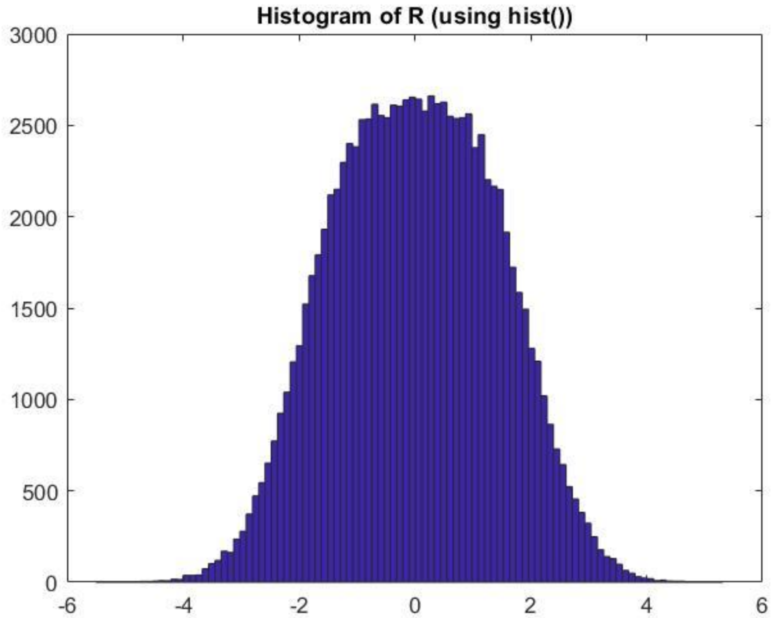


Answer: *Both histograms look similar. But the widths of each bin are different.*

Question-5 Part(a)

Now we change the number of bins to 100 and run the code.

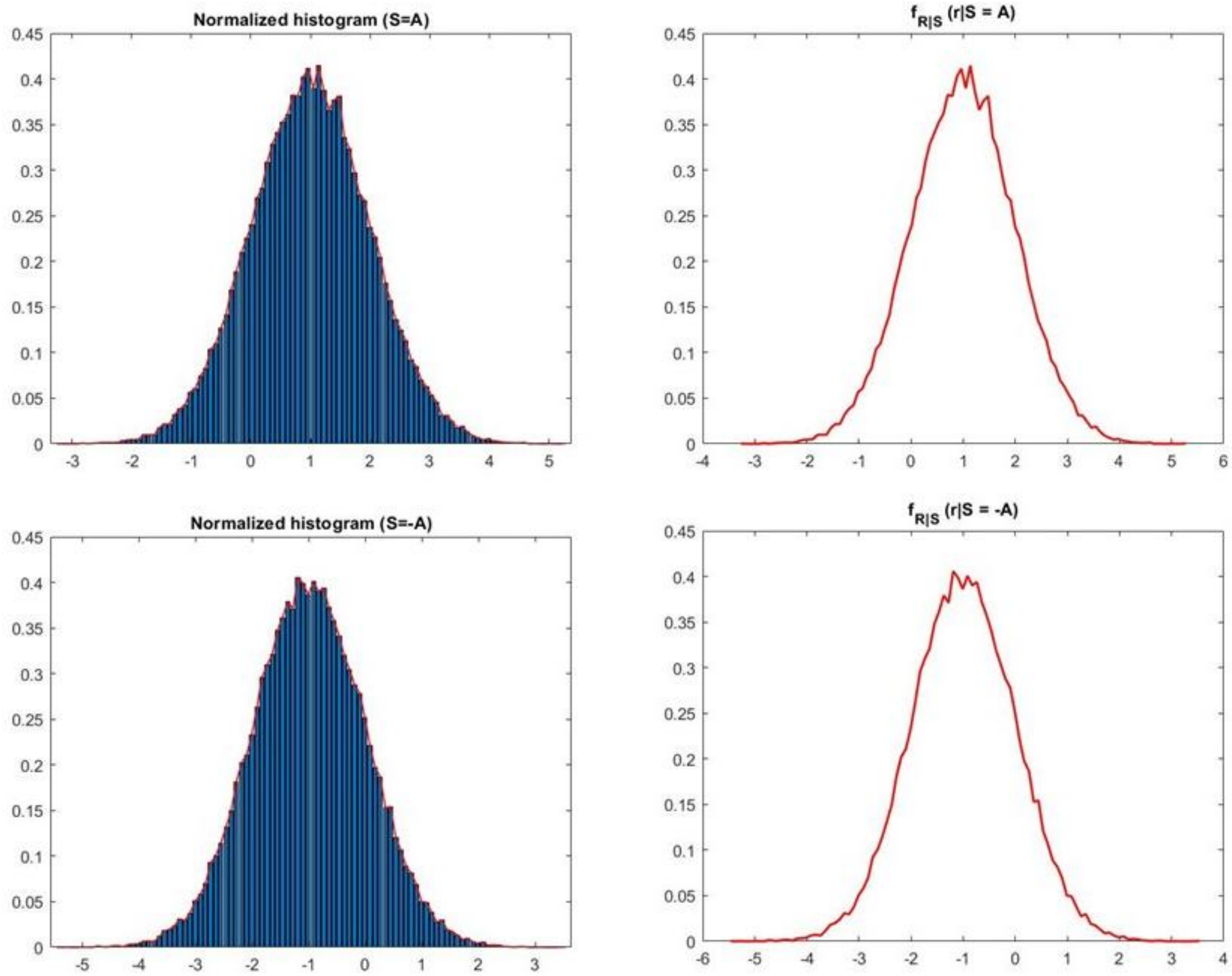




Answer: *When the number of bins=100 two histograms become more similar.*

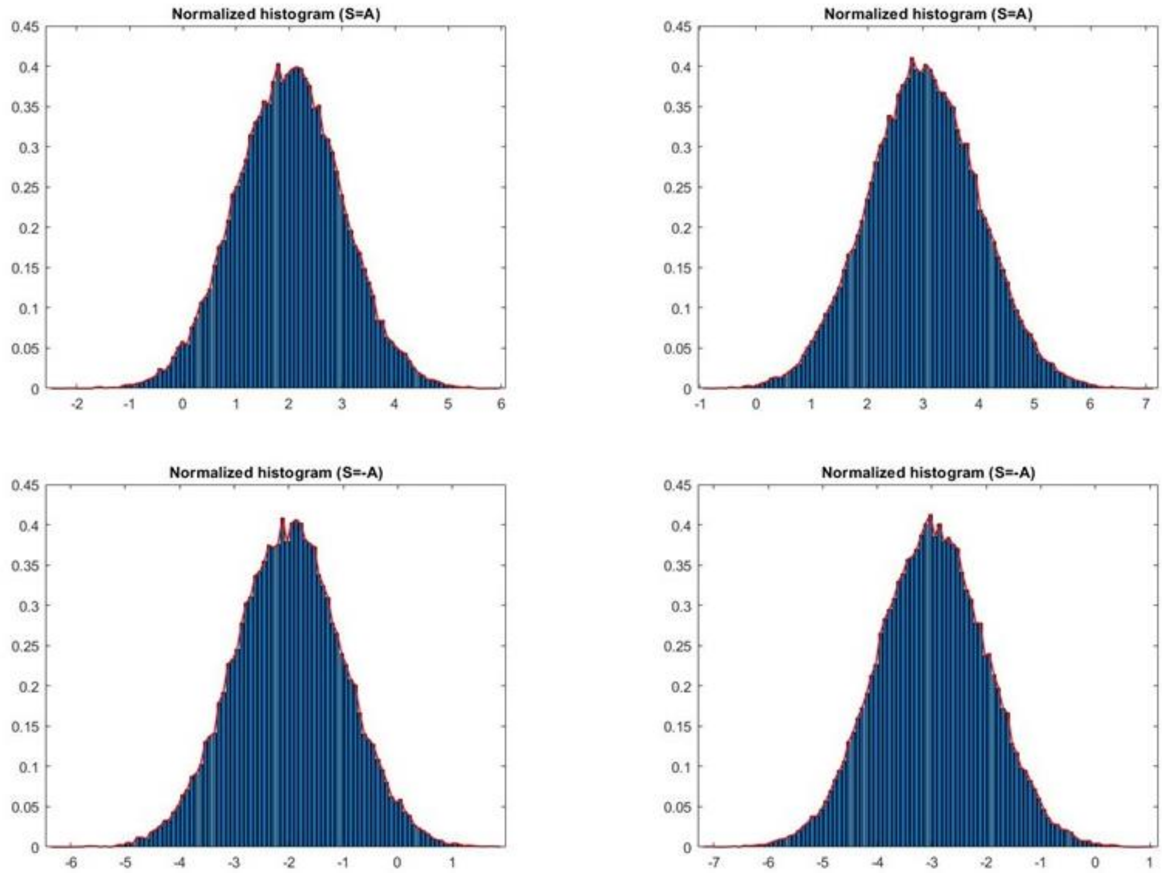
Question-5 Part(b)

- For this part, we select the number of bins as 100 for getting a more accurate pdf.
- First, we find R values when S equals A and S equals -A.
- Next, we generate normalized histogram when S=A and we get $f_{R|S}(r|S = A)$ from that.
- We calculate normalized values for the histogram by dividing the height of each bin by (total number of heights*width of a bin).



Next, we change A to 2 & 3 and run the code for observing the impact.

A = 2 **A = 3**



Answer: *When A is increasing μ is go away from zero. The expected value of $f_{R|S}(r|S = A)$ is nearly equal to A and the expected value of $f_{R|S}(r|S = -A)$ is nearly equal to -A. But the variation of A doesn't affect to the variance(*σ^2*).*

Question-5 Part(c)

The expected value can be got by the below equation.

$$E[X] = \int_{-\infty}^{\infty} x\, f_x(x)\, dx$$

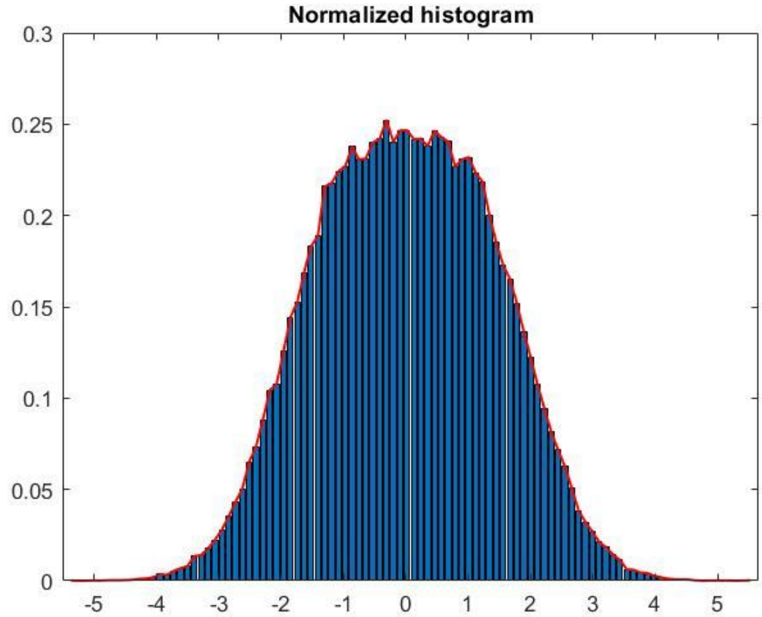
Since we have discrete sets of values, we can calculate the expected values by the below equation.

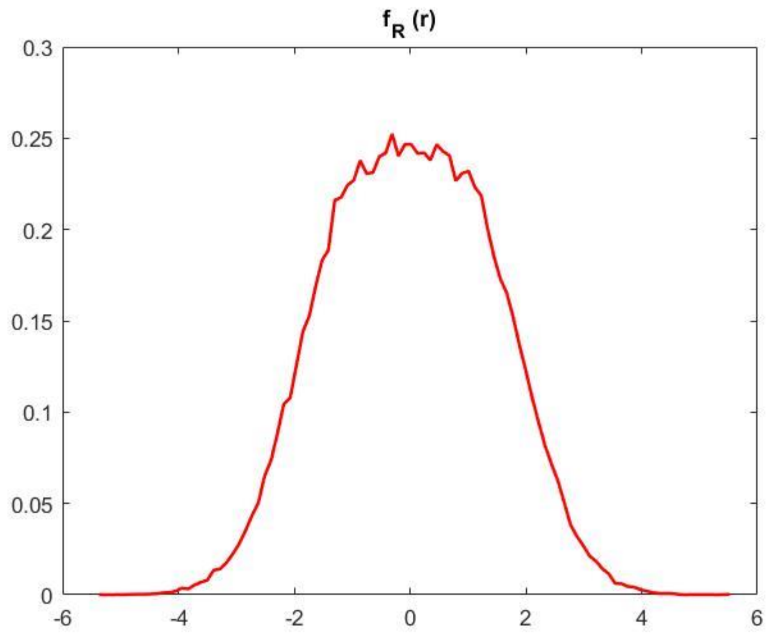
$$E[X] = \sum_{i=1}^{N} x_i \, f_X(x_i) \, \Delta x_i$$

- $E[R \mid S = A] = \sum_{i=1}^{N} r_i \; f_{R|S}(r_i| \, S = A) \, \Delta r_i$
- $E[R \mid S = -A] = \sum_{i=1}^{N} r_i \; f_{R|S}(r_i| \, S = -A) \, \Delta r_i$
- $E[R \,] = \sum_{i=1}^{N} r_i \, f_R(r_i|) \, \Delta r_i$

	E[R|S = A]	**E[R|S = -A]**	**E[R]**
A=1	0.9995	-0.9997	0.0028
A=2	1.9997	-1.9989	0.0076
A=3	3.0055	-3.0011	0.0027

Question-5 Part(d)

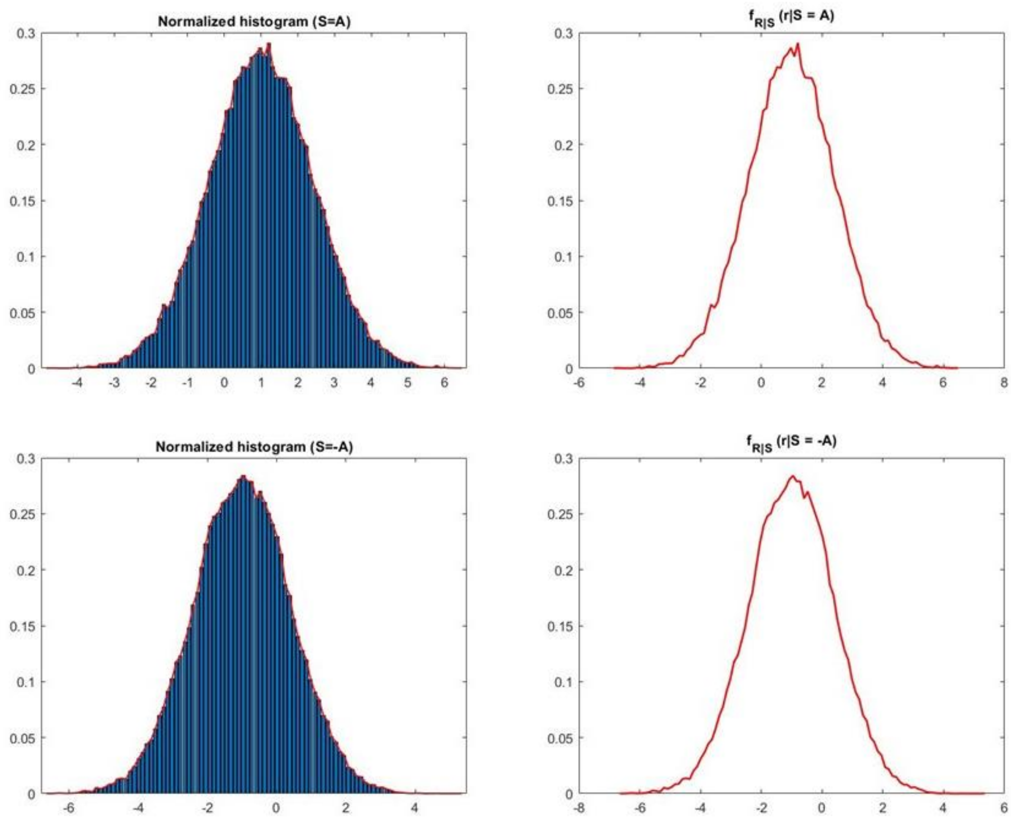


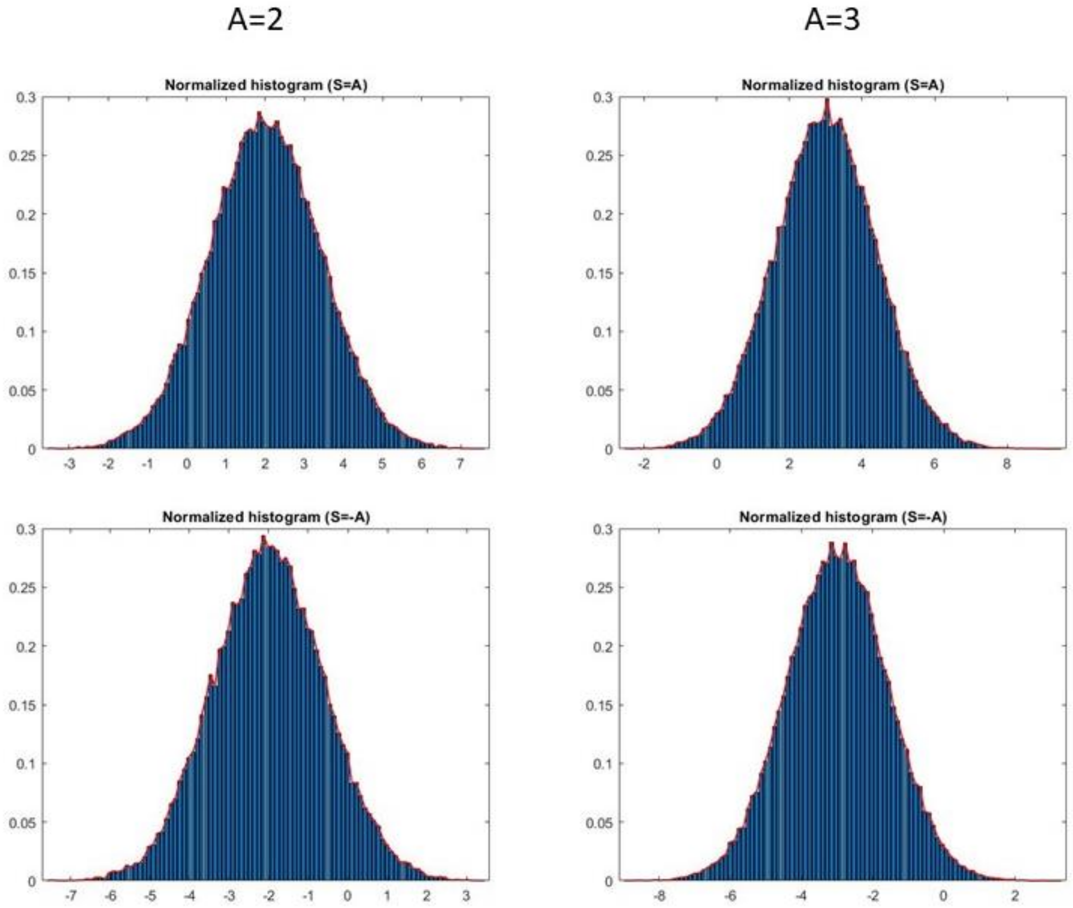


Question-6

First, we generate the interference I and get the received signal R. Next, we run the same code used to Question-5 part (b), part(c) and part(d) with the new R.

Part(b)

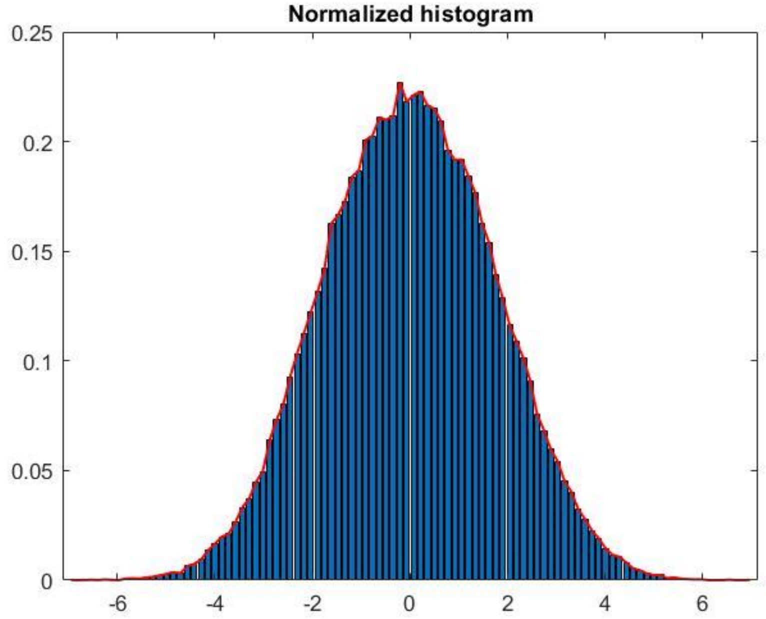


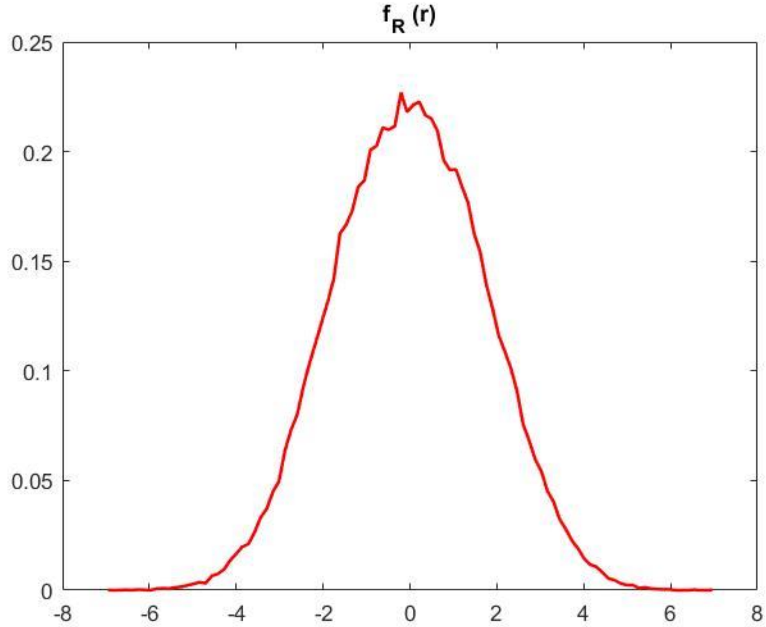


Part(c)

	E[R|S = A]	E[R|S = -A]	E[R]
A=1	0.9908	-1.0067	-0.0085
A=2	2.0044	-1.9958	0.0091
A=3	2.9978	-3.0004	-0.0033

Part(d)



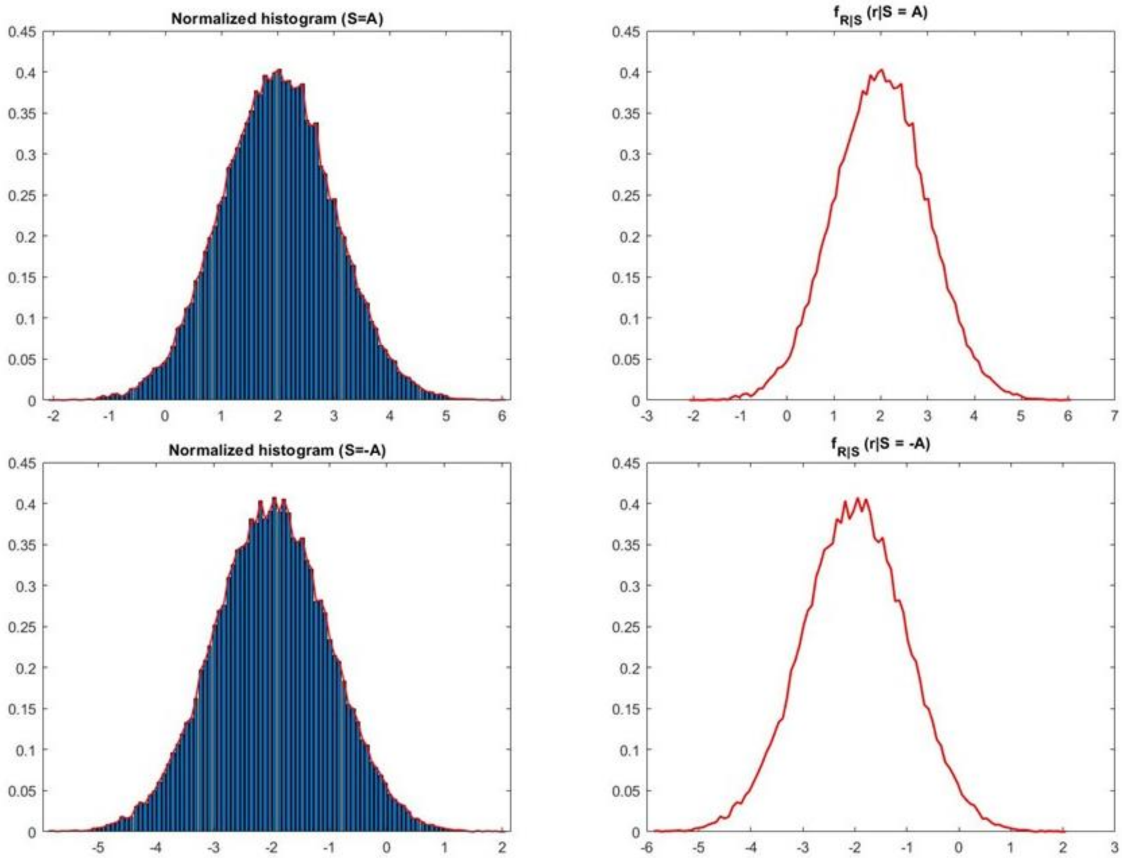


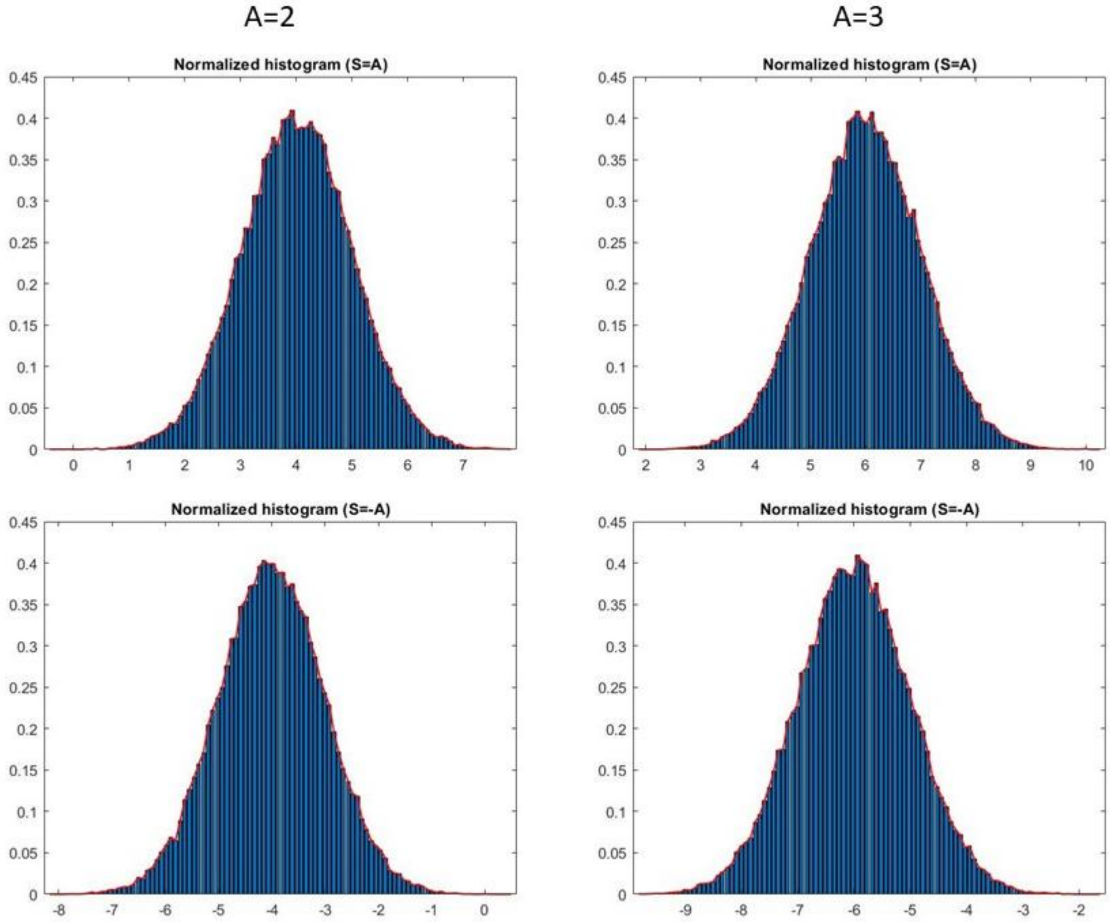
Answer: *When the interference has been added the ranges of the x-axis of the pdfs increase. The reason for that is when the interference is added to the transmitted signal the amplitude of the received signal is increased.*

Question-7

Taking α = 2, we generate the new received signal R. Next, we run the same code used to Question-5 part (b), part(c) and part(d) with the new R.

Part(b)

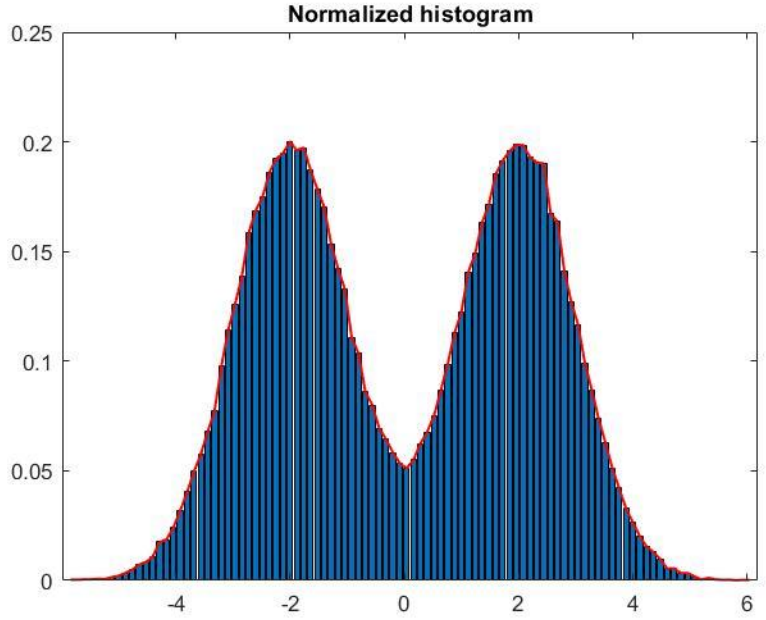


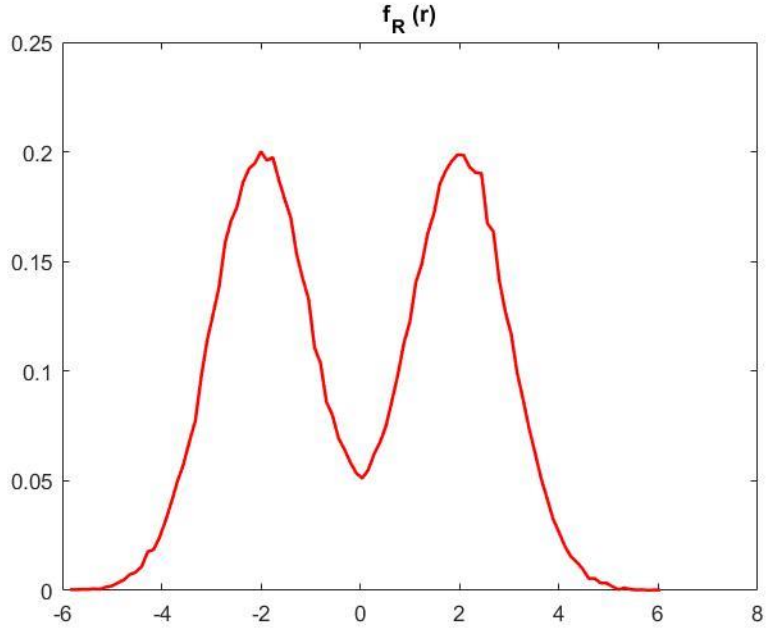


Part(c)

	E[R|S = A]	E[R|S = -A]	E[R]
A=1	1.9943	-2.0005	-0.0048
A=2	4.0052	-4.0095	-2.6949e-04
A=3	5.9975	-6.0007	0.0027

Part(d)





Answer: *When the received signal amplified by a factor signal to noise ratio is increased.*

APPENDIX

Question-1

```
L=1000; %Question 5: L=10,000
%Generating a binary sequence with Pr(D=0)=0.5 & Pr(D=1)=0.5
D=randi([0 1],1,L);

%Generating a stream of rectangular pulses of amplitude A
A=1;
S=zeros(1,L); %Initializing S
for i=1:1:L
    if D(i)==1
        S(i)=A; %if D=1 then S=A
    else
        S(i)=(-1)*A; %if D=0 then S=-A
    end
end
```

Question-2

```
%Generating AWGN sequence of length L, mean=mu and standard deviation=sd
mu=0;
sd=1; %variance=1 then sd=1
N=sd*randn(1,L)+mu;
```

Question-3

```
R=S+N; %Generating R
figure;
stairs([1:L],R); %Plotting R
title('Received Signal (R)');

%First we generate AWGN of variance=4 & sd=2
sd1=2;
N1=sd1*randn(1,L)+mu;

%Next we generate AWGN of variance=9 & sd=3
sd2=3;
N2=sd2*randn(1,L)+mu;

%We plot received signals
R1=S+N1;
R2=S+N2;
figure;
subplot(1,2,1);
stairs([1:L],R1); %Plotting R1
```

```
title('Received Signal (sd=2)');

subplot(1,2,2);
stairs([1:L],R2); %Plotting R2
title('Received Signal (sd=3)');
```

Question-4

```
%Generating decoded sequence
Y=zeros(1,L); %Initializing Y
tau=0; %threshold is 0
for j=1:1:L
    if R(j)>tau
        Y(j)=A;
    else
        Y(j)=(-1)*A;
    end
end

%Plotting transmitted signal and decoded signal
figure;
subplot(2,1,1);
stairs([1:L],S); %Plotting S
ylim([-1.5 1.5])
title('Transmitted Signal (S)');

subplot(2,1,2);
stairs([1:L],Y); %Plotting Y
ylim([-1.5 1.5])
title('Decoded Signal (Y)');

%For comparing we can zoom in S & Y
figure;
subplot(2,1,1);
stairs([1:L],S); %Plotting S
xlim([0 100]),ylim([-1.5 1.5])
title('Transmitted Signal (S)');

subplot(2,1,2);
stairs([1:L],Y); %Plotting Y
xlim([0 100]),ylim([-1.5 1.5])
title('Decoded Signal (Y)');
```

Question-5

```
bins = 10; %Number of bins(In Q5 part(a) this is changed to 100)
Rmax = max(R); %Maximum value of R
Rmin = min(R); %Minimum value of R
marg = linspace(Rmin,Rmax,bins+1); %Margins of bins
%next we have to get the mid value of margins
x = zeros(1,bins); %Mid values of margins
for k=1:1:bins
    x(k)=(marg(k)+marg(k+1))/2;
end

%Next we have to calculate the height of each bin
count = zeros(1,bins); %Height of bins
for m=1:1:L
    for n=1:1:bins
        if R(m)<=marg(n+1)
            count(n)=count(n)+1;
            break
        end
    end
end

%Now we plot the histogram without using hist()
figure;
bar(x,count);
title('Histrogram of R (without using hist())');

%Now we plot the histogram using hist()
figure;
hist(R,bins);
title('Histrogram of R (using hist())');
```

Question-5 Part(b)

```
bins=100;
Rp=[]; %R values gien that S=A
Rn=[]; %R values gien that S=-A
for p=1:1:L
    if S(p)==A
        Rp(end+1)=R(p); %When S=A R(p) should be added to Rp
    else
        Rn(end+1)=R(p); %When S=-A R(p) should be added to Rn
    end
end

%S=A(S is positive)
[np,xp]=hist(Rp,bins); %We get heights and bins using hist()
widthp = (max(Rp)-min(Rp))/(bins); %Calculating the width of a bin
Np=length(Rp); %Total number of heights
yp=np/(Np*widthp); %Normalized values
```

```
figure;
bar(xp,yp); %Plotting the histrogram
title('Normalized histogram (S=A)');
hold on;
plot(xp,yp,'r-','linewidth',1.2); %Plotting f R|S (r|S = A) in same figure

figure;
plot(xp,yp,'r-','linewidth',1.5); %Plotting f R|S (r|S = A)
title('f_R_|_S (r|S = A)');

%S=-A(S s negative)
[nn,xn]=hist(Rn,bins); %We get heights and bins using hist()
widthn = (max(Rn)-min(Rn))/(bins); %Calculating the width of a bin
Nn=length(Rn); %Total number of heights
yn=nn/(Nn*widthn); %Normalized values

figure;
bar(xn,yn); %Plotting the histrogram
title('Normalized histogram (S=-A)');
hold on;
plot(xn,yn,'r-','linewidth',1.2); %Plotting f R|S (r|S = -A) in same figure

figure;
plot(xn,yn,'r-','linewidth',1.5); %Plotting f R|S (r|S = -A)
title('f_R_|_S (r|S = -A)');
```

Question-5 Part(c)

```
%calculating E[R|S=A]
Expa = 0;
for p = 1:bins
 Expa = Expa + (xp(p)*yp(p)*widthp);
end
disp(Expa);

%calculating E[R|S=-A]
Exna = 0;
for q = 1:bins
 Exna = Exna + (xn(q)*yn(q)*widthn);
end
disp(Exna);

%Calculating E[R]
[n,x]=hist(R,bins); %We get heights and bins using hist()
width = (max(R)-min(R))/(bins); %Calculating the width of a bin
Tot=length(R); %Total number of heights
y=n/(Tot*width); %Normalized values

Ex = 0;
for r = 1:bins
 Ex = Ex + (x(r)*y(r)*width);
end
disp(Ex);
```

Question-5 Part(d)

```
figure;
bar(x,y); %Plotting the histrogram
title('Normalized histogram');
hold on;
plot(x,y,'r-','linewidth',1.2); %Plotting f R (r) in same figure

figure;
plot(x,y,'r-','linewidth',1.5); %Plotting f R (r)
title('f_R (r)');
```

Question-6

```
%Generating an interference of length L, mean=mu_i and standard deviation=sd_i
%I is also gaussian distributed
mu_i=0;
sd_i=1; %variance=1 then sd=1
I=sd_i*randn(1,L)+mu_i;

%Getting the received signal
R=S+N+I;

%Next we run the same code used to question 5 part(b), part(c) and part(d) with new R
```

Question-7

```
%Taking alpha as 2
alpha = 2;

%getting the received signal
R = alpha*S + N;

%Next we run the same code used to question 5 part(b), part(c) and part(d) with new R
```