

전미분

$$f(x+dx) - f(x) = f'(x)dx$$

$$\Rightarrow \underline{f(x+dx) = f(x) + f'(x)dx}$$

2변수함수의 전미분

$$L(u+du, v) = L(u, v) + L_u(u, v)du$$

$$L(u, v+dv) = L(u, v) + L_v(u, v)dv$$

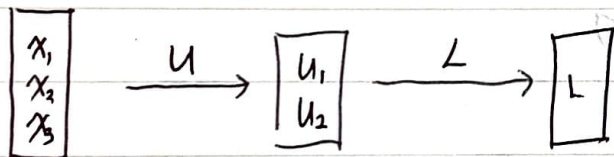
$$L(u+du, v+dv) = L(u, v) + L_u(u, v)du + L_v(u, v)dv$$

$$\underline{L(u+du, v+dv) - L(u, v) = L_u(u, v)du + L_v(u, v)dv}$$

∠에 대한 변화량 $\Rightarrow dL$ 로 표기

$$dL = L_u \cdot du + L_v \cdot dv$$

$$= \frac{\partial L}{\partial u} \cdot du + \frac{\partial L}{\partial v} \cdot dv$$



예: ∠을 x_i 로 표현할 수 있는가?

$$dL = \frac{\partial L}{\partial u_1} \cdot du_1 + \frac{\partial L}{\partial u_2} \cdot du_2$$

양변을 ∂x_i 로 나눈다.

$$\frac{\partial L}{\partial x_i} = \frac{\partial L}{\partial u_1} \cdot \frac{du_1}{\partial x_i} + \frac{\partial L}{\partial u_2} \cdot \frac{du_2}{\partial x_i}$$

$$\therefore \frac{\partial L}{\partial x_i} = \frac{\partial L}{\partial u_1} \frac{du_1}{\partial x_i} + \frac{\partial L}{\partial u_2} \frac{du_2}{\partial x_i} \quad (i=1, 2, 3)$$

시그모이드 함수

• 시그모이드 함수는 짝함수이다

$$\begin{aligned} \text{pf)} \quad f(x) + f(-x) &= \frac{1}{1 + \exp(-x)} + \frac{1}{1 + \exp(x)} \\ &= \frac{1}{1 + \exp(-x)} + \frac{\exp(-x)}{(1 + \exp(x))(\exp(-x))} \\ &= \frac{1}{1 + \exp(-x)} + \frac{\exp(-x)}{1 + \exp(-x)} \\ &= \frac{1 + \exp(-x)}{1 + \exp(-x)} = 1 \end{aligned}$$

$$\therefore \frac{1}{2}(f(x) + f(-x)) = \frac{1}{2}$$

• 시그모이드 함수 미분

$$u = 1 + \exp(-x)$$

$$y = \frac{1}{u}$$

$$\frac{dy}{dx} = \boxed{\frac{dy}{du}} \cdot \boxed{\frac{du}{dx}}$$

$$= \left(\frac{1}{u}\right)' \cdot -\exp(-x)$$

$$= -\frac{1}{u^2} \cdot -\exp(-x)$$

$$\begin{aligned} v &= -x \\ u &= 1 + \exp(v) \end{aligned}$$

$$\frac{du}{dx} = \frac{dv}{dx} \cdot \frac{du}{dv}$$

$$= -1 \cdot \exp(v)$$

$$= -\exp(-x)$$

$$= \frac{\exp(-x)}{(1 + \exp(-x))^2} = \frac{1 + \exp(-x) - 1}{(1 + \exp(-x))^2} = \frac{1}{1 + \exp(-x)} - \frac{1}{(1 + \exp(-x))^2} = y - y^2$$

$$= y(1 - y)$$

• 소프트맥스 맵

입력벡터 (x_1, x_2, x_3)

출력벡터 (y_1, y_2, y_3)

$$\begin{cases} y_1 = \frac{\exp(x_1)}{\exp(x_1) + \exp(x_2) + \exp(x_3)} = g(x_1, x_2, x_3) \\ y_2 = \frac{\exp(x_2)}{g(x_1, x_2, x_3)} \\ y_3 = \frac{\exp(x_3)}{g(x_1, x_2, x_3)} \end{cases}$$

• x 와 y 의 값이 같은 경우 y_i 를 x_i 으로 미분

$$y_1 = \frac{\exp(x_1)}{g(x_1, x_2, x_3)}$$

$$\frac{\partial y_1}{\partial x_1} = \frac{\frac{\exp(x_1)}{g(x_1, x_2, x_3)} \cdot g - \exp(x_1) \cdot \frac{\partial g}{\partial x_1}}{g^2}$$

$$= \frac{\exp(x_1)}{g} \cdot \frac{g - \exp(x_1)}{g} = \frac{\exp(x_1)}{g} \left(1 - \frac{\exp(x_1)}{g}\right) = y_1(1 - y_1)$$

• x 와 y 의 값이 다른 경우

$$y_2 = \frac{\exp(x_2)}{g(x_1, x_2, x_3)}$$

$$\frac{\partial y_2}{\partial x_1} = \frac{g \cdot \frac{\partial}{\partial x_1} \left(\frac{\exp(x_2)}{g} \right) - \exp(x_2) \cdot \frac{\partial g}{\partial x_1}}{g^2} = \frac{g \cdot 0 - \exp(x_2) \cdot h(x_1)}{g^2}$$

$$= - \frac{h(x_2) \cdot h(x_1)}{g^2}$$

$$= - \frac{h(x_2)}{g} \cdot \frac{h(x_1)}{g} = -y_2 \cdot y_1$$

$$\therefore \frac{\partial y_i}{\partial x_j} = \begin{cases} y_i(1 - y_i) & (i=j) \\ -y_i y_j & (i \neq j) \end{cases}$$