전미분

$$f(x+dx) - f(x) = f'(x)dx$$

$$\Rightarrow f(x+dx) = f(x) + f'(x)dx$$

2변수함수의 전미분

$$L(u+du, v) = L(u,v) + L_u(u,v)du$$

$$L(u, v+dv) = L(u,v) + L_v(u,v)dv$$

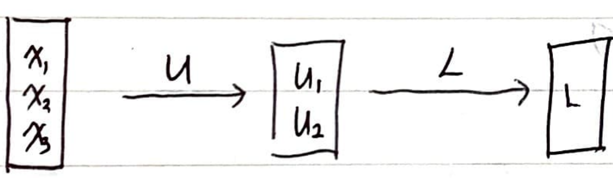
$$L(u+du, v+dv) = L(u,v) + L_u(u,v)du + L_v(u,v)dv$$

$$L(u+du, v+dv) - L(u,v) = L_u(u,v)du + L_v(u,v)dv$$

L에 대한 변화량 ⇒ dL로 표기

$$dL = L_u \cdot du + L_v \cdot dv$$

$$= \frac{\partial L}{\partial u} \cdot du + \frac{\partial L}{\partial v} \cdot dv$$



문제: L을 x_1으로 전미분한 결과는?

$$dL = \frac{\partial L}{\partial u_1} \cdot du_1 + \frac{\partial L}{\partial u_2} \cdot du_2$$

양변을 ∂x_1으로 나눈다.

$$\frac{\partial L}{\partial x_1} = \frac{\partial L}{\partial u_1} \cdot \frac{du_1}{\partial x_1} + \frac{\partial L}{\partial u_2} \cdot \frac{du_2}{\partial x_1}$$

$$\therefore \frac{\partial L}{\partial x_i} = \frac{\partial L}{\partial u_1} \cdot \frac{du_1}{\partial x_i} + \frac{\partial L}{\partial u_2} \cdot \frac{du_2}{\partial x_i} \quad (i = 1, 2, 3)$$

시그모이드 함수

- 시그모이드 함수는 점대칭이다

pf) $$f(x)+f(-x) = \frac{1}{1+\exp(-x)} + \frac{1}{1+\exp(x)}$$

$$= \frac{1}{1+\exp(-x)} + \frac{\exp(-x)}{(1+\exp(x))(\exp(-x))}$$

$$= \frac{1}{1+\exp(-x)} + \frac{\exp(-x)}{1+\exp(-x)}$$

$$= \frac{1+\exp(-x)}{1+\exp(-x)} = 1$$

$$\therefore \frac{1}{2}(f(x)+f(-x)) = \frac{1}{2}$$

- 시그모이드 함수 미분

$$u = 1+\exp(-x)$$

$$y = \frac{1}{u}$$

$$\frac{dy}{dx} = \boxed{\frac{dy}{du}} \cdot \boxed{\frac{du}{dx}}$$

$$v = -x$$
$$u = 1+\exp(v)$$

$$\frac{du}{dx} = \frac{dv}{dx} \cdot \frac{du}{dv}$$
$$= -1 \cdot \exp(v)$$
$$= -\exp(-x)$$

$$= \left(\frac{1}{u}\right)' \cdot -\exp(-x)$$

$$= -\frac{1}{u^2} \cdot -\exp(-x)$$

$$= \frac{\exp(-x)}{(1+\exp(-x))^2} = \frac{1+\exp(-x)-1}{(1+\exp(-x))^2} = \frac{1}{1+\exp(-x)} - \frac{1}{(1+\exp(-x))^2} = y - y^2$$

$$= y(1-y)$$

• 소프트맥스 함수 미분

입력벡터 (x_1, x_2, x_3)

출력벡터 (y_1, y_2, y_3)

$$
\begin{cases}
y_1 = \dfrac{\exp(x_1)}{\exp(x_1)+\exp(x_2)+\exp(x_3)} \quad \exp(x_1)+\exp(x_2)+\exp(x_3) = g(x_1, x_2, x_3) \\
y_2 = \dfrac{\exp(x_2)}{g(x_1, x_2, x_3)} \\
y_3 = \dfrac{\exp(x_3)}{g(x_1, x_2, x_3)}
\end{cases}
$$

• x와 y의 첨자가 같은 경우 y_1을 x_1으로 미분

$$y_1 = \frac{\exp(x_1)}{g(x_1, x_2, x_3)}$$

$$\frac{\partial y_1}{\partial x_1} = \frac{\overset{\exp(x_1)}{(\exp(x_1))'} \cdot g - \exp(x_1) \cdot \overset{\exp(x_1)}{g_{x_1}}}{g^2}$$

$$= \frac{\exp(x_1)}{g} \cdot \frac{g - \exp(x_1)}{g} = \frac{\exp(x_1)}{g}\left(1 - \frac{\exp(x_1)}{g}\right) = y_1(1-y_1)$$

• x와 y의 첨자가 다른 경우

$$y_2 = \frac{\exp(x_2)}{g(x_1, x_2, x_3)}$$

$h \rightarrow \exp$

$$\frac{\partial y_2}{\partial x_1} = \frac{g \cdot h(x_2)_{x_1} - h(x_2) \cdot g_{x_1}}{g^2} = \frac{g \cdot 0 - h(x_2) \cdot h(x_1)}{g^2}$$

$$= -\frac{h(x_2) \cdot h(x_1)}{g^2}$$

$$= -\frac{h(x_2)}{g} \cdot \frac{h(x_1)}{g} = -y_2 \cdot y_1$$

$$\therefore \frac{\partial y_i}{\partial x_j} = \begin{cases} y_i(1-y_i) & (i=j) \\ -y_i y_j & (i \neq j) \end{cases}$$