행렬 : 숫자를 직사각형 모양으로 배열한 것.

EX) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\begin{bmatrix} 1 & 3 & 2 & -1 \\ 5 & 2 & 7 & 6 \end{bmatrix}$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Row : m개

Column : n개 Size $= m \times n$

EX) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$

~~A의 두번째~~ row $[5, 6, 7, 8]$

A의 세번째 Column $\begin{bmatrix} 3 \\ 7 \\ 11 \end{bmatrix}$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

ij-entry : a_{ij}

i th row : $[a_{i1}\ a_{i2} \cdots a_{in}]$ $1 \times n$

j th column : $\begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix}$ $m \times 1$

• A, B 둘다 행렬

$A = B \rightarrow$ 이게 무슨 뜻?

① size same $A : m \times n$ $B : p \times q$ $(m = p, n = q)$

② 같은 위치에 있는 entry가 같아야한다. $(a_{ij} = b_{ij})$

• 행렬 표기법

$A = [a_{ij}]_{m \times n}$

$[A]_{ij} = a_{ij}$

$A, B \in$ matrix

$A+B$

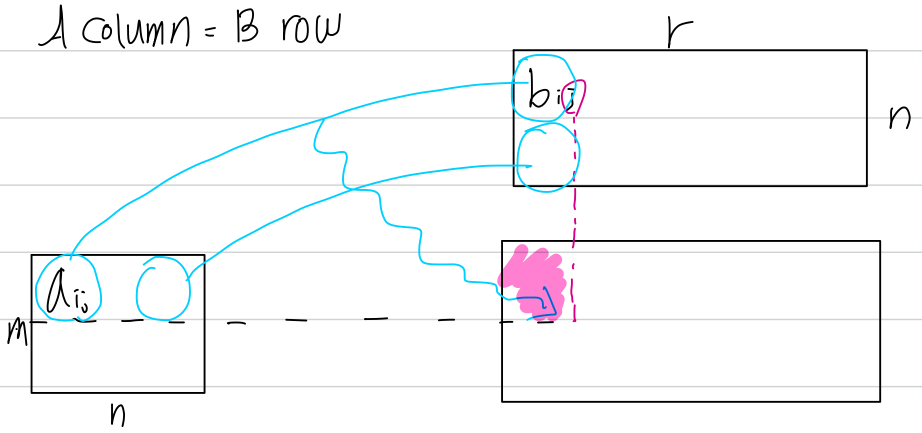
① size same

$A = [a_{ij}]_{m\times n}$ $B = [b_{ij}]_{m\times n}$

$A+B = [a_{ij}+b_{ij}]_{m\times n}$

$A\times B$

A column = B row



$A = [a_{ij}]_{m\times n}$ $B = [b_{ij}]_{n\times r}$

$AB = \left[\sum_{k=1}^{n} a_{ik} b_{kj}\right]_{m\times r}$

• 성질 $(A+B)+C = A+(B+C)$

$(AB)C = A(BC)$

$A(B+C) = AB+AC$

$A(kB) = kAB$

* Transpose

$(A^T)^T = A$

$(kA)^T = k(A)^T$

$(A+B)^T = A^T + B^T$

$(AB)^T = B^T A^T$

$AB = [\underbrace{\textstyle\sum_{k=1}^{n} a_{ik} b_{kj}}_{C_{ij}}]_{m\times s}$

$(AB)^T = [C'_{ij}]_{s\times m}$, $C'_{ij} = C_{ji} = \sum_{k=1}^{n} a_{jk} b_{ki}$

$B^T A^T = \left[\sum_{k=1}^{n} b'_{jk} a'_{kj}\right]_{s\times m}$

$\sum_{k=1}^{n} a_{jk} b_{ki} = \sum_{k=1}^{n} b'_{ik} a'_{kj}$

|| ||

b_{ik} a_{jk}

$\therefore AB = B^T A^T$

Assymmetric matrix

$A = [a_{ij}]_{n\times n}$, $a_{ij} = a_{ji}$

$A + A^T$가 대칭이 됨을 보이시오.

$B = [a_{ij} + a_{ji}]_{n\times n}$ $\quad b_{ij} = b_{ji}$

$\rightarrow b_{ij} = a_{ij} + a_{ji}$

$b_{ji} = a_{ji} + a_{ij}$

A^T : 대칭행렬 (정사각행렬)

$A : m\times n \quad A^T : n\times m$

$AA^T = m\times m$

• 행렬곱의 여러표현법

$A = m\times n \qquad B = n\times r$

$$A = \begin{bmatrix} A_1 \\ \hline A_2 \\ \vdots \\ \hline A_m \end{bmatrix} \qquad AB = \left.\begin{bmatrix} A_1B \\ A_2B \\ A_nB \end{bmatrix}\right\} m개 \quad m\times r$$

$1\times n \; n\times r = 1\times r$

$$B = [B_1 \mid B_2 \mid \cdots \mid B^r] \qquad AB = \overbrace{[AB_1 \mid AB_2 \mid \cdots \mid AB^r]}^{r개} \quad m개 \quad r\times m$$

$m\times n \; n\times 1 = m\times 1$

$$A = [A_1 \mid A_2 \mid \cdots A_n] \quad B = \begin{bmatrix} B_1 \\ \hline B_2 \\ \vdots \\ B_n \end{bmatrix} \quad AB = [A^1][B_1] + [A^2][B_2] + [A^n][B_n]$$

$m\times 1$

$1\times r$

$= m\times r$

Ex)

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 1 \\ 4 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 3 \\ 2 & 0 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 2 & -2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 & 1 \\ 4 & 1 & 5 \end{bmatrix}$$