

Part 2 Linear Combination and Span

$$\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n \in \mathbb{R}^2$$

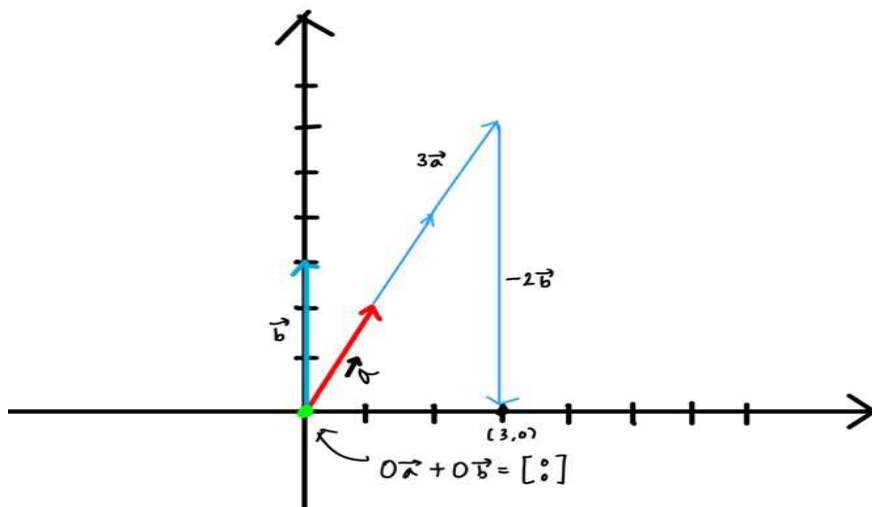
$$c_1 \vec{v}_1 + c_2 \vec{v}_2 + \dots + c_n \vec{v}_n \mid c_1 \sim c_2 \in \mathbb{R}$$

Adding the vectors and just scale up by some scaling factor.

Ex)

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

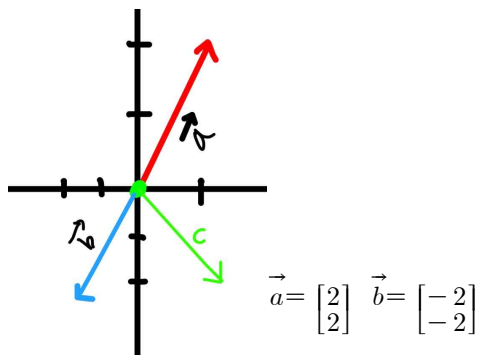
$$0\vec{a} + 0\vec{b} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad 3\vec{a} + (-2)\vec{b} = \begin{bmatrix} 3 - 0 \\ 6 - 6 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$



Linear combinations can represent all vectors in two-dimensional coordinates.

Span of the Vector A and B : $\text{span}(\vec{a}, \vec{b}) = \mathbb{R}^2$

But if two vectors are colinear, it cannot represent all vectors in \mathbb{R}^2



Is it possible to represent \vec{c} by using \vec{a}, \vec{b} ? NO

prove 1)

if $\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$ and $\vec{X} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$: *represent arbitrary point* $X \in R^2$

$$c_1 \vec{a} + c_2 \vec{b} = \vec{x}$$

$$c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$c_1 = x_1$$

$$c_2 = \frac{1}{3}(x_2 - 2x_1)$$

looking to get the point 2,2

$$c_1 = 2$$

$$c_2 = \frac{1}{3}(2 - 4) = \frac{-2}{3}$$

$$\therefore 2\vec{a} - \frac{2}{3}\vec{b} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$