Part 3 Linear Independence & Linear dependence

Linear dependence : one of the vectors in the set can be represented by some combination of the other vectors in the set.

Ex) $\left\{\begin{bmatrix}2\\3\end{bmatrix},\begin{bmatrix}4\\6\end{bmatrix}\right\}$ is it dependent or independent set

so1) linearly dependent set

$c_1\begin{bmatrix}2\\3\end{bmatrix}+c_2\begin{bmatrix}4\\6\end{bmatrix}$

$=c_1\begin{bmatrix}2\\3\end{bmatrix}+2c_2\begin{bmatrix}2\\3\end{bmatrix}$

$=(c_1+2c_2)\begin{bmatrix}2\\3\end{bmatrix}$

$=c_1\begin{bmatrix}2\\3\end{bmatrix}$

Linear Independence : one of the vectors in the set cannot be represented by some combination of the other vectors in the set.

Ex) $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}0\\7\\0\end{bmatrix},\begin{bmatrix}0\\0\\2\end{bmatrix}\right\}$

There is no combination of the vector that can represent other vector.

Linear Independence & dependence

Linear Dependent $\Leftrightarrow$ $c_1v_1+c_2v_2+\ldots+c_nv_n=0$

(for some c_i not all are zero = at least one is non-zero)

prove 1)

$v_1=a_2v_2+a_3v_3+\ldots+a_nv_n$

0=$-v_1+a_2v_2+a_3v_3+\ldots+a_nv_n$(one of the constant non-zero)

prove 2)

Assume $c_1\neq 0$

$v_1+\frac{c_2}{c_1}+\ldots+v_n \bullet \frac{c_n}{c_1}=0$

$-\frac{c_2}{c_1}\bullet v_2+-\frac{c_3}{c_2}\bullet v_3+\ldots-\frac{c_b}{c_1}\bullet v_n=v_1$

=> if c_1 or c_2 non-zero => dependent

=> if c_1 and $c_1$zero => independent

Ex 1)

$$\left\{\begin{bmatrix}2\\1\end{bmatrix},\begin{bmatrix}3\\2\end{bmatrix}\right\}$$

$$c_1\begin{bmatrix}2\\1\end{bmatrix}+c_2\begin{bmatrix}3\\2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$2c_1+3c_2=0$
$c_1+2c_2=0$

$\frac{1}{2}c_2=0$
$c_1=0$

$\therefore$ linearly independent set(It cannot represent vector by combination the other)

Ex 2)

$$\left\{\begin{bmatrix}2\\1\end{bmatrix},\begin{bmatrix}3\\2\end{bmatrix},\begin{bmatrix}1\\2\end{bmatrix}\right\}$$

$$c_1\begin{bmatrix}2\\1\end{bmatrix}+c_2\begin{bmatrix}3\\2\end{bmatrix}+c_3\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$2c_1+3c_2+c_3=0$
$c_1+2c_2+2c_3=0$

Three unknowns, two equations (so we assume $c_3=-1$)

=> $c_2=3$, $c_1=-4$

$$-4\begin{bmatrix}2\\1\end{bmatrix}+3\begin{bmatrix}3\\2\end{bmatrix}+-1\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

c_1 or c_2 or c_3 non-zero => dependent