5. Matrix for solving system of equations by addition and subtraction

#Reduced row echelon

$$x_1 + 2x_2 + x_3 + x_4 = 7$$
$$x_1 + 2x_2 + 2x_3 - x_4 = 12$$
$$2x_1 + 4x_2 \quad + 6x_4 = 4$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 7 \\ 1 & 2 & 2 & -1 & 12 \\ 2 & 4 & 0 & 6 & 4 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 7 \\ 0 & 0 & 1 & -2 & 5 \\ 0 & 0 & -2 & 4 & -10 \end{array}\right]$$

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$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 2 \\ 0 & 0 & 1 & -2 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \; x_1 + 2x_2 + \quad x_4 = 2$$
$$x_3 - 2x_4 = 5$$

$$\Rightarrow \quad x_1 + 2x_2 + \quad x_4 = 2 \qquad x_1, x_3 \text{ = pivot variable / } x_2, x_4 \text{=free variable}$$
$$x_3 - 2x_4 = 5$$

$$\Rightarrow \quad x_1 = 2 - 2x_2 - x_4$$
$$x_3 = 5 + 2x_4$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 5 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$