Part 4 Subspaces & basis of subspaces

#Linear subspace

V=subspace of $R^n = [X_1, X_2 \ \ldots X_n] \mid X_i \in R, 1 \le i \le n$ <=> $V \subset R^n$

-Definition of subspace

V=subspaces of $R^n$

1. contains zero-vector
2. closed under scalar multiple closure.
3. closed under addition closure.

Ex 1)

V=$\left\{\begin{bmatrix}0\\0\\0\end{bmatrix}\right\}$

V a subspace of $R^3$ ?

1. check zero-vector ：Ok.
2. closed under scalar multiple closure ：Ok.
3. closed under addition ：Ok.

Ex 2)

S=$\left\{\begin{bmatrix}X_1\\X_2\end{bmatrix} \in R^2 \mid X_1 \ge 0\right\}$

Is S subspace of $R^2$ ?

① $0 \cdot X_1 + 0 \cdot X_2 = \begin{bmatrix}0\\0\end{bmatrix}$ ：Ok

② if $-1\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}-a\\-b\end{bmatrix}$ ：Not closed under multiplication.

③ $\begin{bmatrix}a\\b\end{bmatrix} + \begin{bmatrix}c\\d\end{bmatrix} = \begin{bmatrix}a+c \mid \ge 0\\b+d\end{bmatrix}$ ：Ok

=> It it not a subspace of $R^2$

U(subspace)=span$\left(\begin{bmatrix}1\\1\end{bmatrix}\right)$

Is U a subspace of $R^2$

① $0\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

② $c_1\begin{bmatrix}1\\1\end{bmatrix}$ ：closed under scalar multiple closure

③ $c_1\begin{bmatrix}1\\1\end{bmatrix}+c_2\begin{bmatrix}1\\1\end{bmatrix}=(c_1+c_2)\begin{bmatrix}1\\1\end{bmatrix}$

$=c_3\begin{bmatrix}1\\1\end{bmatrix}$

closed under addition.

=>U is a subsace of $R^2$

#Basis of subspaces

if V=span($\overrightarrow{v_1},\overrightarrow{v_2}\cdots\overrightarrow{v_n}$) and {$\overrightarrow{v_1},\overrightarrow{v_2}\cdots\overrightarrow{v_n}$} a linearly independent, vectors are minimal subset of subspace. It called basis.

Ex) add $\overrightarrow{v_c}=\overrightarrow{v_1}+\overrightarrow{v_2}$ in basis

There are two subset T={$\overrightarrow{v_1},\overrightarrow{v_2}\cdots\overrightarrow{v_n},\overrightarrow{v_c}$}, V={$\overrightarrow{v_1},\overrightarrow{v_2}\cdots\overrightarrow{v_n}$} and span(T)=span(V), but subset(T)>subset(V). It can be said that subset of T has unnecessary element. In other words, subset T can constitute subspace, not minimal subset.

Ex) S=$\left\{\begin{bmatrix}2\\3\end{bmatrix},\begin{bmatrix}7\\0\end{bmatrix}\right\}$

span(s) = $c_1\begin{bmatrix}2\\3\end{bmatrix}+c_2\begin{bmatrix}7\\0\end{bmatrix}=\begin{bmatrix}X_1\\X_2\end{bmatrix}$ => $c_1=\dfrac{x_2}{3}$, $c_2=\dfrac{x_1}{7}-\dfrac{2}{21}x_2$ =>$x_1,x_2\in R$

To determine linear independent

$c_1\begin{bmatrix}2\\3\end{bmatrix}+c_2\begin{bmatrix}7\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$ $c_1=0, c_2=0$ => linear independent

So S is a basis