Homework 3

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Problem 2:

The equation

$$8x - x^4 = 0$$

has 0 and 2 for its solutions. Write the Newton's iterative scheme for the numerical solution of this equation, simplify the obtained expression. Now assume x_0 = 1.

a. Carry out by hand two steps of the Newton's method (i.e., calculate x_1 and x_2)

$$df = 8 - 4x^3$$

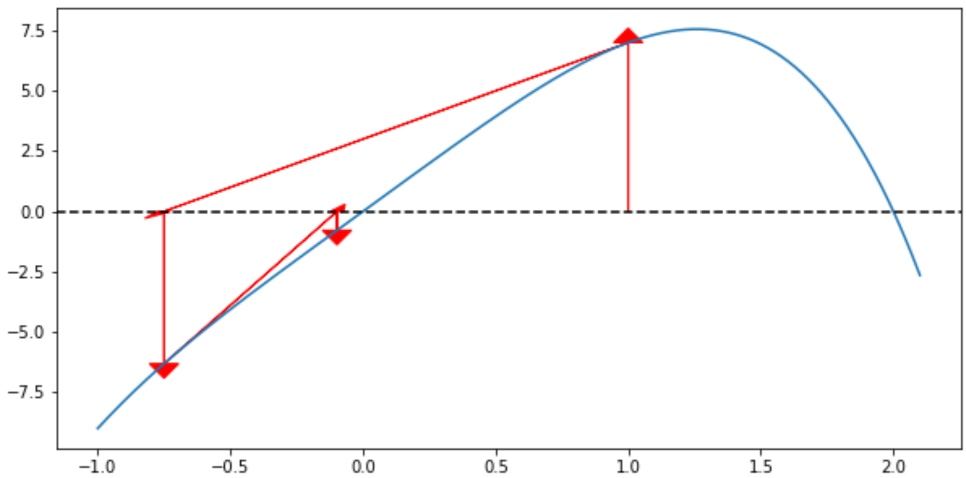
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{7}{4} = 0.75$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.75 - \frac{5.68359}{6.3125} = -0.150371$$

b. Graph the function

$$y = 8x - x^4$$

(you can use Python to obtain the plot, and then copy the result to your homework), show the initial point x_0 on the OX axis, and then show how to obtain x_1 and x_2 geometrically.



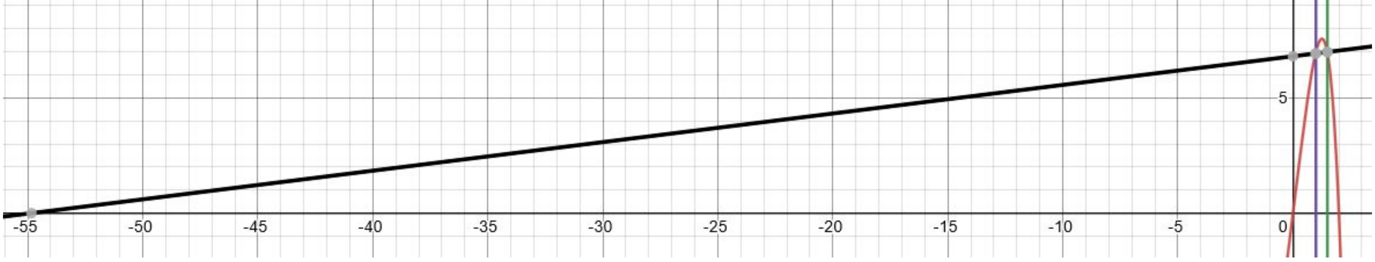
Problem 3:

Consider the same equation $8x - x^4 = 0$ as in the Problem 1.
Now assume that the initial point x_0 = 1.5 and x_1 = 1.

a. Carry out by hand one step of the Secant method (i.e., calculate x_2)

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)} = 1 - \frac{3.5}{0.0625} = -55$$

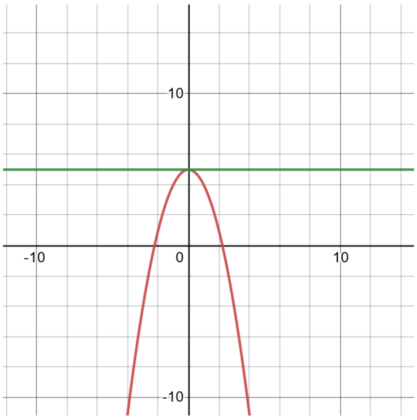
b. Graph the function $y = 8x - x^4$, show the initial points x_0 and x_1 on the OX axis, and then show how to obtain x_2 geometrically.



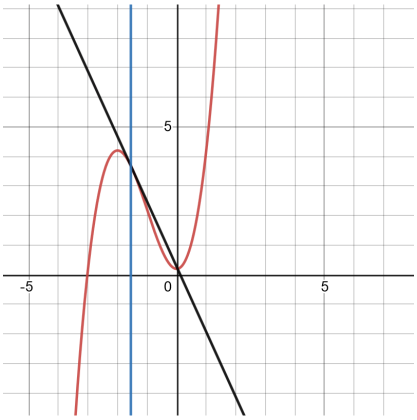
Problem 4:

Give an example (graphically or analytically, one example for each case below) of a function f : R → R having a unique root $x^{\backslash *}$ in R, and example of a point $x0_0$ ∈ R such that when constructing the approximating sequence x_n by the Newton’s method for the equation f(x) = 0 with the initial approximation x_0,

a. x_2 will be undefined;



b. the distance of x^2 from $x^{\backslash *}$ is greater than the distance of x^1 to $x^{\backslash *}$.



Problem 5:

Assume

$$x_{n+1} = x_n + \frac{(2 - e^{x_n})(x_n - x_{n-1})}{e^{x_n} - e^{x_{n-1}}}$$

with x_0 = 0 and x_1 = 1. The sequence x_n convergence (no need to proof) evaluate

$$\lim_{n \to \infty} x_n$$

The sequence x_n is given to us is similar to the secant method applied on the function 2 - e^x. Therefore the limit will converge to 0.693 which is the root of said function.