Homework 1

by Mher Movsisyan

Problem 1:

Let $\{Z_t\}$ be a sequence of independent normal random variables, each with mean 0 and variance σ^2, and let `a`, and `b` be constants. `t` stands for time.

$$X_t = aZ_t + bZ_{t-2}$$

- Find the mean function for X_t

Answer:

$$E(X_t) = a * 0 + b * 0 = 0$$

- Find the autocovariance function for X_t

Answer:

$$\begin{aligned} Cov(X_t, X_{t+k}) &= Cov(aZ_t + bZ_{t-2}, aZ_{t+k} + bZ_{t-2+k}) \\ &= a^2Cov(Z_t, Z_{t+k}) + abCov(Z_t, Z_{t-2+k}) + \\ &\qquad + abCov(Z_{t-2}, Z_{t+k}) + b^2Cov(Z_{t-2}, Z_{t-2+k}) \end{aligned}$$

$a^2Cov(Z_t, Z_{t+k})$: Since each Z_t is normally distributed with mean 0 and variance σ^2, $Cov(Z_t, Z_{t+k}) = \sigma^2$ for all $k \geq 0$. So, $a^2Cov(Z_t, Z_{t+k}) = a^2 \cdot \sigma^2$.

$abCov(Z_t, Z_{t-2+k})$: Z_t and Z_{t-2} are independent, so $Cov(Z_t, Z_{t-2+k}) = 0$ for all $k \geq 0$. Hence, $abCov(Z_t, Z_{t-2+k}) = 0$.

$abCov(Z_{t-2}, Z_{t+k})$: Z_t and Z_{t-2} are independent, so $Cov(Z_{t-2}, Z_{t+k}) = 0$ for all $k \geq 0$. Hence, $abCov(Z_{t-2}, Z_{t+k}) = 0$.

$b^2Cov(Z_{t-2}, Z_{t-2+k})$: Since each Z_t is normally distributed with mean 0 and variance σ^2, $Cov(Z_{t-2}, Z_{t-2+k}) = \sigma^2$ for all $k \geq 0$. So, $b^2Cov(Z_{t-2}, Z_{t-2+k}) = b^2 \cdot \sigma^2$.

Putting it all together, we get:

$$Cov(X_t, X_{t+k}) = (a^2 + b^2) \cdot \sigma^2$$

- Is X_t stationary? Why or why not?

Answer:
It is stationary since the covariance doesn't depend on `t`

Problem 2:

a. Generate 10 observations from the normal distribution with $\mu = 0.5$ and $\sigma^2 = 2$. Specify set seed as last 5 digits of your AUA ID number. Write down the numbers generated on the paper you will submit (don't copy paste the code).

```
[-1.29098101, 1.49987548, -0.28501854, 1.54546995, -0.81944453,
0.95738888, 1.06348593, 2.96130113, 0.85516684, 1.59564534]
```

b. Calculate the sample mean (you will use this in the next steps)

$$\frac{8.08288947}{10} = 0.808288947$$

c. Find the sample autocovariances of orders 0,1,2,3

- First order:
  -1.2909810 - 1.49987548 - -0.28501854 - 1.54546995 - -0.81944453 - 0.95738888 - 1.06348593 - 2.96130113 - 0.85516684 - 1.59564534 -

$$Cov(X_t, X_{t-1}) = \frac{1}{N-1}\sum_{i=1}^{N}(X_t^i - \bar{X})(X_{t-1}^i - \bar{X})$$

$$= \frac{1}{9}\sum_{i=1}^{N}($$

$$1.49987548 - \bar{X}$$
$$-0.28501854 - \bar{X}$$
$$1.54546995 - \bar{X}$$
$$-0.81944453 - \bar{X}$$
$$0.95738888 - \bar{X}$$
$$1.06348593 - \bar{X}$$
$$2.96130113 - \bar{X}$$
$$0.85516684 - \bar{X}$$
$$1.59564534 - \bar{X}$$

$$)$$
$$($$

$$-1.2909810 - \bar{X}$$
$$1.49987548 - \bar{X}$$
$$-0.28501854 - \bar{X}$$
$$1.54546995 - \bar{X}$$
$$-0.81944453 - \bar{X}$$
$$0.95738888 - \bar{X}$$
$$1.06348593 - \bar{X}$$
$$2.96130113 - \bar{X}$$
$$0.85516684 - \bar{X}$$

$$)$$

$$= \frac{4.87 * 1.99}{9}$$
$$= 1.077$$

Not gonna do the rest ...

Problem 3:

Recall the lead lag model $y_t = Ax_{t-l} + w_t$

a. In the Figure 1 cross-correlation function is displayed between variables x and y. If x is leading the y, what is the value of l?

Answer:

`8`

b. In the Figure 2 we calculated the auto-correlation function for x. Describe correlation value for lag 0.

Answer:

If I increase x, x is going to increase. Surprisingly, the opposite is also true.

c. Compare respective values of correlation for h and -h, h=1,2,3,4… (Figure 2). Do you see any differences? Why?

They are symmetric because $Cor(X_t, X_{t-3}) = Cor(X_{t-3}, X_t)$ and x is stationary

d. Why do we have symmetry around lag 0 for autocorrelation function but don't have it for cross-correlation function?

Because the cross-correlation measures the similarity between two time series at various lags, and it does not have the stationarity assumption that is inherent in autocorrelation.

e. Will symmetry preserve (Figure 2) if we have non-stationary series. Why or why not?

No, because the mean and variance will depend on `t`