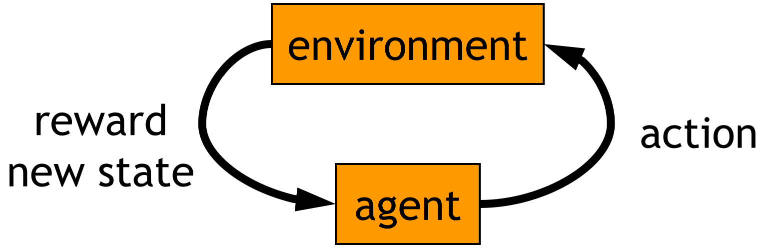
Reinforcement Learning

Peter Bodík

Previous Lectures

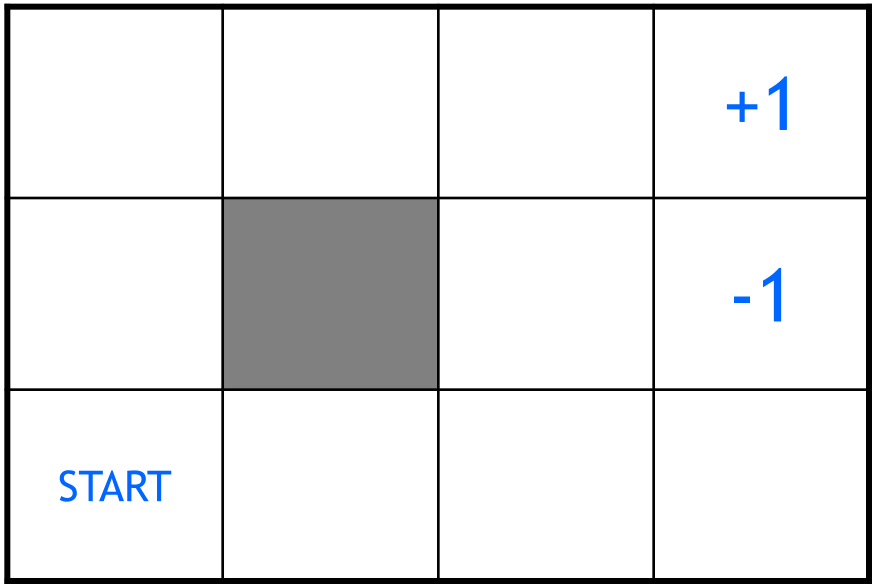
- Supervised learning
  - classification, regression
- Unsupervised learning
  - clustering, dimensionality reduction
- Reinforcement learning
  - generalization of supervised learning
  - learn from interaction w/ environment to achieve a goal



Today

- examples
- defining a Markov Decision Process
  - solving an MDP using Dynamic Programming
- Reinforcement Learning
  - Monte Carlo methods
  - Temporal-Difference learning
- automatic resource allocation for in-memory database
- miscellaneous
  - state representation
  - function approximation, rewards

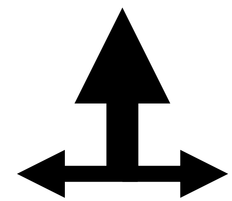
Robot in a room



actions: UP, DOWN, LEFT, RIGHT

UP

80% move UP
10% move LEFT
10% move RIGHT



- reward +1 at [4,3], -1 at [4,2]
- reward -0.04 for each step

- what's the strategy to achieve max reward?
- what if the actions were deterministic?

Other examples

- pole-balancing
- walking robot (applet)
- TD-Gammon [Gerry Tesauro]
- helicopter [Andrew Ng]

- no teacher who would say “good” or “bad”
  - is reward “10” good or bad?
  - rewards could be delayed

- explore the environment and learn from the experience
  - not just blind search, try to be smart about it

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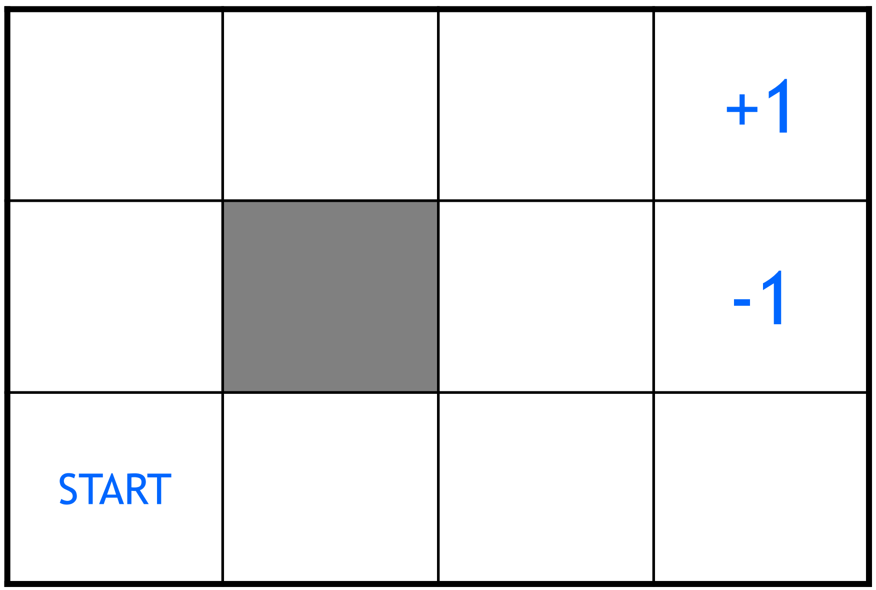

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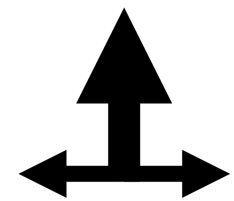
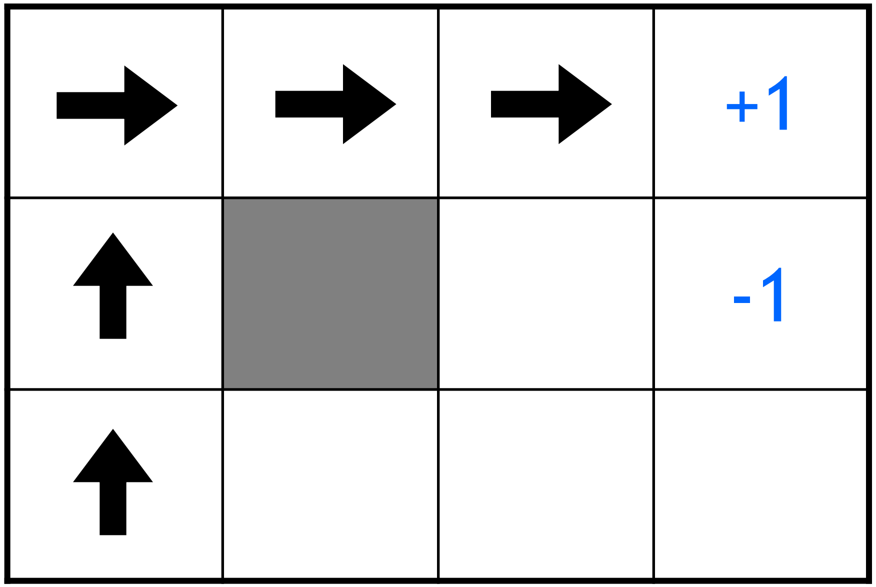


reward +1 at [4,3], -1 at [4,2]
reward -0.04 for each step

- states
- actions
- rewards

- what is the solution?

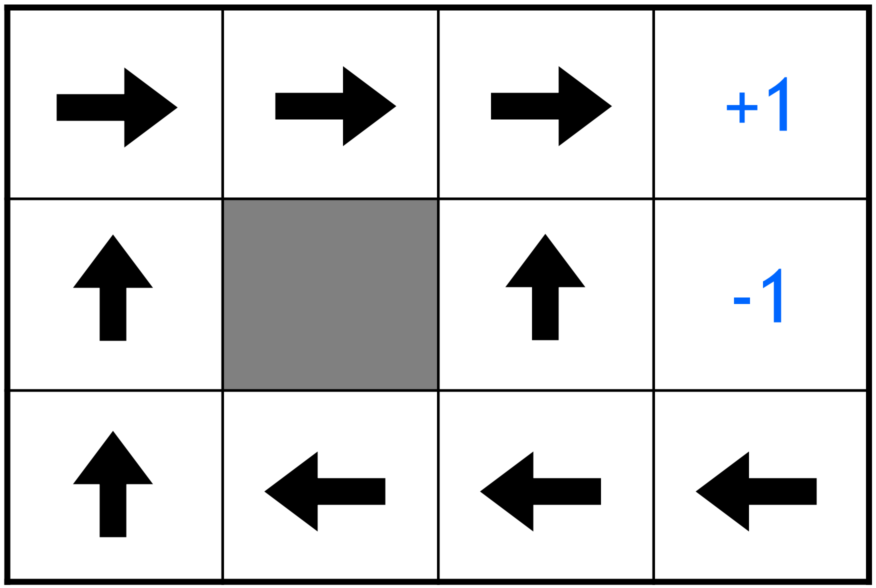
Is this a solution?



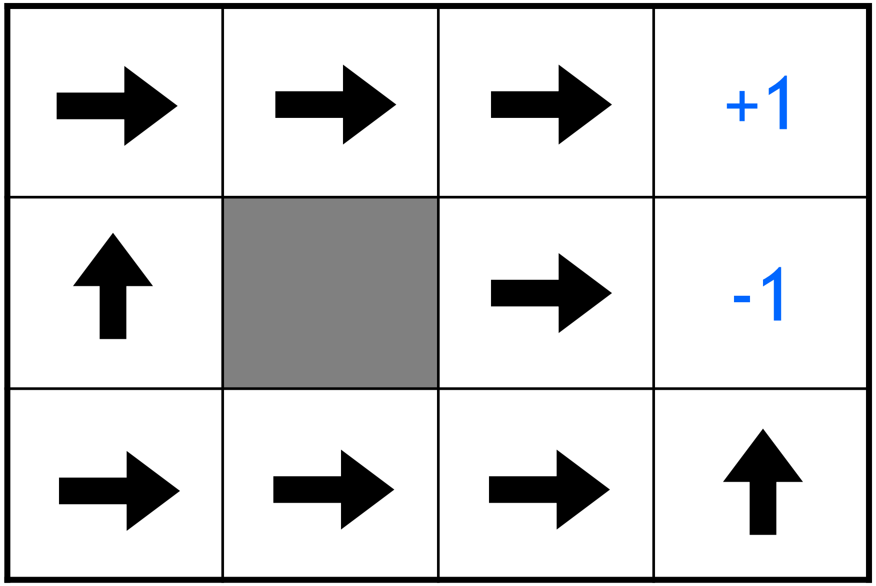
- only if actions deterministic
  - not in this case (actions are stochastic)

- solution/policy
  - mapping from each state to an action

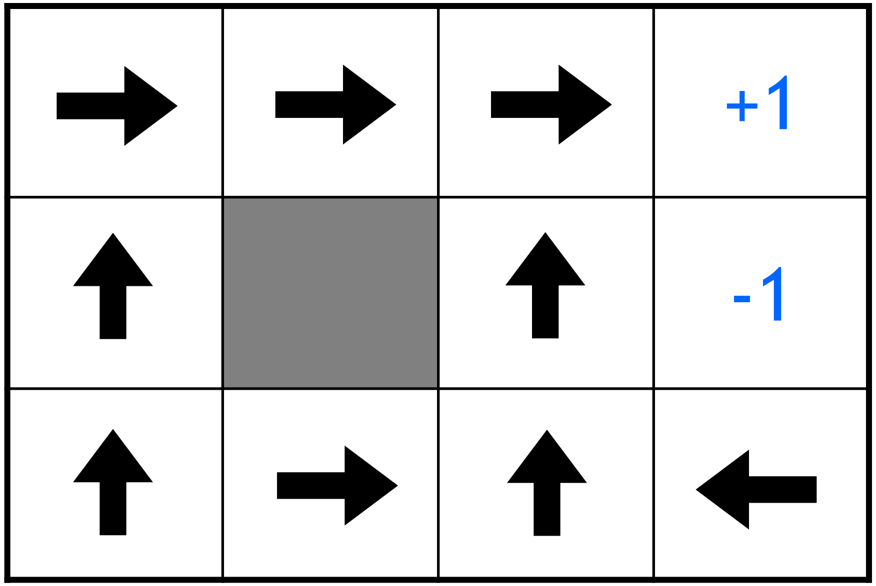
Optimal policy



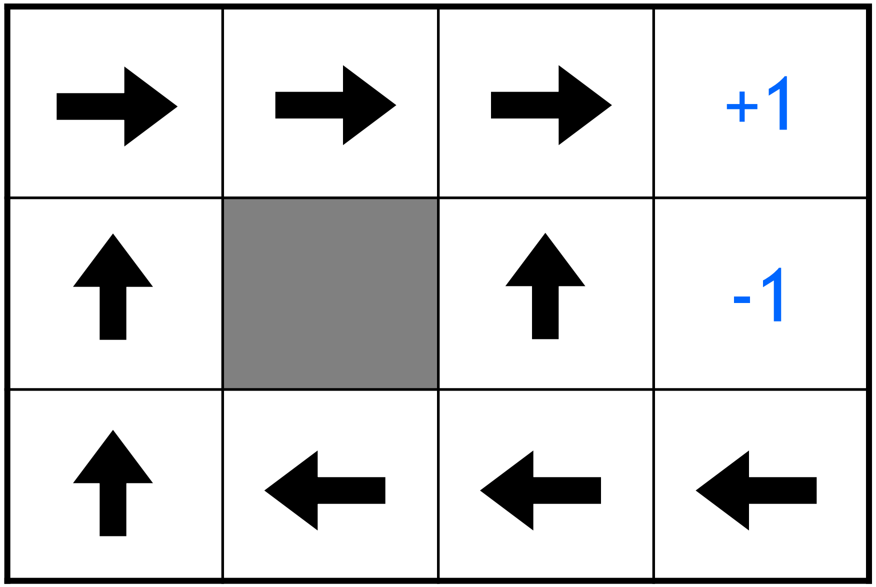
Reward for each step -2



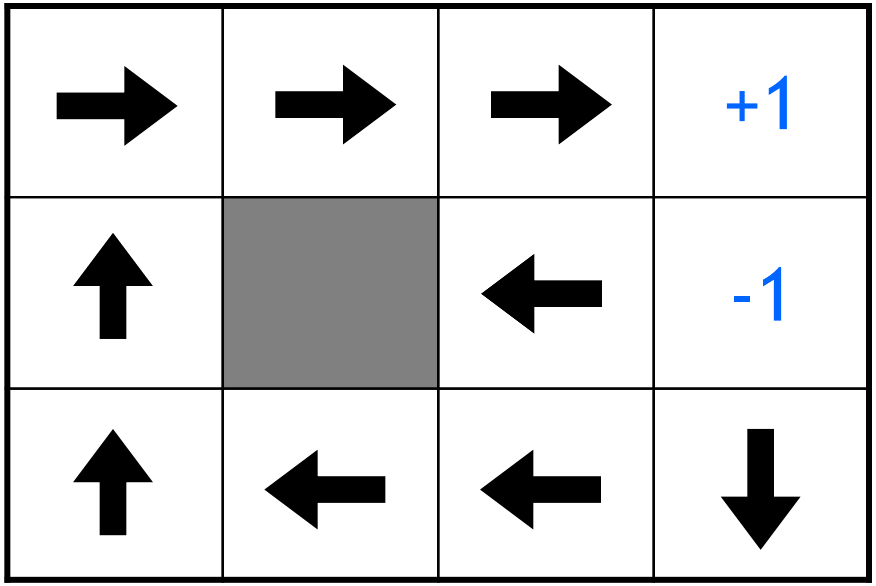
Reward for each step: -0.1



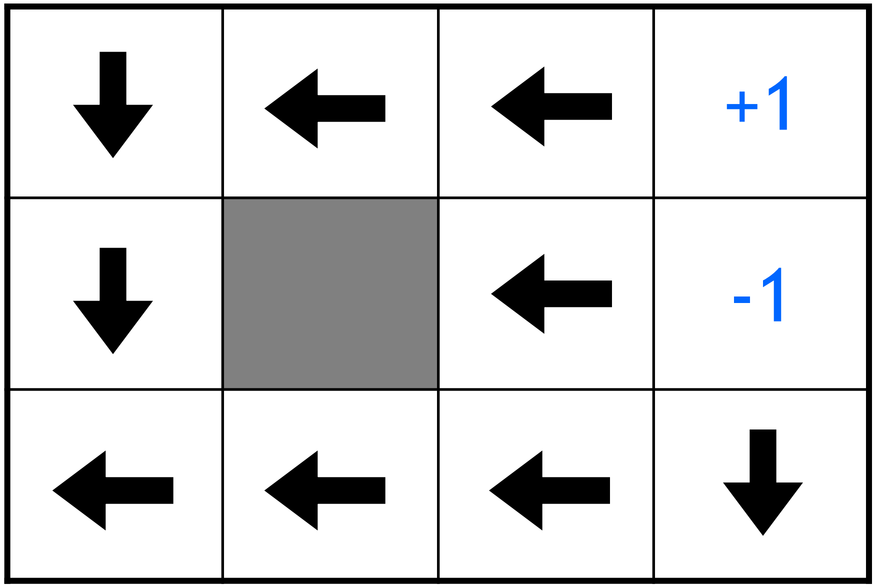
Reward for each step: -0.04



Reward for each step: -0.01

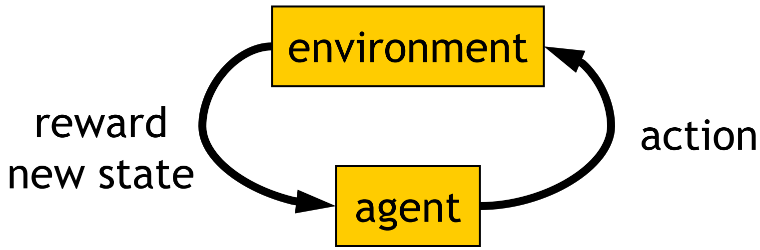


Reward for each step: +0.01



Markov Decision Process (MDP)

- set of states S, set of actions A, initial state S_0
- transition model $P(s'|s,a)$
  - $P(\,[1,2] \mid [1,1], up\,) = 0.8$
  - Markov assumption



- reward function $r(s)$
  - $r(\,[4,3]\,) = +1$
- goal: maximize cumulative reward in the long run

- policy: mapping from S to A
  - $\pi(s)$ or $\pi(s,a)$

- reinforcement learning
  - transitions and rewards usually not available
  - how to change the policy based on experience
  - how to explore the environment

Computing return from rewards

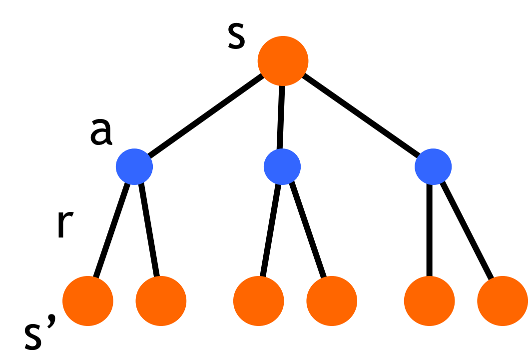
- episodic (vs. continuing) tasks
  - "game over" after N steps
  - optimal policy depends on N; harder to analyze

- additive rewards
  - $V(s_0, s_1, \ldots) = r(s_0) + r(s_1) + r(s_2) + \ldots$
  - infinite value for continuing tasks

- discounted rewards
  - $V(s_0, s_1, \ldots) = r(s_0) + \gamma^* r(s_1) + \gamma^{2*} r(s_2) + \ldots$
  - value bounded if rewards bounded

Value functions

- state value function: $V^{\pi}(s)$
  - expected return when starting in *s* and following π
- state-action value function: $Q^{\pi}(s,a)$
  - expected return when starting in *s*, performing *a*, and following π
- useful for finding the optimal policy
  - can estimate from experience
  - pick the best action using $Q^{\pi}(s,a)$



- Bellman equation

$$V^{\pi}(s) = \sum_{a} \pi(s,a) \sum_{s'} P^{a}_{ss'} \left[r^{a}_{ss'} + \gamma V^{\pi}(s') \right] = \sum_{a} \pi(s,a) Q^{\pi}(s,a)$$

Optimal value functions

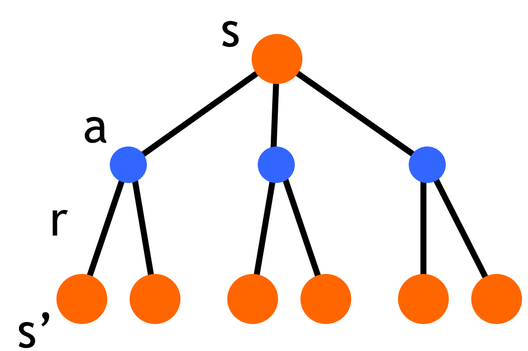
- there's a set of *optimal* policies
  - V^π defines partial ordering on policies
  - they share the same optimal value function

$$V^*(s) = \max_\pi V^\pi(s)$$

- Bellman optimality equation

$$V^*(s) = \max_a \sum_{s'} P^a_{ss'} \left[r^a_{ss'} + \gamma V^*(s') \right]$$

  - system of n non-linear equations
  - solve for V*(s)
  - easy to extract the optimal policy



- having Q*(s,a) makes it even simpler

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

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Dynamic programming

- main idea
  - use value functions to structure the search for good policies
  - need a perfect model of the environment

- two main components
  - policy evaluation: compute V^π from π
  - policy improvement: improve π based on V^π

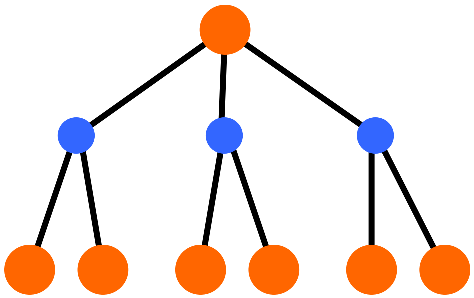
  - start with an arbitrary policy
  - repeat evaluation/improvement until convergence

Policy evaluation/improvement

- policy evaluation: π -> V^π
  - Bellman eqn's define a system of n eqn's
  - could solve, but will use iterative version

$$V_{k+1}(s) = \sum_a \pi(s,a) \sum_{k'} P^a_{ss'} \left[r^a_{ss'} + \gamma V_k(s') \right]$$

  - start with an arbitrary value function V_0, iterate until V_k converges



- policy improvement: V^π -> π'

$$\begin{aligned} \pi'(s) &= \arg\max_a Q^\pi(s,a) \\ &= \arg\max_a \sum_{s'} P^a_{ss'} \left[r^a_{ss'} + \gamma V^\pi(s') \right] \end{aligned}$$

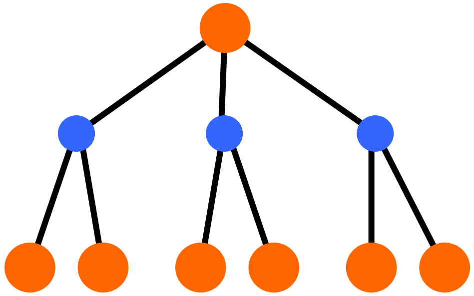
  - π' either strictly better than π, or π' is optimal (if $\pi = \pi'$)

Policy/Value iteration

- Policy iteration

$$\pi_0 \rightarrow^E V^{\pi_0} \rightarrow^I \pi_1 \rightarrow^E V^{\pi_1} \rightarrow^I \dots \rightarrow^I \pi^* \rightarrow^E V^*$$

  - two nested iterations; too slow
  - don't need to converge to V^{π_k}
    - just move towards it



- Value iteration

$$V_{k+1}(s) = \max_a \sum_{s'} P^a_{ss'} \left[r^a_{ss'} + \gamma V_k(s') \right]$$

  - use Bellman optimality equation as an update
  - converges to V*

Using DP

- need complete model of the environment and rewards
  - robot in a room
    - state space, action space, transition model
- can we use DP to solve
  - robot in a room?
  - back gammon?
  - helicopter?
- DP bootstraps
  - updates estimates on the basis of other estimates

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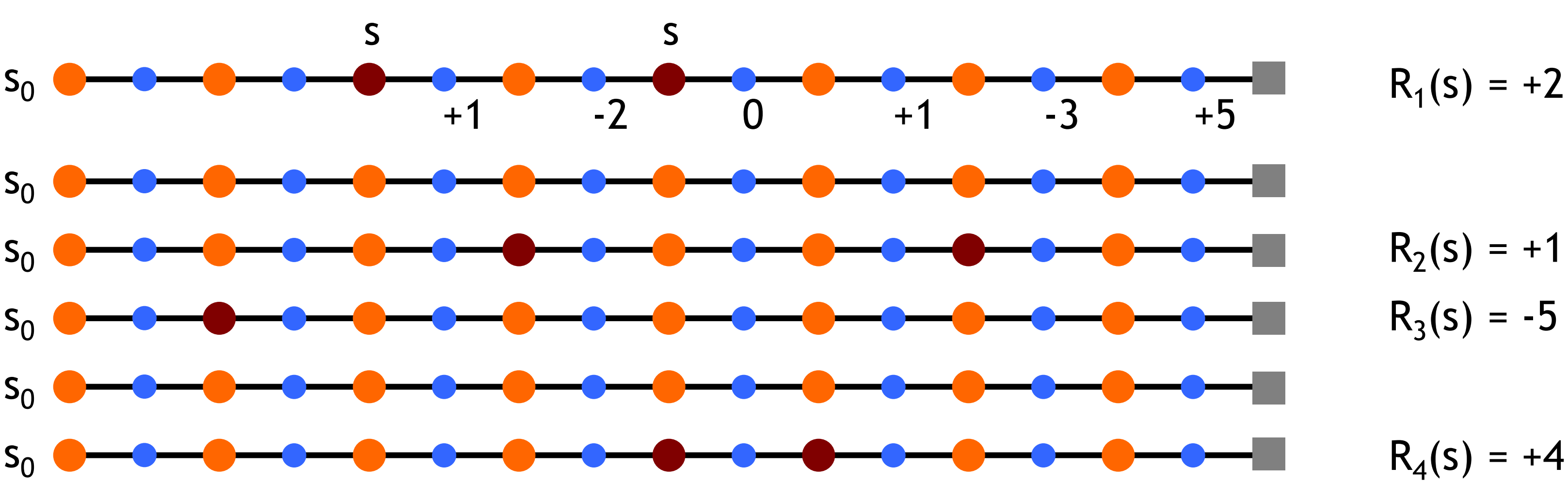
Monte Carlo methods

- don’t need full knowledge of environment
  - just experience, or
  - simulated experience
- averaging sample returns
  - defined only for episodic tasks
- but similar to DP
  - policy evaluation, policy improvement

Monte Carlo policy evaluation

- want to estimate $V^\pi(s)$
  = expected return starting from s and following π
  - estimate as average of observed returns in state s

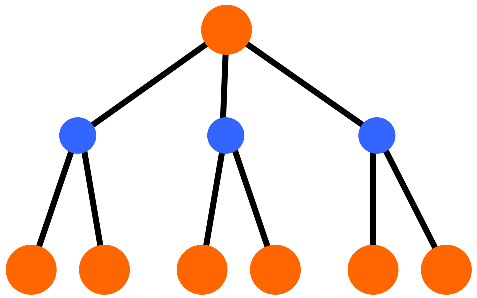
- first-visit MC
  - average returns following the first visit to state s



$V^\pi(s) \approx (2 + 1 - 5 + 4)/4 = 0.5$

Monte Carlo control

- V^π not enough for policy improvement
  - need exact model of environment



- estimate $Q^\pi(s,a)$

$$\pi'(s) = \arg\max_a Q^\pi(s,a)$$

- MC control

$$\pi_0 \xrightarrow{E} Q^{\pi_0} \xrightarrow{I} \pi_1 \xrightarrow{E} Q^{\pi_1} \xrightarrow{I} \dots \xrightarrow{I} \pi^* \xrightarrow{E} Q^*$$

  - update after each episode

- non-stationary environment

$$V(s) \leftarrow V(s) + \alpha\,[R - V(s)]$$

- a problem
  - greedy policy won't explore all actions

Maintaining exploration

- key ingredient of RL
- deterministic/greedy policy won't explore all actions
  - don't know anything about the environment at the beginning
  - need to try all actions to find the optimal one
- maintain exploration
  - use *soft* policies instead: $\pi(s,a)>0$ (for all s,a)
- ε-greedy policy
  - with probability 1-ε perform the optimal/greedy action
  - with probability ε perform a random action

  - will keep exploring the environment
  - slowly move it towards greedy policy: ε -> 0

Simulated experience

- 5-card draw poker
  - s_0: A♣, A♦, 6♠, A♥, 2♠
  - a_0: discard 6♠, 2♠
  - s_1: A♣, A♦, A♥, A♠, 9♠ + dealer takes 4 cards
  - return: +1 (probably)

- DP
  - list all states, actions, compute P(s,a,s')
    - P([A♣,A♦,6♠,A♥,2♠], [6♠,2♠], [A♠,9♠,4]) = 0.00192

- MC
  - all you need are sample episodes
  - let MC play against a random policy, or itself, or another algorithm

Summary of Monte Carlo

- don't need model of environment
  - averaging of sample returns
  - only for episodic tasks
- learn from:
  - sample episodes
  - simulated experience
- can concentrate on "important" states
  - don't need a full sweep
- no bootstrapping
  - less harmed by violation of Markov property
- need to maintain exploration
  - use soft policies

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Temporal Difference Learning

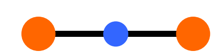
- combines ideas from MC and DP
  - like MC: learn directly from experience (don't need a model)
  - like DP: bootstrap
  - works for continuous tasks, usually faster then MC

- constant-alpha MC:
  - have to wait until the end of episode to update

$$V(s_t) \leftarrow V(s_t) + \alpha \left[R_t - V(s_t)\right]$$

target

- simplest TD
  - update after every step, based on the successor



$$V(s_t) \leftarrow V(s_t) + \alpha \left[r_{t+1} + \gamma V(s_{t+1}) - V(s_t)\right]$$

MC vs. TD

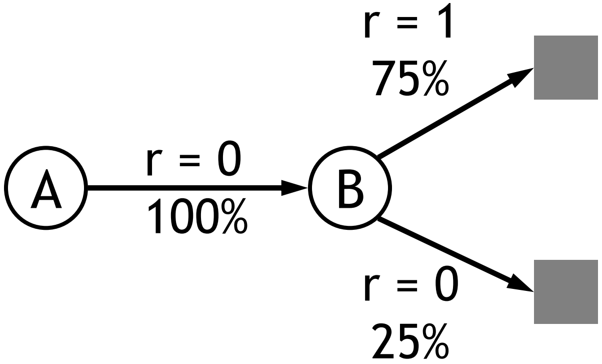
- observed the following 8 episodes:

A – 0, B – 0	B – 1	B – 1	B - 1
B – 1	B – 1	B – 1	B – 0

- MC and TD agree on V(B) = 3/4

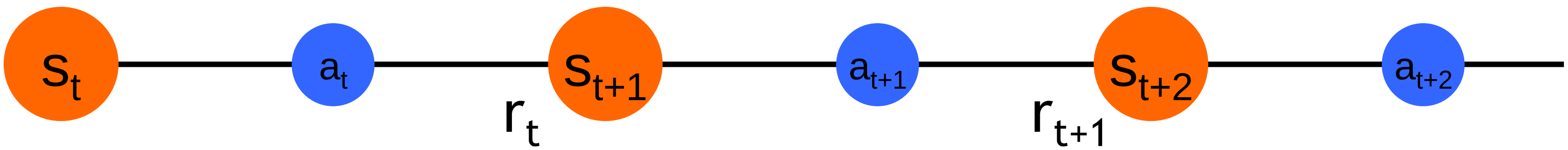
- MC: V(A) = 0
  - converges to values that minimize the error on training data

- TD: V(A) = 3/4
  - converges to ML estimate of the Markov process



Sarsa

- again, need Q(s,a), not just V(s)



$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[r_t + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t) \right]$$

- control
  - start with a random policy
  - update Q and π after each step
  - again, need ε-soft policies

Q-learning

- previous algorithms: on-policy algorithms
  - start with a random policy, iteratively improve
  - converge to optimal

- Q-learning: off-policy
  - use any policy to estimate Q

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t) \right]$$

  - Q directly approximates Q* (Bellman optimality eqn)
  - independent of the policy being followed
  - only requirement: keep updating each (s,a) pair

- Sarsa

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[r_t + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t) \right]$$

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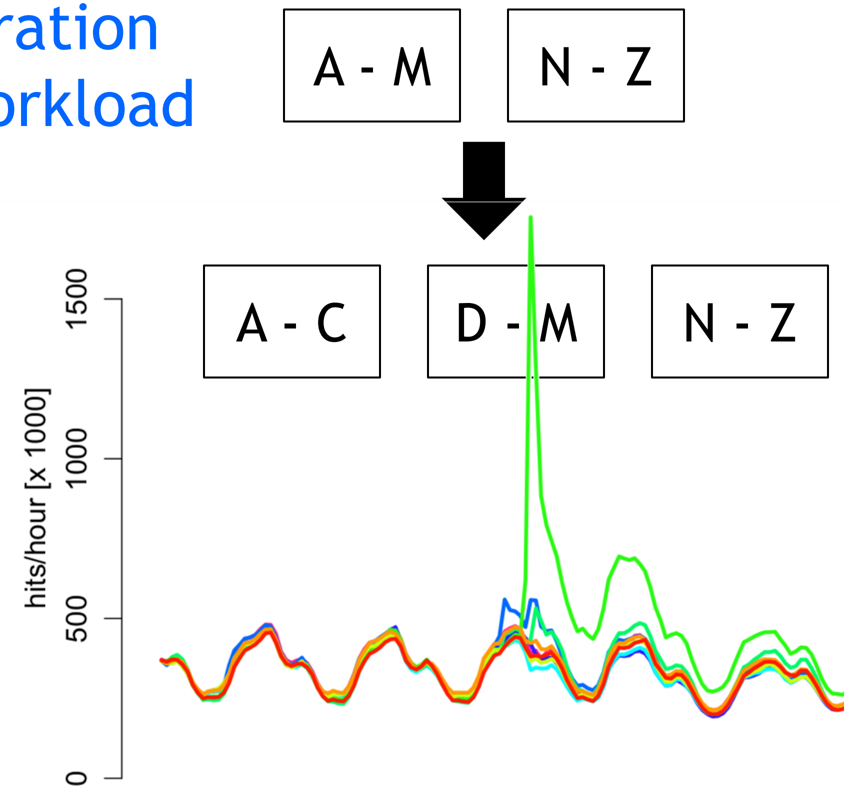
Dynamic resource allocation for a in-memory database

- Goal: adjust system configuration in response to changes in workload
  - moving data is expensive!



- Actions:
  - boot up new machine
  - shut down machine
  - move data from M1 to M2
- Policy:
  - input: workload, current system configuration
  - output: sequence of actions
  - huge space => can’t use a table to represent policy
  - can’t train policy in production system

Model-based approach

- Classical RL is model-free
  - need to explore to estimate effects of actions
  - would take too long in this case

- Model of the system:
  - input: workload, system configuration
  - output: performance under this workload
  - also model *transients*: how long it takes to move data

- Policy can estimate the effects of different actions:
  - can efficiently search for best actions
  - move smallest amount of data to handle workload

Optimizing the policy

- Policy has a few parameters:
  - workload smoothing, safety buffer
  - they affect the cost of using the policy
- Optimizing the policy using a simulator
  - build an approximate simulator of your system
  - **input:** workload trace, policy (parameters)
  - **output:** cost of using policy on this workload
  - run policy, but simulate effects using performance models
  - simulator 1000x faster than real system
- Optimization
  - use hill-climbing, gradient-descent to find optimal parameters
  - see also Pegasus by Andrew Ng, Michael Jordan

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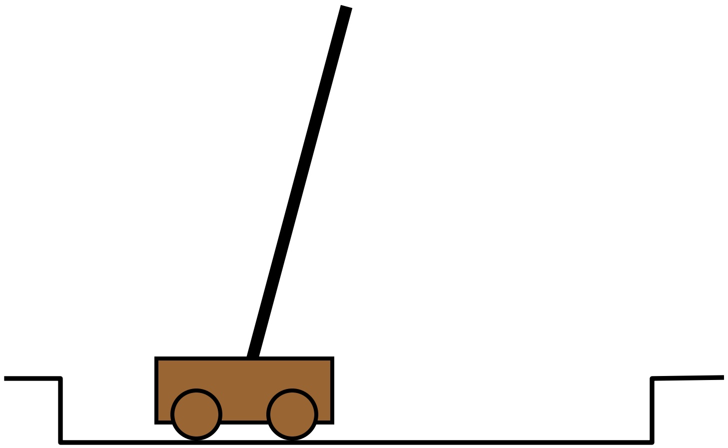
State representation

- pole-balancing
  - move car left/right to keep the pole balanced



- state representation
  - position and velocity of car
  - angle and angular velocity of pole
- what about *Markov property*?
  - would need more info
  - noise in sensors, temperature, bending of pole
- solution
  - coarse discretization of 4 state variables
    - left, center, right
  - totally non-Markov, but still works

Function approximation

- until now, state space small and discrete
- represent V_t as a parameterized function
  - linear regression, decision tree, neural net, …
  - linear regression: $V_t(s) = \vec{\theta}_t^T \vec{\phi}_s = \sum_{i=1}^{n} \theta_t(i)\phi_s(i)$
- update parameters instead of entries in a table
  - better generalization
    - fewer parameters and updates affect “similar” states as well

- TD update

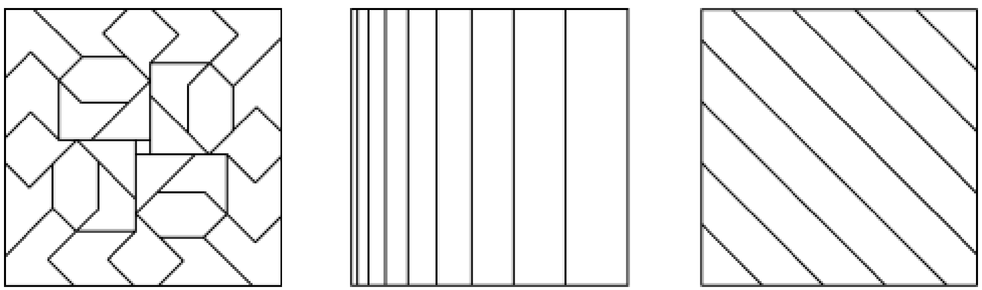
$$V(s_t) \leftarrow V(s_t) + \alpha \left[r_{t+1} + \gamma V(s_{t+1}) - V(s_t) \right]$$

$$V(\underbrace{s_t}_{x}) \mapsto \underbrace{r_{t+1} + \gamma V(s_{t+1})}_{y}$$

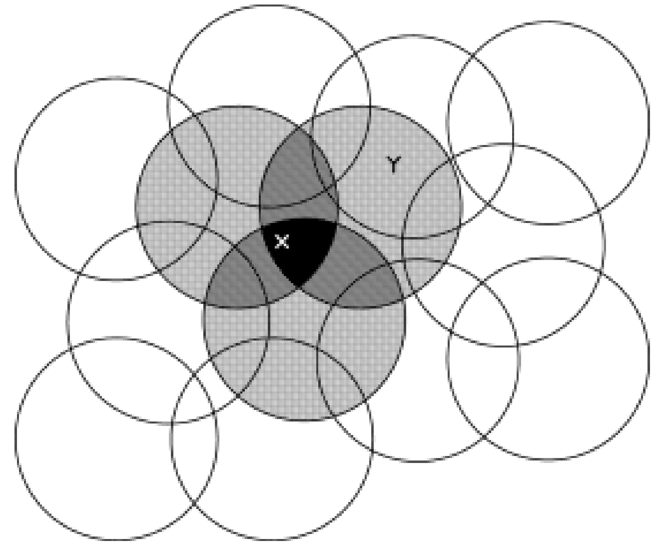
  - treat as one data point for regression
  - want method that can learn on-line (update after each step)

Features

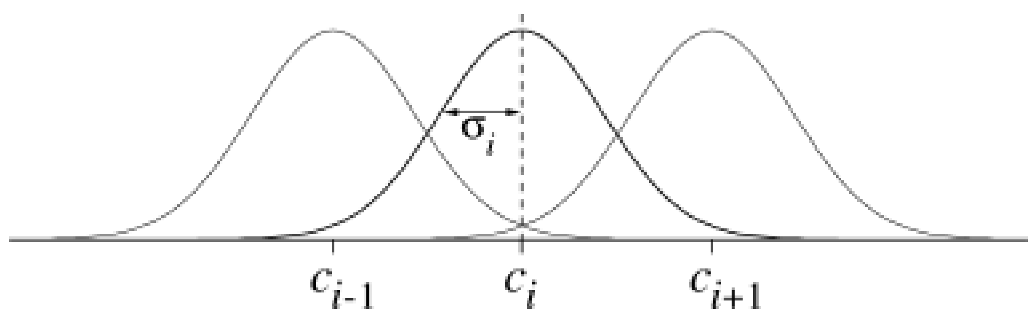
- tile coding, coarse coding
  - binary features



a) Irregular b) Log stripes c) Diagonal stripes



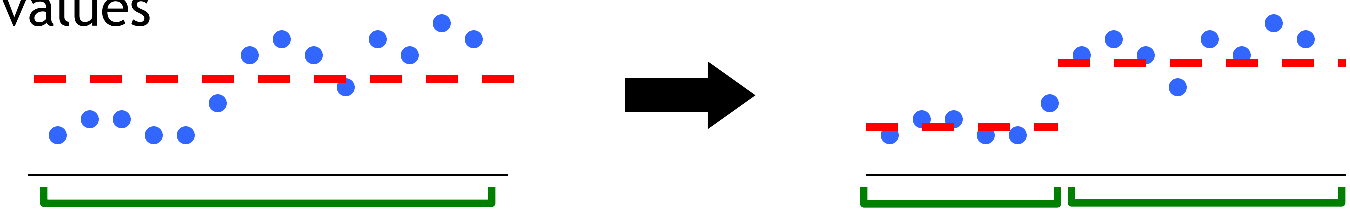
- radial basis functions
  - typically a Gaussian
  - between 0 and 1



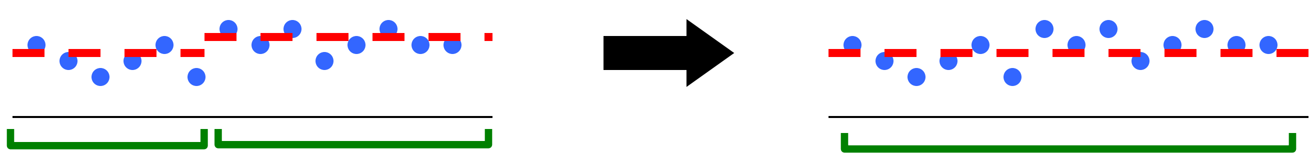
[Sutton & Barto, Reinforcement Learning]

Splitting and aggregation

- want to discretize the state space
  - learn the best discretization during training
- splitting of state space
  - start with a single state
  - split a state when different *parts of that state* have different values



- state aggregation
  - start with many states
  - merge states with similar values

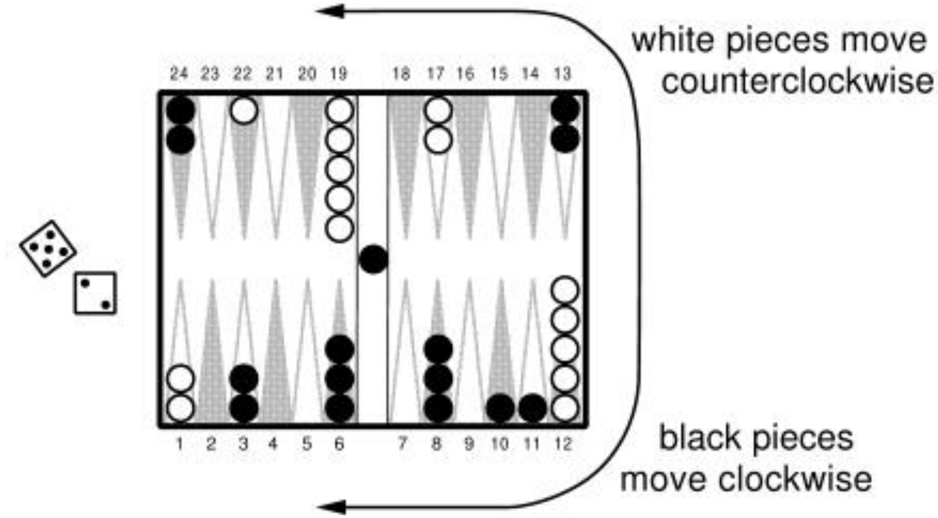


Designing rewards

- robot in a maze
  - episodic task, not discounted, +1 when out, 0 for each step
- chess
  - GOOD: +1 for winning, -1 losing
  - BAD: +0.25 for taking opponent's pieces
    - high reward even when lose
- rewards
  - rewards indicate what we want to accomplish
  - NOT how we want to accomplish it
- shaping

  - positive reward often very "far away"
  - rewards for achieving subgoals (domain knowledge)
  - also: adjust initial policy or initial value function

Case study: Back gammon



- rules
  - 30 pieces, 24 locations
  - roll 2, 5: move 2, 5
  - hitting, blocking
  - branching factor: 400
- implementation
  - use TD(λ) and neural nets
  - 4 binary features for each position on board (# white pieces)
  - no BG expert knowledge
- results
  - TD-Gammon 0.0: trained against itself (300,000 games)
    - as good as best previous BG computer program (also by Tesauro)
    - lot of expert input, hand-crafted features
  - TD-Gammon 1.0: add special features
  - TD-Gammon 2 and 3 (2-ply and 3-ply search)
    - 1.5M games, beat human champion

Summary

- Reinforcement learning
  - use when need to make decisions in uncertain environment
  - actions have delayed effect

- solution methods
  - dynamic programming
    - need complete model

  - Monte Carlo
  - time difference learning (Sarsa, Q-learning)

- simple algorithms
- most work
  - designing features, state representation, rewards

www.cs.ualberta.ca/~sutton/book/the-book.html