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Design of gravity balancing leg orthosis using non-zero free length springs

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Abstract

For gait retraining of stroke victims, there is a need for rehabilitation devices, which can support the weight of leg during walking. Machines that gravity balances the leg are potentially useful as rehabilitation devices. This paper presents gravity balancing designs for two and three links planar chains using non-zero free length springs. Non-zero free length springs are those springs which have initial free length. These springs are cheaper, easily available and can be useful when complete weight balancing is not required. These designs are further optimized for spring connection points and parameters of the spring such as free length and stiffness to achieve greater balancing.

1. Introduction

A machine is said to be gravity balanced if the joint actuator forces are not needed to keep the system in equilibrium. The system behaves as if it is moving in gravity-less environment. Previous works on gravity-balanced machines include designs with counterweights, zero free length springs and auxiliary parallelograms [1,2,4,5]. A method of gravity balancing planar mechanism with springs [2,9], along with parallelogram mechanism to identify the center of mass of the mechanism is used in the hybrid method [8].

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In a machine, gravity balancing can be achieved if the system center of mass remains inertially fixed during motion or if the potential energy of the system remains constant in every configuration. The potential energy of the system can be made to be a constant by adding springs to the system at appropriate places. This paper uses the notion of auxiliary parallelograms to first locate the center of mass and then springs are connected through this point to make the potential energy invariant with configuration.

The theory of gravity balancing can be applied to build a device aimed at subjects who cannot fully lift their leg because of muscle weakness, i.e., muscles are not able to generate enough force to lift the leg. This device will be designed to take away the weight of the leg and be completely passive unlike other devices proposed in [3,6] which are active. The subjects will wear this device as an exoskeleton on the affected leg. All the spring connection points are on the device and not on the human body.

In this paper, we present designs of gravity balancing of one, two and three degrees-of-freedom leg orthoses with non-zero free length springs. We first present a design motivated by one degree-of-freedom hip rotation in the sagittal plane. The second design is for two degrees-of-freedom motion of the leg, namely hip and knee rotation. Finally, the three degrees-of-freedom design with hip, knee and ankle rotation. The non-zero free length designs are compared to the zero free length spring designs.

1.1. Center of mass using auxiliary parallelograms

The underlying philosophy of our study is to locate the center of mass of a multi degrees-of-freedom system using auxiliary parallelograms [1], as illustrated through an example in Fig. 1. The location of center of mass of a three link planar open-chain, from the point O_1, as shown in Fig. 1, is given by

$$r_{o_1c} = s_1\mathbf{b_1} + s_2\mathbf{b_2} + s_3\mathbf{b_3}, \tag{1}$$

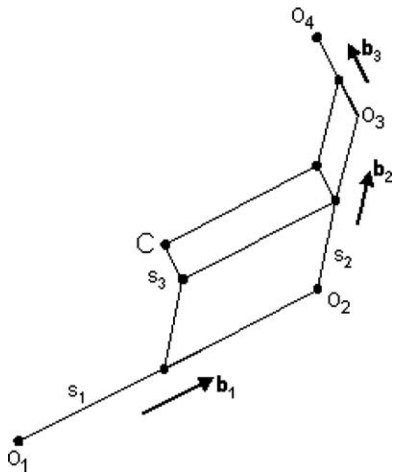

Fig. 1. A three link system showing the auxiliary parallelograms to determine center of mass C [1].

where

$$s_i = \frac{1}{M}\left(\sum_{k=i+1}^{n} m_k l_i + m_i l_{*i}\right), \quad \text{and} \quad M = \sum_{i=1}^{n} m_i,$$

Here, $\mathbf{b_i}$ is the unit vector along link i, $l_i = O_iO_{i+1}$ is the length of link i, M is the total mass of the system. In this example, the number of links $n = 3$, and l_{*i} is the center of mass of link i from point O_i. Note that s_i are factors of geometry and mass distribution of the links and are usually denoted by the terminology "scaled lengths". The three scaled lengths can then be used to form parallelograms in order to identify the center of mass C. For the three link mechanism, the system consists of parallelograms in two layers. The first layer has two parallelograms while the second layer has one. This procedure can be extended to n links as described in [1]. If the machine is connected to the base at the point C, the center of mass becomes an inertially fixed point, and the system becomes gravity balanced.

In this paper, we are considering an alternative approach to gravity balancing. The underlying theory in this paper is to add springs at appropriate places in the machine such that the sum total of the gravitational potential energy and spring potential energy becomes invariant with configuration. This procedure is illustrated through several examples on one, two and three link open-chain designs. The spring potential energy for a spring connected between two points separated by a distance d, is written as $1/2Kd^2$ or $1/2K(d - l_0)^2$. The first case is for the spring with zero free length while the second represents the energy for a spring with free length l_0. Gravity-balanced devices using zero free length springs have been proposed in rehabilitation [7,8]. In the literature, balancing with zero free length has been proposed. However, in practice, zero free length can be obtained only through clever engineering [4,5,7] or manufactured at an increased cost. In order to avoid the complexity and cost, we study gravity balancing devices with non-zero free length springs in this paper.

The organization of this paper is as follows: The next section describes the underlying theory of gravity-balanced machines for one link design with zero and non-zero free length springs. This is followed by a section which compares two link designs with zero and non-zero free length springs. The next section considers three link designs with zero and non-zero free length springs. This is followed by a section focussed on optimization of all the above designs to obtain the performance.

2. One link design

Consider a link connected by a revolute joint to an inertially fixed frame and supported by a spring as shown in Fig. 2. The potential energy of the system can be written as

$$V = V_g + V_s, \tag{2}$$

where

$$V_g = Mgl_{c1}\sin\theta, \tag{3}$$

$$V_s = \frac{1}{2}K(x - x_0)^2. \tag{4}$$

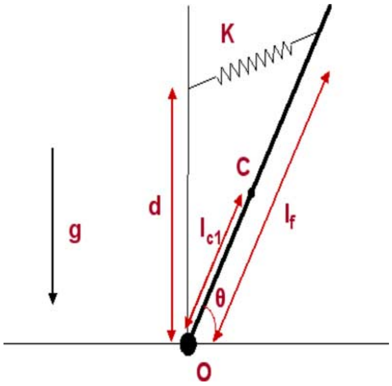

Fig. 2. One link system.

Here,

M = total mass of the link,
l_{c1} = distance of center of mass C from point O,
K = stiffness of the spring,
x_0 = free length of the spring,
x = length of the spring at current angle θ.

On substituting these expressions in V, we get

$$V = Mgl_{c1}\sin\theta + \frac{1}{2}K(x - x_0)^2. \tag{5}$$

For zero free length spring, $x_0 = 0$, and the expression for potential energy takes the form

$$V_0 = Mgl_{c1}\sin\theta + \frac{1}{2}Kx^2, \tag{6}$$

where

$$x^2 = d^2 + l_f^2 - 2dl_f\sin\theta, \tag{7}$$

and d, l_f and l_{c1} are defined in Fig. 2. On substituting, we obtain

$$V_0 = (Mgl_{c1} - Kdl_f)\sin\theta + \frac{1}{2}K(d^2 + l_f^2). \tag{8}$$

For static balancing, $\partial \mathbf{V}_0/\partial\theta = 0$. The potential energy becomes constant when the coefficient of $\sin\theta$ is zero, i.e.,

$$Mgl_{c1} - Kdl_f = 0, \tag{9}$$

$$K = \frac{Mgl_{c1}}{dl_f}. \tag{10}$$

Using non-zero free length spring ($x_0 \neq 0$), the potential energy can be written as

$$V = Mgl_{c1}\sin\theta + \frac{1}{2}K\left(d^2 + l_f^2 - 2dl_f\sin\theta + x_0^2 - 2x_0\sqrt{d^2 + l_f^2 - 2dl_f\sin\theta}\right). \tag{11}$$

Unlike the zero free length case, this equation will not be a constant for any selected value of K. The potential energy of non-zero free length spring system can be written as

$$V = Mgl_{c1}\sin\theta + \frac{1}{2}K\left(d^2 + l_f^2 - 2dl_f\sin\theta + x_0^2 - 2x_0\sqrt{d^2 + l_f^2}\sqrt{1 - \frac{2dl_f\sin\theta}{d^2 + l_f^2}}\right). \tag{12}$$

Using Taylor's Series, if $|\frac{2dl_f\sin\theta}{d^2+l_f^2}| \leqslant 1$, we can make the approximation

$$V = Mgl_{c1}\sin\theta + \frac{1}{2}K\left(d^2 + l_f^2 - 2dl_f\sin\theta + x_0^2 - 2x_0\sqrt{d^2 + l_f^2}\left(1 - \frac{dl_f\sin\theta}{d^2 + l_f^2}\right)\right), \tag{13}$$

$$= Mgl_{c1}\sin\theta + \frac{1}{2}K\left(d^2 + l_f^2 + x_0^2 - 2x_0\sqrt{d^2 + l_f^2} - 2dl_f\sin\theta\left(1 - \frac{x_0}{\sqrt{d^2 + l_f^2}}\right)\right). \tag{14}$$

For gravity balancing, the coefficient of $\sin\theta$ should vanish. So, we get

$$Mgl_{c1} - Kdl_f\left(1 - \frac{x_0}{\sqrt{d^2 + l_f^2}}\right) = 0.$$

This simplifies to

$$K = \frac{Mgl_{c1}}{dl_f\left(1 - \frac{x_0}{\sqrt{d^2+l_f^2}}\right)}.$$

Note that this is true for the approximation $|\frac{2dl_f}{d^2+l_f^2}| \leqslant 1$, since $\sin\theta$ is bounded between -1 and 1. For $x_0 = 0$, this design becomes the same as the zero free length spring design.

3. Two link design

3.1. Design A

A schematic of a two link mechanism supported by two springs is shown in Fig. 3. The fixed end of spring one is on the vertical axis located through O. The second spring connects the center of mass with the joint 2. The two spring lengths x_1 and x_2 satisfy the following relations

$$x_1^2 = (s_1\cos\theta_1 + s_2\cos(\theta_1 + \theta_2))^2 + (d_1 + s_1\sin\theta_1 + s_2\sin(\theta_1 + \theta_2))^2, \tag{15}$$

$$x_2^2 = d_2^2 + s_2^2 - 2d_2s_2\cos\theta_2. \tag{16}$$

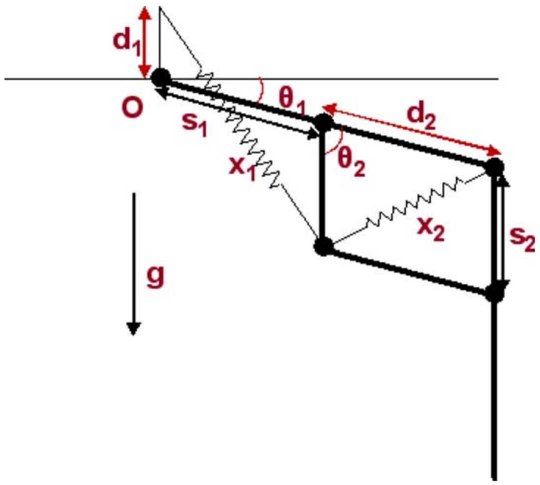

Fig. 3. Two link design A.

All symbols appearing in the two equations are shown in Fig. 3. Using zero free length springs, the total potential energy of the system is given by

$$V = -Mg[s_1 \sin\theta_1 + s_2 \sin(\theta_1 + \theta_2)] + \frac{1}{2}K_1 x_1^2 + \frac{1}{2}K_2 x_2^2, \tag{17}$$

$$= -Mg[s_1 \sin\theta_1 + s_2 \sin(\theta_1 + \theta_2)] + \frac{1}{2}K_1[c_1^2 + 2d_1 s_1 \sin\theta_1 + 2d_1 s_2 \sin(\theta_1 + \theta_2) + 2s_1 s_2 \cos\theta_2] + \frac{1}{2}K_2(c_2^2 - 2d_2 s_2 \cos\theta_2). \tag{18}$$

Here, $c_1 = \sqrt{d_1^2 + s_1^2 + s_2^2}$, $c_2 = \sqrt{d_2^2 + s_2^2}$.

We can find the values of K_1, K_2 that make the coefficients of $\sin\theta_1$, $\sin(\theta_1 + \theta_2)$ and $\cos\theta_2$ to zero. Now, we consider the same design with non-zero free length springs,

$$V = -Mg[s_1 \sin\theta_1 + s_2 \sin(\theta_1 + \theta_2)] + \frac{1}{2}K_1(x_1 - x_{01})^2 + \frac{1}{2}K_2(x_2 - x_{02})^2. \tag{19}$$

Substituting x_1 and x_2 and using Taylor's approximation by assuming

$$\left|\frac{2d_1 s_1 \sin\theta_1 + 2d_1 s_2 \sin(\theta_1 + \theta_2) + 2s_1 s_2 \cos\theta_2}{c_1^2}\right| \leqslant 1 \quad \text{and} \quad \left|\frac{2d_2 s_2 \cos\theta_2}{c_2^2}\right| \leqslant 1,$$

we get

$$\begin{aligned} V = {} & -Mg[s_1 \sin\theta_1 + s_2 \sin(\theta_1 + \theta_2)] \\ & + \frac{1}{2}K_1\left[c_1^2 + x_{01}^2 - 2x_{01}c_1 + (2d_1 s_1 \sin\theta_1 + 2d_1 s_2 \sin(\theta_1 + \theta_2) + 2s_1 s_2 \cos\theta_2)\left(1 - \frac{x_{01}}{c_1}\right)\right] \\ & + \frac{1}{2}K_2\left[c_2^2 + x_{02}^2 - 2x_{02}c_2 - 2d_2 s_2 \cos\theta_2\left(1 - \frac{x_{02}}{c_2}\right)\right]. \end{aligned} \tag{20}$$

Table 1
Comparison of two link designs

Design A (zero free length spring)	Design A (non-zero free length spring)	Design B
$K_1 = \frac{Mg}{d_1}$	$K_1 = \frac{Mg}{d_1\left(1-\frac{x_{01}}{c_1}\right)}$	$K_1 = \frac{Mg}{d_1\left(\frac{x_{01}}{c_1}-1\right)}$
$K_2 = \frac{Mg}{d_1 d_2}$	$K_2 = \frac{Mg}{d_1 d_2\left(1-\frac{x_{02}}{c_2}\right)}$	$K_2 = \frac{Mg}{d_1 d_2\left(1-\frac{x_{02}}{c_2}\right)}$

Now, we can find values of K_1, K_2 by putting the coefficients of $\sin\theta_1$, $\sin(\theta_1+\theta_2)$ and $\cos\theta_2$ to zero. These design conditions are summarized in Table 1. This design becomes the same as the design with zero free length spring if we make x_{01} and x_{02} zero.

3.2. Design B

Fig. 4 shows another two link design where the spring connection point is below the hip level. Considering the design with non-zero free length springs,

$$x_1^2 = (s_1\cos\theta_1 + s_2\cos(\theta_1+\theta_2))^2 + (-d_1 + s_1\sin\theta_1 + s_2\sin(\theta_1+\theta_2))^2, \tag{21}$$

$$x_2^2 = d_2^2 + s_2^2 + 2d_2 s_2\cos\theta_2. \tag{22}$$

Substituting x_1 and x_2 in the potential energy V, we get

$$\begin{aligned} V = &-Mg[s_1\sin\theta_1 + s_2\sin(\theta_1+\theta_2)] + \frac{1}{2}K_1\Big[c_1^2 + x_{01}^2 - 2d_1 s_1\sin\theta_1 - 2d_1 s_2\sin(\theta_1+\theta_2) \\ &+2s_1 s_2\cos\theta_2 - 2x_{01}\sqrt{c_1^2 - 2d_1 s_1\sin\theta_1 - 2d_1 s_2\sin(\theta_1+\theta_2) + 2s_1 s_2\cos\theta_2}\Big] \\ &+\frac{1}{2}K_2\left[c_2^2 + x_{02}^2 - 2d_2 s_2\cos\theta_2 - 2x_{02}\sqrt{c_2^2 + 2d_2 s_2\cos\theta_2}\right], \end{aligned} \tag{23}$$

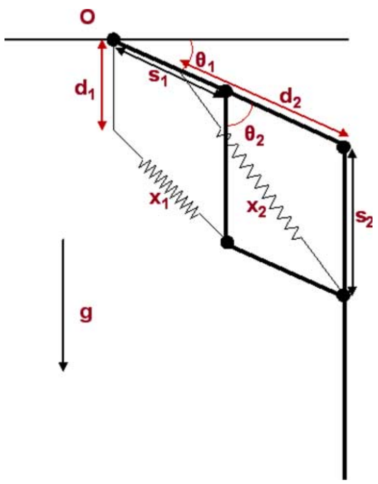

Fig. 4. Two link design B.

Using Taylor's approximation by assuming

$$\left|\frac{-2d_1s_1\sin\theta_1 - 2d_1s_2\sin(\theta_1+\theta_2) + 2s_1s_2\cos\theta_2}{c_1^2}\right| \leqslant 1 \quad \text{and} \quad \left|\frac{2d_2s_2\cos\theta_2}{c_2^2}\right| \leqslant 1,$$

we get

$$\begin{aligned} V = &-Mg[s_1\sin\theta_1 + s_2\sin(\theta_1+\theta_2)] \\ &+\frac{1}{2}K_1\left[c_1^2 + x_{01}^2 - 2x_{01}c_1 + (-2d_1s_1\sin\theta_1 - 2d_1s_2\sin(\theta_1+\theta_2) + 2s_1s_2\cos\theta_2)\left(1-\frac{x_{01}}{c_1}\right)\right] \\ &+\frac{1}{2}K_2\left[c_2^2 + x_{02}^2 - 2x_{02}c_2 + 2d_2s_2\cos\theta_2\left(1-\frac{x_{02}}{c_2}\right)\right]. \end{aligned} \tag{24}$$

Now, we can find values of K_1, K_2 by putting the coefficients of $\sin\theta_1$, $\sin(\theta_1 + \theta_2)$ and $\cos\theta_2$ to zero. These design conditions are summarized in Table 1.

3.3. Comparison of two designs

It can be seen from Table 1 that if $x_{01} = 0$ and $x_{02} = 0$, the design A with zero free length springs is the same as with non-zero free length springs while it is not the case with design B as design B is not gravity balanced with zero free length springs.

Please note from Table 1 that the first spring of designs A and B are different. In the case of design B, first spring can be a compression spring. In case of design A, all springs are extension springs.

4. Three link design

4.1. Design A

A schematic of a three link mechanism supported by four springs is shown in Fig. 5. The fixed end connection of spring 1 is on a vertical axis located on the first link.

$$\begin{aligned} x_1^2 = &(s_1\cos\theta_1 + s_2\cos(\theta_1+\theta_2) + s_3\cos(\theta_1+\theta_2+\theta_3))^2 + (d_1 + s_1\sin\theta_1 + s_2\sin(\theta_1+\theta_2) \\ &+ s_3\sin(\theta_1+\theta_2+\theta_3))^2, \end{aligned} \tag{25}$$

$$x_2^2 = d_2^2 + s_2^2 - 2d_2s_2\cos\theta_2, \tag{26}$$

$$x_3^2 = d_3^2 + s_3^2 - 2d_3s_3\cos(\theta_2+\theta_3), \tag{27}$$

$$x_4^2 = d_4^2 + s_3^2 - 2d_4s_3\cos\theta_3. \tag{28}$$

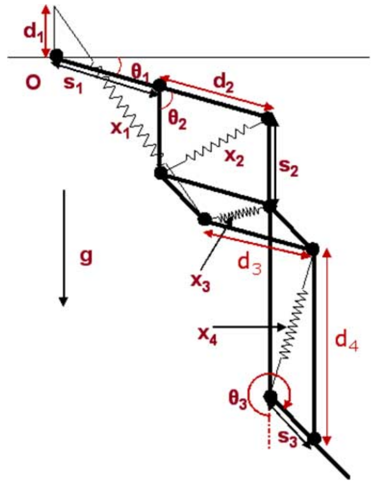

Fig. 5. Three link design A.

Using zero free length springs, the total potential energy of the system is given by

$$V = -Mg[s_1 \sin\theta_1 + s_2 \sin(\theta_1+\theta_2) + s_3 \sin(\theta_1+\theta_2+\theta_3)] + \frac{1}{2}K_1x_1^2 + \frac{1}{2}K_2x_2^2 + \frac{1}{2}K_3x_3^2 + \frac{1}{2}K_4x_4^2. \tag{29}$$

$$\begin{aligned} V = {} & -Mg[s_1 \sin\theta_1 + s_2 \sin(\theta_1+\theta_2) + s_3 \sin(\theta_1+\theta_2+\theta_3)] + \frac{1}{2}K_1[p_1^2 + 2d_1s_1\sin\theta_1 \\ & + 2d_1s_2\sin(\theta_1+\theta_2) + 2d_1s_3\sin(\theta_1+\theta_2+\theta_3) + 2s_1s_2\cos\theta_2 + 2s_2s_3\cos\theta_3 \\ & + 2s_1s_3\cos(\theta_2+\theta_3)] + \frac{1}{2}K_2(p_2^2 - 2d_2s_2\cos\theta_2) + \frac{1}{2}K_3[p_3^2 - 2d_2s_3\cos(\theta_2+\theta_3)] \\ & + \frac{1}{2}K_4[p_4^2 - 2(l_2 - s_2)s_3\cos\theta_3], \end{aligned} \tag{30}$$

where $p_1 = \sqrt{d_1^2 + s_1^2 + s_2^2 + s_3^2}$, $p_2 = \sqrt{d_2^2 + s_2^2}$, $p_3 = \sqrt{d_3^2 + s_3^2}$, $p_4 = \sqrt{d_4^2 + s_3^2}$.

Now, we can find values of K_1, K_2, K_3 and K_4 by putting the coefficients of $\sin\theta_1$, $\sin(\theta_1+\theta_2)$, $\sin(\theta_1+\theta_2+\theta_3)$, $\cos(\theta_2+\theta_3)$, $\cos\theta_3$ and $\cos\theta_2$ to zero. These design conditions are summarized in Table 2. Now, we consider the same design with non-zero free length springs,

$$V = -Mg[s_1 \sin\theta_1 + s_2 \sin(\theta_1+\theta_2) + s_3 \sin(\theta_1+\theta_2+\theta_3)] + \frac{1}{2}K_1(x_1 - x_{01})^2 + \frac{1}{2}K_2(x_2 - x_{02})^2 + \frac{1}{2}K_3(x_3 - x_{03})^2 + \frac{1}{2}K_4(x_4 - x_{04})^2. \tag{31}$$

After using Taylor's approximation by assuming

$$\left|\frac{2d_1s_1\sin\theta_1 + 2d_1s_2\sin(\theta_1+\theta_2) + 2d_1s_3\sin(\theta_1+\theta_2+\theta_3) + 2s_1s_2\cos\theta_2 + 2s_2s_3\cos\theta_3}{p_1^2} + \frac{2s_1s_3\cos(\theta_2+\theta_3)}{p_1^2}\right| \leqslant 1,$$

$$\left|\frac{2d_2s_2\cos\theta_2}{p_2^2}\right| \leqslant 1, \quad \left|\frac{2d_3s_3\cos(\theta_2+\theta_3)}{p_3^2}\right| \leqslant 1 \quad \text{and} \quad \left|\frac{2d_4s_3\cos\theta_3}{p_4^2}\right| \leqslant 1,$$

Table 2
Comparison of three link designs

Design A (zero free length spring)	Design A (non-zero free length spring)	Design B
$K_1 = \frac{Mg}{d_1}$	$K_1 = \frac{Mg}{d_1\left(1-\frac{x_{01}}{p_1}\right)}$	$K_1 = \frac{Mg}{d_1\left(\frac{x_{01}}{p_1}-1\right)}$
$K_2 = \frac{Mg}{d_1 d_2}$	$K_2 = \frac{Mg}{d_1 d_2\left(1-\frac{x_{02}}{p_2}\right)}$	$K_2 = \frac{Mg}{d_1 d_2\left(1-\frac{x_{02}}{p_2}\right)}$
$K_3 = \frac{Mg}{d_1 d_3}$	$K_3 = \frac{Mg}{d_1 d_3\left(1-\frac{x_{03}}{p_3}\right)}$	$K_3 = \frac{Mg}{d_1 d_3\left(1-\frac{x_{03}}{p_3}\right)}$
$K_4 = \frac{Mg}{d_1 d_4}$	$K_4 = \frac{Mg}{d_1 d_4\left(1-\frac{x_{04}}{p_4}\right)}$	$K_4 = \frac{Mg}{d_1 d_4\left(1-\frac{x_{04}}{p_4}\right)}$

we get

$$\begin{aligned} V = &-Mg[s_1\sin\theta_1 + s_2\sin(\theta_1+\theta_2) + s_3\cos(\theta_1+\theta_2+\theta_3)] \\ &+\frac{1}{2}K_1\Bigg[p_1^2 + x_{01}^2 - 2x_{01}p_1 + (2d_1s_1\sin\theta_1.+2d_1s_2\sin(\theta_1+\theta_2) + 2d_1s_3\sin(\theta_1+\theta_2+\theta_3) \\ &+ 2s_1s_2\cos\theta_2 + 2s_2s_3\cos\theta_3 + 2s_1s_3\cos(\theta_2+\theta_3))\left(1-\frac{x_{01}}{p_1}\right)\Bigg] \\ &+\frac{1}{2}K_2\left[p_2^2 + x_{02}^2 - 2x_{02}p_2 - 2d_2s_2\cos\theta_2\left(1-\frac{x_{02}}{p_2}\right)\right] \\ &+\frac{1}{2}K_3\left[p_3^2 + x_{03}^2 - 2x_{03}p_3 - 2d_3s_3\cos(\theta_2+\theta_3)\left(1-\frac{x_{03}}{p_3}\right)\right] \\ &+\frac{1}{2}K_4\left[p_4^2 + x_{04}^2 - 2x_{04}p_4 - 2d_4s_3\cos\theta_3\left(1-\frac{x_{04}}{p_4}\right)\right], \end{aligned} \tag{32}$$

Now, we can find values of K_1, K_2, K_3 and K_4 by putting the coefficients of $\sin\theta_1$, $\sin(\theta_1+\theta_2)$, $\sin(\theta_1+\theta_2+\theta_3)$, $\cos(\theta_2+\theta_3)$, $\cos\theta_3$ and $\cos\theta_2$ to zero. These design conditions are summarized in Table 2.

4.2. Design B

Fig. 6 shows a second three link design where the spring connection point is below the hip level. Considering the design with non-zero free length springs,

$$\begin{aligned} x_1^2 = &(s_1\cos\theta_1 + s_2\cos(\theta_1+\theta_2) + s_3\cos(\theta_1+\theta_2+\theta_3))^2 \\ &+ (-d_1 + s_1\sin\theta_1 + s_2\sin(\theta_1+\theta_2) + s_3\sin(\theta_1+\theta_2+\theta_3))^2, \end{aligned} \tag{33}$$

$$x_2^2 = d_2^2 + s_2^2 + 2d_2s_2\cos\theta_2, \tag{34}$$

$$x_3^2 = d_3^2 + s_3^2 + 2d_3s_3\cos(\theta_2+\theta_3), \tag{35}$$

$$x_4^2 = d_4^2 + s_3^2 + 2d_4s_3\cos\theta_3. \tag{36}$$

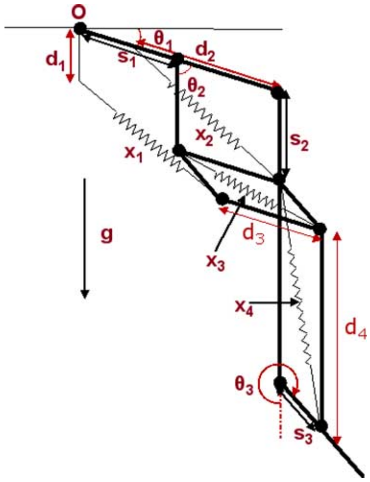

Fig. 6. Three link design B.

After using Taylor's approximation by assuming

$$\left|\frac{-2d_1s_1\sin\theta_1-2d_1s_2\sin(\theta_1+\theta_2)-2d_1s_3\sin(\theta_1+\theta_2+\theta_3)+2s_1s_2\cos\theta_2+2s_2s_3\cos\theta_3}{p_1^2}+\frac{2s_1s_3\cos(\theta_2+\theta_3)}{p_1^2}\right|\leqslant 1$$

$$\left|\frac{2d_2s_2\cos\theta_2}{p_2^2}\right|\leqslant 1,\quad \left|\frac{2d_3s_3\cos(\theta_2+\theta_3)}{p_3^2}\right|\leqslant 1\quad\text{and}\quad \left|\frac{2d_4s_3\cos\theta_3}{p_4^2}\right|\leqslant 1,$$

we get

$$\begin{aligned}
V &= -Mg[s_1\sin\theta_1+s_2\sin(\theta_1+\theta_2)+s_3\sin(\theta_1+\theta_2+\theta_3)]\\
&\quad+\frac{1}{2}K_1\Big[p_1^2+x_{01}^2-2x_{01}p_1+(-2d_1s_1\sin\theta_1-2d_1s_2\sin(\theta_1+\theta_2)-2d_1s_3\sin(\theta_1+\theta_2+\theta_3)\\
&\quad+2s_1s_2\cos\theta_2+2s_2s_3\cos\theta_3+2s_1s_3\cos(\theta_2+\theta_3))\left(1-\frac{x_{01}}{p_1}\right)\Big]\\
&\quad+\frac{1}{2}K_2\left[p_2^2+x_{02}^2-2x_{02}p_2+2d_2s_2\cos\theta_2\left(1-\frac{x_{02}}{p_2}\right)\right]\\
&\quad+\frac{1}{2}K_3\left[p_3^2+x_{03}^2-2x_{03}p_3+2d_3s_3\cos(\theta_2+\theta_3)\left(1-\frac{x_{03}}{p_3}\right)\right]\\
&\quad+\frac{1}{2}K_4\left[p_4^2+x_{04}^2-2x_{04}p_4+2d_4s_3\cos\theta_3\left(1-\frac{x_{04}}{p_4}\right)\right].
\end{aligned}\tag{37}$$

Now, we can find values of K_1, K_2, K_3 and K_4 by putting the coefficients of $\sin\theta_1$, $\sin(\theta_1+\theta_2)$, $\sin(\theta_1+\theta_2+\theta_3)$, $\cos(\theta_2+\theta_3)$, $\cos\theta_3$ and $\cos\theta_2$ to zero. These design conditions are summarized in Table 2.

4.3. Comparison of two designs

It can be seen from Table 2 that if $x_{01} = 0$, $x_{02} = 0$, $x_{03} = 0$ and $x_{04} = 0$, then design A with zero free length springs is the same as with non-zero free length springs while it is not the case with design B as design B is not gravity balanced with zero free length springs.

Please note from Table 2 that the first spring of designs A and B are different. In the case of design B first spring could be a compression spring. In case of design A, all springs are extension springs.

5. Design optimization

The following inertia and geometric parameters are considered in the design of leg orthosis in MKS units: $l_1 = 0.4322$ m, $l_2 = 0.421$ m, $m_1 = m_{\text{thigh}} = 7.39$ kg, $m_{\text{shank}} = 3.11$ kg, $m_{\text{foot}} = 0.97$ kg, $m_2 = m_{\text{shank}} + m_{\text{foot}} = 4.08$ kg, total mass $M = 11.47$ kg, $l_{c1} = 0.41 * l_1$, $l_{c2} = 0.44 * l_2$ and $l_{c3} = 0.4 * l_3$. Please note that m_2 consists of mass of the shank and the foot for a two link design but for a three link design, masses will be treated separately. All parameters are for a normal user and are taken from anthropomorphic data. Using these values, stiffness of springs are derived for different designs.

We have considered the exact (with zero free length spring) and nominal design (with non-zero free length spring) of one, two and three link leg orthosis. In this section we perform the optimization of these designs.

The design parameters selected are d_i, x_{0i}, and K_i where $i = 1, 2, 3, 4$ depending on number of links. The design objective is to minimize the total applied torque due to gravity and springs over its range of motion. The optimization function is selected as

Table 3
Design parameters for one link design

Design	d (m)	x_0 (m)	K (N/m)
No spring	0	0	0
Nominal	0.05	0.2	1099.3
Optimal	0.05	0.15	907.67

Table 4
Design parameters for two link design A

Design	d_1 (m)	x_{01} (m)	K_1 (N/m)	d_2 (m)	x_{02} (m)	K_2 (N/m)
No spring	0	0	0	0	0	0
Nominal	0.05	0.1	3493.6	0.165	0.08	6397
Optimal	0.2	0.34	1750.2	0.164	0.0825	1328

$$f(p) = \sum_{\theta_i} \sum_{i=1}^{n} \alpha_i \left(\frac{\partial V}{\partial \theta_i} \right)^2. \tag{38}$$

A grid is created over the range of joint variable. The function is then optimized to get the optimal parameters. This optimal design torque is compared to a nominal design with non-zero free length springs and a design without the springs. This optimization is performed using the Matlab with the command '*fmincon*'. This is based on SQP method for optimization of function with constraints. The details of this command can be looked in Matlab manual available online. The values α_i's are taken to be 1 so as to give equal weightage to the torque at each joint.

The parameter d_i for the nominal design is chosen such that it satisfies the Taylor's series approximations made to simplify the potential energy expressions for the above designs, while x_{0i} is chosen based on the range of motion such that the springs are either always in extension or always in compression. K_i is chosen to make the potential energy of the nominal system constant. The optimal design parameters for all the designs are listed in Tables 3–7.

One link design: Fig. 7 shows that reduction in the peak torque is about 95% in both nominal and optimal design.

Table 5
Design parameters for two link design B

Design	d_1 (m)	x_{01} (m)	K_1 (N/m)	d_2 (m)	x_{02} (m)	K_2 (N/m)
No spring	0	0	0	0	0	0
Nominal	0.05	0.5	2848.5	0.31	0.08	5268
Optimal	0.05	0.5	2850	0.31	0.08	5200

Table 6
Design parameters for three link design A

Design	d_1 (m)	x_{01} (m)	K_1 (N/m)	d_2 (m)	x_{02} (m)	K_2 (N/m)	d_3 (m)	x_{03} (m)	K_3 (N/m)	d_4 (m)	x_{04} (m)	K_4 (N/m)
No spring	0	0	0	0	0	0	0	0	0	0	0	0
Nominal	0.05	0.14	4307	0.165	0.1	7854	0.165	0.1	9190.8	0.355	0.2	1240
Optimal	0.2	0.35	1746.7	0.165	0.0625	1201	0.165	0.1	600.7	0.355	0.2	701

Table 7
Design parameters for three link design B

Design	d_1 (m)	x_{01} (m)	K_1 (N/m)	d_2 (m)	x_{02} (m)	K_2 (N/m)	d_3 (m)	x_{03} (m)	K_3 (N/m)	d_4 (m)	x_{04} (m)	K_4 (N/m)
No spring	0	0	0	0	0	0	0	0	0	0	0	0
Nominal	0.05	0.4	5571	0.165	0.1	7854	0.165	0.1	9190.8	0.355	0.2	1240
Optimal	0.156	0.39	810	0.164	0.15	2000	0.152	0.1	460	0.353	0.2	700

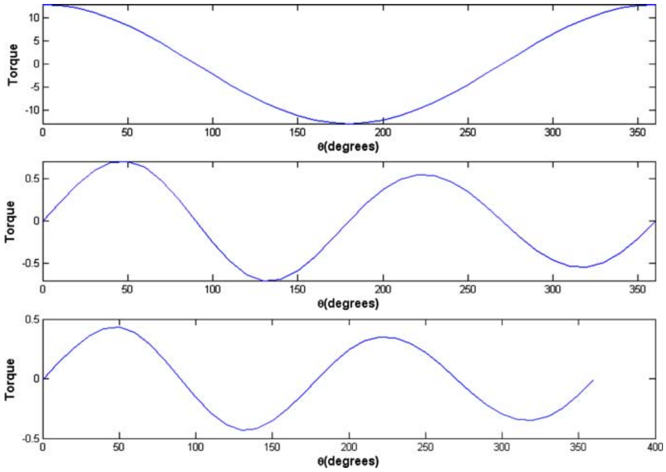

Fig. 7. Joint torque for one link system: (a) without springs, (b) nominal design and (c) optimal design.

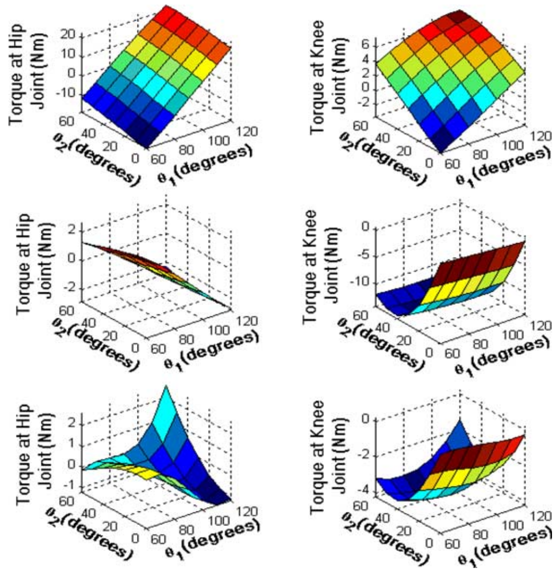

Fig. 8. Joint torque for two link design A: (a) without springs, (b) nominal and (c) optimal.

Two link design A: The peak torque increases at the knee in case of nominal design but there is 90% decrease at the hip, while in optimal design there is 90% and 50% decrease at the hip and the knee respectively (Fig. 8).

Two link design B: The peak torque remains almost same at the knee in case of nominal and optimal design but there is 90% decrease at the hip in both designs (Fig. 9).

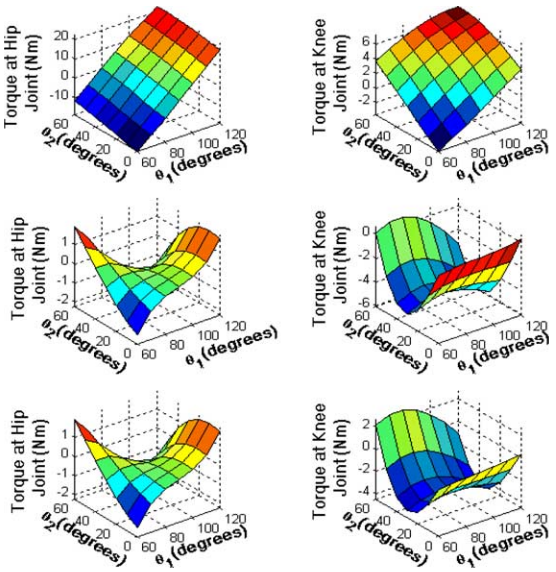

Fig. 9. Joint torque for two link design B: (a) without springs, (b) nominal and (c) optimal.

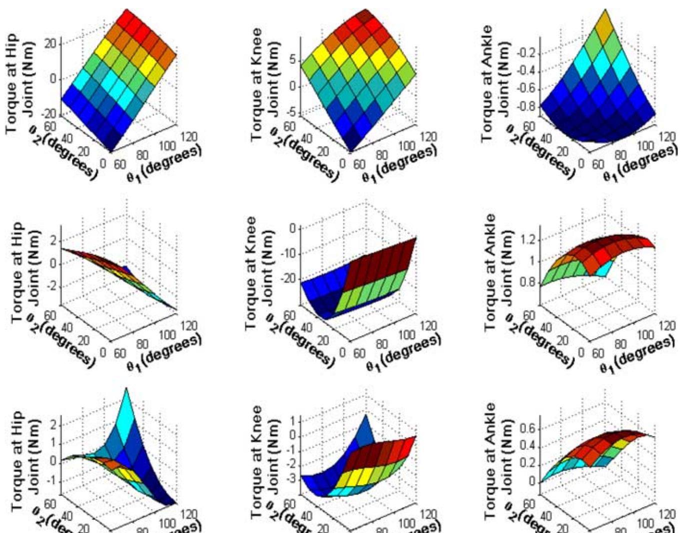

Fig. 10. Joint torque for three link design A: (a) without springs, (b) nominal and (c) optimal.

Three link design A: The peak torque increases at the knee and the ankle in case of nominal design but there is 90% decrease at the hip, while in optimal design there is 90% and 70% decrease at the hip and the knee respectively (Fig. 10).

Three link design B: The peak torque decreases by 75% at the hip in both designs (Fig. 11).

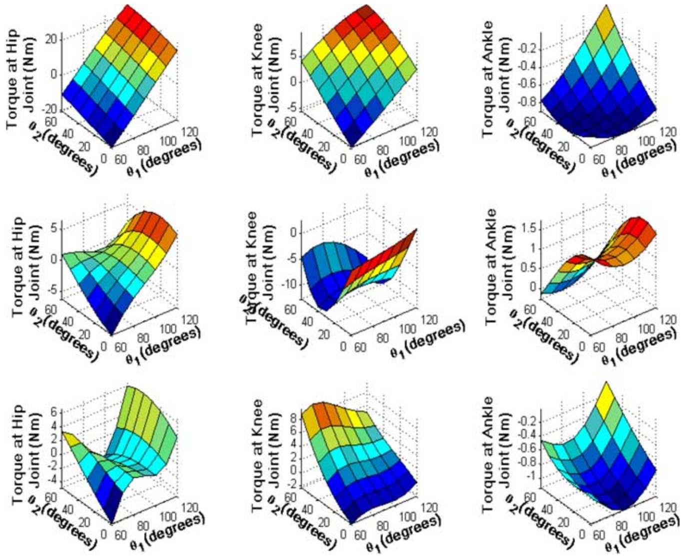

Fig. 11. Joint torque for three link design B: (a) without springs, (b) nominal and (c) optimal.

6. Conclusion

This paper presented one, two and three links design of gravity balancing leg orthosis using non-zero free length springs. These designs require one, two and four springs respectively to take away the weight of the leg. This design may prove to be useful to subjects who can support a part of their body weight. Both zero and non-zero free length spring designs were discussed. Optimization was performed, which gave design parameters that minimize the torque at the joints in the range of motion. The torques at the joints in nominal and optimal designs were compared and torques at the joint were considerably less than the torque at the joints in design without springs.

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