A Coordinated Adaptive Traffic Control Strategy Based on Phase— Time Network

Farzana R. Chowdhury, Peirong (Slade) Wang, Pengfei (Taylor) Li*, Ph.D., P.ENG, M.ASCE

Abstract— Successful traffic signal system implementation will maximize available resources for effective coordinated operations during under saturated conditions, demand variations, and near-saturation conditions. These strategies will allow signal systems to react quickly to anticipated volume surges, to manage queues better, and to recover quickly from congestion by implementing coordination settings such as using early return, late arrival, and platoon identification schemes. These operational guidelines could also be used to take advantage of many currently available traffic monitoring and signal system fine-tuning capabilities to postpone the possibility of oversaturation on many highways and major arterials.

Earlier, coordination has been used in actuated traffic control systems and fixed timed traffic-controlled system. However, in this paper we are going to do time-based coordination in an adaptive traffic control system using in the phase time network. This indicates the uniqueness of this work where we used an adaptive controller in a coordinated intersection to analyze the performance of an isolated intersection using the phase time network model.

I. INTRODUCTION

Traffic signal performance and arterial traffic progression are important for everyday traffic operations. In tradition, the traffic signal performance is evaluated based on end-to-end arterial travel times or speed. With the advancement of modern technology, the adaptive signal control system allows the transit and emergency vehicle to notify the signal controller before approaching the intersection for priority service. The traditional traffic control system adjusts the signal timing by increasing the green time according to the request, which may cause degradation of the overall performance of the system. Traffic progression and delay situations can be improved largely by using traffic signal coordination as it gives the benefit between an adaptive traffic control system and a traditional time-based coordination system by reducing the number of stops hence reducing the overall arterial travel time. In the coordinated system, split time and cycle time can switch according to the existing traffic condition, and phasing sequence and offset time are adjusted to accommodate the progressive flow. This offset adjustment is made by shortening or increasing intermediate phase time. However, this might create a temporary decrease in the overall capacity and generate

additional delays. Therefore, combining coordination with an adaptive signal control system can help to minimize the additional delay due to the signal plan transition.

Successful traffic signal system implementation will maximize available resources for effective coordinated operations under saturated conditions, demand variations, and near-saturation conditions. These strategies will allow signal systems to react quickly to anticipated volume surges, to manage queues better, and to recover quickly from congestion by implementing coordination settings such as early return, late arrival, and platoon identification schemes. These operational guidelines could also be used to take advantage of many currently available traffic monitoring and signal system fine-tuning capabilities to postpone the possibility of oversaturation on many highways and major arterials.

Earlier, coordination has been used in actuated traffic control systems and fixed timed traffic-controlled system. However, in this paper, we are going to do coordination in an adaptive traffic control system using the phase time network. This indicates the uniqueness of this work where we used an adaptive controller in a coordinated intersection to analyze the performance of an intersection using the phase time network model.

The framework of the phase-time network model is derived from the intersection automation model developed by Li et al. [1]. According to the phase-time network model, traffic signal optimization over time is viewed as the shortest path problem. The advantage of the phase-time network over the traditional "Ring-Barrier" representation is that all the nonlinear hard constraints inherent to the traffic signal mandates can be "prebuilt" into the phase-time network model and therefore the model is a linear and full-scale representation of traffic signal operations. The definition of signal phases in this paper refers to groups of compatible vehicle movements within intersections, which is often referred to as "stages" in Europe.

Farzana R. Chowdhury, Peirong (Slade) Wang, are with the department of civil engineering, The University of Texas at Arlington, TX 76019, USA.Email: farzana.chowdhury@mavs.uta.edu/ Email: peirong.wang@mavs.uta.edu/

^{*}Pengfei (Taylor) Li. Corresponding Author, is with The Department of Civil Engineering, The University of Texas at Arlington, Arlington, TX 76019, USA.: Email: taylor.li@uta.edu

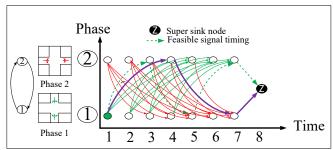


Fig.1. Using phase-time network model to represent traffic signal operations (adapted from Li et al. (2017))

As shown in Fig. 1 the phase-time network is constructed as a forward, acyclic network. The outbound arcs at any phase-time node are defined according to the feasible next phase and valid range of split (green+ yellow+ all red). Without loss of generality, one second as the traffic signal control resolution is adopted. For the sake of argument, let us assume the valid split range of Phase 1 is from 2s to 5s. Therefore at most four outbound phase-time arcs, which are noted as (i, t, j, s) can be constructed from the phase-time node (1,t),t=1,2,3,...,7. As an example, the list of outbound arcs at (1,1) include (1,1,2,3),(1,1,2,4),(1,1,2,5),(1,1,2,6) and each outbound arc can be interpreted as "Phase 1 starts at t=1, after yellow and all-red, it turns over the green to phase 2 at $t=\tau$ ($\tau=3,4,5,6$)".

II. LITERATURE REVIEW

An adaptive traffic signal system is developed to respond to real-time changes in traffic demand which can adjust the signal timing plans in real-time based on current traffic conditions, road capacity, and some real-time performance objectives. The deployment of adaptive signal systems have received significant attention around the world. However, SCOOT [2] (Split, Cycle and Offset Optimization Technique) and SCAT [3] (Sydney coordinated adaptive traffic system) are considered as the pioneer of adaptive traffic signal control systems. In SCOOT, a road network model is embedded within the online system and run continuously in real-time to monitor the effect of incremental changes of splits, offsets, and cycle time at individual intersections. However, unlike SCOOT, SCAT does not have any analytical optimization model; instead, a heuristic algorithm is applied to the adjustment of cycle length, split and offset and minimizes the overall stop delay and maximizes the overall throughput by reducing queue formation.

OPAC [4] (Optimization Policies for Adaptive Control) is another adaptive traffic signal control developed by Gartner initially as a part of real-time traffic adaptive control system (RT-TRACS) developed by Federal Highway Administration USA. Later, the system is extended for network. It uses online data from the network and minimizes the intersection control delay by using dynamic programming at a predefined time interval. Another widely discussed adaptive traffic signal control system is the Real-time Hierarchical Optimized Distributed Effective System or RHODES developed by Mirchandani et al. [5].

The best way to evaluate the performance of a coordinated signal control system is to check bandwidth efficiency or progression bandwidth. For traffic dynamic flow models and traffic progression problems, a mixed-integer linear programming (MILP) approach is used. Coordination in signal timing plan gives the benefit between an adaptive traffic control system and a traditional time-based coordination system. Network under coordination reduces the number of stops result in a reduction of arterial travel time. A good number of literature are available for coordinated traffic single control as well. He et al [6, 7] develop a mixed-integer linear program (MILP) formulation with the fixed phasing sequences, namely PAMSCOD, to serve special vehicles and pedestrians through coordination when the conflicting situation arises in both the actuated approaches and any priority requesting approaches. He et al. [8] further integrates multimodal priority control elements, including emergency vehicles, transit buses, commercial trucks, and pedestrians. They also consider background traffic coordination and vehicle actuation.

For efficient multimodal traffic signals control, Mehdi et al.[9] present a MILP formulation which assumes the presence of a pre-determined optimized signal coordination plan and aim to generate a virtual coordination priority request with the connected vehicle environment. The formulation aims to minimize the priority request delay by giving an optimal signal schedule over a rolling horizon of two cycles. To solve the MILP formulation, a search algorithm is designed to allocate minimum green times to the phases. Jia Hu et al. [10] propose a transit progression logic to enable the bus and signal coordination as well as the cooperation with the connected vehicle environment, namely TSPCV-C.

Some research scholars have carried out research to provide better progressions in the coordinated traffic signal control system. Park et al. [11, 12] proposed a stochastic method for optimizing signal parameters by combining a GA-SOM interface with simulation optimization. Pei et al. [13] applied traffic signal survey data as baseline information for designing signal coordination control systems in urban trunk roads. Day et al. [14-17] make a quantitative analysis of coordination for both fully actuated and fixed traffic control systems based on individual signal phase transitions. The selected MOEs include equivalent hourly volumes, green durations, volume overcapacity (v/c) ratios, and highway capacity manual arrival types. Later, Day et al. design a few new diagrams based on traffic signal events to demonstrate the performance of traffic signal systems commonly referred to as the "Purdue Coordination Diagram" or PCD nowadays.

Little et al. [18] formulate the MILP formulation MAX BAND to maximize a bandwidth linearly dependent on the offsets, the continuous variables in the MILP formulation. Some intermediate integer variables are used to make the offsets constrained. Here, the basic assumption is the local traffic signal timings (cycle, splits) and travel time ranges between intersections are given. The MAX BAND is further expanded to MULTIBAND by Gartner et al. [4, 19], which

increases the flexibility of MAX BAND by allowing various bandwidths between intersections. Both the MAX BAND and MULTIBAND are considered as the base for many network traffic controls and coordination. Zhang et al. [20] proposed AM-BAND, also considered as an asymmetrical version of MULTIBAND that generates a progression band considering progression line, which separates the inbound and outbound band into two independent elements. Arsava et al [21] proposed another extension of the MULTIBAND is the ODNETBAND for traffic networks to maximize the weighted bands based on the network origin-destination demands. Li et al. present a new vehicle-oriented MILP formulation to model the intersection automation for a future scenario where the traffic approaching intersections is heterogeneous and mixed of human-driving vehicles and connected automated vehicles [22]. Unlike the MAXBAND or its variants, all the above MILP traffic control models are optimized for isolated intersections.

The literature review section summarizes that a good number of works have been conducted previously using an adaptive traffic control system and actuated coordinated traffic control system. However, none of the previous researchers used coordination in an adaptive traffic control system. This indicates the uniqueness of this work, where we used an adaptive controller in a coordinated manner. Without the loss of generality, time based adaptive coordination is used in this paper to analyze the performance. Which means the interactions are not connected to each other, rather they are based on the same back ground clock. Once a stable back ground clock can be established for one intersection, automatically it will coordinate all the intersections of the corridor. Operational effectiveness depends on the detector configurations and parameter settings, controller timing, and coordination settings.

III. METHODOLOGY

A. Construction of phase time network for coordinated adaptive traffic signal control system (CATSC)

To develop a coordinated adaptive traffic signal control system, initially, we are going to construct a phase time network and then add coordination to that. And to maintain coordination, we are going to assign a dynamic arc cost.

In this section, a MILP formulation for the phase-time network is presented. The objective of this problem is to find a feasible network that can service in coordination while it does not change the basic traffic signal configurations (e.g., cycle time, split and offset) developed for the background traffic.

TABLE I. PARAMETERS AND VARIABLES FOR THE NEW MILP FORMULATION

Notations for road network and space-time network		
$G_{\alpha}(N_{\alpha},A_{\alpha})$	Road network	

L(i,j) (i,t),(j,s),	The storage capacity of $(i, j) \in A_0$ $(i, t).(i, s) \in N. \forall (i, t, i, s) \in A: if (i, j) \in A_0. s = t +$
(i,t),(j,s), (i,t,j,s)	$(i, t), (j, s) \in N, \forall (i, t, j, s) \in A; if(i, j) \in A_o, s = t + FFTT_{(i, j)}; if i = j (waiting arc), s = t + 1$
(i,t,j,s)	
(· · · · · · · · · · ·	
v_1 , v_2	
v_1 , v_2	v_1 : Set of regular vehicles; v_2 : Set of CAVs.
	v_1 : Set of regular vehicles; v_2 : Set of CAVs.
	1 0 , 1
v, V	1 0 , 1
v, V	Vehicle $v \in V$; $v = v_1 + v_2$
o(v), d(v)	· • • • • • • • • • • • • • • • • • • •
o(v), a(v)	Origin and destination of v
$c_{(i,j)},\mathbb{C}$	$c_{(i,j)}$: total free-flow path travel time if (i,j) is the last
m(p,i,j)	link of v 's path; otherwise 0 ; $\mathbb{C} = \{c_{(i,j)}\}, \forall v \in V$ Mapping matrix from a traffic control phase p to its corresponding controlled signal link(s); always equal
	to 1 if (i, j) is a regular link otherwise can be either 1
	if p is the current green or 0 otherwise.
	F 8 -
Notations for ph	ase-time network

$\Psi(P,T)$	Phase-time network
$\phi(v,i,j),\phi$	$\phi(v,i,j)=1$ if (i,j) is the last link of v's path; 0 otherwise.
p, P	Set of phases, $P = \{p\}$
$(p,t),(p^{\prime},t^{\prime})$	Nodes in Ψ , p , $p' \in P$, t , $t' \in T$
$(p_o,0),(p_z,H)$	Origin (current phase) and destination vertex (ending phase) in Ψ
(p,t,p',t')	A phase-time edge in Ψ , representing: "phase p starts green at t , after yellow and all-red clearance, turns over green to phase p' at t' "; note $p \neq p'$ because there are no waiting arcs in the phase time networks
c_0	The constant cost for each phase transition

Variables	
$x_{(v,i,t,j,s)} \in X$	Equal to 1 if v enters link (i,j) at t and leaves at s;
	otherwise 0
$y_{(p,\tau,p',h)} \in Y$	Equal to 1 if and only if the phase-time arc (p, τ, p', h)
	is selected, otherwise 0. When $y_{(p,\tau,p',h)} = 1$, it can be
	interpreted as MI phase p starts at τ and turns over
	green to p' after yellow and all-red clearance

Even though the decision variables have 4 indices, they are two-dimension, arc-time-indexed variables. The new MILP formulation considering the phase-time network is described as follows.

 $\text{Min } Z_1 = \sum_{v \in V} \sum_{s \leq H} \sum_{1 \leq t} \sum_{(i,j) \in A_0} ((s-t_0^v) \times \varphi_{(v,i,j)} \times x_{(v,i,t,j,s)} - \sum_{v \in V} \sum_{(i,j) \in A_0} c_{(v,i,j)} + \\ \sum_{(p,\tau,p',h) \in \Psi} c_{o_{\left(p,\tau,p',h\right)}} \times y_{(p,\tau,p',h)} \ \, (1)$

The MILP formulation based on the phase-time network is described as follows.

Constraints for Traffic Dynamics

In this formulation, we have three constraints for traffic dynamics, capacity constraint for regular link, capacity constraint for signalized (control) link and constraint for the storage capacity of road link.

Regular link capacity constraints

$$\textstyle \sum_{v \in V} x_{(v,i,t,j,s)} \leq SR_{(i,j)}, t, s \sim [1,H], \forall (i,j) \in (A_0 - A_s), p \in P \ (2a)$$

(2a) indicates that the number of vehicles that are entering the link (i, j) should be within the flow capacity of the link (i, j).

Control link capacity constraints

$$\textstyle \sum_{v \in V} x_{(v,i,t,j,s)} \leq m(p,i,j) \times SR_{(i,j)} \times \sum_{(p,\tau,p',h) \in \Psi} y_{(p,\tau,p',h)}, t, s \sim [1,H], \forall (i,j) \in A_s, p \in P \text{ (2b)}$$

If the capacity of (i,j) is controlled phase p, then M(p,i,j)=1. When phase p is green, $y_{(p,\tau,p',h)}=1$. Therefore, the RHS of (2) is equal to $SR_{(i,j)}$; when p is red, the RHS of (2) is equal to 0.

Road link storage constraint

$$\left(\sum_{0 \leq t \leq t} \sum_{v \in V} \chi_{(v,t,i,t+FFTT_{(i,j)})} \right) - \left(\sum_{0 \leq t \leq t} \sum_{v \in V} \sum_{(j,t,t+FFTT_{(j,i)}) \in A} \chi_{(v,j,t,t+FFTT_{(i,j)})} \right) \leq L_{(i,j)}, \forall (i,j), (j,i) \in A_o, t \sim [1,\mathcal{H}]$$

(3) define that the number of vehicles (i.e., the difference between cumulative arrivals and cumulative departures at any time) on a link must less than the link's storage capacity. The first term on the left side of (3) represents the cumulative vehicles arriving at i at t while the second term represents the cumulative vehicles departing j at t. The difference is the number of vehicles on (i,j) at t.

Flow conservation constraint at space-time network

The space-time flow conservation constraints guarantee vehicles to move from their origins to destinations along their defined paths. Constraints (4) are as follows:

$$\sum_{(i,t,j,s)\in A} \left(x_{(v,i,t,j,s)}\right) - \sum_{(j,s,i,s')\in A} \left(x_{(v,j,s,i,s')}\right) = \begin{cases} -1; \ (j,s) = o_v \\ 1; \ (j,s) = d_v \ , \forall v \in V, \forall (j,s) \in N \end{cases} \tag{4}$$
 0; otherwise

(4) ensures all vehicles cross the intersections in a feasible solution.

<u>Constraints for traffic control modeling in phase-time</u> network flow conservation constraint at phase-time network

A feasible traffic signal timing plan can be tracked as phase P with time, and it can be represented by a path from origin phase $(p_o, 0)$ to the destination phase (p_z, T) in phase time network. Therefore, the flow conservations can be formulated as in (5).

$$\Sigma_{(p, r, p', h) \in \Psi} \mathcal{Y}_{(p, r, p', h)} - \Sigma_{(p', h, p, h') \in \Psi} \mathcal{Y}_{(p', h, p, h')} = \begin{cases} -1; \ (p', h) = (p_o, 0), \\ 1; \ (p', h) = (p_z, H), \ for \ \forall (p', h) \in \Psi \ (5) \\ 0; \ otherwise \end{cases}$$

Objective function (1) aims to find a feasible schedule to service all heterogeneous TSP requests with the minimum transitions.

B. Coordinated Phase-Time Network

Coordination in signal timing plan gives the benefit between an adaptive traffic control system and a traditional time-based coordination system. If a network is controlled under the coordination mode, it will reduce the number of stops hence the arterial travel time can be reduced. Apart from the fully adaptive phase-time network, a coordinated phase-time network is developed for CATSC. In this modified network as shown in Fig.2 ϕ 4 is the coordination phase and the phasing sequence follows as ϕ 1 \rightarrow ϕ 2 \rightarrow ϕ 3 \rightarrow ϕ 4.

To maintain the coordination and phasing sequence, it is considered that the cost of regular arc is M1 (M=1) if no background traffic is considered. This means, if the phase-time network does not follow the phasing sequence and coordination; a specified penalty will be included for each violation. To be mentioned here, this penalty is adjusted in network construction not in object function. If a phase violates the phasing sequence, (e.g. when $\phi 3$ starts earlier than phase $\phi 2$, within a cycle or if phase $\phi 1$ ends at phase $\phi 4$) a penalty M2 (M2=100) is given and M3 is the penalty for violating phasing sequence. Since the optimization problem is to minimize the total cost, given such penalty and reward, the coordination and phasing sequence will be maintained for the optimal network. The below figure demonstrates the coordination and phasing sequence. The penalty and reward values here are set for the sake of argument and they are subject to be modified later.

IV. NUMERICAL EXPERIMENT

Numerical experiment is conducted within a demonstrative example network to validate the proposed formulations in the previous section. Fig. 3 represents the layout for numerical experiment for the proposed model. As shown in the figure, a network with 28 nodes and 28 links is considered for the numerical experiment. Among those 28 links, 20 are normal links represented in solid lines with travel time 5s, and the other 8 are control links in dash lines of different colors with travel time 2s.

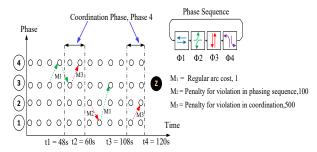


Fig. 2. Coordinated phase-time network with arc cost

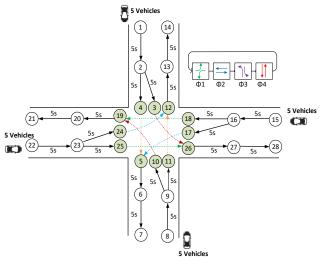


Fig. 3. Layout for numerical experiment

The traffic signal is controlled by 4 phases, the minimum green is 3 s, maximum green is 10 s, and all-red clearance plus yellow is set as 2 s.

The coordinated phase-time network will not prohibit flexible phasing sequence or flexible cycle length (i.e., the phase-time network model is still fully connected). However, violating phasing sequence and length will incur additional high penalties (or cost). As a result, the final least-cost path phase time network (i.e., the optimal traffic control plan) will avoid such violations unless try such penalties can be balanced by the saved control delays. Within a horizon of 120 s, a total of 20 vehicles plan to cross the intersections, and they are released into the network with little competition for green lights when arriving at intersections. As mentioned earlier, \$\phi 4\$ is the coordination phase and the phasing sequence follows as $\phi 1 \rightarrow \phi 2 \rightarrow \phi 3 \rightarrow \phi 4$.

For solving the MILP formulation proposed in methodology section, GUROBI optimizer in used using python interface. For solving the network with 28 nodes and 20vehicles, it took around 45s initially since only one core was used. However, when all the cores were used for the analysis, the optimization result is obtained within a second which indicates the computational efficiency of the model.

Fig. 4 represents the optimum solution for the proposed coordinated adaptive network. In the solution at time $t_1 = 52 \, s$, $\phi 2$ ends at coordinated phase $\phi 4$ and stays there till $t_2 = 60 \, s$. Similarly, at time $t_3 = 90 \, s$ $\phi 3$ ends at supper sink phase to keep the coordination. In the formulation we considered a fixed phasing sequence which is $\phi 1 \rightarrow \phi 2 \rightarrow \phi 3 \rightarrow \phi 4$. Also since $\phi 4$ is the coordination phase, it can go to any other phases. The result also shows that there is no violation of phasing sequence hence no additional penalty is added in the optimum solution.

Fig. 5 represents the trajectory of individual vehicles. The figure shows that all the main line through and main line left turning vehicles can pass immediately in the coordination mode and side street vehicles has to wait few seconds to get green.

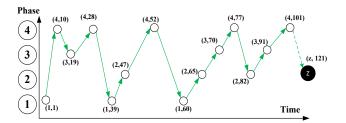


Fig. 4. Optimum signal scheduling for coordinated adaptive phase-time network

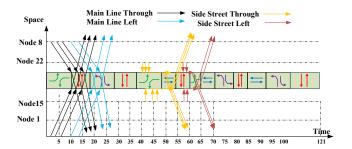


Fig. 5. Trajectories at intersection for coordinated adaptive network

V. CONCLUSION

In this paper, a framework for coordination in an adaptive traffic control system is developed using the space-time and phase-time network model. In the formulation we considered the multi commodity network flow model from road network to space-time network in conjunction with cumulative arrival and departure curves on each link. In order to observe the proven traffic flow principles, more constraints are formulated for link flow capacity, link storage capacity and flow conservations are for each vehicle in both space time and phase time network.

The proposed MILP formulation can be solved using GUROBI optimizer in the python interface. The numerical experiment justifies the validation of the proposed formulation. In the numerical experiment, we considered an isolated four phase intersection where 20 vehicles are allowed to enter the network in 120s time horizon. We added some penalty for the violation of phasing sequence and violation of coordination. The solution can be used to estimate the queue propagation for the coordinated adaptive network using the individual vehicle's trajectories.

In the future, the same formulation will be applied for multiple intersections with larger traffic to estimate the overall performance of the corridor. In addition, a micro simulation environment will be set up to compare the obtained optimization result in a real-time scenario. Recommended signal timing and detector settings may be used as a standard design procedure to provide system adaptability through existing signal control equipment already in the field.

REFERENCES

- [1] P. T. Li and X. Zhou, "Recasting and optimizing intersection automation as a connected-and-automated-vehicle (CAV) scheduling problem: A sequential branch-and-bound search approach in phase-time-traffic hypernetwork," *Transportation Research Part B: Methodological*, vol. 105, pp. 479-506, 2017.
- [2] P. Hunt, D. Robertson, R. Bretherton, and R. Winton, "SCOOT-a traffic responsive method of coordinating signals," 0266-7045, 1981.
- P. Lowrie, "Scats, sydney co-ordinated adaptive traffic system: A traffic responsive method of controlling urban traffic," 1990.
- [4] N. H. Gartner, "OPAC: Strategy for demand-responsive decentralized traffic signal control," *IFAC Proceedings Volumes*, vol. 23, no. 2, pp. 241-244, 1990.
- [5] P. Mirchandani and L. Head, "A real-time traffic signal control system: architecture, algorithms, and analysis," *Transportation Research Part C: Emerging Technologies*, vol. 9, no. 6, pp. 415-432, 2001.
- [6] Q. He, "Robust-intelligent traffic signal control within a vehicleto-infrastructure and vehicle-to-vehicle communication environment." 2010.
- [7] Q. He, K. L. Head, and J. Ding, "PAMSCOD: Platoon-based arterial multi-modal signal control with online data," *Transportation Research Part C: Emerging Technologies*, vol. 20, no. 1, pp. 164-184, 2012.
- [8] Q. He, K. L. Head, and J. Ding, "Multi-modal traffic signal control with priority, signal actuation and coordination," *Transportation research part C: emerging technologies*, vol. 46, pp. 65-82, 2014.
- [9] M. Zamanipour, K. L. Head, Y. Feng, and S. Khoshmagham, "Efficient priority control model for multimodal traffic signals," *Transportation Research Record*, vol. 2557, no. 1, pp. 86-99, 2016.
- [10] J. Hu, B. B. Park, and Y.-J. Lee, "Coordinated transit signal priority supporting transit progression under connected vehicle technology," *Transportation Research Part C: Emerging Technologies*, vol. 55, pp. 393-408, 2015.
- [11] B. B. Park, N. M. Rouphail, J. P. Hochanadel, and J. Sacks, "Evaluating reliability of TRANSYT-7F optimization schemes," *Journal of transportation engineering*, vol. 127, no. 4, pp. 319-326, 2001.
- [12] B. B. Park, N. M. Rouphail, and J. Sacks, "Assessment of stochastic signal optimization method using microsimulation," *Transportation Research Record*, vol. 1748, no. 1, pp. 40-45, 2001
- [13] Y. Pei, M. Sun, and X. Dong, "'Research on the design of signal coordination control system in intersections of urban main roads," *J. Transp. Eng. Inf*, vol. 2, no. 2, pp. 41-46, 2004.
- [14] C. M. Day and D. M. Bullock, "Computational efficiency of alternative algorithms for arterial offset optimization," *Transportation Research Record*, vol. 2259, no. 1, pp. 37-47, 2011.
- [15] C. M. Day et al., "Evaluation of arterial signal coordination: Methodologies for visualizing high-resolution event data and measuring travel time," *Transportation Research Record*, vol. 2192, no. 1, pp. 37-49, 2010.
- [16] C. M. Day, H. Premachandra, T. M. Brennan Jr, J. R. Sturdevant, and D. M. Bullock, "Operational evaluation of wireless magnetometer vehicle detectors at signalized intersection," *Transportation research record*, vol. 2192, no. 1, pp. 11-23, 2010.
- [17] C. M. Day, E. J. Smaglik, D. M. Bullock, and J. R. Sturdevant, "Quantitative evaluation of fully actuated versus nonactuated coordinated phases," *Transportation Research Record*, vol. 2080, no. 1, pp. 8-21, 2008.
- [18] J. D. Little, M. D. Kelson, and N. H. Gartner, "MAXBAND: A versatile program for setting signals on arteries and triangular networks," 1981.
- [19] N. H. Gartner, S. F. Assmann, F. Lasaga, and D. L. Hous, "MULTIBAND--a variable-bandwidth arterial progression scheme," *Transportation Research Record*, no. 1287, 1990.

- [20] C. Zhang, Y. Xie, N. H. Gartner, C. Stamatiadis, and T. Arsava, "AM-band: an asymmetrical multi-band model for arterial traffic signal coordination," *Transportation Research Part C: Emerging Technologies*, vol. 58, pp. 515-531, 2015.
- [21] T. Arsava, Y. Xie, and N. Gartner, "OD-NETBAND: An Approach for Origin-Destination Based Network Progression Band Optimization," *Transportation Research Record*, vol. 2672, no. 18, pp. 58-70, 2018.
- [22] P. Li, P. Mirchandani, and X. Zhou, "Solving simultaneous route guidance and traffic signal optimization problem using spacephase-time hypernetwork," *Transportation Research Part B: Methodological*, vol. 81, pp. 103-130, 2015.