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Virtual Platoon based CAVs Cooperative Driving at Unsignalized Intersection

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Abstract— **The emergence of connected and automated vehicles (CAVs) poses a promising future of transportation systems. Complicated traffic conditions at intersections lead to challenges for cooperative driving of CAVs while ensuring safety and efficiency. This paper proposes a conflict-free cooperative control method for CAVs at unsignalized intersections. Based on the conflict points inside the intersection, we project each conflict vehicles cluster onto one virtual lane and construct multiple virtual platoons for ensuring safety. To maximize the intersection utilization, a packing problem is proposed for developing optimal car-following order, which is formulated as a mixed-integer linear programming (MILP). Considering the practical conditions at the intersection, we remove the redundant constraints to obtain nearly global-optimal solutions in real time. Numerical simulations validate that the proposed method saves 5.5%-9.3% total evacuation time under normal traffic flow.**

I. Introduction

The unbalanced growth of road capacity and the number of vehicles bring great challenges to traffic mobility optimization. Encouragingly, connected and automated vehicles (CAVs) provide full control of individual vehicles and exchange necessary driving information in real time via vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication technology, which can be exploited to improve the efficiency and safety of traffic networks [1].

As the bottleneck where different traffic flow meets, intersection plays a critical role in traffic mobility optimization. According to the official statistics, 36% of the vehicle crashes were intersection-related in the USA [2]. However, by leveraging the advantages of CAVs, many works aim to solve the space-time conflict so that the vehicles can share the limited space resources inside the intersection. Optimizing the vehicle velocity and traffic signal timing at traditional signalized intersections is one of the focuses. The platoon-based optimal control framework is proposed to adaptively change the vehicle velocity under the fixed traffic signal cycle [3]–[5]. Conversely, to schedule traffic signal timing adapted to real-time traffic, the self-adaptive simulation method optimizes the traffic signal timing based on the vehicle motion data obtained via V2I [6]–[8]. However, the traffic signal brings green time loss and unnecessary fuel consumption because of frequent start-up behavior at the signalized intersection.

For traffic efficiency consideration, researchers started to focus on unsignalized intersection coordination. A popular method is to formulate conflict-free intersection crossing as an optimization problem, where collision avoidance is always guaranteed by several hard constraints. The evacuation time [9], network throughput [10], and passenger comfort [11] are usually chosen as the optimization objectives to be minimized or maximized. This method could significantly improve the efficiency and safety of the intersection with the help of the centralized coordination. However, the coordination needs to collect global vehicle information and solves the optimization problem constantly, which means heavy computation and communication burden.

To solve the problem, the virtual platoon method is proposed in [12]. Specifically, the two-dimensional vehicles are transformed onto a one-dimension virtual lane to avoid collision. The centralized process of scheduling vehicles at the intersection equals a decentralized car-following process of forming the virtual platoon. Additionally, the car-following order in virtual platoons lies in the First-in-First-out (FIFO) principle, which is not the optimal order in most cases. Therefore, methods like graphs [13], Monte Carlo tree search [14], and spanning tree [15] have been proposed to reconstruct the car-following order to further improve the efficiency of traffic mobility. However, existing virtual platoon methods construct just one virtual lane, which is not the perfect choice since vehicles on conflict lanes are prohibited to exist concurrently inside the intersection. The potential conflicts only occur at the conflict points between two lanes rather than the whole trajectory along the lanes. Thus constructing one virtual lane has the optimization upper bound of the intersection utilization.

This paper presents a conflict-free cooperative control method based on multiple virtual platoons for CAVs at unsignalized intersections. Conflict vehicles are allowed to exist concurrently inside the intersection. This method is formulated as a bilevel problem in which the upper level is a scheduling problem aimed at finding the optimal car-following order in multiple virtual platoons. The lower level is a decentralized control problem for CAVs to reach the desired car-following status. The contributions of this paper are summarized as follows:

- A multiple virtual platoons method is proposed to transform all collisions at the intersection into inter-vehicle space between vehicles on multiple virtual lanes.

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- A packing problem, formulated as a mixed-integer linear programming (MILP), is presented to obtain the optimal car-following order in virtual platoons and maximize the intersection throughput.
- An efficient optimization algorithm is proposed to simplify the MILP and obtain the solutions in real time.

The remainder of this paper is organized as follows. Section II details the construction of virtual platoons and the packing problem at the intersection. Section III solves the packing problem in real time with an optimization algorithm. Section IV discusses the main simulations results. Finally, Section V gives the concluding remarks and presents the future research.

II. Problem Formulation

In this paper, the 4-way unsignalized isolated intersection with exclusive left-turn, straight, and right-turn lanes per road is considered, as shown in Fig. 1. The lanes are marked as $l_0 - l_{11}$ in the clockwise direction, roughly forming potential conflict points $c_0 - c_7$ inside the intersection. Considering vehicles from different directions covering all conflict points inside the intersection as shown in Fig. 1. The conflict lanes are marked in the same color as their shared conflict point.

The intersection is separated into two zones, i.e., merging zone (MZ) and cooperative zone (CZ). The former represents the overlap area of all lanes while the latter provides vehicles with enough time to self-organize to satisfy the desired speed and position. The length of MZ and the radius of CZ are denoted as d_m and d_c ($d_c > d_m$), respectively. The centralized coordination device is equipped at MZ to implement limited computation and transmit necessary information to orchestrate vehicles achieve conflict-free intersection crossing. We assume that all vehicles are CAVs equipped with V2I and V2V devices and have completed lane changing and overtaking maneuvers, which means they would drive in strict accordance with their lanes. Besides, vehicles are supposed to obey the arrangement from the upper level for maximizing the intersection utilization.

A. Construction of Multiple Virtual Platoons

The incoming vehicles are indexed according to the arriving order at CZ and we denote N as the total number of the vehicles, i.e, $i \in \mathcal{N} = \{1, \cdots, N\}$. Let $\mathcal{S}$ and $\mathcal{T}$ denote straight lanes set and right-turn lanes set, respectively. Then we define $\mathcal{I} = \mathcal{S} \cup \mathcal{T}$. Additionally, the conflict points set $\mathcal{C}_j$ denotes the conflict points along lane l_j. For example, $\mathcal{C}_0 = \varnothing$, $\mathcal{C}_1 = \{0, 7, 6\}$, $\mathcal{S} = \{1, 4, 7, 10\}$, $\mathcal{T} = \{2, 5, 8, 11\}$, $\mathcal{I} = \{1, 2, 4, 5, 7, 8, 10, 11\}$, as shown in Fig. 1.

For maximal intersection utilization consideration, we construct multiple virtual lanes based on the conflict points. The constructions of virtual lanes l_{c_3} and l_{c_4} based on the conflict points c_3 and c_4 are shown in Fig. 2(a). The conflict point is at the end of the virtual lane so the absolute position p_{ij}^k of vehicle i on virtual lane l_{c_k} denotes its distance to the conflict point c_k.

$$p_{ij}^k = \begin{cases} d_i + m_j^k, & k \in \mathcal{C}_j, \\ \infty, & otherwise, \end{cases} \tag{1}$$

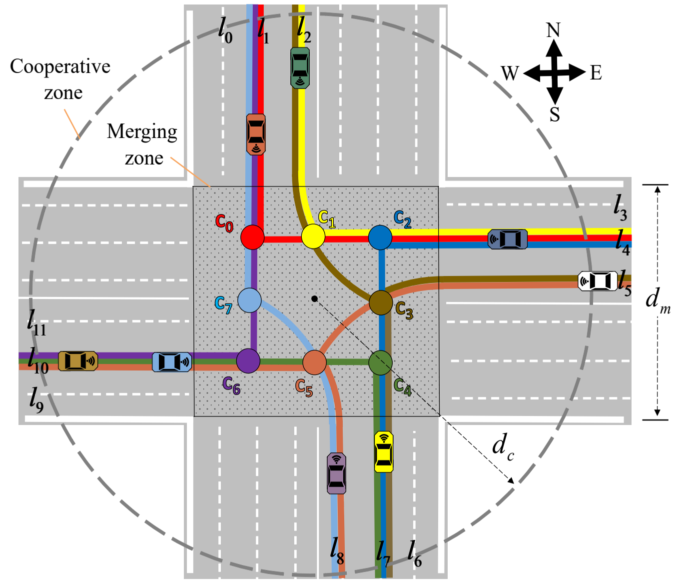

Fig. 1. 4-way unsignalized intersection.

where d_i represents the distance to MZ. m_j^k denotes the extra distance on lane l_j from MZ to conflict point c_k.

Since each lane has at least two conflict points at MZ, one vehicle will be projected on different virtual lanes concurrently. However, the relative distance between vehicle positions on different virtual lanes is based solely on its fixed lane. Thus, any p_{ij}^k ($k \in \mathcal{C}_j$) can exclusively denote the positions of vehicle i on different virtual lanes.

The ultimate multiple virtual lanes $l_{c_0} - l_{c_7}$ are shown in Fig. 2(b). Collisions between vehicles at the intersection involve the rear-end collision and the lateral collision. The former occurs between vehicles on the same lane while the latter occurs at the conflict points. Therefore, all vehicles on the same virtual lane l_{c_k} are either on the same lane or have potential collision at the conflict point c_k. Then the rear-end collision and the lateral collision are described uniformly as the inter-vehicle space on each virtual lane. As a result, all kinds of collisions at the intersection can be avoided by orchestrating adjacent vehicles on each virtual lane to keep a safe car-following distance, i.e., forming a stable platoon on each virtual lane.

Different from the real vehicle platoon, overlap of vehicles on the virtual lane may not mean collision happens. However, if the vehicles still overlap at the end of the virtual lane, they must have a lateral collision at the conflict point. To form a stable virtual platoon on each virtual lane l_{c_k}, vehicle i intends to move cooperatively with its preceding vehicle j on the virtual lane, i.e., traveling at a harmonized longitudinal speed and desired car-following distance D_j.

$$\begin{cases} \lim_{t\to\infty} ||v_i(t) - v_j(t)|| = 0 \\ \lim_{t\to\infty} (p_i(t) - p_j(t) - D_j) = 0 \end{cases}, i, j \in \mathcal{N}, \tag{2}$$

where $v_i(t)$ and $p_i(t)$ denote the vehicle velocity and position on the virtual lane, respectively.

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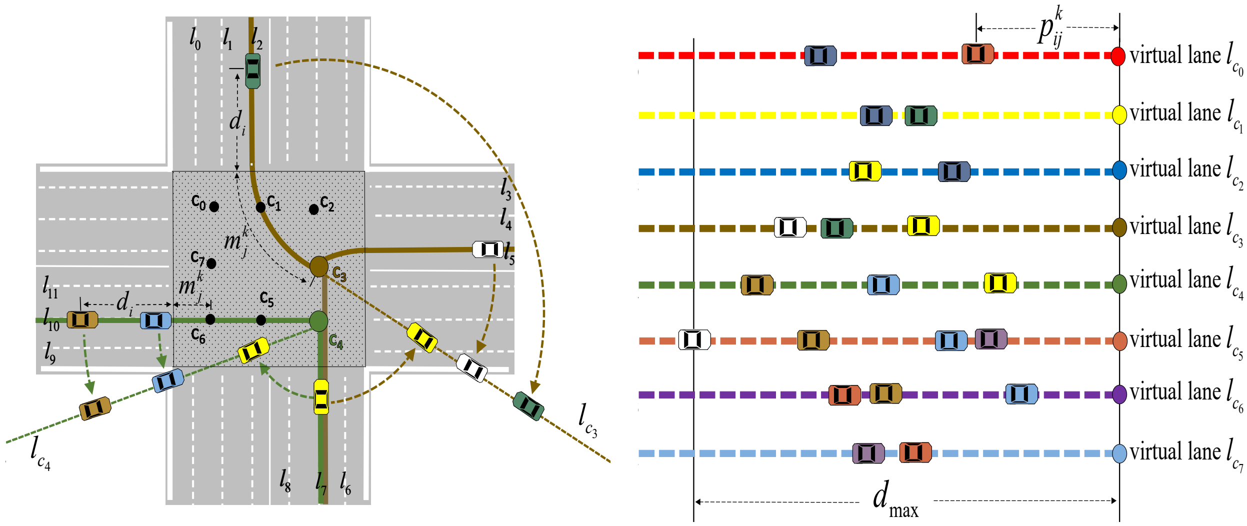

(a) Constructions of virtual lanes c_3 and c_4.

(b) Multiple virtual lanes.

Fig. 2. Multiple virtual lanes constructions.

Note that right-turn lanes have no conflict point so that the vehicles with right-turn maneuvers only have the risk of rear-end collision, which can be avoided by keeping a safe time headway between adjacent vehicles on their lane. Therefore, vehicles with right-turn maneuvers are ignored in the constructions of the virtual lanes.

B. Construction of Packing Problem

Definition (virtual platoons harmonized status): The virtual platoons harmonized status is defined as the target car-following status where all vehicles reach desired car-following distance and harmonized speed with their preceding one on the virtual lane. Hereafter, we will use harmonized status to denote virtual platoons harmonized status for the sake of simplicity.

Please note that the optimal scheduling problem in this paper is based on the platoon where desired car-following distance and harmonized speed are the prerequisites. Then the car-following order in virtual platoons directly determines the total evacuation time. In harmonized status, all vehicles have the same harmonized longitudinal speed. Thus, the time error $\Delta t \propto \Delta d$, which means the time error can be linearly represented by the distance error. The time error arriving at the same conflict point is proportional to the car-following distance in virtual platoons. The total evacuation time that all vehicles leave the intersection is proportional to the maximal distance difference d_{max} on whole virtual lanes as shown in Fig. 2(b). Based on that, the optimal car-following order in virtual platoons that maximizes the intersection utilization is equivalent to the optimal harmonized status where vehicles on virtual lanes keep a safe car-following distance d_s and the maximal distance difference d_{max} is minimized.

Taking the vehicle and its safe car-following distance d_s together as a rectangular item as shown in Fig. 3. Subsequently, the optimal harmonized status problem is transformed into a well-developed packing problem. The packing problem is an optimization problem of scheduling a set of rectangular items into a larger rectangular container to minimize the waste [16]. There are two principles of the packing problem, i.e., no rectangular items overlap, and the rectangular container is minimized. The former corresponds to the safe car-following distance while the latter corresponds to minimizing the maximal distance difference d_{max} in harmonized status.

Because of fixed trajectory and multiple conflict points along the lane, vehicle in harmonized status is solely allowed to move horizontally and be represented as the composed of several identical rectangular items. Therefore, the ultimate packing problem is equivalent to a one-dimension irregular packing problem.

To obtain the global-optimal solutions, this paper solves the packing problem with the optimization method. The decision variable of the optimization problem is the vehicle distance x_{ij} to the first conflict point for vehicle i on lane l_j as shown in Fig. 3. The optimization problem of the packing problem is formulated as follows. Hereafter, we will use bold letters to express vectors.

$$\min_{\mathbf{x}} \; d_{max}, \tag{3a}$$

$$\text{s.t. } x_{ij} \geq 0, \quad \forall i \in \mathcal{N}, j \in \mathcal{I}, \tag{3b}$$

$$\mathbf{e}_j x_{ij} + \mathbf{h}_j \leq \mathbf{e} d_{max}, \quad \forall i \in \mathcal{N}, j \in \mathcal{I}, \tag{3c}$$

$$|\mathbf{e}_j x_{ij} + \mathbf{h}_j - (\mathbf{e}_n x_{mn} + \mathbf{h}_n)| \circ \mathbf{e}_j \circ \mathbf{e}_n \geq \mathbf{e} d_s \circ \mathbf{e}_j \circ \mathbf{e}_n \\ , \forall i < m \in \mathcal{N}, j, n \in \mathcal{I}. \tag{3d}$$

The objective function (3a) aims to minimize the maximal distance difference d_{max} in harmonized status. Constraints (3b) and (3c) ensure that all vehicles are involved in harmonized status. $\mathbf{e}$ represents the vector of ones, and $\mathbf{e}_j$ is the vector representation of $\mathcal{C}_j$, e.g., $\mathbf{e}_1 = [1, 0, 0, 0, 0, 0, 1, 1]^T$ when $\mathcal{C}_1 = \{0, 7, 6\}$. Vector $\mathbf{h}_j$ defines the relative distance between vehicle positions on different virtual lanes l_{c_k} ($k \in \mathcal{C}_j$), compared with x_{ij}. Taking lane l_1 as an example,

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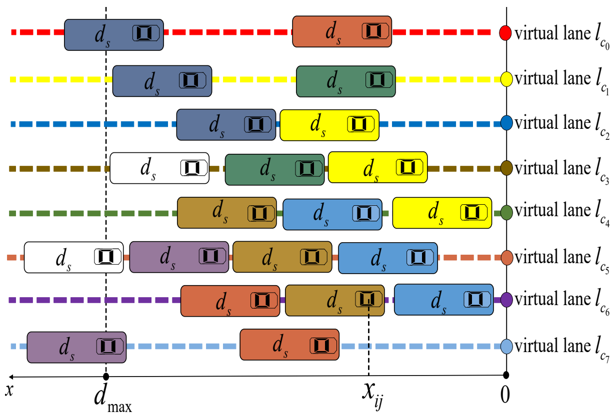

Fig. 3. Packing problem in harmonized status.

$\mathbf{h}_1 = [0,0,0,0,0,0,m_1^6 - m_1^0, m_1^7 - m_1^0]^T$. Thus, $\mathbf{e}_j x_{ij} + \mathbf{h}_j$ is the vector representation of vehicle positions on all virtual lanes. Constraint (3d) guarantees that vehicle i on lane l_j will keep a safe distance with its behind vehicle m on lane l_n in harmonized status. Here, $\circ$ denotes Hadamard product, i.e.,

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \circ \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_1 b_1 \\ a_2 b_2 \\ a_3 b_3 \end{bmatrix}. \tag{4}$$

Since constraint (3d) is a nonconvex constraint that introduces non-linearity to the problem. We add a binary variable $\theta_{in}^q = \{0,1\}$ to linearize it and the qth $(q > 0)$ constraint for vehicle i on lane l_n is further transformed as follows.

$$\begin{aligned} &[\mathbf{e}_j x_{ij} + \mathbf{h}_j - (\mathbf{e}_n x_{mn} + \mathbf{h}_n)] \circ \mathbf{e}_j \circ \mathbf{e}_n \\ &\qquad \geq [\mathbf{e} d_s - \mathbf{e} M (1 - \theta_{in}^q)] \circ \mathbf{e}_j \circ \mathbf{e}_n, \quad (5a) \\ &-[\mathbf{e}_j x_{ij} + \mathbf{h}_j - (\mathbf{e}_n x_{mn} + \mathbf{h}_n)] \circ \mathbf{e}_j \circ \mathbf{e}_n \\ &\qquad \geq (\mathbf{e} d_s - \mathbf{e} M \theta_{in}^q) \circ \mathbf{e}_j \circ \mathbf{e}_n, \quad (5b) \end{aligned}$$

where M is a large number. From the practical point of view, if vehicle i and vehicle m have potential conflict on c_k $(k \in \mathcal{C}_j \cap \mathcal{C}_n)$, θ_{in}^q denotes the relative order between vehicle i and vehicle m on virtual lane l_{c_k} in harmonized status. Specifically, $\theta_{in}^q = 1$ indicates that, compared with arriving order at CZ, vehicle m overtakes vehicle i on virtual lane l_{c_k} in harmonized status. $\theta_{in}^q = 0$ means vehicle i is still in front of vehicle m on virtual lane l_{c_k} in harmonized status.

The optimal solutions to the packing problem are not unique, so we choose the energy-saving one among them, which means the solution with fewer overtaking maneuvers. Then, the objective function (3a) is updated as follows.

$$\min_{\mathbf{x},\theta} \quad d_{max} + \gamma \sum_{i \in \mathcal{N}, n \in \mathcal{I}} \theta_{in}^q, \tag{6}$$

where γ is a small value representing the conversion factor.

III. Solution Approach

A. The Optimization Algorithm

Because of the huge number of possible car-following orders, the solutions to the packing problem may not be obtained in real time. However, the optimization algorithm contains some potential redundant constraints, which consume unnecessary computational effort. Thus, based on the practical conditions at the intersection, we simplify the optimization algorithm by removing redundant constraints to obtain nearly global-optimal solutions in real time.

Firstly, vehicles at CZ have completed overtaking on real lane so that adjacent vehicles with the same trajectory just need to keep a safe inter-vehicle space. Subsequently, the maximal distance d_{max} on whole virtual lanes is reflected by the position of the first and the last vehicles on each virtual lane. Then the constraints (3b)-(3c) and (5a)-(5b) can be updated as follows.

$$\begin{aligned} & x_{ij} \geq 0, \quad \forall i \in \mathcal{N}_f, j \in \mathcal{I}, && (7a) \\ & x_{ij} + d_s \leq x_{mn}, \quad \forall i < m \in \mathcal{N}, j = n \in \mathcal{I}, && (7b) \\ & \mathbf{e}_j x_{ij} + \mathbf{h}_j \leq \mathbf{e} d_{max}, \quad \forall i \in \mathcal{N}_l, j \in \mathcal{I}, && (7c) \\ & [(\mathbf{E}_{ij} - \mathbf{E}_{mn}) - \mathbf{e} d_s + \mathbf{e} M (1 - \theta_{in}^q)] \circ \mathbf{e}_j \circ \mathbf{e}_n \geq \mathbf{0}, \\ & \qquad \forall i < m \in \mathcal{N}, j, n \in \mathcal{I}, j \neq n, && (7d) \\ & [-(\mathbf{E}_{ij} - \mathbf{E}_{mn}) - \mathbf{e} d_s + \mathbf{e} M \theta_{in}^q] \circ \mathbf{e}_j \circ \mathbf{e}_n \geq \mathbf{0}, \\ & \qquad \forall i < m \in \mathcal{N}, j, n \in \mathcal{I}, j \neq n, && (7e) \end{aligned}$$

where $\mathbf{E}_{ij} = \mathbf{e}_j x_{ij} + \mathbf{h}_j$. $\mathcal{N}_f$ and $\mathcal{N}_l$ are vehicle sets which contain the first and the last vehicles on each virtual lane.

Secondly, for vehicles on compatible lanes without any conflict point, constraints (7d)-(7e) become redundant. Thus, the above constraints can be updated as follows.

$$\begin{aligned} & [(\mathbf{E}_{ij} - \mathbf{E}_{mn}) - \mathbf{e} d_s + \mathbf{e} M (1 - \theta_{in}^q)] \circ \mathbf{e}_j \circ \mathbf{e}_n \geq \mathbf{0}, \\ & \qquad \forall i < m \in \mathcal{N}, j, n \in \mathcal{I}, j \neq n, \mathcal{C}_j \cap \mathcal{C}_n \neq \varnothing, && (8a) \\ & [-(\mathbf{E}_{ij} - \mathbf{E}_{mn}) - \mathbf{e} d_s + \mathbf{e} M \theta_{in}^q] \circ \mathbf{e}_j \circ \mathbf{e}_n \geq \mathbf{0}, \\ & \qquad \forall i < m \in \mathcal{N}, j, n \in \mathcal{I}, j \neq n, \mathcal{C}_j \cap \mathcal{C}_n \neq \varnothing. && (8b) \end{aligned}$$

Thirdly, if vehicle m on lane l_n is determined not to overtake vehicle i on virtual lane, it is unnecessary for vehicles r $(r > m)$ on lane l_n to verify the distance to vehicle i through constraints (8a)-(8b). Thus, we predefine overtaking tolerance variable ω on each virtual lane, which denotes that the top ω vehicles behind vehicle i in arriving order are allowed to overtake it in harmonized status. Therefore, constraints (8a)-(8b) can be updated as follows.

$$\begin{aligned} & [(\mathbf{E}_{ij} - \mathbf{E}_{mn}) - \mathbf{e} d_s + \mathbf{e} M (1 - \theta_{in}^q)] \circ \mathbf{e}_j \circ \mathbf{e}_n \geq \mathbf{0}, \\ & \forall i < m \in \mathcal{N}, j, n \in \mathcal{I}, j \neq n, \mathcal{C}_j \cap \mathcal{C}_n \neq \varnothing, q \leq \omega, && (9a) \\ & [-(\mathbf{E}_{ij} - \mathbf{E}_{mn}) - \mathbf{e} d_s + \mathbf{e} M \theta_{in}^q] \circ \mathbf{e}_j \circ \mathbf{e}_n \geq \mathbf{0}, \\ & \forall i < m \in \mathcal{N}, j, n \in \mathcal{I}, j \neq n, \mathcal{C}_j \cap \mathcal{C}_n \neq \varnothing, q \leq \omega, && (9b) \\ & [-(\mathbf{E}_{ij} - \mathbf{E}_{mn}) - \mathbf{e} d_s] \circ \mathbf{e}_j \circ \mathbf{e}_n \geq \mathbf{0}, && (9c) \\ & \forall i < m \in \mathcal{N}, j, n \in \mathcal{I}, j \neq n, \mathcal{C}_j \cap \mathcal{C}_n \neq \varnothing, q = \omega + 1. \end{aligned}$$

Especially, $\omega = 0$ means overtaking maneuver on any virtual lane is forbidden, i.e., all vehicles should obey the arriving order to cross the intersection, which is similar to the FIFO principle. Note that a small value of ω may alter the global-optimal solutions since it eliminates the possible car-following order condition where one vehicle is overtaken by the vehicle from $\omega + 1$ far away on one virtual lane in harmonized status. Nonetheless, the optimal values of

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objective function when $\omega > 0$ is very close to the global-optimal ones under normal traffic flow at the intersection.

Eventually, the primal optimization algorithm is formulated as the MILP with objective function (6), constraints (7a)-(7c), and constraints (9a)-(9c). The solution to the packing problem reflects the optimal car-following relationships in virtual platoons that maximizes the intersection utilization and minimizes the total evacuation time.

Centralized coordination collects the arriving order and the driving lanes information of vehicles to implement the optimization algorithm and transforms the optimal solutions to vehicles at CZ. The solutions contain the car-following order and the desired inter-vehicle distance. Once the solutions are determined, vehicles no longer need to communicate with the centralized coordination but with the neighboring vehicles to get the information of their preceding one, which has the potential to decrease the communication burden.

B. Decentralized Control at Discrete Time

Due to the inevitable communication interval, the discrete-time state-space model of vehicle i is obtained as

$$\mathbf{x}_i(k+1) = \begin{bmatrix} 1 & -t_d \\ 0 & 1 \end{bmatrix} \mathbf{x}_i(k) + \begin{bmatrix} -\frac{1}{2}t_d^2 \\ t_d \end{bmatrix} u_i(k), \tag{10}$$

where $x_i(k) = [p_i(k), v_i(k), u_i(k)]^T$, $p_i(k)$, $v_i(k)$, and $u_i(k)$ denote the position, velocity and the acceleration of the vehicle i on the virtual lane at time t_k ($t_k = kt_d$), respectively. t_d is the communication time interval and $k = \lfloor t/t_d \rfloor$.

The acceleration of vehicle i is set as the linear combination of the position error $\Delta d_{ij}(k)$ and speed error $\Delta v_{ij}(k)$ with its preceding vehicle j.

$$\begin{aligned} u_i(k+1) &= k_p \Delta d_{ij}(k) + k_v \Delta v_{ij}(k) \\ &= k_p(p_i(k) - p_j(k) - D_i) + k_v(v_i(k) - v_j(k)), \end{aligned} \tag{11}$$

where k_p and k_v are the feedback gains of position error and speed error, respectively. In this paper, the control laws k_p and k_v of all vehicles are the same. Note that one vehicle has at least two vehicles on different virtual lanes needed to follow. However, as long as the vehicle feedback gains k_p and k_v are appropriate, the vehicle only needs to follow one of them and the ultimate system will be stable.

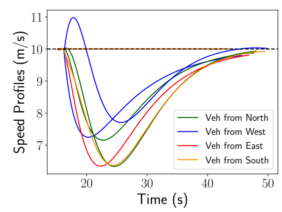
IV. Numerical Simulations

To validate the effectiveness of the proposed method, we present the simulation framework using an open-source traffic system simulation tool SUMO [17]–[19] and solve the packing problem with Gurobi Mathematical Optimization Solver [20]. We employ the typical intersection scenario as shown in Fig. 1. The simulations spawn vehicles by Poisson distribution with an expectation of 540 vehicles per hour on each lane. The main parameters of the simulations are tabulated in Table I.

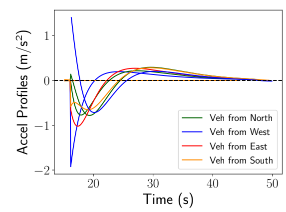
We count the total computation time of solving the packing problem and the results show that the maximum time cost on communication is $1.678s$. Fig. 4 exhibits the motion data of 8 vehicles approaching the intersection as shown in Fig. 1. Fig. 4(a) and Fig. 4(b) show that vehicles adjust their accelerations to avoid potential collision and follow their preceding one during CZ. And they eventually reach the harmonized status with a consensus speed and acceleration, which means they achieve the desired harmonized status in finite time. Furthermore, in Fig. 4(c), the horizontal axes represent the vehicle position and the vertical axes represent the time. Fig. 4(c) represents the driving trajectory of the vehicles at MZ and the driving trajectories in the same color indicate the incoming vehicles from the same direction. What's more, the star points along the driving trajectory are the potential conflict points at MZ. From Fig. 4(c), we can see that the driving trajectories do not intersect at the

TABLE I
Simulation Parameters

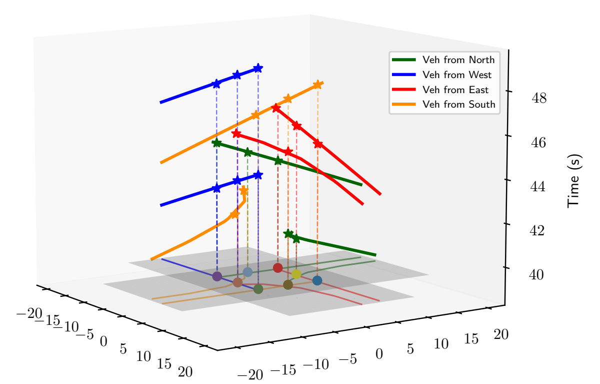
Simulation Parameters	Symbols	Value
length of overall intersection	d	$400m$
length of merging zone	d_m	$30m$
radius of cooperative zone	d_c	$200m$
safe car-following distance	d_s	$20m$
extra distance on straight lane	m_1^0, m_1^7, m_1^6	$10m$,$15m$,$20m$
extra distance on turning lane	m_2^1, m_2^3	$12m$,$18m$
harmonized longitudinal speed	v_d	$10m/s$
initial speed	v_{int}	$10m/s$
maximal speed	v_{max}	$20m/s$
minimal speed	v_{min}	0
maximal acceleration	a_{max}	$2.6m/s^2$
minimal comfortable acceleration	a_{min}	$-2m/s^2$
overtaking tolerance	ω	1
feedback gain for position error	k_p	0.05
feedback gain for speed error	k_v	-0.55
time interval	t_d	0.1



(a) Speed profiles at CZ.



(b) Accel profiles at CZ.



(c) Position profiles at MZ. Each line represents the vehicle's driving trajectories and the star points along the driving trajectory are the potential conflict points at MZ.

Fig. 4. Motion data of vehicles.

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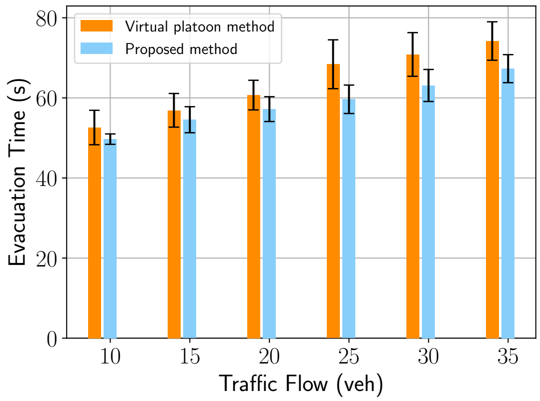

Fig. 5. Total evacuation time under different traffic flow.

conflict points, which means all vehicles pass each conflict point orderly, i.e., all vehicles on different lanes cross the intersection without any collision.

Fig. 5 demonstrates the performance of the proposed method under different traffic flow compared with the original virtual platoon method proposed in [12]. Each traffic flow consists of ten-times simulations, and the standard deviations of the results are also illustrated as error bar in Fig. 5. Due to the FIFO principle and constructing just one virtual platoon based on lanes, the passing order and the intersection utilization in the original virtual platoon method is the local optimal solution. In contrast, the proposed method organizes virtual platoons based on the conflict points and obtains nearly global-optimal car-following order in virtual platoons. Fig. 5 shows that the proposed method outperforms the original virtual platoon method and saves $5.5\% - 9.3\%$ total evacuation time under normal traffic flow. Besides, the error bar indicates that the proposed method has better adaptability to different traffic flow compared with the original virtual platoon method.

V. Conclusion and Future work

Motivated by the virtual platoon method, this paper proposed a conflict-free cooperative control method for CAVs at unsignalized intersections. Specifically, we construct multiple virtual platoons based on the conflict points to describe the vehicle collisions at the intersection. Then the optimal car-following order in virtual platoons for maximal intersection utilization is modeled as a packing problem and formulated as the MILP. To obtain the solutions in real time, an efficient optimization algorithm is further proposed to simplify the MILP by removing redundant constraints. Eventually, the vehicles are orchestrated with a decentralized control method. A series of simulations are carried out to validate that the proposed method prevents vehicles from collision and saves more total evacuation time compared with the original virtual platoon method.

Unsolved topics for future work include the sensitivity analysis of overtaking tolerance ω and the optimal cooperative driving of CAVs at multiple adjacent intersections.

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