2022 IEEE 25th International Conference on Intelligent Transportation Systems (ITSC)
October 8-12, 2022, Macau, China

Heterogeneous Traffic Intersection Coordination Based on Distributed Model Predictive Control with Invariant Safety Guarantee

Xiao Chen[1] and Jonas Mårtensson[2]

2022 IEEE 25th International Conference on Intelligent Transportation Systems (ITSC) | 978-1-6654-6880-0/22/$31.00 ©2022 IEEE | DOI: 10.1109/ITSC55140.2022.9922184

***Abstract*— This paper considers the heterogeneous traffic intersection where both Human Driven Vehicles (HDVs) and Connected and Automated Vehicles (CAVs) exist. In such a dynamic environment, CAVs must act in a way such that safety is guaranteed at all times, which is challenging due to the unpredictable nature of human behavior. To guarantee safety, in this paper we consider the worst-case behavior of HDVs by constructing the forward reachable set and ensuring collision avoidance against the forward reachable set within the CAV's planning horizon. To ensure safety at all times, a maximal invariant safe set is designed and used as a terminal constraint such that within this set there is always admissible control for CAVs to react against all possible future behavior of other vehicles safely. Finally, we propose to solve the intersection coordination problem within a Distributed Model Predictive Control (DMPC) framework where all pairwise safety constraints among CAVs are decoupled by prioritization. As a result, each CAVs solves a Mixed Integer Quadratic Programming (MIQP) problem considering collision avoidance with all CAVs of higher priority and with all HDVs. We give theoretical proof of the recursive feasibility of our proposed DMPC formulation and practical invariant safety guarantees. The resulting solution is evaluated in simulation and shows that our coordination framework can provide invariant safe coordination in a heterogeneous traffic intersection.**

I. Introduction

In recent years, connected and automated vehicle (CAV) technology has attracted large interest from both industry and academia. By removing the error-prone human factor, the deployment of CAVs is expected to drastically improve both the safety and efficiency of our transportation system [1]. Due to the computation and communication capabilities, CAVs can establish vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication. This allows them to identify unsafe regions and resolve potential conflicts by minimizing unnecessary stops and avoiding potential accidents. This opens up the opportunity for new solutions to many challenging traffic coordination problems.

In particular, the study of intersection coordination of CAVs hosts great potential and challenge. Since traffic intersection constitutes one of the primary sources of traffic congestion [2] and often hosts numerous types of potentially unsafe scenarios. These unsafe scenarios, together with the complication of intensive interactions among traffic participants acting in an intersection, make it more challenging for the system designer to provide safety guarantees for road users.

Xiao Chen and Jonas Mårtensson are with the Division of Decision and Control Systems, School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology, SE-10044 Stockholm, Sweden. Emails: `xiao2@kth.se` `jonas1@kth.se`

The vast majority of existing work on the topic of intersection coordination for CAVs can be classified as either centralized or distributed approaches. Interested readers are referred to the surveys [3] and [4] for a general review of some recent advancements in this field. In the centralized approach [5], [6], [7], [8], a central node is responsible for designing conflict resolution plans for all approaching vehicles, assuming complete knowledge of their states. Although this approach may significantly reduce conflicts and provide optimal coordination, it does not scale well with the number of vehicles due to the curse of dimensionality, which often results in high computation times. To resolve these problems, distributed frameworks were proposed in [9], [10], [11], [12], in which vehicles solve individual optimal control problems for conflict resolution based on local state information received from neighboring vehicles for the benefit of computational efficiency and the possibility of parallelization.

A common assumption made by the majority of the studies is that the traffic environment under consideration is homogeneous with CAVs only. In reality, it is more likely to have a heterogeneous traffic intersection where CAVs and HDVs co-exist. This fact, backed up by recent evidence of fatal accidents involving CAVs [13], emphasizes the need for the system designer to develop a coordination framework for CAVs with invariant safety guarantees in heterogeneous environments. The main challenge in such a setting is the anticipation of HDVs because human behavior can not be fully predicted. As a result, heterogeneous intersection coordination can be viewed as a multi-agent coordination task in a dynamic environment. It is essentially an infinite horizon optimal control problem with time-varying state constraints. Model Predictive Control (MPC) is a promising approach to solving the infinite horizon problem sub-optimally through iteration over a finite horizon. But under time-varying state constraints, it is generally challenging to provide recursive feasibility guarantees [14], [15]. Fail-safe trajectory planning based on safe invariant sets was introduced in [16], [17], which considers the worst-case predictions of the surrounding vehicles to form a fail-safe vehicle trajectory with formal safety guarantees. Inspired by these studies, we propose to use the forward reachable set for safety constraints within the finite horizon and to use the maximal safe invariant set as a terminal constraint in the MPC formulation to provide an invariant safety guarantee at all times. In addition, for computational efficiency and avoidance of deadlock situations, we employ a DMPC strategy similar to [18], [11], [12], where the pairwise safety constraints between any two

CAVs are decoupled through prioritization. As a result, the conflict resolution constraints are introduced only for lower-priority agents which are responsible for avoiding collisions with vehicles of higher priority.

In summary, the work presented in this paper features the following contributions:

1) A DMPC framework with constraint decoupling is proposed for the heterogeneous traffic intersection environment with the benefit of computational efficiency and avoidance of deadlock situations;
2) The introduction of forward reachable sets to formulate safety constraints and the use of the maximal invariant safe set as a terminal constraint to handle the safety issue raised by the presence of HDVs;
3) A theoretical proof for the recursive feasibility property of the proposed DMPC framework and the implied practical invariant safety property;
4) An efficient way of computing the maximal invariant safe set in the intersection coordination problem.

The remainder of the paper is organized as follows. In Section II we explain the fundamental traffic scenarios with the corresponding safety constraints and formulate the DMPC coordination framework with invariant safety guarantee under the heterogeneous traffic setting. In Section III we explain the simulation setup and evaluate our proposed coordination approach with simulation results. Finally, we provide some concluding remarks on our work in Section IV

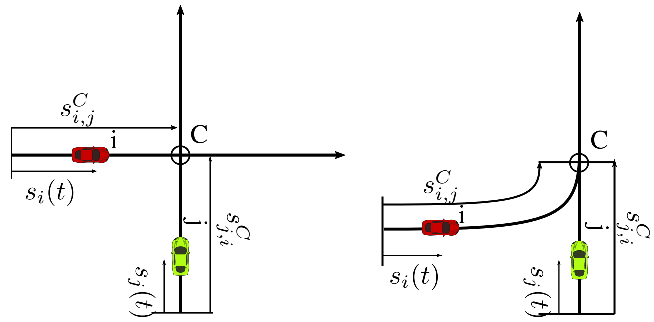
II. Problem formulation

A. Scenario description

A general traffic intersection consists of two possible types of conflicting motion. Here we denote them as the crossing scenario and the merging scenario Fig.1. Common for both scenarios, there is a conflicting area C in which two driving lanes coincide. We choose to model this conflicting area C as a single point in space for simplicity. Considering two vehicles i and j with a conflicting point C, we define $s^C_{i,j}$ as the distance to C measured from the road aligned coordinate of i, and $s^C_{j,i}$ as the distance to C measured from the road aligned coordinate of j, as can be seen in Fig.1. In the crossing scenario Fig.1a, the driving lanes separate after C, while in the merging scenario Fig.1b the driving lanes merge into one after C. Any type of complex intersection can be modeled as a combination of these two fundamental types of scenarios.

Let $\mathbb{A} = \{1, ..., A\}$ be the set of indices for all CAVs in an intersection. The index i also indicates the computational priority of CAVs in which lower index indicates a higher priority. Similarly, the set of indices for all HDVs is denoted as $\mathbb{H} = \{1, ..., H\}$, All HDVs have the highest priority for every $i \in \mathbb{A}$

For each $i \in \mathbb{A}$, we further define $\mathbb{CA}_i = \{j : j \in \mathbb{A}, j < i, \text{ i and j in crossing scenario}\}$, $\mathbb{MA}_i = \{j : j \in \mathbb{A}, j < i, \text{ i and j in merging scenario}\}$ as the set of indices of higher prioritized CAVs j in crossing- respective merging scenario with $i \in \mathbb{A}$. Similarly, we define $\mathbb{CH}_i = \{j : j \in \mathbb{H}, \text{ i and j in crossing scenario}\}$, $\mathbb{MH}_i = \{j : j \in \mathbb{H}, \text{ i and j in merging scenario}\}$ as the set of indices of HDVs j in crossing- respective merging scenario with $i \in \mathbb{A}$.



(a) Crossing scenario (b) Merging scenario

Fig. 1: The fundamental types of motion in a general traffic intersection.

The intersection coordination scenario considered in this work are restricted by the following assumption.

Assumption 1: The state of all vehicles at current time can be perfectly measured and shared to CAVs with no communication delay.

B. Vehicle model

In this work, we restrict to the longitudinal motion only. The dynamic model considered for each vehicle $i \in \mathbb{A}$ or $i \in \mathbb{H}$ is a second order linear differential equation given as follows:

$$\begin{aligned} \dot{s}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t), \end{aligned} \tag{1}$$

where $s_i(t)$ is the distance traveled of vehicle i measured from its road aligned coordinate frame, see Fig.1. $v_i(t)$ is the velocity and $u_i(t)$ is the input acceleration.

Both $v_i(t)$ and $u_i(t)$ are restricted by the following constraints

$$\begin{aligned} &0 \le v_i(t) \le v_{max} \\ &u_i^{min} \le u_i(t) \le u_i^{max}, \end{aligned} \tag{2}$$

where v_{max} is the maximal permitted speed limit and u_i^{min} and u_i^{max} are the minimum deceleration and maximum acceleration for vehicle i. Here we assume that u_i^{min} and u_i^{max} are chosen so model (1) acts as an under-approximation of the true vehicle dynamic for $i \in \mathbb{A}$, such that any state reachable with (1) is reachable by the actual vehicle. In opposite, for $i \in \mathbb{H}$, we assume u_i^{min} and u_i^{max} are chosen so model (1) is an over-approximation of the true vehicle dynamic. This is done so the safety guarantees based on reachability analysis in this work is achieved in a conservative fashion.

C. Distance measure and safety constraints

Considering the ego vehicle $i \in \mathbb{A}$ and a neighboring vehicle $j \in \mathbb{CA}_i$ or $\mathbb{CH}_i$ or $\mathbb{MA}_i$ or $\mathbb{MH}_i$, we denote $D_{i,j}(t)$

as the distance measure between i and j, which is a function of $s_i(t)$ and $s_j(t)$. To ensure safety, we require that

$$D_{i,j}(t) \geq d_{safe}, \forall t \geq 0 \tag{3}$$

where d_{safe} is a predefined fixed safety distance.

Due to the distinction between the crossing and merging scenarios, we treat the definition of $D_{i,j}(t)$ and the resulting safety constraints for the crossing and merging scenarios separately in the follow sections.

1) Crossing scenario: For the crossing scenario, i.e., $j \in \mathbb{CA}_i$ or $\mathbb{CH}_i$, we define

$$D^{cross}_{i,j}(t) = |s_i(t) - s^C_{i,j}| + |s_j(t) - s^C_{j,i}| \tag{4}$$

as the sum of relative distances to the common conflicting point C.

For safety, we require $D^{cross}_{i,j}(t) \geq d_{safe}, \forall t \geq 0$. By introducing binary variable $B_{i,j} \in \{0, 1\}$ and a sufficient large constant M, we remove the absolute-value operator and derive the following safety constraints

$$\text{if } -d_{safe} \leq s_j(t) - s^C_{j,i} < 0 :$$
$$s_i(t) \leq s^C_{i,j} - d_{safe} + MB_{i,j} \tag{5a}$$
$$s_i(t) \geq s^C_{i,j} + d_{safe} - s^C_{j,i} + s_j(t) - M(1 - B_{i,j}) \tag{5b}$$
$$\text{if } 0 \leq s_j(t) - s^C_{j,i} < d_{safe} :$$
$$s_i(t) \leq s^C_{i,j} - d_{safe} - s^C_{j,i} + s_j(t) + MB_{i,j} \tag{5c}$$
$$s_i(t) \geq s^C_{i,j} + d_{safe} - M(1 - B_{i,j}) \tag{5d}$$

where we use the fact that $D^{cross}_{i,j}(t) \geq d_{safe}$ is only active when $|s_j(t) - s^C_{j,i}| \leq d_{safe}$, and that $s_i(t)$ is non-decreasing according to (1) and (2). Clearly, when $B_{i,j} = 1$, Ego vehicle i is to precede neighboring vehicle j enforced by (5b),(5d) and leaving (5a), (5c) redundant.. The opposite is true when $B_{i,j} = 0$.

2) Merging scenario: For merging scenario, i.e., $j \in \mathbb{CA}_i$ or $\mathbb{CH}_i$, we define

$$D^{merge}_{i,j}(t) = \begin{cases} |s_i(t) - s^C_{i,j}| + |s_j(t) - s^C_{j,i}|, \\ \quad \text{if } s_i(t) < s^C_{i,j} \wedge s_j(t) < s^C_{j,i} \ . \\ \\ \max\{(s_i(t) - s^C_{i,j}), (s_j(t) - s^C_{j,i})\} \\ - \min\{(s_i(t) - s^C_{i,j}), (s_j(t) - s^C_{j,i})\}, \\ \quad \text{otherwise.} \end{cases} \tag{6}$$

where the distance measure is defined same as in the crossing scenario before vehicle i and j has passed point C, after either i or j has passed C the distance measure is defined as the difference between the distance traveled as i and j are now in same direction, see Fig.1b.

By requiring $D^{merge}_{i,j}(t) \geq d_{safe}, \forall t \geq 0$, we can derive the following safety constraints using the binary variable $B_{i,j} \in \{0, 1\}$ and a sufficient large constant M:

$$\text{if } -d_{safe} \leq s_j(t) - s^C_{j,i} < 0 :$$
$$s_i(t) \leq s^C_{i,j} - d_{safe} + MB_{i,j} \tag{7a}$$
$$s_i(t) \geq s^C_{i,j} + d_{safe} - s^C_{j,i} + s_j(t) - M(1 - B_{i,j}) \tag{7b}$$
$$\text{if } s_j(t) - s^C_{j,i} \geq 0 :$$
$$s_i(t) \leq s^C_{i,j} - d_{safe} - s^C_{j,i} + s_j(t) + MB_{i,j} \tag{7c}$$
$$s_i(t) \geq s^C_{i,j} + d_{safe} - s^C_{j,i} + s_j(t) - M(1 - B_{i,j}) \tag{7d}$$

D. Invariant safe DMPC

Consider any $i \in \mathbb{A}$ at a current time instance t^0. The task for vehicle i is to plan its motion such that the safety constraints (5) and (7) are satisfied against any conflicting vehicle $j \in \mathbb{CA}_i$ or $\mathbb{CH}_i$ or $\mathbb{MA}_i$ or $\mathbb{MH}_i$,$\forall t \geq t^0$. This is an infinite horizon optimal control problem, which is typically impossible to solve directly. In this paper, we propose to use the MPC formulation which solves the infinite horizon problem in an iterative fashion with receding horizon. For this, a planning horizon of N steps with a discretization of Δt for each step is defined. Safety during the fixed planning horizon is ensured by imposing the safety constraints (5) and (7) at all discretization steps $k = 1, ..., N$. Two problems exist in such formulation: Firstly, the safety constraint (5) and (7) is based on the fact that $s_j(t)$ is given. Due to the fact that HDVs are non-cooperative, the only available information for vehicle i to perform motion planning is the current state measurement of the HDVs. Secondly, the goal of the infinite horizon optimal control problem is to guarantee (5) and (7) $\forall t \geq t^0$. Terminal constraint at $k = N$ needs to be designed such that fulfillment of terminal constraint will imply invariant safety at all times. In what follows, we will address these two problems

To ensure safety against the worst case scenario, we need to consider all possible future behaviors of the HDVs. This motivates us to use the forward reachable set to form the safety constraints. We denote $u([t_0, t_f])$ as the input sequence from t_0 to t_f. Given the initial state $x^0 = [s^0 \ v^0]^T$ and an admissible input sequence $u([t_0, t_f]) \in [u^{min}, u^{max}]$, we denote $\chi(t_f, x^0, u([t_0, t_f]))$ as the solution of (1) subject to (2) at time t_f. Using these notations, the forward reachable set is defined as:

Definition 2.1 (Forward reachable set): Given the initial vehicle state $x^0 = [s^0 \ \ v^0]^T$ at t_0, a forward reachable set at time t_h, denoted as $\mathcal{R}(x^0, t_h)$, is defined to be the set of solutions $\chi(t_h, x^0, u([t_0, t_f]))$ for all admissible input sequences $u([t_0, t_h]) \in [u^{min}, u^{max}]$.

Given the vehicle model (1) and constraints (2), the forward reachable set of vehicle i can be expressed and computed as follows:

$$\mathcal{R}_i(x^0, t_h) = \{[s \ v]^T : s \in [s^{break}_i(x^0, t_h), s^{acc}_i(x^0, t_h)], \\ v \in [v^{break}_i(x^0, t_h), v^{acc}_i(x^0, t_h)]\} \tag{8}$$

where $s_i^{break}(x^0, t_h)$, $v_i^{break}(x^0, t_h)$ respectively $s_i^{acc}(x^0, t_h)$, $v_i^{acc}(x^0, t_h)$ are obtained as follows:

$$s_i^{break}(x^0, t) = \begin{cases} s_i^0 + v_i^0 t - \frac{u_i^{min} t^2}{2} & \text{if } 0 \le t \le T_i^{break} \\ s^{break}(T_i^{break}) & \text{if } t > T_i^{break} \end{cases} \tag{9}$$

$$v_i^{break}(x^0, t) = \begin{cases} v_i^0 - u_i^{min} t & \text{if } 0 \le t \le T_i^{break} \\ 0 & \text{if } t > T_i^{break} \end{cases} \tag{10}$$

$$s_i^{acc}(x^0, t) = \begin{cases} s_i^0 + v_i^0 t + \frac{u_i^{max} t^2}{2} & \text{if } 0 \le t \le T_i^{acc} \\ s_i^{acc}(T_i^{acc}) + v_{max}(t - T_i^{acc}) & \text{if } t > T_i^{acc} \end{cases} \tag{11}$$

$$v_i^{acc}(x^0, t) = \begin{cases} v_i^0 + u_i^{max} t & \text{if } 0 \le t \le T_i^{acc} \\ v_{max} & \text{if } t > T_i^{acc} \end{cases} \tag{12}$$

where $T_i^{break} = \frac{v_i^0}{u_i^{min}}$ is the time it takes for vehicle to break until full stop and $T_i^{acc} = \frac{(v_{max} - v_i^0)}{u_i^{max}}$ is the time it takes for vehicle to accelerate to v_{max}.

For each conflicting vehicle j, we formulate the safety constraints (5) and (7) based on forward reachable set $\mathcal{R}_j(x_j^0, t_h)$ through the following definition of a safe state:

Definition 2.2 (Crossing safe state): The set of safe states for ego vehicle i at time t_h against conflicting vehicle j in a crossing scenario with initial state x_j^0 denoted as $\mathcal{F}_{i,j}^C(x_j^0, t_h)$ is the set of $s_i(t_h)$ in which safety constraint (5) is satisfied for all $s_j(t_h) \in \mathcal{R}(s_j^0, v_j^0, t_h)$.

Definition 2.3 (Merging safe state): The set of safe states for ego vehicle i at time t_h against conflicting vehicle j in a merging scenario with initial state x_j^0 denoted as $\mathcal{F}_{i,j}^M(x_j^0, t_h)$ is the set of $s_i(t_h)$ in which safety constraint (7) is satisfied for all $s_j(t_h) \in \mathcal{R}(s_j^0, v_j^0, t_h)$.

Notice that neither safe state set $\mathcal{F}^C$ nor $\mathcal{F}^M$ contains v_i, since the safety constraints (5) and (7) are based on s_i only, i.e., distance traveled of vehicle i.

To impose safety at all times, we use the notion of safety invariant set introduced in [16], [17] and define the following modified version of maximal invariant safe set

Definition 2.4 (Maximal invariant safe set): The maximal invariant safe set of ego vehicle i at time t_h against conflicting vehicle j with initial state x_j^0 denoted as $\mathcal{S}_{i,j}(x_j^0, t_h)$ is defined as the set of all state $x_i(t_h) = \begin{bmatrix} s_i(t_h) & v_i(t_h) \end{bmatrix}^T$ such that at time t_h, $s_i(t_h) \in \mathcal{F}(x_j^0, t_h)$, $v_i(t_h) \in [0, v_{max}]$ and at $\forall \tau \ge t_h$, $\exists u_i([t_h, \tau]) \in [u^{min}, u^{max}]$ s. t. $\chi(\tau, x_i(t_h), u([t_h, \tau]) \in \mathcal{F}(x_j^0, \tau)$

Given $x_i(t_h) \in \mathcal{S}_{i,j}(x_j^0, t_h)$ implies that there always is an admissible input sequence such that all future states $x_i(\tau)$, $\forall \tau \ge t_h$ starting from $x_i(t_h)$ can stay in the safe state set $\mathcal{F}(x_j^0, \tau)$. We denote the complement of $\mathcal{S}_{i,j}(x_j^0, t_h)$ as $\overline{\mathcal{S}_{i,j}}(x_j^0, t_h)$. Then, as a consequence, $\forall x_i(t_h) \in \overline{\mathcal{S}_{i,j}}(x_j^0, t_h)$, $\nexists u([t_h, \tau]) \in [u^{min}, u^{max}]$ such that $\chi(\tau, x_i(t_h), u([t_h, \tau]) \in \mathcal{F}(x_j^0, \tau)$, $\forall \tau \ge t_h$.

For every $i \in \mathbb{A}$, at any planning time instance t_0, given the initial state x_j^0 of all conflicting vehicle j and with the help of definition 2.2, 2.3, 2.4, we formulate the MPC problem for vehicle i as follows:

$$\min \quad \sum_{k=0}^{N-1} u_{i,k}^2 - W[1\ 0]x_{i,N} \tag{13a}$$

$$\text{s. t.} \quad x_{i,k+1} = Ax_{i,k} + Bu_{i,k}, \ k = 0, ..., N-1 \tag{13b}$$

$$x_{i,0} = x_i^0, \tag{13c}$$

$$(2), \tag{13d}$$

$$[1\ 0]x_i(k) \in \mathcal{F}^C(x_j^0, k), \forall j \in \mathbb{CA}_i, k = 1, ..., N \tag{13e}$$

$$[1\ 0]x_i(k) \in \mathcal{F}^C(x_j^0, k), \forall j \in \mathbb{CH}_i, k = 1, ..., N \tag{13f}$$

$$[1\ 0]x_i(k) \in \mathcal{F}^M(x_j^0, k), \forall j \in \mathbb{MA}_i, k = 1, ..., N \tag{13g}$$

$$[1\ 0]x_i(k) \in \mathcal{F}^M(x_j^0, k), \forall j \in \mathbb{MH}_i, k = 1, ..., N \tag{13h}$$

$$x_{i,N} \in \bigcap_{\forall j \in \mathbb{MH}_i, \mathbb{MA}_i} S_{i,j}^M(x_j^0, N) \tag{13i}$$

$$x_{i,N} \in \bigcap_{\forall j \in \mathbb{CH}_i, \mathbb{CA}_i} S_{i,j}^C(x_j^0, N) \tag{13j}$$

Here $x_{i,k} = [s_{i,k}\ v_{i,k}]^T$ and $u_{i,k}$ are the state and input of vehicle i at discrete time instance k with the initial condition $x_{i,0} = x_i^0$. The first term in the objective function (13a) is to minimize control effort while the second term is to maximize control progress. A fixed weight W is introduced to balance the contribution of the two terms. The constraint (13b) is the discrete-time dynamics obtained by the discretization of (1). Constraints (13e)-(13h) are the safety constraints to ensure that $s_{i,k}$ is in the set of safe state at time instance k. The terminal constraints (13i)-(13j) ensure that the terminal state $x_{i,N}$ will stay in the maximal invariant safe set to guarantee invariant safety at all times, here $S_{i,j}^C$ and $S_{i,j}^M$ denote the maximal invariant safe set for merging respective crossing scenario specifically. Observe that in (13) only conflicting CAVs with higher priority are considered due to the definition of $\mathbb{CA}_i$ and $\mathbb{MA}_i$, so the pairwise safety constraint between any two conflicting CAVs is decoupled through prioritization. This prevents any deadlock situation and the resulting DMPC formulation (13) can be fully parallelized.

Theorem 2.1: Given that the optimization problem (13) is initially feasible for all $i \in \mathbb{A}$, with the fixed priority $\mathbb{A}$, the DMPC formulation (13) has the recursive feasibility property when applied iteratively with a receding horizon. As a result, the closed-loop solution of the DMPC is invariantly safe for all $t \ge 0$

Proof: Assume that at time k, an optimal solution of problem (13) is obtained with the resulting optimal state sequence denoted as $\{x_k^*, x_{k+1}^*, ..., x_{k+N}^*\}$. At the next planning instance $k+1$, we consider the shifted sequence $\{x_{k+1}^*, ..., x_{k+N}^*, x_{k+N+1}\}$ as a candidate solution to (13) at $k+1$. Here we assume $x_{k+N+1} \in S(k+N+1)$, which is the overall maximal safe invariant set at $k+N+1$. Now since $x_{k+1}^*, ..., x_{k+N}^*$ is the optimal solution at earlier time instance k, they are in the set of safe states at $k+1, ..., k+N$ which ensures safety constraint satisfaction against all possible behaviors of conflicting vehicles at $k+1, ..., k+N$, according to definition 2.2 and 2.3. So safety constraints (13e)-(13h) are again satisfied by $\{x^*_{k+1}, ..., x^*_{k+N}, x_{k+N+1}\}$. It remains to show that there is an admissible control that leads x^*_{k+N} into x_{k+N+1}. Since $x^*_{k+N} \in S(k+N)$ then $\exists u([k+N, \tau]) \in [u^{min}, u^{max}]$ such that $\chi(\tau, x^*_{k+N}, u([k+N, \tau]) \in \mathcal{F}(\tau)$ for all $\tau \geq (k+N)$. Assume now that $\nexists u([(k+N),(k+N+1)]) \in [u^{min}, u^{max}]$ such that $\chi((k+N+1), x^*_{k+N}, u([(k+N),(k+N+1)])) \in S(k+N+1)$. In other words, assume that any admissible control $u([(k+N),(k+N+1)]) \in [u^{min}, u^{max}]$ leads x^*_{k+N} to the $\overline{S}(k+N+1)$, due to the fact that $S(k+N+1)$ is the maximal safe invariant set at $k+N+1$. This implies that there is no admissible control sequence starting from $x^*(k+N)$ that will lead to a future state $x(\tau) \in \mathcal{F}(\tau)$ for $\tau \geq k+N+1$. This contradicts the fact that $x^*_{k+N} \in S(k+N)$. In conclusion, it is feasible to construct $\{x^*_{k+1}, ..., x^*_{k+N}, x_{k+N+1}\}$ which is feasible at $k+1$. So the DMPC problem (13) is recursively feasible at all time steps. As a result, the closed loop solution of the DMPC is invariantly safe due to the property of the maximal invariant safe set. ■

Remark 1: For theorem 2.1, we assume that the true form of the maximal invariant safe set $S_{i,j}$ is used as final constraint. In practice, to make (13) solvable, a piece-wise linear under-approximation $\hat{S}_{i,j}$ is often used in place of $S_{i,j}$. In such case, it is possible that for $x^*_{i,k+N} \in \hat{S}_{i,j}(k+N)$, the admissible control will lead to $x_{i,k+N+1} \in S_{i,j}(k+N+1) \setminus \hat{S}_{i,j}(k+N+1)$. In such situation, (13) becomes infeasible at $k+1$. But due to the fact that the solution in previous step k still guarantees invariant safety at all times, we can use the previous solution to reconstruct a safe backup control sequence under which the practical invariant safety guarantee is preserved.

Remark 2: Since the pairwise safety constraint between any two CAVs is decoupled through prioritization, for vehicle i, one can think of using the optimal prediction $\{x^*_{j,k}, x^*_{j,k+1}, ..., x^*_{j,N}\}$, obtained by conflicting CAVs j with higher priority, to formulate the safety constraints instead of (13e) and (13g). This is true in a homogeneous static environment without HDVs, since the prediction $\{x^*_{j,k}, x^*_{j,k+1}, ..., x^*_{j,N}\}$ will be properly followed. Employ in addition the invariant safe set as final constraints, and the recursive feasibility and invariant safety property can be deduced similarly as in (13). However, in a heterogeneous setting, such a proposal will ruin the feasibility and invariant safety property of (13) due to the fact that the presence of HDVs renders a dynamic environment for CAVs under which CAVs with higher priority will not follow their previous prediction if the behavior of HDVs vary between time steps. So the DMPC formulation (13) is required.

E. Computation of Maximal Safe Invariant Set

For any ego vehicle $i \in \mathbb{A}$ and a conflicting vehicle j, it remains to show how the maximal safe invariant set $S_{i,j}(x^0_j, t_h)$ is obtained. We state the result for the crossing and the merging scenarios in separate sections.

1) The crossing scenario: Given the initial state $x^0_j = [s^0_j \ v^0_j]^T$ for vehicle j, we know that $x_j(t) \in \mathcal{R}(x^0_j, t)$, in other words, $s_j(t) \in [s^{break}_j(x^0_j, t), s^{acc}_j(x^0_j, t)]$. Assume for the moment that vehicle i decides to let vehicle j pass first, i.e., $B_{i,j} = 0$. Then safety constraints (5b) and (5d) will become redundant leaving only (5a) and (5c) active. The states of vehicle i, $x_i(t_h) = [s_i(t_h), \ v_i(t_h)]^T$ at time t_h that constitute the maximal invariant safe set $\mathcal{S}^C_{i,j}(x^0_j, t_h)$ need to satisfy (5a) and (5c) for any $s_j(t) \in [s^{break}_j(t), s^{acc}_j(t)]$ at time $t \geq t_h$ according to definition 2.4. Since (5a) and (5c) entail upper bound for $s_i(t)$, $s^{break}_j(t)$ will become the limiting factor.

Using the fact that distance traveled $s_i(t)$ at time t of ego vehicle i can be written as

$$s_i(t) = s_i(t_h) + v_i(t_h)t + \int_{t_h}^{t}\int_0^T u_i(\tau)d\tau dT \quad \text{s. t.,} \quad (2)$$

we can rewrite (5a) and (5c) as follows:

$$\begin{aligned} &\text{if } s^C_{j,i} - d_{safe} \leq s^{break}_j(t) \leq s^C_{j,i}: \\ &\quad s_i(t_h) \leq s^C_{i,j} - d_{safe} - v_i(t_h)t - \int_{t_h}^{t}\int_0^T u_i(\tau)d\tau dT && \text{(14a)} \\ &\text{if } s^C_{j,i} \leq s^{break}_j(t) - s^C_{j,i} \leq s^C_{j,i} + d_{safe}: \\ &\quad s_i(t_h) \leq s^C_{i,j} - d_{safe} - s^C_{j,i} + s^{break}_j(t) - v_i(t_h)t \\ &\quad - \int_{t_h}^{t}\int_0^T u_i(\tau)d\tau dT && \text{(14b)} \\ &\text{s. t.,} \quad (2) && \text{(14c)} \end{aligned}$$

We see that given any $v_i(t_h) \in [0, v_{max}]$, the maximal invariant safe set can be obtained by deriving the maximal upper bound for $s_i(t_h)$, which is achieved by minimizing $\int_{t_h}^{t}\int_0^T u_i(\tau)d\tau dT$. This is exactly when ego vehicle i takes the full breaking action with initial time $t^0 = t_h$ and initial state $x^0 = [0 \ v_i(t_h)]^T$. This will result in the following description for the maximal invariant safe set denoted as $\mathcal{S}^{CA}_{i,j}$ superscript CA indicate the condition that i decide to cross after j.

$$\begin{aligned} &\text{if } s^C_{j,i} - d_{safe} \leq s^{break}_j(t) \leq s^C_{j,i}: \\ &\quad s_i(t_h) \leq s^C_{i,j} - d_{safe} - s^{break}_i(x^0, t) && \text{(15a)} \\ &\text{if } s^C_{j,i} \leq s^{break}_j(t) - s^C_{j,i} \leq s^C_{j,i} + d_{safe}: \\ &\quad s_i(t_h) \leq s^C_{i,j} - d_{safe} - s^C_{j,i} + s^{break}_j(t) - s^{break}_i(x^0, t) && \text{(15b)} \end{aligned}$$

The form of $\mathcal{S}^{CA}_{i,j}$ is dependent on $s^{break}_j(t)$, $t \geq t_h$. As a result, we need to consider the following cases:

If $s^{break}_j(T^{break}_j) < s^C_{j,i}$ then (15a) need to be satisfied for $t \in [t_h, \infty)$. For $\mathcal{S}^{CA}_{i,j}$ we get

$$s_i(t_h) \leq s^C_{i,j} - d_{safe} - s^{break}_i(x^0, T^{break}_i) \quad (16)$$

If $s^{break}_j(T^{break}_j) \geq s^C_{j,i}$, constraint (15b) becomes active, we need to know $\frac{d(s^{break}_j(t) - s^{break}_i(x^0,t))}{dt}$ in order to find the permissible upper bound. The value of

$\frac{d(s_j^{break}(t)-s_i^{break}(x^0,t))}{dt}$ depends on the relation between u_i^{min}, u_j^{min}, due to the limited space and for simplicity, in what follows, we will assume that $u_i^{min} = u_j^{min}$, $u_i^{max} = u_j^{max}$, the results for the other cases can be obtained in similar fashion with slightly more complex structure. We state that

$$\frac{d(s_j^{break} - s_i^{break})}{dt} \begin{cases} \begin{matrix} < 0, t \leq T_i^{break} \\ = 0, t \geq T_i^{break}, \end{matrix} & \text{if } v_i(t_h) > v_j(t_h) \\ \\ = 0, \ t \geq 0, \quad \text{if } v_i(t_h) = v_j(t_h) & \\ \\ \begin{matrix} > 0, t \leq T_j^{break} \\ = 0, t \geq T_j^{break}, \end{matrix} & \text{if } v_i(t_h) < v_j(t_h) \end{cases} \tag{17}$$

The proof of this is omitted. We introduce the notation of T^I, T^C, T^O such that $s_j^{break}(T^I) = s_{j,i}^C - d_{safe}$, $s_j^{break}(T^C) = s_{j,i}^C$ and $s_j^{break}(T^O) = s_{j,i}^C + d_{safe}$. The following cases are considered:

If $s_j^{break}(t_h) < s_{j,i}^C$ and $s_{j,i}^C \leq s_j^{break}(T_j^{break}) < s_{j,i}^C + d_{safe}$, then (15b) is required for $t \in [T^C, \infty)$. Using (17), the time at which the minimum is obtain, denoted as t^* can be found to be $t^* = T_i^{break}$, $\forall v_i(t_h) \geq v_j(t_h)$ and $t^* = T^C$, $\forall v_i(t_h) < v_j(t_h)$.

If $s_{j,i}^C \leq s_j^{break}(t_h) < s_{j,i}^C + d_{safe}$ and $s_{j,i}^C \leq s_j^{break}(T_j^{break}) < s_{j,i}^C + d_{safe}$, then (15b) is required for $t \in [t_h, \infty)$. Using (17), we get $t^* = T_i^{break}$, $\forall v_i(t_h) \geq v_j(t_h)$ and $t^* = t_h$, $\forall v_i(t_h) < v_j(t_h)$.

If $s_j^{break}(t_h) < s_{j,i}^C$ and $s_j^{break}(T_j^{break}) \geq s_{j,i}^C + d_{safe}$, then (15b) is required for $t \in [T^C, T^O]$. Using (17), we get $t^* = T^O$, $\forall v_i(t_h) \geq v_j(t_h)$ and $t^* = T^C$, $\forall v_i(t_h) < v_j(t_h)$.

If $s_{j,i}^C \leq s_j^{break}(t_h) < s_{j,i}^C + d_{safe}$ and $s_j^{break}(T_j^{break}) \geq s_{j,i}^C + d_{safe}$, then (15b) is required for $t \in [t_h, T^O]$. Using (17), we get $t^* = T^O$, $\forall v_i(t_h) \geq v_j(t_h)$ and $t^* = t_h$, $\forall v_i(t_h) < v_j(t_h)$.

Given t^*, according to (15b), the maximal invariant safe set $\mathcal{S}_{i,j}^{CA}$ can be described as

$$s_i(t_h) \leq s_{i,j}^C - d_{safe} - s_{j,i}^C + s_j^{break}(t^*) - s_i^{break}(x^0, t^*)$$

Following the same analysis we can derive the maximal invariant safe set $\mathcal{S}_{i,j}^{CB}$ when ego vehicle i decides to cross before j.

if $s_{j,i}^C \leq s_j^{acc}(t_h) < s_{j,i}^C + d_{safe}$ then the maximal invariant safe set $\mathcal{S}_{i,j}^{CB}$ is described as

$$s_i(t_h) \geq s_{i,j}^C + d_{safe} - s_i^{acc}(x^0, t_h)$$

if $s_{j,i}^C - d_{safe} \leq s_j^{acc}(t_h) < s_{j,i}^C$ then $t^* = t_h$, $\forall v_i(t_h) \geq v_j(t_h)$ and $t^* = T^C$, $\forall v_i(t_h) < v_j(t_h)$.

if $s_j^{acc}(t_h) < s_{j,i}^C - d_{safe}$ then $t^* = T^I$, $\forall v_i(t_h) \geq v_j(t_h)$ and $t^* = T^C$, $\forall v_i(t_h) < v_j(t_h)$.

Given t^*, the maximal invariant safe set $\mathcal{S}_{i,j}^{CB}$ is described as

$$s_i(t_h) \geq s_{i,j}^C + d_{safe} - s_{j,i}^C + s_j^{acc}(t^*) - s_i^{acc}(x^0, t^*)$$

.

The overall maximal invariant safe set for the crossing scenario denoted as $\mathcal{S}_{i,j}^C$ can then be obtained as $\mathcal{S}_{i,j}^C = \mathcal{S}_{i,j}^{CB} \cup \mathcal{S}_{i,j}^{CA}$. Here the binary variable $B_{i,j}$ together with the large constant M can be used to form the union operator as was done in (5).

2) The merging scenario: We denote $\mathcal{S}_{i,j}^{MB}$ as the merging invariant safe set when i decides to merge before j. There are following cases:

If $s_j^{acc}(t_h) < s_{j,i}^C - d_{safe}$ then $t^* = T^I$, $\forall v_i(t_h) \geq v_j(t_h)$ and $t^* = T_i^{acc}$, $\forall v_i(t_h) < v_j(t_h)$.

If $s_j^{acc}(t_h) \geq s_{j,i}^C - d_{safe}$ then $t^* = t_h$, $\forall v_i(t_h) \geq v_j(t_h)$ and $t^* = T_i^{acc}$, $\forall v_i(t_h) < v_j(t_h)$.

Given t^*, the maximal invariant safe set $\mathcal{S}_{i,j}^{MB}$ can be described as

$$s_i(t_h) \geq s_{i,j}^C + d_{safe} - s_{j,i}^C + s_j^{acc}(t^*) - s_i^{acc}(x^0, t^*)$$

.

We denote $\mathcal{S}_{i,j}^{MA}$ as the merging invariant safe set when i decides to merge after j. There are following cases:

If $s_j^{break}(T_j^{break}) < s_{j,i}^C$, then for $\mathcal{S}_{i,j}^{MA}$ we have

$$s_i(t_h) \leq s_{i,j}^C - d_{safe} - s_i^{break}(x^0, T_i^{break}) \tag{18}$$

.

If $s_j^{break}(t_h) < s_{j,i}^C$ and $s_j^{break}(T_j^{break}) \geq s_{j,i}^C$, we get $t^* = T_i^{break}$, $\forall v_i(t_h) \geq v_j(t_h)$ and $t^* = T^C$, $\forall v_i(t_h) < v_j(t_h)$.

If $s_j^{break}(t_h) \geq s_{j,i}^C$ and $s_j^{break}(T_j^{break}) \geq s_{j,i}^C$, we get $t^* = T_i^{break}$, $\forall v_i(t_h) \geq v_j(t_h)$ and $t^* = t_h$, $\forall v_i(t_h) < v_j(t_h)$.

Given t^*, the maximal invariant safe set $\mathcal{S}_{i,j}^{MA}$ can be described as

$$s_i(t_h) \leq s_{i,j}^C - d_{safe} - s_{j,i}^C + s_j^{break}(t^*) - s_i^{break}(x^0, t^*)$$

The overall maximal invariant safe set for the merging scenario denoted as $\mathcal{S}_{i,j}^M$ can then be obtained as $\mathcal{S}_{i,j}^M = \mathcal{S}_{i,j}^{MB} \cup \mathcal{S}_{i,j}^{MA}$. For implementation purpose, piece-wise linear under-approximation $\hat{\mathcal{S}}_{i,j}^M$, $\hat{\mathcal{S}}_{i,j}^C$ are used in (13) instead of the obtained true maximal invariant safe sets $\mathcal{S}_{i,j}^M$, $\mathcal{S}_{i,j}^C$. The resulting optimization problem is MIQP which can be efficiently solved by modern solver.

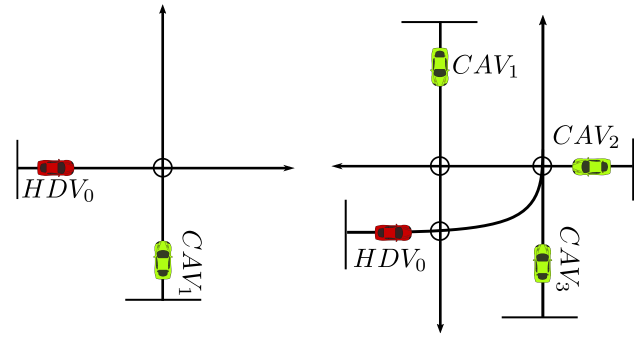
III. Simulation Results

The proposed DMPC coordination framework (13) together with the simulation are all implemented in Matlab. The MIQP problem, as a result of our DMPC formulation, is modeled using Yalmip [19] and solved by GUROBI.

A. Simulation Setup

To demonstrate the effectiveness of our proposed DMPC framework (13) in a heterogeneous intersection coordination task, two intersection scenarios are designed in simulation as can be seen in Fig.2. Common for both scenarios, we set $N = 10$, $\Delta t = 0.1$, $v_{max} = 25$, $u_{min} = 5$, $u_{max} = 3$, and $d_{safe} = 10$.

For the purpose of showing the invariant safety property of our proposed DMPC in comparison with a baseline



(a) Two vehicle crossing scenario.

(b) General intersection with four vehicles.

Fig. 2: Simulation scenarios for validation purpose.

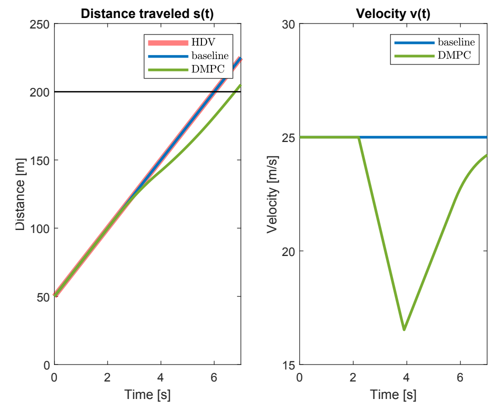


Fig. 3: Comparison of simulation results between DMPC and baseline in the two vehicle scenario

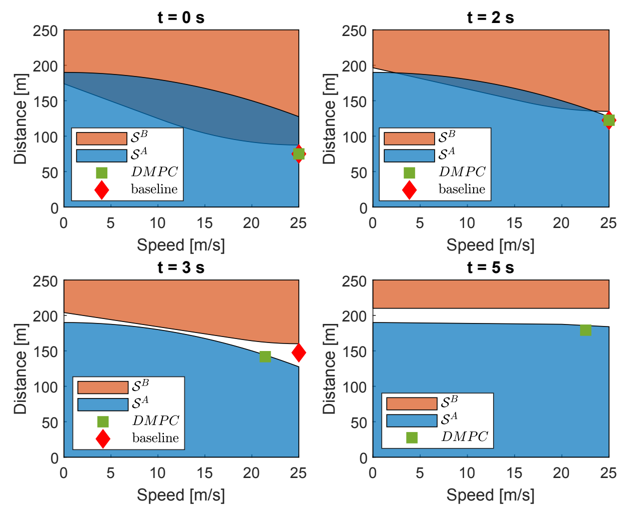


Fig. 4: Maximal invariant safe set $\mathcal{S}^B$ and $\mathcal{S}^F$ together with the terminal state x_N obtained by DMPC and baseline. Here the red diamond is x_N for baseline and the green square is x_N for DMPC

controller, a simple two-vehicle case with one HDV and one CAV is designed, see Fig.2a. The baseline controller has the same formulation as in (13) but without the terminal invariant safety constraint. We assign the HDV with index 0 and the CAV with index 1, the initial condition are set as $s_0(0) = s_1(0) = 50$, $v_0(0) = v_1(0) = 25$ and $s^C_{0,1} = s^C_{1,0} = 200$. For the HDV, we let it drive at $v_0(t) = 25$ at all times.

To demonstrate that our proposed DMPC can handle general intersection coordination tasks with invariant safety guarantees, we design a general intersection layout with both merging and crossing scenarios, where one HDV and three CAVs are placed together with their identity index as in Fig.2b. For HDV_0 and CAV_1 we set $s_{0,1} = 200$ and $s_{1,0} = 220$. Similarly, we set $s_{0,2} = 200 + 10\pi$, $s_{2,0} = 200$; $s_{0,3} = 200 + 10\pi$, $s_{3,0} = 220$, $s_{1,2} = 200$, $s_{2,1} = 220$, $s_{2,3} = 200$, and $s_{3,2} = 220$. Feasible initial conditions for all vehicles are chosen at random. For the HDV, we let it accelerate from initial velocity up until and remain at v_{max} to resemble an aggressive driver.

B. Simulation Results

The simulation results for the two-vehicle scenario is shown in Fig.3 and Fig.4. As can be seen in Fig.3, the baseline solution fails to anticipate the HDV and approaches the intersection at full speed, leading to collision at the intersection point $s^C_{0,1} = s^C_{1,0} = 200$. In comparison, the DMPC anticipates the worst-case behavior of the HDV and slows down before approaching the intersection, avoiding potential collision with the HDV. Looking at the safe invariant set and the corresponding terminal state at distinct time instances in Fig.4, we see that by including the safe invariant set as terminal constraint, the DMPC is always able to place $x_N \in \mathcal{S}^B \cup \mathcal{S}^A$. So the recursive feasibility property is validated in this result. More specifically, we see that the DMPC chooses to yield for HDV by choosing $x_N \in \mathcal{S}^A$, which explains why the DMPC slows down while approaching the intersection in Fig.3. In comparison, we see that the baseline solution has $x_N \notin \mathcal{S}^A \cup \mathcal{S}^B$ at $t = 3$, which then leads to an infeasible situation at $t = 5$ so the collision becomes unavoidable.

For the general intersection scenario of 4 vehicles, we display the distance traveled in each individual vehicle's perspective, as can be seen in Fig.5. If we follow the trajectory of HDV_0 in the first plot, when it passed the first intersection which conflicts with CAV_1 as can be seen in Fig.2b, CAV_1 chose to yield for HDV_0 by staying behind its blue shaded safety region. When HDV_0 continued through its second intersection which has a crossing conflict with CAV_2 and a merging conflict with CAV_3 as can be seen in Fig.2b, CAV_2 decided to resolve the conflict by passing before HDV_0 and stayed in front its orange shaded safety region while CAV_3 was far behind HDV_0 and maintained safety with regard to its green shaded safety region even after merge. We can interpret all the other plots in the same fashion and conclude that the DMPC (13) on each CAV enables them to safely traverse the intersection under the presence of an aggressive HDV. The minimum safety distance is maintained as an indication of the invariant safety property of (13). Like in the two-vehicle scenario, all CAVs maintain their terminal state within the maximal invariant safe set. These results are not shown due to limited space.

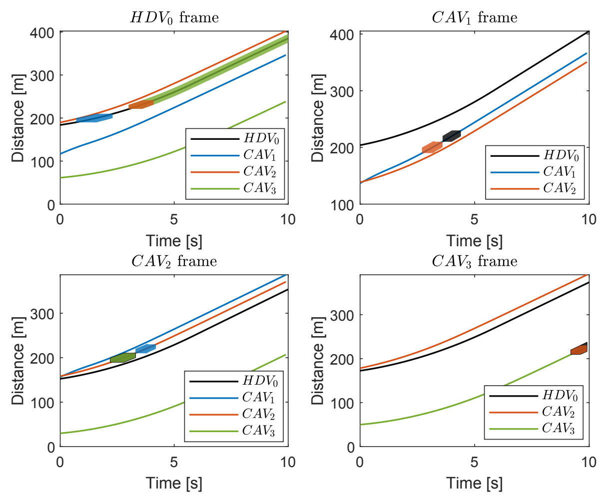


Fig. 5: Simulation result of the four vehicles scenario, the distance traveled $s_j(t)$ of conflicting vehicle j is aligned in the local frame of ego vehicle i in each subplot, The color shaded areas along ego vehicle trajectory indicate the safety region in time and space for the corresponding conflicting vehicle j which shares the same color.

IV. Conclusions

In this work, we have developed a DMPC framework to handle the traffic coordination task of CAVs in a general heterogeneous traffic intersection. To guarantee invariant safety, we propose to use the forward reachable set for safety constraint and to use the maximal invariant safe set as a terminal constraint in the MPC formulation. Recursive feasibility and practical invariant safety guarantees are shown both theoretically and validated in simulation.

In the simulation, we demonstrate both the coordination capability in a general intersection and also in comparison to a baseline controller without terminal constraints under which the simulation results suggest that our proposed framework can generate invariant safe vehicle coordination in a heterogeneous environment, while the baseline controller fails to generate collision-free results.

In future research, we would like to incorporate proper treatment for measurement uncertainty and communication delay into our framework and deploy the controller on physical vehicle platform for experimental validation. Besides, we will look into the following research aspects: Priority assignment task for a continuous traffic setting and potential deadlock situation caused by human driver; Improving the performance of the controller by reducing conservatism in the current formulation; Improving the computational efficiency by separating the passing order decision into a higher planning layer.

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