Exercise 1 (8 points)

Children are happy if (and only if) someone makes them happy. Parents make their children happy. Anna is the mother of Paola.

- (a) Represent the above sentences in first order logic.
- (b) Transform them in CNF, and tell whether there are any non-Horn clauses.
- (c) Is Paola a happy child? If she is, prove it using resolution. Otherwise, consider adding some knowledge in order to prove it (except for the trivial addition of happyChild(Paola)) and show the resolution proof.

Tell whether the following FOL sentences correctly represent the corresponding English ones. Explain the answer and provide the correct formula if the one given is incorrect.

```
d1) "Daniele and Francesco have the same fiscal number \exists n \ [CF(n) \land Has\_cf(Daniele, n) \land Has\_cf(Francesco, n)]
```

```
d2) "Everyone's fiscal code is made of 16 characters \forall x \ \forall n \ [(Person(x) \land CF(n)) \Rightarrow (Has\_cf(x,n) \land Char(n,16))]
```

Exercise 2 (4 points)

Describe forward and backward reasoning in horn clauses; provide a simple example of each of the two reasoning algorithms.

Exercise 3 (4 points)

A matrix that stores natural numbers can be represented in PROLOG as a list of lists of numbers.

- (a) Write a PROLOG program GETD that, given in input a matrix M and an index I, returns the value V of the element in position [I,I]. Rows and columns are counted starting from 1. If the index is out of bounds the program should fail. For example, GETD([[11,12,13],[21,22,23],[31,32,33]],2,V) returns V=22.
- (b) Write a PROLOG program GETDIAG that, given in input a matrix M, returns a list L containing all the elements in the main diagonal. For example, GETDIAG([[11,12,13],[21,22,23],[31,32,33]],L) returns L=[11,22,33].
- (c) [EXTRA QUESTION tbd only after having answered everything else] Write a PROLOG program that, given in input a matrix M, checks whether it is diagonal (all the elements not belonging to the main diagonal are 0).