SAPIENZA
UNIVERSITÀ DI ROMA

Artificial Intelligence

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Lab 6:
Propositional Logic

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*The slides have been prepared using the textbook material available on the web, and the slides of the previous editions of the course by Prof. Luigia Carlucci Aiello, Prof. Daniele Nardi and Dott. Fabio Previtali.

Exercises

Let α, β, γ be three propositional predicates, tell whether or not:

$$\phi(\alpha, \beta, \gamma) = [(\alpha \land \beta) \Rightarrow \gamma \models (\alpha \Rightarrow \gamma) \lor (\beta \Rightarrow \gamma)]$$

Let

$\phi_1(\alpha, \beta, \gamma) = (\alpha \land \beta) \Rightarrow \gamma$

$\phi_2(\alpha, \beta, \gamma) = (\alpha \Rightarrow \gamma) \lor (\beta \Rightarrow \gamma)$

Exercises

The entailment is true. The truth table is as follows:

$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	γ	$\phi_1(\alpha, \beta, \gamma)$	$\phi_2(\alpha, \beta, \gamma)$	$\phi(\alpha, \beta, \gamma)$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	1	1	1	1

Exercises

Tell whether the following propositional formula is valid:

$$\phi(A, B) = (A \land B) \lor (\neg A \land \neg B)$$

Exercises

Tell whether the following propositional formula is valid:

$$\phi(A, B) = (A \wedge B) \vee (\neg A \wedge \neg B)$$

FALSE. The truth table is as follows:

A	**B**	$\boldsymbol{\phi}(\mathbf{A}, \mathbf{B})$
0	0	1
0	1	0
1	0	0
1	1	1

Exercises

Tell which one among the following formulae is a good representation of the sentence.

If John studies and his father works, then his grandfather is happy.

(1) $(Study \land Work) \Rightarrow Happy$
(2) $Study \land Work \land Happy$
(3) $\neg Study \lor \neg Work \lor Happy$
(4) $(Study \lor Work) \Rightarrow Happy$

Exercises

(1) $(Study \wedge Work) \Rightarrow Happy$
correct

(2) $Study \wedge Work \wedge Happy$
incorrect

(3) $\neg Study \vee \neg Work \vee Happy$
correct, logically equivalent to 1. Why?

(4) $(Study \vee Work) \Rightarrow Happy$
incorrect

Exercises

Consider a knowledge base consisting of the conjunction of the following propositions:

$$\neg A \Rightarrow B$$
$$B \Rightarrow A$$
$$A \Rightarrow (C \wedge D)$$

1 Tell whether the knowledge base is consistent. In the positive case provide a model

2 Transform the above propositions into a new knowledge base written in conjunctive normal form

3 Which of the clauses in your new knowledge base - if any - are not Horn clauses? Justify your answer

Exercises

Recall: A knowledge base is consistent if it admits at least one model.

The knowledge base is **consistent** because there are two models:

$$\{A, B, C, D\} \text{ and } \{A, C, D\}$$

Exercises

The new knowledge base written in **CNF** is as follows:

$$A \vee B$$
$$\neg B \vee A$$
$$\neg A \vee C$$
$$\neg A \vee D$$

$A \vee B$ is **NOT** a Horn clause, because it has more than one positive literal.

Exercises

Derive $A \wedge C \wedge D$ using Resolution

Exercises

Derive $A \wedge C \wedge D$ using Resolution

Clausal form including the negated thesis:

$$\{A \vee B\}_1, \{\neg B \vee A\}_2, \{\neg A \vee C\}_3, \{\neg A \vee D\}_4, \{\neg A \vee \neg C \vee \neg D\}_5$$

Proof by **resolution**

From (1) and (2) $\Rightarrow \{A\}_6$
From (3) and (6) $\Rightarrow \{C\}_7$
From (4) and (6) $\Rightarrow \{D\}_8$
From (5) and (6) $\Rightarrow \{\neg C \vee \neg D\}_9$
From (7) and (9) $\Rightarrow \{\neg D\}_{10}$
From (8) and (10) $\Rightarrow \{\}$

Exercises

If I leave and go on vacation, then I am happy
If I leave then I go on vacation
I leave

Question: Can I derive, *I go on vacation and I am happy?*

Exercises

$$\Gamma = \{\{(L \wedge V) \Rightarrow H\}, \{L \Rightarrow V\}, \{L\}\} \vdash_R (V \wedge H)$$

Negate the thesis:

Transform into clausal form:

$$\{\neg L, \neg V, H\}_1, \{\neg L, V\}_2, \{L\}_3, \{\neg V, \neg H\}_4$$

From (1) and (2) $\Rightarrow \{\neg L, H\}_5$
From (3) and (5) $\Rightarrow \{H\}_6$
From (4) and (6) $\Rightarrow \{\neg V\}_7$
From (2) and (7) $\Rightarrow \{\neg L\}_8$
From (3) and (8) $\Rightarrow \{\}$

Exercises

Consider the following propositional formulae:

$$P \Rightarrow (Q \Leftrightarrow R)$$
$$Q \Rightarrow (P \vee R)$$
$$R \Rightarrow (Q \wedge \neg P)$$

1 Convert them into Conjunctive Normal Form

2 Tell whether or not the resulting set of clauses is Horn

3 Tell whether or not the resulting set of clauses is satisfiable, in the positive case show a model

Exercises

I'm happy iff I won the lottery or my girlfriend is with me
If it is raining my girlfriend is not with me
It is raining and I am happy

Question: Can I derive, *I am happy iff I won the lottery?*