Heuristic Search[1]

Lecture 4

[1] The slides have been prepared using the textbook material available on the web, and the slides of the previous editions of the course by Prof. Luigia Carlucci Aiello

Summary

Russell & Norvig Chapter 3, Sec. 5–6

- Best-first search
- A*
- Heuristics
- IDA*
- SMA*

Best-first search

Idea: use an *evaluation function* for each node
– estimate of "desirability"

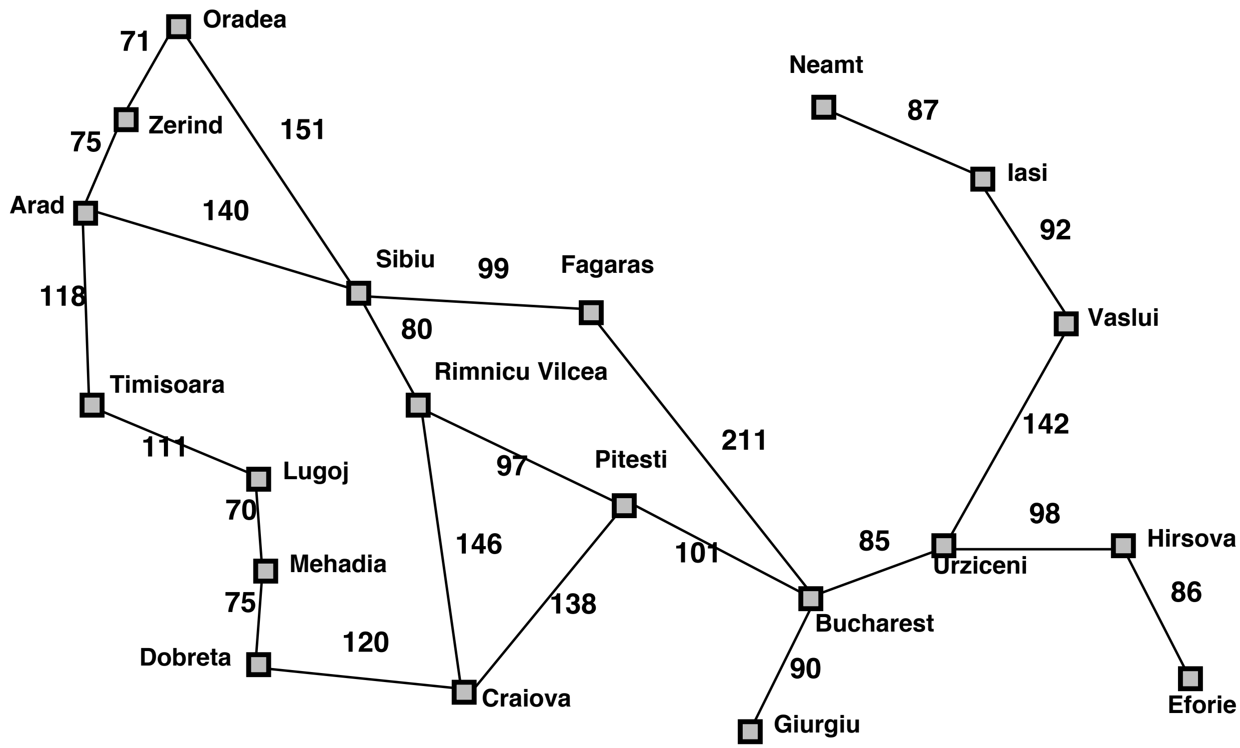
$\Rightarrow$ Expand most desirable unexpanded node

Implementation:
$fringe$ is a queue sorted in decreasing order of desirability

Special cases:
"best-first" search
A* search

best? "Best-first" vs greedy

Romania with step costs in km



Straight–line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy "Best-first" search

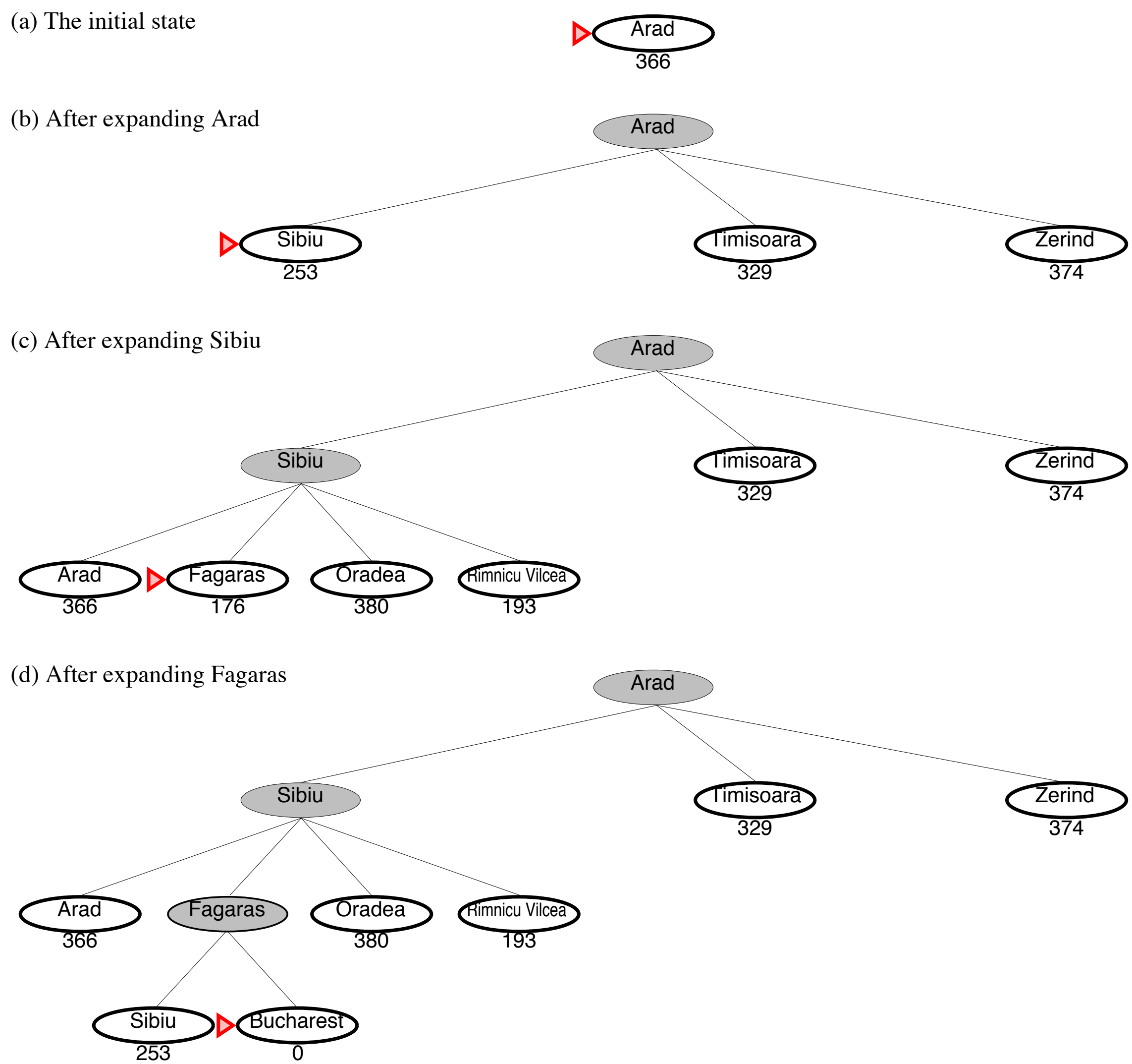
Evaluation function $h(n)$ (heuristic)
$= \text{estimate of cost from } n \text{ to the closest goal}$

E.g., $h_{\mathrm{SLD}}(n) = $ straight-line distance from n to Bucharest

"Best-first" search expands the node that *appears* to be closest to goal

A remark about heuristics: Usually a good h greatly improves the search (by expanding less nodes), but knowing ideal h amounts to knowing the solution ...

Greedy search example



Properties of "best-first" search

Complete No–can get stuck in loops, e.g.,
Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow$
Complete in finite space with repeated-state checking

Time $O(b^m)$, but a good heuristic can give dramatic improvement

Space $O(b^m)$—keeps all nodes in memory

Optimal No

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

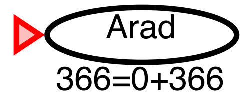
$g(n)$ = cost so far to reach n
$h(n)$ = estimated cost to goal from n
$f(n)$ = estimated total cost of path through n to goal

A* search uses an *admissible* heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the *true* cost from n.
(Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)

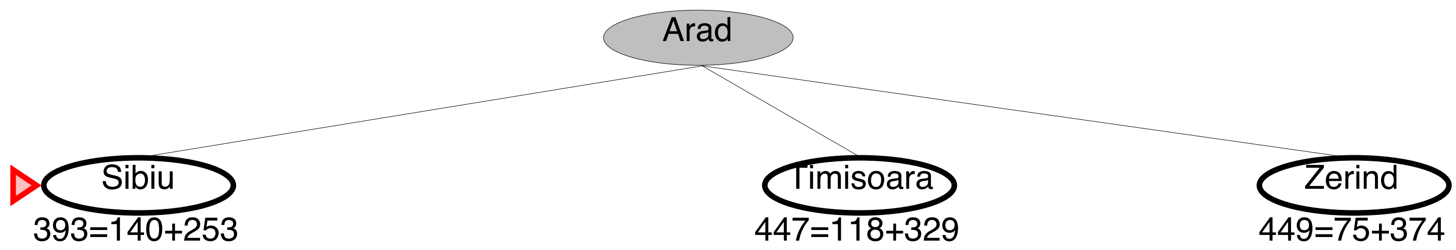
E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

Key property: A* search is optimal

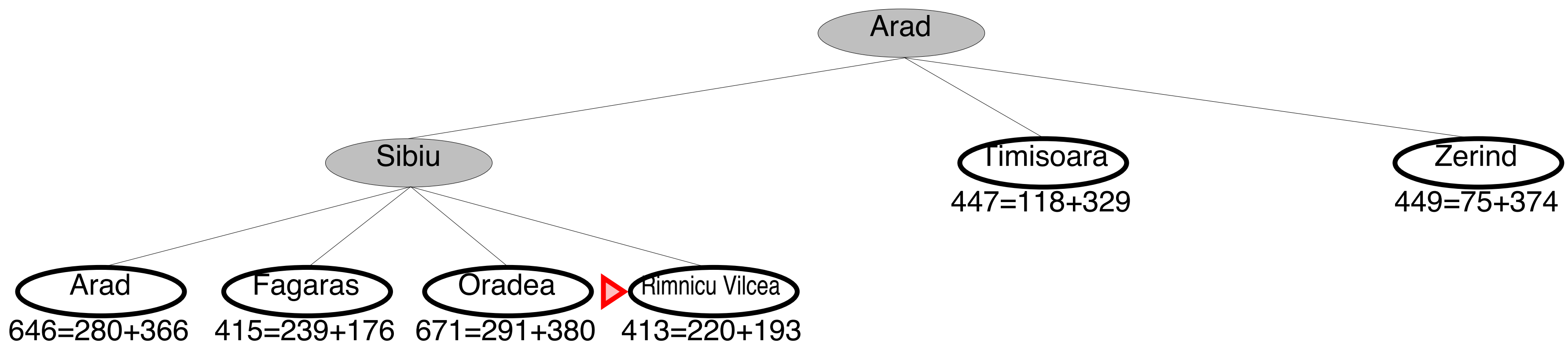
A* search example



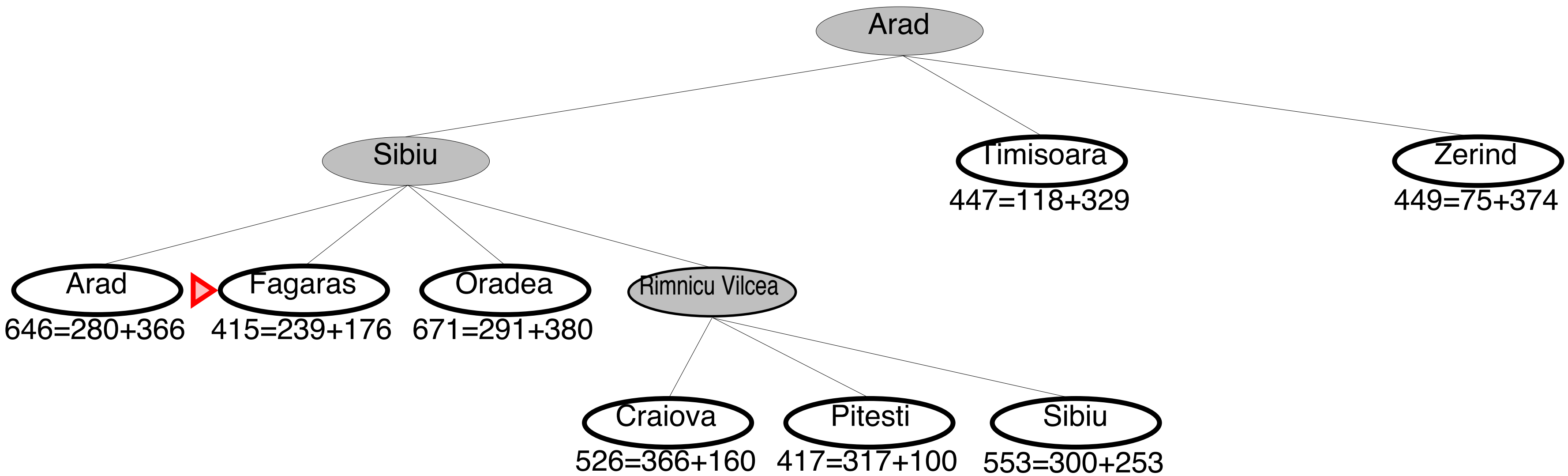
A* search example



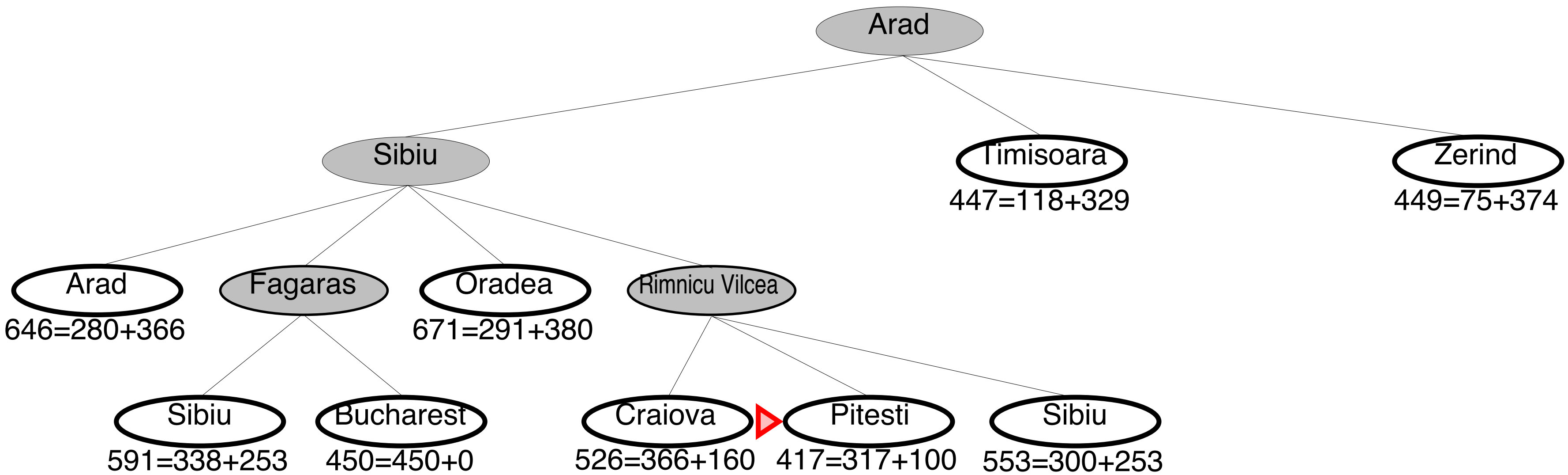
A* search example



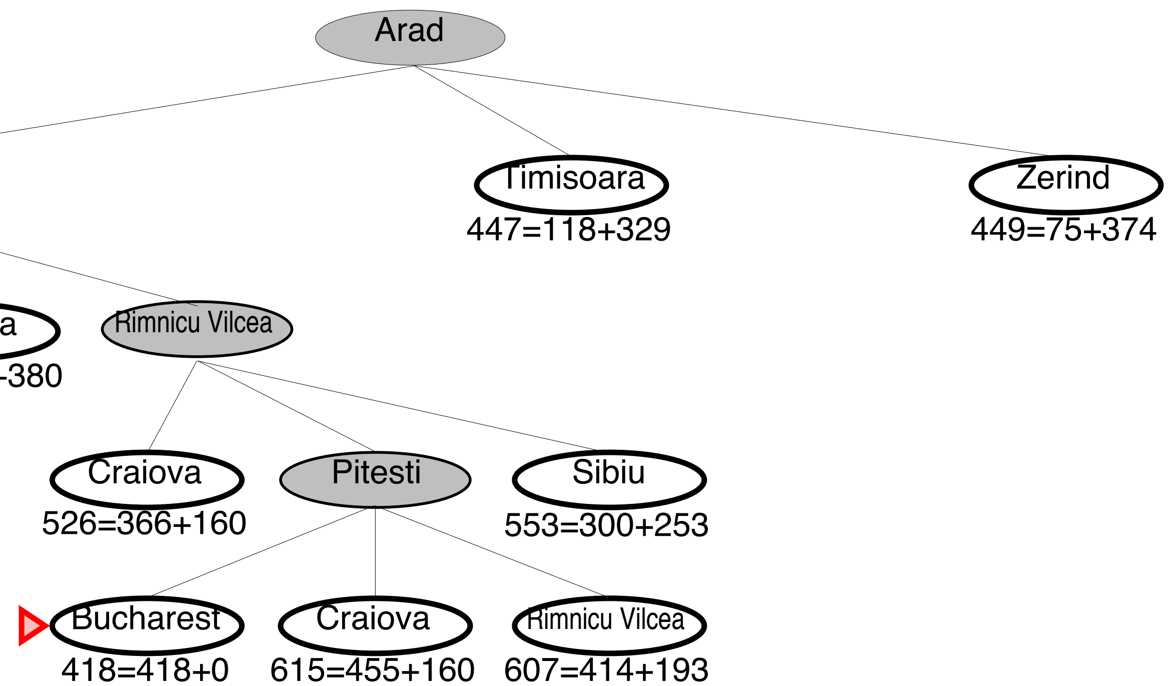
A* search example



A* search example

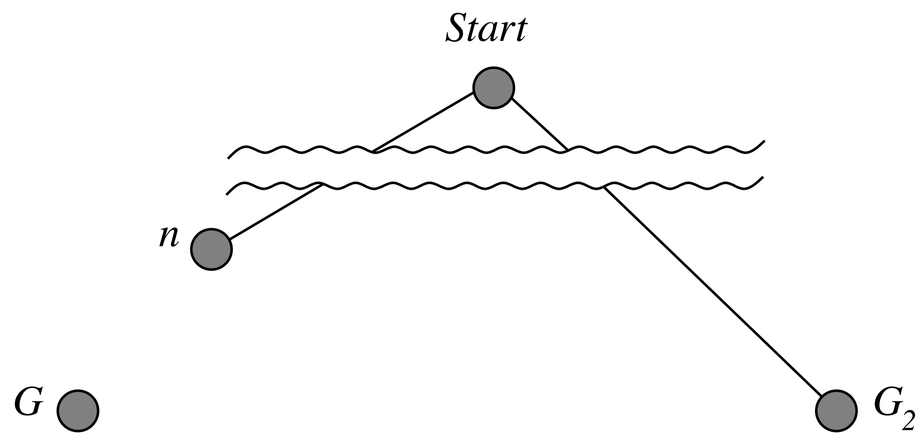


A* search example



Optimality of A* (standard proof)

Let G_2 be some suboptimal goal in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G.



$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
&> g(G) && \text{since } G_2 \text{ is suboptimal} \\
&\geq f(n) && \text{since } h \text{ is admissible}
\end{aligned}
$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Consistent Heuristics

For some admissible heuristics, f can *decrease* in a path. This is a problem when the search space is a **graph**.

A heuristics is **consistent** if

$$h(n) \leq c(n, a, n') + h(n')$$

This guarantees that f is not decreasing (monotonic)

$$f(n') = g(n') + h(n') = g(n) + c(n, a, n') + h(n') \geq g(n) + h(n)$$

Pathmax

Usually, admissible heuristics are consistent.

In alternative, pathmax can be build for a heuristics:
Instead of $f(n') = g(n') + h(n')$, take

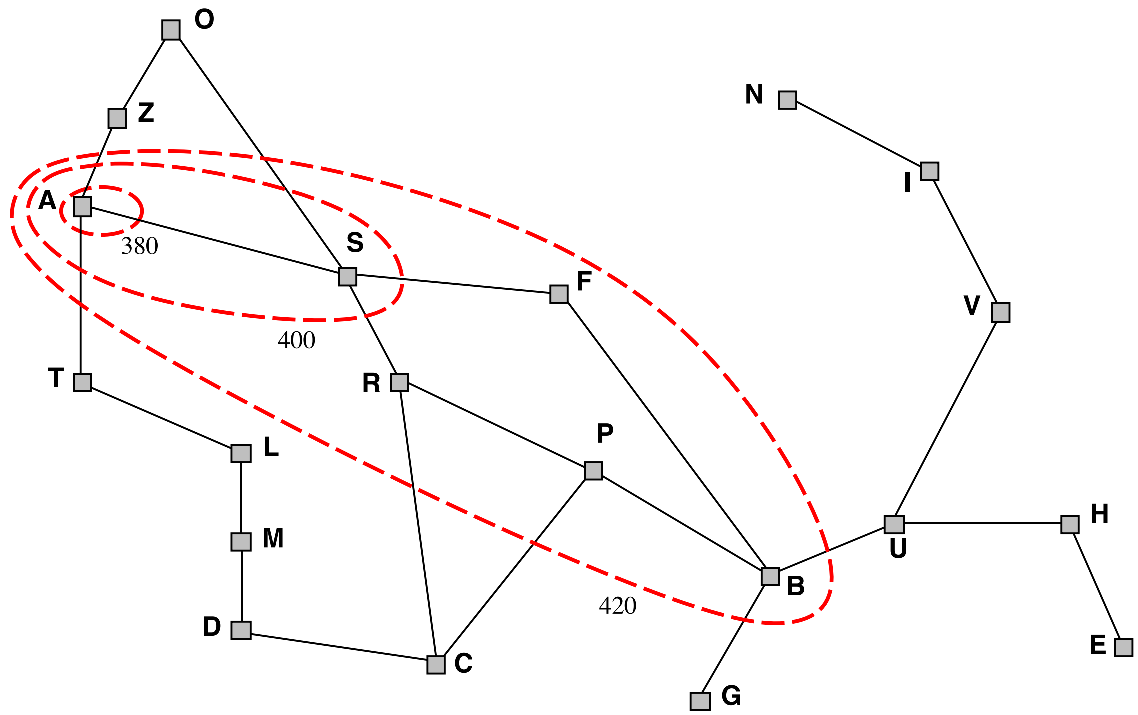
$$f(n') = max(g(n') + h(n'), f(n))$$

Using **pathmax**, f is always <u>non decreasing</u>

Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value*

Gradually adds "f-contours" of nodes (cf. uniform search)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A*

Complete Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time Exponential in [relative error in $h \times$ length of soln.]

Space Keeps all nodes in memory

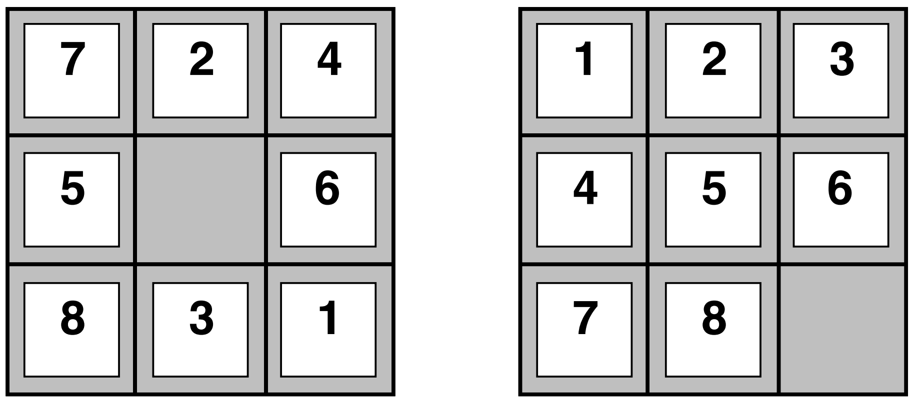
Optimal Yes—cannot expand f_{i+1} until f_i is finished

A* expands all nodes with $f(n) < C^*$
A* expands some nodes with $f(n) = C^*$
A* expands no nodes with $f(n) > C^*$

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



Start State Goal State

$h_1(S) = 7$
$h_2(S) = 4+0+3+3+1+0+2+1 = 14$

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 *dominates* h_1 and is better for search

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
A$^*(h_1)$ = 539 nodes
A$^*(h_2)$ = 113 nodes
$d = 24$ IDS $\approx$ 54,000,000,000 nodes
A$^*(h_1)$ = 39,135 nodes
A$^*(h_2)$ = 1,641 nodes

The nodes expanded by A* with a dominant h is always less.

Effective branching factor

$N =$ total number of nodes expanded by A*
$d =$ the depth of the solution
b^* is the branching factor of a uniform tree of depth d with $N+1$ nodes.

$$N + 1 = 1 + b^* + (b^*)^2 + \cdots + (b^*)^d.$$

Example: if A* finds a solution at depth 5 with 52 nodes, then the effective branching factor is 1,92.

Effective branching factor: 8-puzzle

$d = 14$ IDS $=$ 2,83 ebf
A$^*(h_1) =$ 1,44 ebf
A$^*(h_2) =$ 1,23 ebf
$d = 24$ IDS $=$ too many nodes
A$^*(h_1) =$ 1,48 ebf
A$^*(h_2) =$ 1,26 ebf

Relaxed problems

Admissible heuristics can be derived from the *exact* solution cost of a *relaxed* version of the problem

If the rules of the 8-puzzle are relaxed so that a tile can move *anywhere*, then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to *any adjacent square*, then $h_2(n)$ gives the shortest solution

Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Combination of heuristics

What if we have several heuristics not dominating each other?

let $h_1 \dots h_m$ a collection of such heuristics, define

$$h(n) = max(h_1(n), \dots, h_m(n)).$$

h is admissible and dominates $h_1 \dots h_m$

Looking for heuristics

- Learning by looking at the “meta-level computation tree.
- Finding relaxed problems (via a formal problem specification)
- Pattern DataBases (use solutions to subproblems)
- Learning the heuristics by successfully solving several instances of the problem

Limitations of: A^*

◇ the memory to store the $fringe$ may be exponential.

◇ this does not come up with good h

Countermeasures:

- Use good, but not admissible heuristics
- Live with sub-optimal solutions

◇ space problem remains (as in breadth first).

Limiting the memory requirement: IDA^*

$\diamondsuit$ Iterative Deepening search where f $(g + h)$ is fixed

$\diamondsuit$ IDA^* is complete and optimal as A^* using linear memory

$\diamondsuit$ the next limit can be chosen by increasing f of a fixed amount or looking at the values of the successors

Weakness: same sources of inefficiencies as uniform-cost search

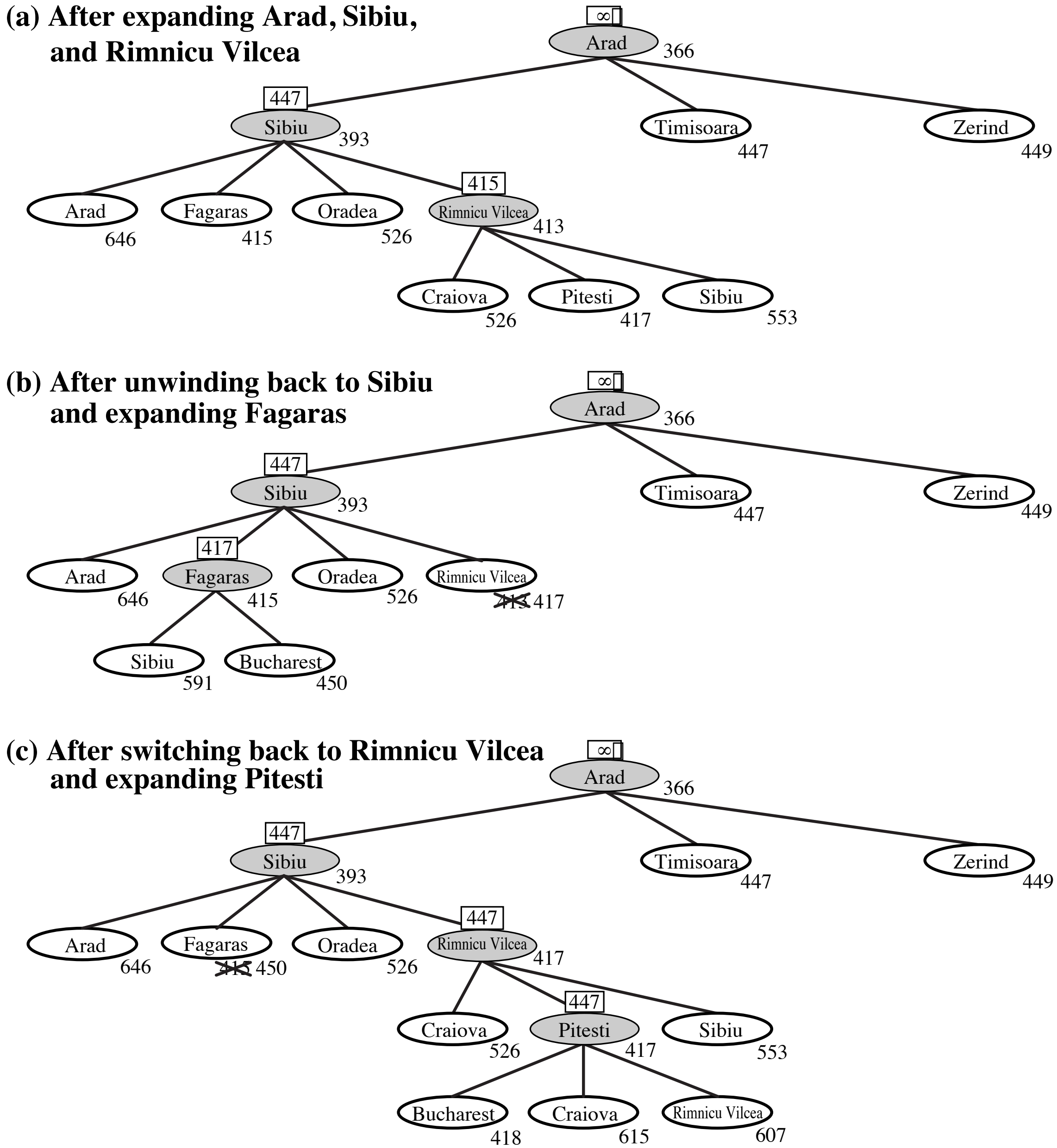
Recursive Best First Search: RBFS

♢ Stores the f of the unexpanded nodes and goes deep till f of the current node becomes worse of one of the previous alternatives

♢ Backs up recording the best f of the nodes discarded by backtracking

♢ more efficient than IDA^* but sometimes it wastes time in re-generating discarded nodes.

♢ space complexity is linear $O(bd)$


(a) After expanding Arad, Sibiu, and Rimnicu Vilcea
∞
Arad 366
447
Sibiu 393
Timisoara 447
Zerind 449
Arad 646
Fagaras 415
Oradea 526
415
Rimnicu Vilcea 413
Craiova 526
Pitesti 417
Sibiu 553
(b) After unwinding back to Sibiu and expanding Fagaras
∞
Arad 366
447
Sibiu 393
Timisoara 447
Zerind 449
Arad 646
417
Fagaras 415
Oradea 526
Rimnicu Vilcea ~~413~~ 417
Sibiu 591
Bucharest 450
(c) After switching back to Rimnicu Vilcea and expanding Pitesti
∞
Arad 366
447
Sibiu 393
Timisoara 447
Zerind 449
Arad 646
Fagaras ~~415~~ 450
Oradea 526
447
Rimnicu Vilcea 417
Craiova 526
447
Pitesti 417
Sibiu 553
Bucharest 418
Craiova 615
Rimnicu Vilcea 607

SMA*

The problem with IDA* and RBFS is that they use *too little* memory.

- in IDA* at each iteration only the current cost limit for f is kept,
- in RBFS the cost of the nodes in the depth first search are recorded

SMA* (Simplified Memory-Bounded A*) can use all the available memory for the search.

Idea: better remember a node that re-generate it when needed

Basic intuition of SMA*

Like A* till there is memory available.

When a new node must be generated the node with the highest f is discarded (**forgotten node**), while keeping its cost f in the parent node.

A discarded node will be re-generated *only* when *all other paths* are worse than the forgotten node.

Properties of SMA*

- It uses all the available memory.
- It avoids to repeat states until the available memory is exhausted.
- It is complete if the available memory allows the *shallowest* solution path to be recorded.
- It is optimal if the available memory allows the *shallowest* optimal solution path to be recorded. Otherwise it returns the best solution attainable with the available memory.
- The search is optimally efficient, when the available memory can store the whole search

Usage of SMA*

SMA* is typically the best heuristic search method

In difficult problems, SMA* **flounders** when it keeps re-generating states that have already been discarded.

Memory limitations lead to unacceptable computing time (as for **trashing** as in pagination).

Summary

◇ Heuristics make the difference

◇ Finding good heuristics is key to success, but ideal heuristics means no search!

◇ A* is optimal but uses too much memory

◇ SMA* Best HS (compromise between memory and time)