Non-monotonic Reasoning

Common Sense

Outline

- ♢ Non-monotonic Logics (RN 12.6 BL 11.1)
- ♢ Closed World Assumption CWA (BL 11.2)
- ♢ Negation as Failure
- ♢ Stable Model Semantics (GL88)
- ♢ Default Logic (BL 11.4)

Monotonicity

A formalism is **monotonic** when the addition of new knowledge can only increase the number of conclusions

If $\Gamma \subseteq \Delta$ and $\Gamma \vdash A$ then $\Delta \vdash A$, or also $Cn(\Gamma) \subseteq Cn(\Delta)$

where $Cn(KB)$ denotes the set of logical consequences of the KB.

Non-monotonic Reasoning

Conversely, in a non-monotonic formalism, the addition of new information can invalidate some of the previously derivable conclusions

Non-monotonicity is a property of the formalism, rather than of the reasoning

Incomplete Representation (lack of information)

Common Sense Reasoning

Negative Information

$flight(c_1, c_2)$ indicates a connection between two cities

$\forall xyz.(flight(x, y) \wedge flight(y, z) \Rightarrow flight(x, z))$

In classical logic we cannot formally derive that two cities are not connected

Assumption: when we cannot formally derive the existence of a connection, that connection doesn't exist

For example, if we cannot derive $flight(Roma, Orte)$,
we assume $\neg flight(Roma, Orte)$

Non-monotonicity: if we add $flight(Roma, Orte)$ we cannot derive any longer that $\neg flight(Roma, Orte)$ holds

Universal and General Assertions

Violins have 4 cords

- **Universal**: Assertive properties that hold for all the instances
- **General**: Properties that generally hold

Violins have 4 cords, but when they have some trouble

Non-monotonicity: If a violin “looses a cord,” it remains a violin with three cords

Exceptions

Birds fly:

$\forall x.bird(x) \Rightarrow flies(x)$

In this form the rule doesn't allow for exceptions, but kiwis do not fly

$\forall x.bird(x) \wedge \neg kiwi(x) \Rightarrow flies(x)$

In addition, to kiwis there are several other exceptions:

$\forall x.bird(x) \wedge \neg kiwi(x) \wedge ... \Rightarrow flies(x)$

In general, not all exceptions are known

Weak Universal Assertions

- **General**:
  - **Normality**: birds fly
  - **Prototipicality**: violins have four strings
  - **Statistics**: italians eat pasta
- **Persistence**:
  - **Inertia**: marital status persists
  - **Time**: observed color of object

Lack of information

- **Lack of knowledge (to the contrary)**:
  - **Familiarity**: elder brother
  - **Group Knowledge**: children learn quickly
- **Conventions**:
  - **Conversation**: AI today in B2
  - **Representation**: Speed limit in country roads

Closed World Reasoning

Basic Idea:

There are many more false things than true things. If something is true and relevant, it has been put into the KB. So, if something is not present in the KB, it can reasonably be assumed to be false.

- Closed World Assumption
- Negation as Failure

Closed World Assumption

Closed World Assumption: CWA (Reiter '78).

$$CLOSURE(KB) = KB \cup \{\neg L | L \text{ is a positive literal and } KB \not\models L\}$$

$$CLOSURE(KB) = KB^{+}$$

KB^{+} contains all the sentences of the form $\neg flight(Roma, Orte)$

Answers to queries are obtained from KB^{+} instead of KB

Complete Knowledge

CWA produces a **complete** KB
(i.e. for all α $KB^+ \models \alpha$ or $KB^+ \models \neg\alpha$)

This holds both for atoms and formulas.

In particular, if $KB^+ \models A \vee B$, then $KB^+ \models A$ or $KB^+ \models B$

Hence, the answer to a query α can be obtained by verifying whether the literals in α are implied by KB^+.

Consistency of the CWA

CWA is consistent on definite KBs

If KB contains **incomplete** knowledge:

$KB \models A \vee B$, but $KB \not\models A$ and $KB \not\models B$

KB^+ contains both $\neg A$ and $\neg B$, hence we have an inconsistency

Generalized CWA

GCWA

$KB^* = KB \cup \{\neg L | L$ is a positive literal
and there is no positive clause C
such that $KB \models L \vee C$ and $KB \not\models C\}$

Further restrictions are possible (CCWA, ECWA).

Negation as failure

Negation as (finite) failure (P is a KB and A an atom)

If from P we do not prove A then from P we deduce $\neg A$

- Predicate completion
- Stable model semantics

Semantics of negation in logic programs

Semantics of stable models

The operational model of logic programming is extended to treat negation as finite failure

Answer Set Programming

An extension of Logic Programming to disjunctive programs and computes the answer according the stable model semantics

Minimal Models

The basic idea is to find a **canonical/preferred** model where to verify the truth of formulas.

Horn clauses have the property that the **minimal** model is unique (No disjunction)

$p(1)$
$q(2)$
$p(x) \Rightarrow q(x)$

Minimal model: $M = \{p(1), q(1), q(2)\}$
Another model: $M' = \{p(1), q(1), q(2), p(2)\}$

Negation in the clause body

$A \wedge \neg B \ \Rightarrow \ C \equiv \neg A \vee B \vee C$

Classical negation in the clause body introduces a disjunction, and we loose the uniqueness of the minimal model

Negation as failure is weaker than negation

$not\, Q \ \Rightarrow \ P$

is not considered equivalent to : $P \vee Q$

the model $M = \{P\}$ is preferred

Grounding

Given a logic program Π, that is, a set of clauses:

$A \leftarrow L_1, \ldots, L_m$ with $m \geq 0$

each clause is replaced with its **ground instances**.

NB: $L_1, \ldots, L_m \; \Rightarrow \; A$

Negation Elimination

Given a set M of atoms of Π, the **reduct** Π_M is obtained by eliminating:

- all the clauses that in the body have a negated atom of M;
- all the negative literals not in M.

Stable Models

The reduct is a program that has only Horn clauses and a unique minimal model. If this coincides with M, it is called **stable set**.

$Theorem$: Every stable set of a program Π is a minimal Herbrand model of Π

The **semantics of stable models** states that:
if a program has **only one stable model** this is the canonical/preferred model of the program.

Stable Models: An example

$P \leftarrow not\ Q$

has a unique stable model $\{P\}$.

in fact, if $M = \{P\}$ the reduct becomes P;

While if $M = \{Q\}$ the reduct becomes the empty program, whose minimal model is $\{\} \neq \{Q\}$.

Stable Models: An example

$p(1,2)$
$q(x) \leftarrow p(x,y), not\ q(y)$

has a unique stable model $\{p(1,2), q(1)\}$.

Stable Models: An example

$P \leftarrow not\ Q$
$Q \leftarrow not\ P$

has two stable models $\{P\}$ and $\{Q\}$.

while

$P \leftarrow not\ P$

has no stable models.

Default Logic

Reiter 1980, "variation" on the inference rules:

$$\frac{prec : just}{concl}$$

where PRECondition, JUSTification, and CONCLusion are propositional or first order sentences

The intuitive reading of a **default rule** is the following:

> If PREC is provable and
> the hypotheses in JUST can be consistently assumed,
> then we can conclude CONCL.

Also in this case a **fix point construction** is required.

Default Theory

A default theory is a pair $\langle W, D \rangle$ such that:

- W is an axiomatizable theory;
- D is a set of default rules: $\frac{\alpha : \beta}{\gamma}$, where α, β, γ are formulas

Extension (first definition)

ε is an extension of a default theory $\langle \mathcal{W}, \mathcal{D} \rangle$

iff for every sentence π

$$\pi \in \varepsilon \text{ iff } \mathcal{W} \cup \{\gamma \mid \langle \alpha : \beta/\gamma \rangle \in \mathcal{D}, \alpha \in \varepsilon, \neg\beta \notin \varepsilon\} \models \pi$$

A default rule is applicable if the default assumption (justification) remains valid after the extension has been built.

Example

Let $\langle W, D \rangle$ be a default theory where:

$W = \{q\}$
$D = \{\frac{q : p}{r}\}$.

Let $S = \{q, r\}$

S is an extension

Example: TWEETY

Let's rewrite the default:

$$\frac{Bird(x) : Flies(x)}{Flies(x)}$$

Default where justifications and conclusions coincide are called **normal**.

It can be proved that a theory where all the defaults are normal always has an extension

Example: NIXON 1

Quackers (usually) are pacifist, republicans (usually) are not.

$$\frac{Quacker(x) : Pacifist(x)}{Pacifist(x)}$$

$$\frac{Republican(x) : \neg Pacifist(x)}{\neg Pacifist(x)}$$

Example: NIXON 2

Nixon is a republican quacker

$$W = \{Quacker(nixon), Republican(nixon)\}$$

Can we conclude that Nixon is pacifist?

$$S = Cn(\{Quacker(nixon), Republican(nixon), Pacifist(nixon)\}).$$

S is an extension.

Example: NIXON 3

$$S' = Cn(\{Quacker(nixon), Republican(nixon),$$
$$\neg Pacifist(nixon)\}).$$

S' is an extension.

Hence the default theory has two distinct extensions: one where Nixon – being a normal quacker – is pacifist, and one where Nixon –being a normal republican – is not pacifist.

Cautious and brave conclusions

Given a default theory a conclusion is

- *Cautious* if it belongs to **all** extensions
- *Brave* if it belongs to **some** extension.

In the Nixon example, the fact that Nixon is pacifist is a brave conclusion.

As an alternative

$$\frac{Quacker(x) : Pacifist(x) \land \neg Political(x)}{Pacifist(x)}$$

$$\forall x Republican(x) \;\Rightarrow\; Political(x)$$

NOTE: If we write the default that usually republicans are political, we again get two extensions

Pinguins do not fly

$$\frac{Bird(x) : Flies(x)}{Flies(x)}$$
$$\frac{Penguin(x) : \neg Flies(x)}{\neg Flies(x)}$$

If we add $Penguin(chilly)$ we get two extensions

To get only one extension we can use a single default rule:

$$\frac{Bird(x) : Flies(x) \wedge \neg Penguin(x)}{Flies(x)}$$

In general, knowing that penguins are a subclass of birds, we'd like that the default on penguins is "stronger" than that on birds

Example

Consider a default as: $\dfrac{q : q}{q}$.

In this pathological case $\{q\}$ is also an extension according to the simplified definition, while $\{\}$ is more reasonable.

Summary of NMR

Semantic characterization:
"minimization of the extension" of predicates

- CWA (GCWA, ...)
- Circumscription

Deductive Characterization:
a form of "defeasible" implication

- Default Logic
- Autoepistemic Logic

MKNF combines default reasoning with negation as failure.