Artificial Intelligence a.y. 2018/19

Knowledge Representation: recap exercises

Modeling in logic

Giovanni loves nature. Nature lovers preserve the environment. People who preserve the environment do not leave garbage in the woods. People who preserve the environment do not light fires in the woods.

1) Represent the above sentences in FOL.
2) Show that Giovanni does not leave garbage and does not light fires in the woods using modus ponens.
3) Transform the above sentences CNF and show that Giovanni does not leave garbage and does not light fires in the woods using resolution.

Solution:

Vocabulary: $G, LovesN(.), CaresN(.), LitterW(.), LightFW(.)$

1 - $Loves(G, N)$
2 - $\forall x\ LovesN(x)\ \Rightarrow\ CaresN(x)$
3 - $\forall x\ CaresN(x)\ \Rightarrow\ \neg LitterW(x)$
4 - $\forall x\ CaresN(x)\ \Rightarrow\ \neg LightFW(x)$

Modus Ponens

5 - $CaresN(G)$ MP: from 1 and 2 $\sigma = \langle x = G \rangle$
6 - $\neg LitterW(G)$ MP: from 5 and 3 $\sigma = \langle x = G \rangle$
7 - $\neg LightFW(G)$ MP: from 5 and 4 $\sigma = \langle x = G \rangle$

CNF

1 - $LovesN(G)$

2 - $\neg LovesN(x) \vee CaresN(x)$

3 - $\neg CaresN(x) \vee \neg LitterW(x)$

4 - $\neg CaresN(x) \vee \neg LightFW(x)$

5 - $LitterW(G) \vee LightFW(G)$

Resolution

6 - $\neg CaresN(G) \vee LightFW(G)$ 5 and 4 $\sigma = \langle x, G \rangle$
7 - $\neg CaresN(G)$ 6 and 3 $\sigma = \langle x, G \rangle$
8 - $\neg LovesN(G)$ 7 and 2 $\sigma = \langle x, G \rangle$
9 - $\{\}$ 8 and 1

NB: Solution with single predicate to represent together **leave garbage in the woods** and also **light fires in the woods also ok**.

Students at Sapienza

Tell whether the following sentences are correctly represented by the corresponding FO formula and provide a suitable explanation.

1. Every university student of “La Sapienza” has a **matricola** number
   $\forall x \; \forall y \; [Student(x) \wedge Univ(x, LS) \; \Rightarrow \; Matricola(x, y)]$
2. Every student doing the master thesis has a thesis advisor
   $\forall x \, \forall t \, \exists r \; [DoingThesis(x) \wedge Thesis(x, t) \Rightarrow Advisor(x, r)]$
3. No student ever won “Turing Award”
   $\forall x \; [Student(x) \; \Rightarrow \; \neg Winner(x, TA)]$
4. A student from DIAG won the Sapienza Best PhD
   $\exists x \; [Student(x) \wedge At(x, DIAG) \; \Rightarrow \; Winner(x, SBP)]$

Solution:

1. Wrong: $\forall x\ \exists y\ [Studente(x) \wedge Univ(x, LS) \Rightarrow Matricola(x, y)]$
2. Correct
3. Correct
4. Wrong: $\exists x\ [Student(x) \wedge At(x, DIAG) \wedge Winner(x, SBP)]$

Giraffes

Let $G(x), F(x), Z(x)$, and $M(x)$ be the predicates "x is a giraff", "x is 15 feet or higher", "x is in this zoo", and "x belongs to me", respectively. Suppose that the universe of discourse is the domain of animals. Express each of the following statements in First Order Logic using $G(x), F(x), Z(x)$, and $M(x)$.

a) No animals, except giraffes, are 15 feet or higher.
b) The animals in the zoo belong to me.
c) I have no animals less than 15 feet high.
d) All animals in this zoo are giraffes.

Express a,b,c,d in FOL and tell whether (d) follows from (a), (b), and (c). Prove this using resolution.

Solution:

(a) $\forall x \; (G(x) \vee \neg F(x))$

(b) $\forall x (Z(x) \;\Rightarrow\; M(x))$

(c) $\forall x \; (M(x) \;\Rightarrow\; F(x))$

(d) $\forall x \; (Z(x) \;\Rightarrow\; G(x))$

Negate (d): $\exists x \; \neg(Z(x) \;\Rightarrow\; G(x))$

CNF

(a) $G(x) \vee \neg F(x)$
(b) $\neg Z(x) \vee M(x))$
(c) $\neg M(x) \vee F(x))$
(d) $Z(c)$ con c costante di skolem
(e) $\neg G(c)$

Resolution

(f) $M(c)$ da (b) e (d)
(g) $F(c)$ da (f) e (c)
(h) $G(c)$ da (g) e (a)
(i) $[]$ da (h) ed (e)

Rectangles

Let

$Rectangle(r)$ be "r is a rectangle";
$Square(q)$ be "r is a square";
$Side(s, r)$ be "s is a side of r";
$Parallel(s_1, s_2)$ be "the sides s_1 and s_2 are parallel";
$Right(s_1, s_2)$ be "the angle in between s_1 and s_2 is right";
$len(s)$ be the function that gives the length of s.

Represent as first order formulas the following sentences.

1. A rectangle has four sides, two pairs of parallel sides, and four right angles.
2. A square is a rectangle whose sides have the same length.

Solution:

1. $\forall r\ Rectangle(r) \iff [\exists s_1 \exists s_2 \exists s_3 \exists s_4 (s_1 \neq s_2) \wedge (s_2 \neq s_3) \wedge (s_3 \neq s_4) \wedge (s_4 \neq s_1) \wedge Side(s_1, r) \wedge Side(s_2, r) \wedge Side(s_3, r) \wedge Side(s_4, r) \wedge Parallel(s_1, s_2) \wedge Parallel(s_3, s_4) \wedge Right(s_1, s_2) \wedge Right(s_2, s_3) \wedge Right(s_3, s_4) \wedge Right(s_4, s_1)]$

2. $\forall r\ Square(r) \iff [Rectangle(r) \wedge (\forall s_1 \forall s_2 (Side(s_1, r) \wedge Side(s_2, r)) \rightarrow len(s_1) = le(s_2))]$

Scalar multiplication

Write a Prolog program that given two vectors of arbitrary length returns their scalar product, and fails if they are of different length.

Forbidden elements

Write a Prolog program that given a binary tree and a list of **forbidden** members, verifies that there exists a path from the root to a leaf that contains at most one occurrence of a forbidden number.