Artificial Intelligence a.y. 2019/20
Sample Exam Exercises: Knowledge Representation

Disclaimer: the exercises are taken from the exams of past academic years and adapted to the syllabus of the current academic year. The exam text of the KR section typically may include several of them.

1. **Exercise**

   Tell, for each one of the following propositional formulas if it is valid, contradictory, neither one:
   1) $(A \rightarrow \neg B) \rightarrow ((A \rightarrow B) \rightarrow \neg A)$
   2) $\neg((A \rightarrow \neg B) \rightarrow ((A \rightarrow B) \rightarrow \neg A))$
   3)$((A \rightarrow (B \wedge B)) \rightarrow C)) \rightarrow C$

   Support your answer with truth tables.

   Solution:

   1) is valid

A	B	$(A \rightarrow \neg B) \rightarrow ((A \rightarrow B) \rightarrow \neg A)$
0	0	1
0	1	1
1	0	1
1	1	1

   2) the second formula is the negation of the first one, hence it is contradictory, its truth table is

A	B	$(\neg((A \rightarrow \neg B) \rightarrow ((A \rightarrow B) \rightarrow \neg A))$
0	0	0
0	1	0
1	0	0
1	1	0

   3) is satisfiable, not valid

A	B	C	$((A \rightarrow (B \wedge B)) \rightarrow C)) \rightarrow C$
0	0	0	1
0	1	0	1
1	0	0	0
1	1	0	1
0	0	1	1
0	1	1	1
1	0	1	1
1	1	1	1

2. **Exercise**

Let A, B, C be propositional symbols. Given

$$KB = \{A \rightarrow C, \quad B \rightarrow C, \quad A \vee B\}$$

tell whether the formula C can be derived from KB in each of the following cases:
a. using Modus Ponens;
b. using Resolution.

Both for a and b, in case of positive answer show the derivation, in case of negative answer explain why.

Note that, in case a, the explanation that MP cannot be used because it is not complete for non-Horn formulas is not acceptable as it is wrong: non-complete method in fact doesn't mean that the method can never be successful!

Solution:
a. C cannot be derived with MP as we only know that $A \vee B$ is true, but we do not know which one of them A or B is true, hence we can neither apply MP to A and $A \rightarrow C$ nor to B and $B \rightarrow C$.

b. C can be derived with Resolution:

1. $\neg A \vee C$
2. $\neg B \vee C$
3. $A \vee B$
4. $\neg C$ negated thesis
5. $B \vee C$ from 1. and 3.
6. C from 2. and 5.
7. $\{ \}$ from 4. and 6.

3. **Exercise**

(a) If α is satisfiable and β is unsatisfiable, then $\alpha \wedge \beta$ is always
A) satisfiable
B) valid
C) unsatisfiable
D) we cannot say.

(b) If α is satisfiable and β is satisfiable, then $\alpha \wedge \beta$ is always
A) satisfiable
B) valid
C) unsatisfiable
D) we cannot say.

Choose the right answers and motivate your choices.

Solution:

(a) $\alpha \wedge \beta$ is always unsatisfiable, because there are no interpretations that make β true and so it is for $\alpha \wedge \beta$. For example, if $\beta = (A \wedge \neg A)$, then $\alpha \wedge \beta$ is unsatisfiable whatever is α.

(b) We cannot say, in fact, for example:
If $\alpha = A$ and $\beta = B$ then $\alpha \wedge \beta = A \wedge B$ is satisfiable.
If $\alpha = A$ and $\beta = \neg A$ then $\alpha \wedge \beta = A \wedge \neg A$ is contradictory.

4. **Exercise**

Given the following propositional symbols:
ST to indicate that John studies;
SY to indicate that John is silly;
LU to indicate that John is lucky;
PS to indicate that John passes the exam of Artificial Intelligence.

If John studies and is not silly, he passes the exam of Artificial Intelligence.
If John is not lucky and is silly, he does not pass the exam of Artificial Intelligence.

(a) Represent the above sentence in the propositional logic.
(b) Tell which one, among the following sets, is a model, and which one is not a model, for the above formulas.
$\{ST, SY, PS\}$; $\{LU, PS\}$; $\{\}$; $\{LU, SY, PS\}$.
(c) Specify which formulas, different from PS, need to be added to derive that John passes the exam of Artificial Intelligence.

Solution:

(a) $(ST \wedge \neg SY) \rightarrow PS$
$(\neg LU \wedge SY) \rightarrow \neg PS$

(b) $\{ST, SY, PS\}$ is not a model;
$\{LU, PS\}$ is a model;
$\{\}$ is a model;
$\{LU, SY, PS\}$ is a model.

(c) To derive PC it is sufficient to add $ST \wedge \neg SY$.

5. **Exercise**

Given the following symbols and sentences:
C to indicate that Gianni is a climber;
F to indicate that Gianni is fit;
L to indicate that Gianni is lucky:
E to indicate that Gianni climbs mount Everest.

If Gianni is a climber and he is fit, he climbs mount Everest.
If Gianni is not lucky and he is not fit, he does not climb mount Everest.
Gianni is fit.

(a) Formalize the above sentences in propositional logic.

(b) Tell if the KB buit in (a) is consistent, and tell if some of the following sets are models for the above sentences:
$\{\ \}$; $\{C, L\}$; $\{L, E\}$; $\{F, C, E\}$; $\{L, F, E\}$.

(c) Verify, using resolution, if it it follows form the KB that Gianni climbs mount Everest.

Solution:

(a) (a) $(C \wedge F) \rightarrow E$
    (b) $(\neg L \wedge \neg F) \rightarrow \neg E$
    (c) F

(b) The KB is consistent: it has at least a model, as the following check shows.
    (a) $\{\ \}$ is not a model (it models 1 and 2 but not 3);
    (b) $\{C, L\}$ is not a model (it models 1 and 2 but not 3);
    (c) $\{L, E\}$ is not a model (it models 1 and 2 but not 3);
    (d) $\{F, C, E\}$ is a model;
    (e) $\{L, F, E\}$ is a model.

6. **Exercise**

Assuming that lower case letters indicate constants and function symbols, while upper case letters denote variables, tell whether the expressions:

$f(X, g(Y, Z))$ and $f(g(a, b), g(b, a)$

unify, and in this case provide the most general unifier. Illustrate step by step, the execution of the unification algorithm.

7. **Exercise**

For each of the following pairs of formulas check whether or not the second one in the pair is a Skolemized form of the first one. In the negative case, tell why and provide a correct Skolemized form.

(a) (a) $\forall x \exists y [P(x, y) \wedge Q(y, x)]$
    (b) $P(x, a) \wedge Q(a, x)$

(b) (a) $\forall x \exists y (Q(x, y) \wedge \exists z \forall t P(z, t))$
    (b) $Q(x, f(x)) \wedge P(g(t), t)$

(c) (a) $\forall x \exists y \exists z [P(x, y) \wedge P(z, x) \rightarrow P(y, z)]$
    (b) $P(x, f(x)) \wedge P(f(x), x) \rightarrow P(f(x), f(x))$

Solution:

(a) No, because a Skolem constant is used in a place where the existential quantifier follows an universal one. A correct form is $P(x, f(x)) \wedge Q(f(x), x)$.

(b) No, because a Skolem function with parameter t is used in a place where the existential quantifier does not follow an universal quantifier for t. A correct form $Q(x, f(x)) \wedge P(g(x), t)$ or also $Q(x, f(x)) \wedge P(a, t)$.

(c) No, because we need a different Skolem function for the two distinct existential quantifiers. A correct form is $P(x, f(x)) \wedge P(g(x), x) \rightarrow P(f(x), g(x))$.

8. **Exercise**

For each one of the following formulas from 1 to 6, tell the sentence from A to F it represents (if any):

1. $\exists x(black(x) \wedge dog(x))$	A. Every dog is black.
2. $\exists x(black(x) \rightarrow dog(x))$	B. There is something black, or a dog.
3. $\exists x(black(x) \vee dog(x))$	C. Everything that is black is a dog.
4. $\forall x(black(x) \wedge dog(x))$	D: Everything is a black dog.
5. $\forall x(black(x) \rightarrow dog(x))$	E. No dog is black.
6. $\forall x(black(x) \vee dog(x))$	F. There is a black dog.

Tell whether there are formulas or sentences that do not find any correnspondance, and in case tell which one.

Solution:
1 F
3 B
4 D
5 C

No correspondence for the formulas 2 and 6, and for the sentences A and E.

9. **Exercise**

Discuss the following statements:

1) $[(\forall x A(x)) \wedge (\forall x B(x))] \leftrightarrow [\forall y(B(y) \wedge A(y))]$ is a valid formula in first order logic.
2) $[(\exists x A(x)) \wedge (\exists x B(x))] \leftrightarrow [(\exists y(B(y) \wedge A(y))]$ is a valid formula in first order logic.

Solution:
The first one is a valid formula of FOL for obvious reasons: If A is satisfied by every element of the domain and B is satisfied by every element of the domain, then A and B are satisfied by every element of the domain, and vice versa. So the formula is true in all interpretations, hence it is valid.

The second formula is not valid: interpret A as 'being odd' and B as 'being even' in the domain of natural numbers; left side true, right side false, hence the formula is not valid.

The formula is satisfiable: consider an interpretation where A and B are interpreted with the same predicate. The formula is true in this interpretation.

10. **Exercise**

Tell the formulas that adequately represent the sentence:

"All the students that take the exam of Artificial Intelligence pass it"

1) $\forall x\ Student(x) \land Exam(x, IntArt) \rightarrow Pass(x, IntArt)$
2) $\forall x\ Student(x) \land Exam(x, IntArt) \land Pass(x, IntArt)$
3) $\forall x \exists y\ Student(x) \land y = IntArt \land Exam(x, y) \rightarrow Pass(x, y)$
4) $\forall x\ Student(x) \rightarrow Exam(x, IntArt) \rightarrow Pass(x, IntArt)$

Motivate each answer.

Solution:
The formulas 1), 3) and 4) are equivalent with one another and adequately represent the sentence. The formula 2) is not adequate as it asserts that all the individuals in the domain are students, take the exam of Int Art, and pass it.

11. **Exercise**

Let $Rose(x), Thorn(x), Has(x, y), Dangerous(x)$ be unary and binary predicate symbols.
Express in FOL the following sentences:

(a) There is no rose without a thorn.
(b) Thorns are dangerous.
(c) Whoever has something dangerous is dangerous.

Show, using resolution, that roses are dangerous.

Solution:

The three sentences can be expressed as:
1) $\forall x(Rose(x) \rightarrow \exists y(Thorn(y) \land Has(x, y)))$
2) $\forall t(Thorn(t) \rightarrow Dangerous(t))$
3) $\forall x((\exists y(Has(x, y) \land Dangerous(y))) \rightarrow Dangerous(x))$

while the last sentence as:
$\forall r(Rose(r) \rightarrow Dangerous(r))$

The transformation of the formulas of KB into clauses (where $F()$ is a Skolem function) gives :
1a) $\neg Rose(x) \lor Thorn(F(x))$
1b) $\neg Rose(x) \lor Has(x, F(x))$
2) $\neg Thorn(t) \lor Dangerous(t)$
3)$\neg Has(x, y) \lor \neg Dangerous(y) \lor Dangerous(x)$
The negation of (4) (where R is a Skolem constant) gives :
4a) $Rose(R)$
4b)$\neg Dangerous(R)$
The empty clause can be derived:
5) $\neg Rose(x) \lor Dangerous(F(x))$ Res. 1a,2 $\langle t/F(x)\rangle$
6) $\neg Rose(x) \lor \neg Dangerous(F(x)) \lor Dangerous(x)$ Res. 1b,3 $\langle y/F(x)\rangle$
7) $\neg Rose(x) \lor Dangerous(x)$ Res. 5,6
8) $Dangerous(R)$ Res. 4a,7 (x R)
9) $\{\}$ Res. 4b,8

12. **Exercise**

Consider the following first order formulas:

I. $\forall x\ Equal(x, x)$
II. $\forall x \forall y\ (Equal(x, y) \rightarrow Equal(y, x))$
III. $\forall x \forall y \forall z\ ((Equal(x, y) \wedge Equal(y, z)) \rightarrow Equal(x, z))$

Starting from I. II. and III. prove by refutation with resolution the following:

$$\forall x \forall y \forall z\ ((Equal(x, y) \wedge \neg Equal(y, z)) \rightarrow \neg Equal(x, z))$$

Solution:
Transform into normal form the original formulas plus the negation of the thesis (A, B, C are Skolem constants):
1) $Equal(x_1, x_1)$
2) $Equal(x_2, x_3) \vee \neg Equal(x_3, x_2)$
3) $Equal(x_4, x_5) \vee \neg Equal(x_4, x_6) \vee \neg Equal(x_6, x_5)$
4) $Equal(A, B)$
5) $\neg Equal(B, C)$
6) $Equal(A, C)$

We can get the empty clause, for instance, as follows:
7) $Equal(B, A)$ resolution from (2) and (4) (substitution $\{x_3/A, x_2/B\}$)
8) $\neg Equal(A, x_5) \vee Equal(B, x_5)$ resolution from(3) and (7) (substitution $\{x_6/A, x_4/B\}$)
9) $Equal(B, C)$ resolution from (6) and (8) (substitution $\{x_5/C\}$)
10) $\{\}$ resolution from (5) and (9)