Exercises on First Order Logic[1]

Exercises FOL

[1] The slides have been prepared using the textbook material available on the web, and the slides of the previous editions of the course by Prof. Luigia Carlucci Aiello

Exercise

Tell which one among the following formulas is a good representation of the sentence.

An elephant is happy if all its children can fly

1 $\forall x\ (elephant(x) \Rightarrow ((\forall y\ child(y,x) \Rightarrow fly(y)) \Rightarrow happy(x)))$

2 $\forall x\ ((elephant(x) \wedge happy(x)) \Rightarrow \forall y\ (child(y,x) \Rightarrow fly(y)))$

3 $\forall x\ (elephant(x) \Rightarrow ((\neg\exists y(child(y,x) \wedge \neg fly(y))) \Rightarrow happy(x)))$

4 $\forall x\ (elephant(x) \Rightarrow \forall y\ ((child(y,x) \wedge fly(y)) \Rightarrow happy(x)))$

Exercise - Solution

The "*all its children can fly*" part needs to be read as:

"*if it's a child of the elephant, then it can fly*"

This will be inside another if-then statement:

"*if all the children of an elephant can fly, then the elephant is happy*"

Then, we need a third implication to say that if something is an elephant, then, if all its children can fly, it is happy.

Exercise - Solution

1 $\forall x\ (elephant(x) \Rightarrow ((\forall y\ child(y,x) \Rightarrow fly(y)) \Rightarrow happy(x)))$
correct

2 $\forall x\ ((elephant(x) \wedge happy(x)) \Rightarrow \forall y\ (child(y,x) \Rightarrow fly(y)))$
incorrect, as it states that only children of happy elephants can fly.

3 $\forall x\ (elephant(x) \Rightarrow ((\neg \exists y (child(y,x) \wedge \neg fly(y))) \Rightarrow happy(x)))$
correct, logically equivalent to 1.

4 $\forall x\ (elephant(x) \Rightarrow \forall y\ ((child(y,x) \wedge fly(y)) \Rightarrow happy(x)))$
incorrect, as it states that an elephant is happy if all elements in the domain are its children and can fly.

Exercise

Choose a suitable vocabulary of constant and predicate symbols then represent the following sentences in FOL:

- *Steve likes easy classes*
- *Violin classes are not easy*
- *Every class of Percussion is easy*
- *The class of "Afro Drums" is a class of Percussion*
- *The class of "Violin for pre-school Kids" is a class of Violin*

Can we infer that *Steve likes the class of Afro Drums*?

1 using **Modus Ponens**

2 using **Resolution**

Exercise - Solution

The vocabulary:

- ◇ $EC(x)$ easy class
- ◇ $VC(x)$ Violin Class
- ◇ $PC(x)$ Percussion Class
- ◇ $likes(x, y)$ y likes x
- ◇ $Steve, AD, VK$ the three constants

Exercise - Solution

FOL knowledge base:

1. $\forall x\ EC(x) \Rightarrow likes(x, Steve)$
2. $\forall x\ VC(x) \Rightarrow \neg EC(x)$
3. $\forall x\ PC(x) \Rightarrow EC(x)$
4. $PC(AD)$
5. $VC(VK)$

Exercise - Solution 1

Using $\sigma = \{x/AD\}$ and (3) and (4), we have:

$$PC(AD) \Rightarrow EC(AD), \text{ hence } EC(AD).$$

Analogously, we have using $\sigma = \{x/AD\}$:

$$EC(AD) \Rightarrow likes(AD, Steve), \text{ hence } likes(AD, Steve)$$

Exercise - Solution 2

Transform KB and negated thesis into clauses:

$$\{\neg EC(x) \vee likes(x, Steve)\}_1, \{\neg VC(x) \vee \neg EC(x)\}_2,$$
$$\{\neg PC(x) \vee EC(x)\}_3, \{PC(AD)\}_4, \{VC(VK)\}_5,$$
$$\{\neg likes(AD, Steve)\}_6$$

From (3) and (4) with $\sigma = \{x/AD\} \Rightarrow \{EC(AD)\}_7$
From (1) and (7) with $\sigma = \{x/AD\} \Rightarrow \{likes(AD, Steve)\}_8$
From (6) and (8) $\Rightarrow \{\}$

Exercise

Let Gov , $Univ$, Lca, $Bill$ be constant symbols, and let $prof(x)$, $stud(x)$, $unhappy(x)$, $cut\_fund(x,y)$, $fail\_exam(x,y)$ be unary and binary predicate symbols.

Given:

1. *If the Government cuts the funds to Universities, professors are unhappy*
2. *If professors are unhappy all students fail their exams*
3. *The Government cuts the funds to Universities*
4. *Lca is a professor and Bill is a student*

1 Express in FOL the above KB

2 Prove that *Bill fails Lca's exam* using both **Modus Ponens** and **Resolution**

Solution 1

FOL knowledge base:

1. $cut\_fund(Gov, Univ) \Rightarrow \forall x\ (prof(x) \Rightarrow unhappy(x))$
2. $\forall x\, \forall y((prof(x) \land unhappy(x) \land stud(y)) \Rightarrow fail\_exam(x, y))$
3. $cut\_fund(Gov, Univ)$
4. $prof(Lca)$
5. $stud(Bill)$

Exam 13/02/2015 - Solution 2

Generalized Modus Ponens:

Using (1) and (3), we have: $\{\forall x \ (prof(x) \Rightarrow unhappy(x))\}_6$

Using (4) and (6) with $\sigma = \{x/Lca\}$, we have: $\{unhappy(Lca)\}_7$

Using (4) and (5) and (7), we have:

$$\{prof(Lca) \wedge unhappy(Lca) \wedge stud(Bill)\}_8$$

Using (2) and (8) with $\sigma = \{x/Lca, \ y/Bill\}$, we have:

$$fail\_exam(Lca, Bill)$$

Exam 13/02/2015 - Solution 2

Resolution:

$$\{\neg cut\_fund(Gov, Univ) \lor \neg prof(x) \lor unhappy(x))\}_1,$$
$$\{\neg prof(x) \lor \neg unhappy(x) \lor \neg stud(y) \lor fail\_exam(x, y)\}_2,$$
$$\{cut\_fund(Gov, Univ)\}_3, \{prof(Lca)\}_4, \{stud(Bill)\}_5,$$
$$\{\neg fail\_exam(Lca, Bill)\}_6$$

From (1) and (3) $\Rightarrow \{\neg prof(x) \lor unhappy(x)\}_7$
From (2) and (7) $\Rightarrow \{\neg prof(x) \lor \neg stud(y) \lor fail\_exam(x, y)\}_8$
From (4) and (8) with $\sigma = \{x/Lca\}$

$$\Rightarrow \{\neg stud(y) \lor fail\_exam(Lca, y)\}_9$$

From (5) and (9) with $\sigma = \{y/Bill\} \Rightarrow \{fail\_exam(Lca, Bill)\}_{10}$
From (6) and (10) $\Rightarrow \{\}$

Exercise - Unification

Tell whether or not the following pairs of expressions unify. Describe the unification process step by step:

$$f(g(a, X), g(X, b)) = f(g(a, b, c, d))$$
$$f(g(a, X), g(Y, Y)) = f(g(a, b), g(f(a), f(Z)))$$
$$f(cons(cons(a, b))) = f(cons(cons(a, nil))$$

Exercise - Skolemization

- Is $P(c)$ the Skolemized version of $\exists x\ P(x)$?
- Is $\forall x\ P(c, x)$ the Skolemized version of $\forall x\, \exists y\ P(y, x)$?
- Is $\forall x\ P(f(x), x)$ the Skolemized version of $\forall x\, \exists y\ P(y, x)$?
- Is $P(c_1, c_2)$ the Skolemized version of $\exists x\, \exists y\ P(x, y)$?
- Is $\forall y\ P(c_1, y, f(y))$ the Skolemized version of $\exists x\, \forall y\, \exists z\ P(x, y, z)$?
- Is $P(x, y, f(y))$ the Skolemized version of $\forall x\, \forall y\, \exists z\ P(x, y, z)$?

Exercise - Solution

- $\diamond$ Is $P(c)$ the Skolemized version of $\exists x\ P(x)$?
  correct
- $\diamond$ Is $P(c, x)$ the Skolemized version of $\forall x\, \exists y\ P(y, x)$?
  incorrect, see next Skolemization
- $\diamond$ Is $P(f(x), x)$ the Skolemized version of $\forall x\, \exists y\ P(y, x)$?
  correct
- $\diamond$ Is $P(c_1, c_2)$ the Skolemized version of $\exists x\, \exists y\ P(x, y)$?
  correct
- $\diamond$ Is $P(c, y, f(y))$ the Skolemized version of $\exists x\, \forall y\, \exists z\ P(x, y, z)$?
  correct
- $\diamond$ Is $P(x, y, f(y))$ the Skolemized version of $\forall x\, \forall y\, \exists z\ P(x, y, z)$?
  incorrect, z depends on (x, y) hence $f(y) \Rightarrow f(x, y)$

Exercise

Given:

1. *The textbooks of class CA are easy*
2. *The textbooks of class CB are difficult*
3. *Mary studies (all and only) easy books*
4. *Mary passes the exam of a class if she studies at least a textbook for that class*
5. *Russel&Norvig is a textbook for class CA*
6. *Tenenbaum is a textbook for class CB*

1 Translate the sentences in FOL, in CNF and tell if it is Horn

2 Prove, using **Resolution**, that *Mary passes an exam*, by adding the appropriate knowledge (if needed)

Exercise - Solution 1

A straightforward translation is:

1. $\forall x\ text(CA, x) \Rightarrow easy(x)$
2. $\forall x\ text(CB, x) \Rightarrow \neg easy(x)$
3. $\forall x\ study(Mary, x) \Leftrightarrow easy(x)$
4. $\forall x\ [\exists y\ text(x, y) \wedge study(Mary, y)] \Rightarrow pass(Mary, x)$
5. $text(CA, Russel\&Norvig)$
6. $text(CB, Tenenbaum)$

Exercise - Solution 1

A straightforward translation in CNF is:

1. $\neg text(CA, x) \lor easy(x)$
2. $\neg text(CB, x) \lor \neg easy(x)$

3.1. $\neg study(Mary, x) \lor easy(x)$

3.2. $study(Mary, x) \lor \neg easy(x)$

4. $\neg text(x, y) \lor \neg study(Mary, y) \lor pass(Mary, x)$
5. $text(CA, Russel\&Norvig)$
6. $text(CB, Tenenbaum)$

It is Horn (at most one positive atom).

Exercise - Solution 2

Knowledge base for the **Resolution**:

$$\{\neg text(CA, x) \vee easy(x)\}_1,$$

$$\{\neg text(CB, x) \vee \neg easy(x)\}_2,$$

$$\{\neg study(Mary, x) \vee easy(x)\}_{3.1},$$

$$\{study(Mary, x) \vee \neg easy(x)\}_{3.2},$$

$$\{\neg text(x, y) \vee \neg study(Mary, y) \vee pass(Mary, x)\}_4,$$

$$\{text(CA, Russel\&Norvig)\}_5,$$

$$\{text(CB, Tenenbaum)\}_6,$$

$$\{\neg pass(Mary, z)\}_7$$

Exercise - Solution 2

From (4) and (7) with $\sigma = \{z/x\}$:

$$\{\neg text(z, y) \vee \neg study(Mary, y)\}_8$$

From (5) and (8) with $\sigma = \{z/CA;\ y/Russel\&Norvig\}$:

$$\{\neg study(Mary, Russel\&Norvig)\}_9$$

From (3.2) and (9) with $\sigma = \{x/Russel\&Norvig\}$:

$$\{\neg easy(Russel\&Norvig)\}_{10}$$

From (1) and (5) with $\sigma = \{x/Russel\&Norvig\}$:

$$\{easy(Russel\&Norvig)\}_{11}$$

From (10) and (11) $\Rightarrow \{\}$