SAPIENZA
UNIVERSITÀ DI ROMA

Artificial Intelligence

2023/2024 Prof: Sara Bernardini

Lab 8:
FOL Resolution

Francesco Argenziano
email: argenziano@diag.uniroma1.it

*The slides have been prepared using the textbook material available on the web, and the slides of the previous editions of the course by Prof. Luigia Carlucci Aiello, Prof. Daniele Nardi and Dott. Fabio Previtali.

Exercises: FOL unification

Tell whether or not the following pairs of expressions unify. Describe the unification process step by step:

$$f(g(a, X), g(X, b)) = f(g(a, b, c, d))$$
$$f(g(a, X), g(Y, Y)) = f(g(a, b), g(f(a), f(Z)))$$
$$f(cons(cons(a, b))) = f(cons(cons(a, nil))$$

$f(cons(car(X), cdr(Y)), Z, X)$ and $f(Z, Z, cons(car(X), cdr(a)))$

$f(g(x, a), g(b, a))$ and $f(y, y)$

$P(g(x, a), f(b, a))$ and $P(g(f(b, y), y), f(z, y))$

Exercises: FOL Resolution

Horses run faster than rabbits. Dogs run faster than rabbits. Because of its name, we know that Fury is either a horse or a dog. Bunny is a rabbit. Arrow is a greyhound.

- (a) Represent the above formulae in first order logic.
- (b) Transform them in CNF and prove by resolution that Fury is faster than Bunny. Or else show that it is not logically entailed.
- (c) Is Arrow faster than Bunny? If it is, prove it. Otherwise, specify whether you can add some knowledge in order to prove it (except for the trivial addition of $faster(Arrow, Bunny)$).

Exercises: FOL Resolution

(a)

$\forall x\, y\, horse(x) \wedge rabbit(y) \Rightarrow faster(x, y)$

$\forall x\, y\, dog(x) \wedge rabbit(y) \Rightarrow faster(x, y)$

$horse(F) \vee dog(F)$

$rabbit(B)$

$greyhound(A)$

(b) CNF:

$\neg horse(x), \neg rabbit(y), faster(x, y)$

$\neg dog(x), \neg rabbit(y), faster(x, y)$

$horse(F) \vee dog(F)$

$rabbit(B)$

$greyhound(A)$

GOAL: $\neg faster(F, B)$

(c)

$\forall x\, greyhound(x) \Rightarrow dog(x)$

Exercises: FOL Resolution

Given:

1. *The textbooks of class CA are easy*
2. *The textbooks of class CB are difficult*
3. *Mary studies (all and only) easy books*
4. *Mary passes the exam of a class if she studies at least a textbook for that class*
5. *Russel&Norvig is a textbook for class CA*
6. *Tenenbaum is a textbook for class CB*

1 Translate the sentences in FOL, in CNF and tell if it is Horn

2 Prove, using **Resolution**, that *Mary passes an exam*, by adding the appropriate knowledge (if needed)

Exercises: FOL Resolution

A straightforward translation is:

1. $\forall x \; text(CA, x) \Rightarrow easy(x)$
2. $\forall x \; text(CB, x) \Rightarrow \neg easy(x)$
3. $\forall x \; study(Mary, x) \Leftrightarrow easy(x)$
4. $\forall x \; [\exists y \; text(x, y) \wedge study(Mary, y)] \Rightarrow pass(Mary, x)$
5. $text(CA, Russel\&Norvig)$
6. $text(CB, Tenenbaum)$

Exercises: FOL Resolution

A straightforward translation in CNF is:

1. $\neg text(CA, x) \vee easy(x)$

2. $\neg text(CB, x) \vee \neg easy(x)$

3.1. $\neg study(Mary, x) \vee easy(x)$

3.2. $study(Mary, x) \vee \neg easy(x)$

4. $\neg text(x, y) \vee \neg study(Mary, y) \vee pass(Mary, x)$

5. $text(CA, Russel\&Norvig)$

6. $text(CB, Tenenbaum)$

It is Horn (at most one positive atom).

Exercises: FOL Resolution

Knowledge base for the **Resolution**:

$$\{\neg text(CA, x) \vee easy(x)\}_1,$$

$$\{\neg text(CB, x) \vee \neg easy(x)\}_2,$$

$$\{\neg study(Mary, x) \vee easy(x)\}_{3.1},$$

$$\{study(Mary, x) \vee \neg easy(x)\}_{3.2},$$

$$\{\neg text(x, y) \vee \neg study(Mary, y) \vee pass(Mary, x)\}_4,$$

$$\{text(CA, Russel\&Norvig)\}_5,$$

$$\{text(CB, Tenenbaum)\}_6,$$

$$\{\neg pass(Mary, z)\}_7$$

Exercises: FOL Resolution

From (4) and (7) with $\sigma = \{z/x\}$:

$$\{\neg text(z, y) \vee \neg study(Mary, y)\}_8$$

From (5) and (8) with $\sigma = \{z/CA;\ y/Russel\&Norvig\}$:

$$\{\neg study(Mary, Russel\&Norvig)\}_9$$

From (3.2) and (9) with $\sigma = \{x/Russel\&Norvig\}$:

$$\{\neg easy(Russel\&Norvig)\}_{10}$$

From (1) and (5) with $\sigma = \{x/Russel\&Norvig\}$:

$$\{easy(Russel\&Norvig)\}_{11}$$

From (10) and (11) $\Rightarrow \{\}$

Exercises: FOL Resolution

I. $\forall x\ Equal(x, x)$
II. $\forall x \forall y\ (Equal(x, y) \rightarrow Equal(y, x))$
III. $\forall x \forall y \forall z\ ((Equal(x, y) \wedge Equal(y, z)) \rightarrow Equal(x, z))$

Starting from I. II. and III. prove by refutation with resolution the following:

$$\forall x \forall y \forall z\ ((Equal(x, y) \wedge \neg Equal(y, z)) \rightarrow \neg Equal(x, z))$$

Exercises: FOL Resolution

Transform into normal form the original formulas plus the negation of the thesis (A, B, C are Skolem constants):

1) $Equal(x_1, x_1)$
2) $Equal(x_2, x_3) \vee \neg Equal(x_3, x_2)$
3) $Equal(x_4, x_5) \vee \neg Equal(x_4, x_6) \vee \neg Equal(x_6, x_5)$
4) $Equal(A, B)$
5) $\neg Equal(B, C)$
6) $Equal(A, C)$

We can get the empty clause, for instance, as follows:

7) $Equal(B, A)$ resolution from (2) and (4) (substitution $\{x_3/A, x_2/B\}$)
8) $\neg Equal(A, x_5) \vee Equal(B, x_5)$ resolution from(3) and (7) (substitution $\{x_6/A, x_4/B\}$)
9) $Equal(B, C)$ resolution from (6) and (8) (substitution $\{x_5/C\}$)
10) $\{\}$ resolution from (5) and (9)

Exercises: FOL Resolution

Let $Rose(x), Thorn(x), Has(x, y), Dangerous(x)$ be unary and binary predicate symbols. Express in FOL the following sentences:

(a) There is no rose without a thorn.

(b) Thorns are dangerous.

(c) Whoever has something dangerous is dangerous.

Show, using resolution, that roses are dangerous.

Exercises: FOL Resolution

The three sentences can be expressed as:
1) $\forall x(Rose(x) \rightarrow \exists y(Thorn(y) \wedge Has(x, y)))$
2) $\forall t(Thorn(t) \rightarrow Dangerous(t))$
3) $\forall x((\exists y(Has(x, y) \wedge Dangerous(y))) \rightarrow Dangerous(x))$

while the last sentence as:
$\forall r(Rose(r) \rightarrow Dangerous(r))$

The transformation of the formulas of KB into clauses (where $F()$ is a Skolem function) gives :
1a) $\neg Rose(x) \vee Thorn(F(x))$
1b) $\neg Rose(x) \vee Has(x, F(x))$
2) $\neg Thorn(t) \vee Dangerous(t)$
3)$\neg Has(x, y) \vee \neg Dangerous(y) \vee Dangerous(x)$
The negation of (4) (where R is a Skolem constant) gives :
4a) $Rose(R)$
4b)$\neg Dangerous(R)$
The empty clause can be derived:
5) $\neg Rose(x) \vee Dangerous(F(x))$ Res. 1a,2 $\langle t/F(x)\rangle$
6) $\neg Rose(x) \vee \neg Dangerous(F(x)) \vee Dangerous(x)$ Res. 1b,3 $\langle y/F(x)\rangle$
7) $\neg Rose(x) \vee Dangerous(x)$ Res. 5,6
8) $Dangerous(R)$ Res. 4a,7 (x R)
9) $\{\}$ Res. 4b,8

Exercises: FOL Resolution

1. All hounds howl at night.
2. Anyone who has any cats will not have any mice.
3. Light sleepers do not have anything which howls at night.
4. John has either a cat or a hound.
5. (Conclusion) If John is a light sleeper, then John does not have any mice.

Exercises: FOL Resolution

1. $\forall x\ (HOUND(x) \rightarrow HOWL(x))$
2. $\forall x\ \forall y\ (HAVE\ (x,y) \wedge CAT\ (y) \rightarrow \neg \exists z\ (HAVE(x,z) \wedge MOUSE\ (z)))$
3. $\forall x\ (LS(x) \rightarrow \neg \exists y\ (HAVE\ (x,y) \wedge HOWL(y)))$
4. $\exists x\ (HAVE\ (John,x) \wedge (CAT(x) \vee HOUND(x)))$
5. $LS(John) \rightarrow \neg \exists z\ (HAVE(John,z) \wedge MOUSE(z))$

1. $\neg HOUND(x) \vee HOWL(x)$
2. $\neg HAVE(x,y) \vee \neg CAT(y) \vee \neg HAVE(x,z) \vee \neg MOUSE(z)$
3. $\neg LS(x) \vee \neg HAVE(x,y) \vee \neg HOWL(y)$

4.
   1. $HAVE(John,a)$
   2. $CAT(a) \vee HOUND(a)$

5.
   1. $LS(John)$
   2. $HAVE(John,b)$
   3. $MOUSE(b)$

Exercises: FOL Resolution

[1.,4.(b):]	6.	$CAT(a) \vee HOWL(a)$
[2,5.(c):]	7.	$\neg HAVE(x,y) \vee \neg CAT(y) \vee \neg HAVE(x,b)$
[7,5.(b):]	8.	$\neg HAVE(John,y) \vee \neg CAT(y)$
[6,8:]	9.	$\neg HAVE(John,a) \vee HOWL(a)$
[4.(a),9:]	10.	$HOWL(a)$
[3,10:]	11.	$\neg LS(x) \vee \neg HAVE(x,a)$
[4.(a),11:]	12.	$\neg LS(John)$
[5.(a),12:]	13.	$\square$