大数据计算及应用(三)

Finding Similar Items: Locality Sensitive Hashing

Slides adapted from http://www.mmds.org

Agenda

High dim. data
- Locality sensitive hashing
- Clustering
- Dimensionality reduction

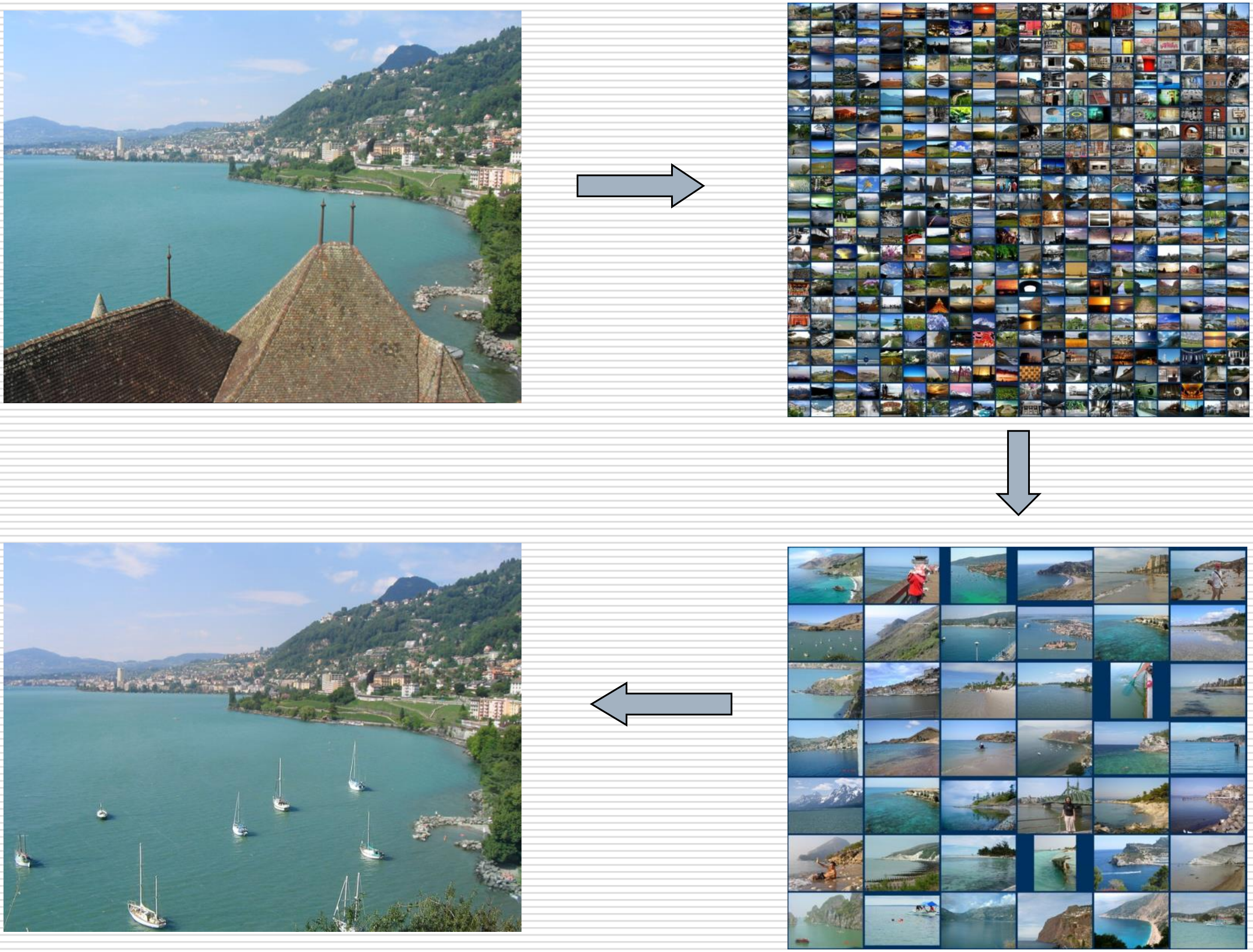
Graph data
- PageRank, SimRank
- Community Detection
- Spam Detection

Infinite data
- Filtering data streams
- Web advertising
- Queries on streams

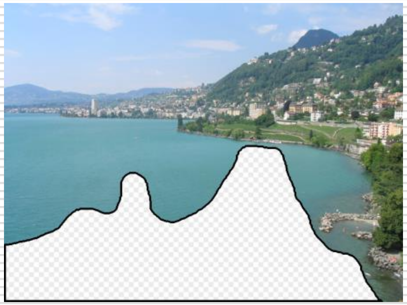
Machine learning
- SVM
- Decision Trees
- Perceptron, kNN

Apps
- Recommender systems
- Association Rules
- Duplicate document detection

Scene Completion Problem

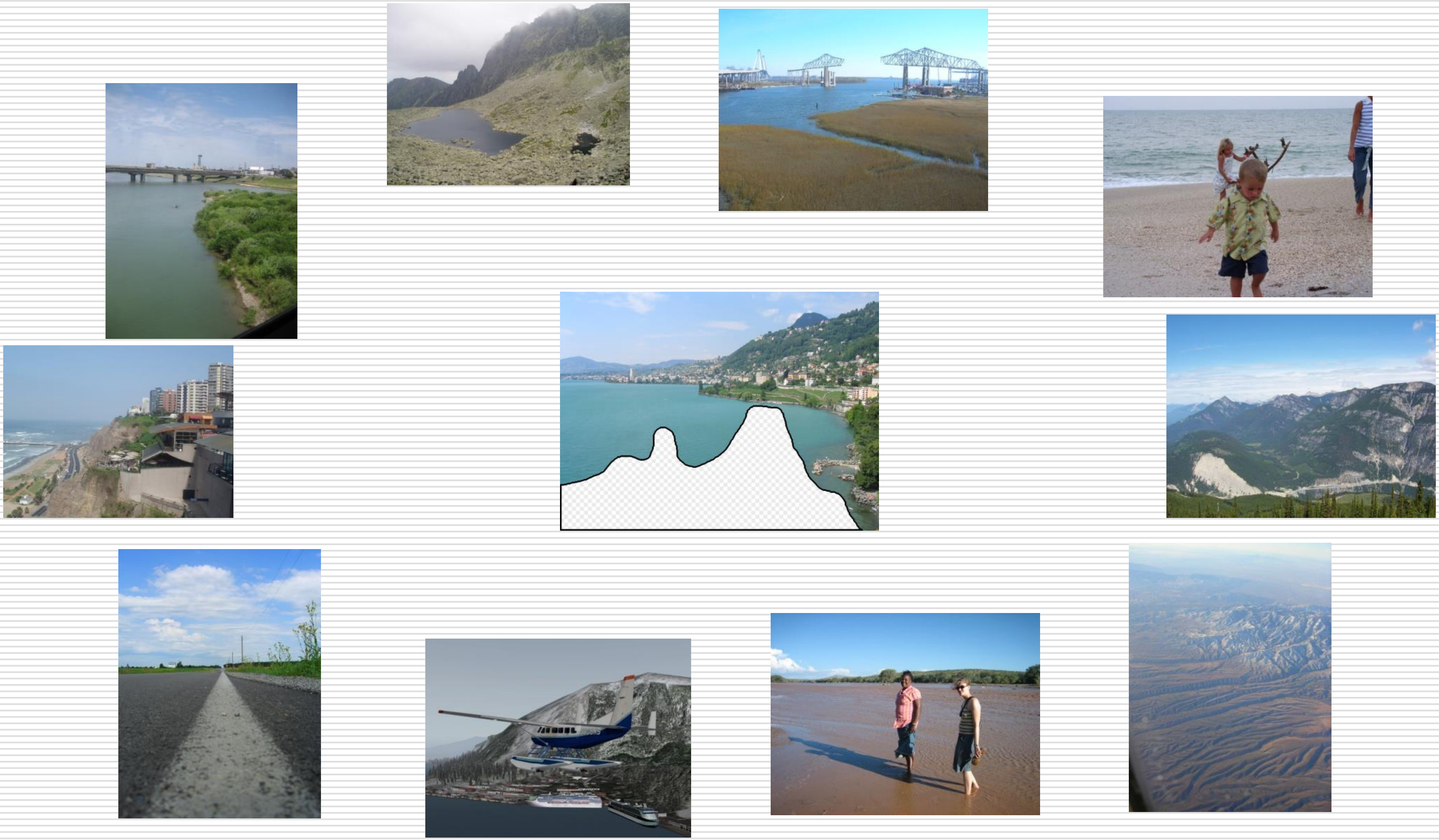


Scene Completion Problem



Scene Completion Problem

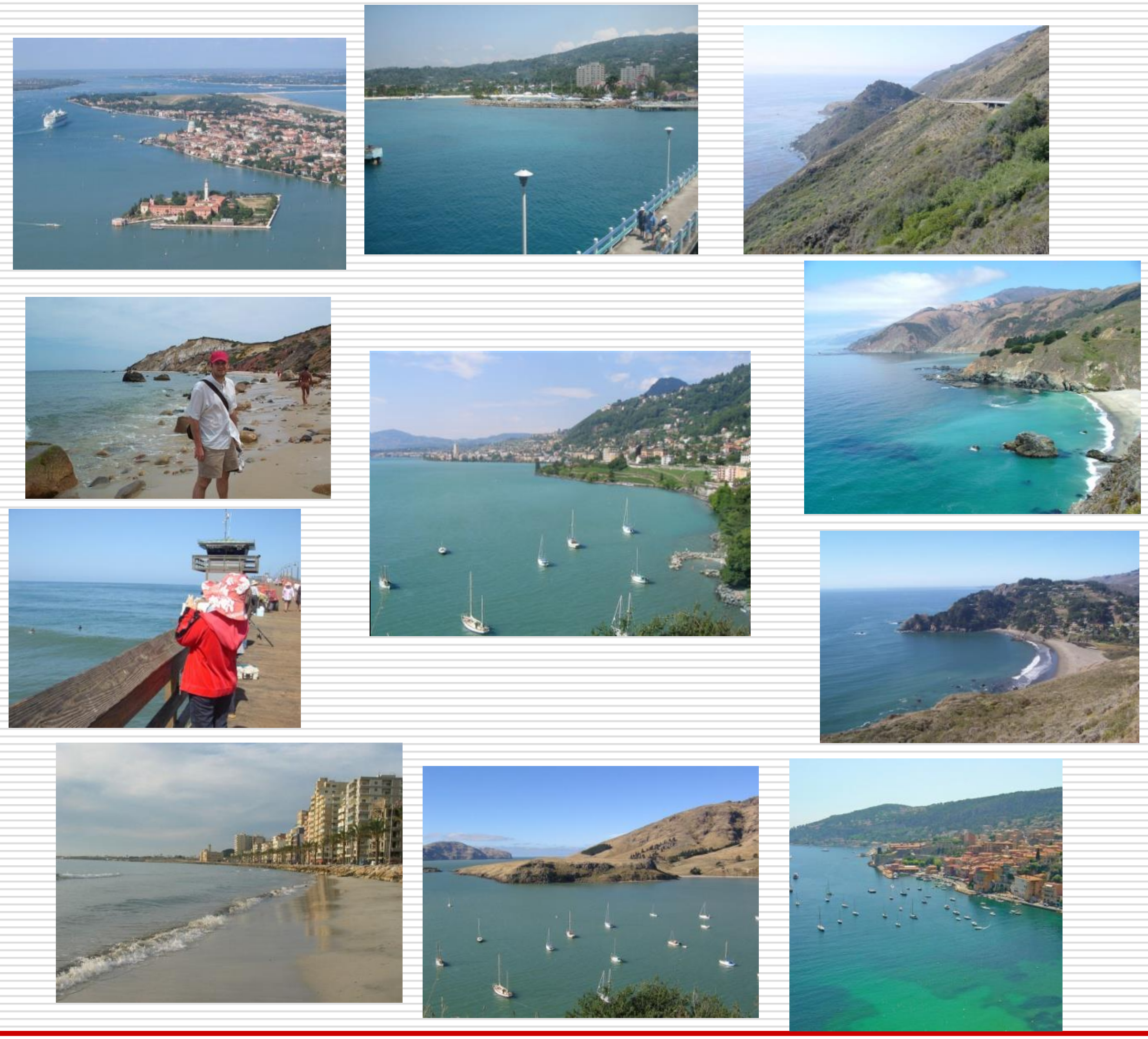
[Hays and Efros, SIGGRAPH 2007]



10 nearest neighbors from a collection of 2.3M images

[Hays and Efros, SIGGRAPH 2007]

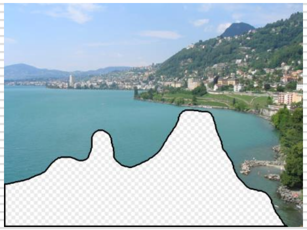
Scene Completion Problem



10 nearest neighbors from a collection of 2.3M images

A Common Metaphor

- Many problems can be expressed as finding "similar" sets:
  - Find near-neighbors in high-dimensional space
- Examples:
  - Pages with similar words
    - For duplicate detection, classification by topic
  - Customers who purchased similar products
    - Products with similar customer sets
  - Images with similar features
    - Users who visited similar websites



Problem for Today's Lecture

- Given: High dimensional data points $x_1, x_2, \ldots$
  - For example: Image is a long vector of pixel colors

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \rightarrow [1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0]$$

img1	*img2*
1	1
0	1
1	1
0	1
0	0
1	1
0	1
1	1
0	0

Is *img1* similar to *img2*?

Problem for Today's Lecture

- Given: High dimensional data points $x_1, x_2, \ldots$
  - Another example:

    Document is a long vector of words existence

	doc1	*doc2*
angle	1	1
before	0	1
country	1	1
end	0	1
food	0	0
good	1	1
house	0	1
live	1	1
use	0	0

Is *doc1* similar to *doc2*?

Problem for Today's Lecture

- Goal: Find all pairs of documents (x_i, x_j) that are within some distance threshold $d(x_i, x_j) \leq s$

n high time complexity: O(n^2) ⇨ O(n)

precision?

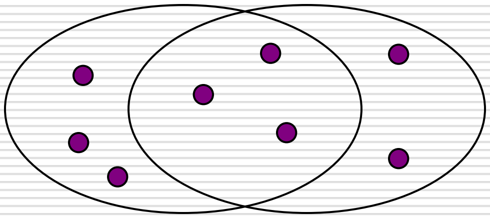
large space requirement

i

doc1	doc2		docN
1	1		1
0	1		1
1	1		1
0	1		1
0	0	......	0
1	1		1
0	1		1
1	1		1
0	0		0
......	......		......

Distance Measures

- Goal: Find near-neighbors in high-dim. space
  - We formally define "near neighbors" as points that are a "small distance" apart
- For each application, we first need to define what "distance" means
- Today: Jaccard distance/similarity
  - The Jaccard similarity of two sets is the size of their intersection divided by the size of their union: $sim(C_1, C_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$
  - Jaccard distance: $d(C_1, C_2) = 1 - |C_1 \cap C_2| / |C_1 \cup C_2|$



3 in intersection
8 in union
Jaccard similarity= 3/8
Jaccard distance = 5/8

Encoding Sets as Bit Vectors

- Encode sets using 0/1 (bit, boolean) vectors
  - One dimension per element in the universal set
- Interpret set intersection as bitwise AND, and set union as bitwise OR

- Example: $C_1 = 10111$; $C_2 = 10011$
  - Size of intersection = 3; size of union = 4,
  - Jaccard similarity (not distance) = 3/4
  - Distance: $d(C_1,C_2) = 1 - (\text{Jaccard similarity}) = 1/4$

Task: Finding Similar Documents

- Goal: Given a large number (N in the millions or billions) of documents, find "near duplicate" pairs
- Applications:
  - Mirror websites, or approximate mirrors
    - Don't want to show both in search results
  - Similar news articles at many news sites
    - Cluster articles by "same story"
- Problems:
  - Many small pieces of one document can appear out of order in another
  - Documents are so large or so many that they cannot fit in main memory
  - Too many documents to compare all pairs

3 Essential Steps for Similar Docs

1. *Shingling:* Convert documents to sets

2. *Min-Hashing:* Convert large sets to short signatures, while preserving similarity

3. *Locality-Sensitive Hashing:* Focus on pairs of signatures likely to be from similar documents
   - Candidate pairs!

Problem for Today's Lecture

precision? Shingling

n

high time complexity: $O(n^2)$ ⇨ $O(n)$

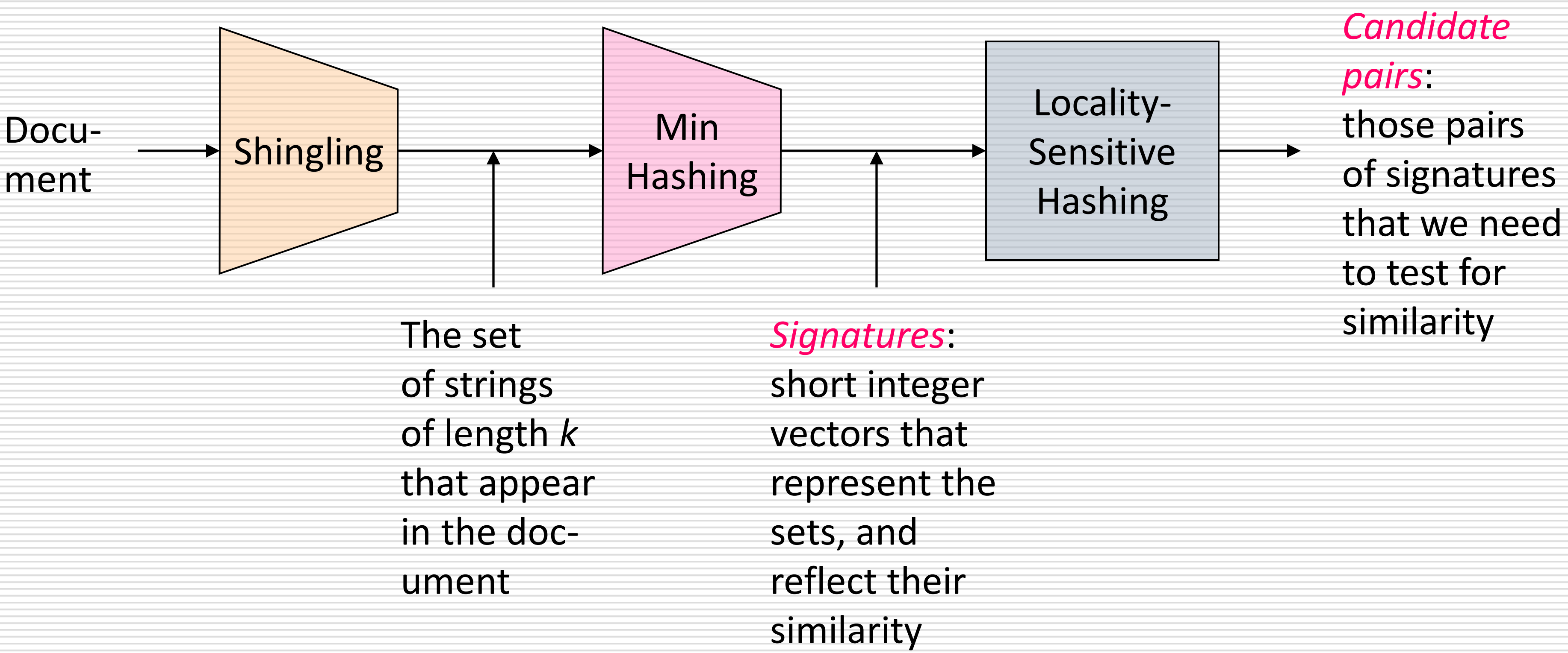
LSH

large space requirement: MinHash

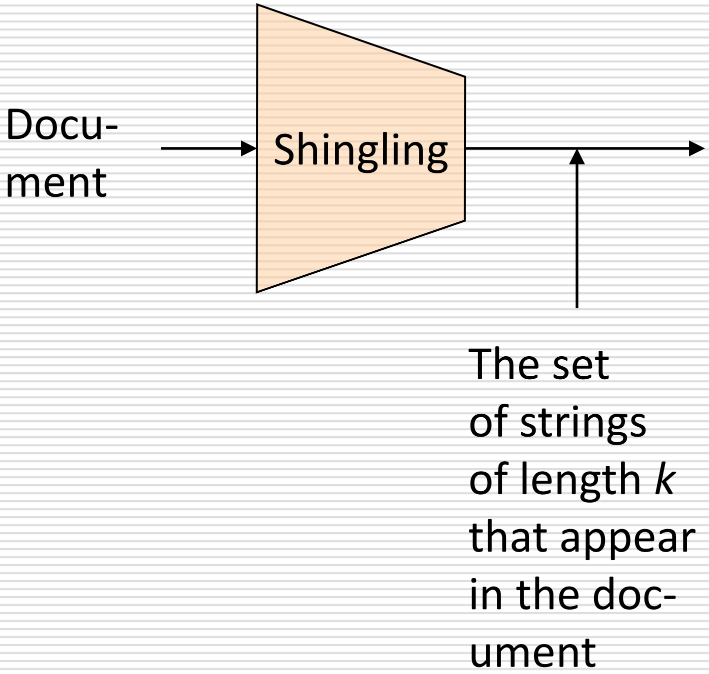
i

doc1	doc2		docN
1	1		1
0	1		1
1	1		1
0	1		1
0	0	......	0
1	1		1
0	1		1
1	1		1
0	0		0
......	......		......

The Big Picture



The Big Picture



- Step 1: *Shingling:* Convert documents to sets

Documents as High-Dim Data

- Step 1: *Shingling:* Convert documents to sets

- Simple approaches:
  - Document = set of words appearing in document
  - Document = set of "important" words
  - Don't work well for this application. Why?

- Need to account for ordering of words!
- A different way: Shingles!

Define: Shingles (Grams)

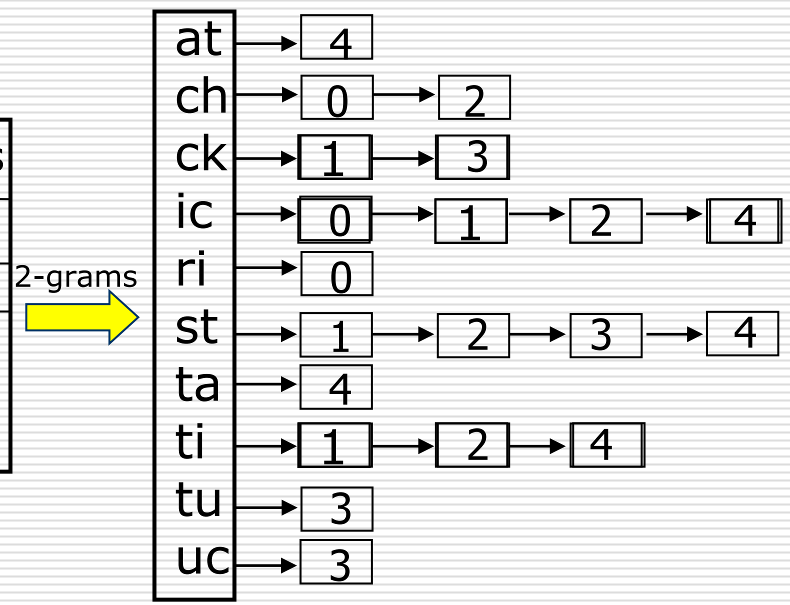
- A *k*-shingle (or *k*-gram) for a document is a sequence of *k* tokens that appears in the doc
  - Tokens can be characters, words or something else, depending on the application
  - Assume tokens = characters for examples

- Example: k=2; string S_1 = abcab
  Set of 2-shingles: $S(S_1)$ = {ab, bc, ca}

Define: Shingles (Grams)

- Application: similar string search

id	strings
0	rich
1	stick
2	stich
3	stuck
4	static



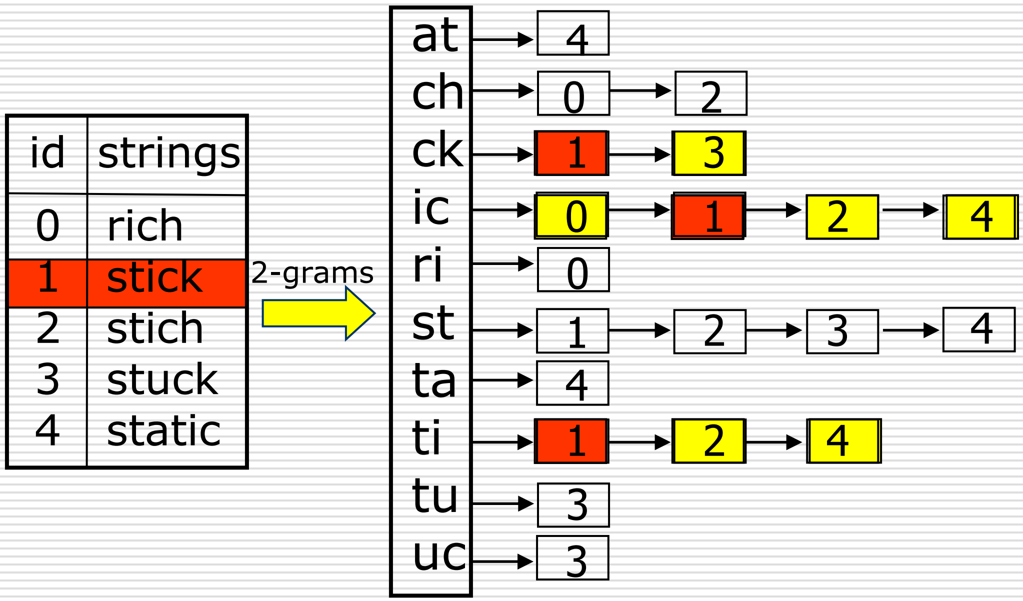
Inverted list

Define: Shingles (Grams)

- Application: similar string search

Query: “shtick”

sh ht ti ic ck



Inverted list

Compressing Shingles

- To compress long shingles, we can hash them to (say) 4 bytes
- Represent a document by the set of hash values of its *k*-shingles
  - Idea: Two documents could (rarely) appear to have shingles in common, when in fact only the hash-values were shared
- Example: k=2; document D_1= abcab
  Set of 2-shingles: $S(D_1)$ = {ab, bc, ca}
  Hash the singles: $h(D_1)$ = {1, 5, 7}

Similarity Metric for Shingles

- **Document D_1 is a set of its k-shingles $C_1=S(D_1)$**
- Equivalently, each document is a 0/1 vector in the space of *k*-shingles
  - Each unique shingle is a dimension
  - Vectors are very sparse

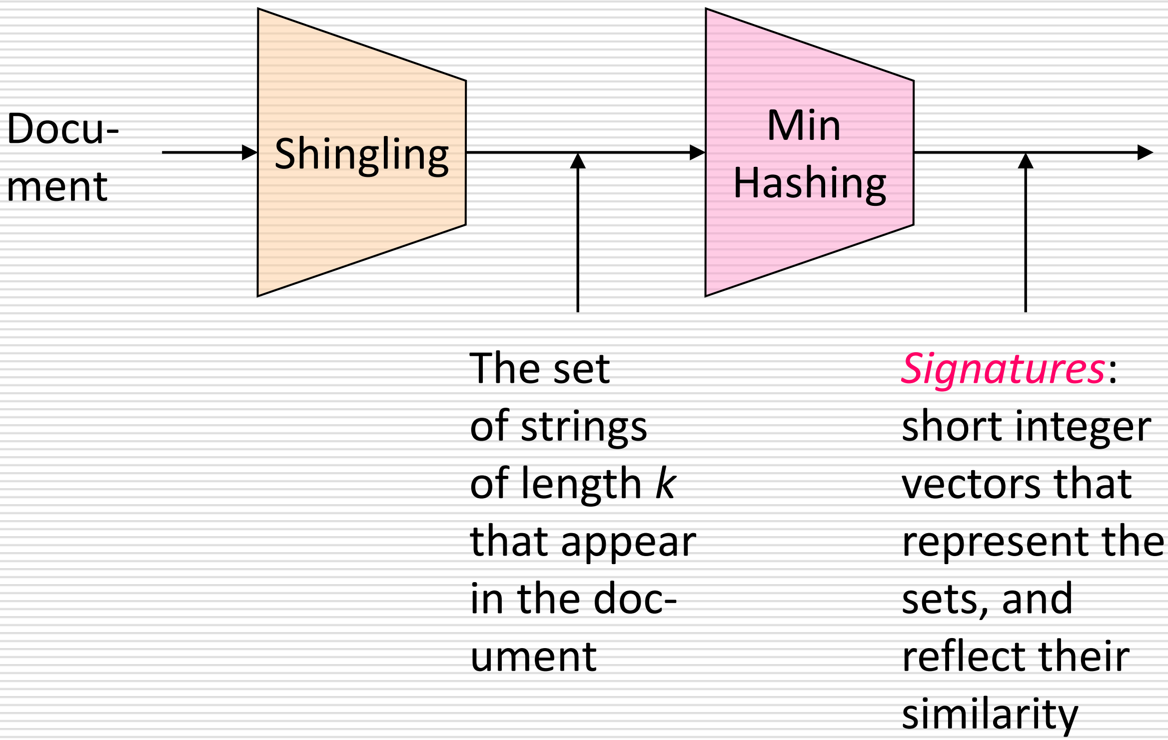
	doc1	*doc2*
go out	1	1
like travel	0	1
Every day	1	1
……	……	

Working Assumption

- Documents that have lots of shingles in common have similar text, even if the text appears in different order

- Caveat: You must pick k large enough, or most documents will have most shingles
  - $k = 5$ is OK for short documents
  - $k = 10$ is better for long documents

The Big Picture



- Step 2: *Minhashing:* Convert large sets to short signatures, while <u>preserving similarity</u>

Problem for Today's Lecture

precision? Shingling

large space requirement: MinHash

i

doc1	doc2		docN
1	1		1
0	1		1
1	1		1
0	1		1
0	0	......	0
1	1		1
0	1		1
1	1		1
0	0		0
......	......		......

From Sets to Boolean Matrices

- Rows = elements (shingles)
- Columns = sets (documents)
  - 1 in row *e* and column *s* if and only if *e* is a member of *s*
  - Column similarity is the Jaccard similarity of the corresponding sets (rows with value *1)*
  - Typical matrix is sparse!
- Each document is a column:
  - Example: sim(C_1 ,C_2) = ?
    - Size of intersection = 3; size of union = 6, Jaccard similarity (not distance) = 3/6
    - d(C_1,C_2) = 1 – (Jaccard similarity) = 3/6

Documents

Shingles				
	1	1	1	0
	1	1	0	1
	0	1	0	1
	0	0	0	1
	1	0	0	1
	1	1	1	0
	1	0	1	0
	C_1	**C_2**		

Outline: Finding Similar Columns

- So far:
  - Documents $\rightarrow$ Sets of shingles
  - Represent sets as boolean vectors in a matrix
- Next goal: Find similar columns while computing small signatures
  - Similarity of columns == similarity of signatures

Outline: Finding Similar Columns

- Next Goal: Find similar columns, Small signatures
- Naïve approach:
  - 1) Signatures of columns: small summaries of columns
  - 2) Examine pairs of signatures to find similar columns
    - Essential: Similarities of signatures and columns are related
  - 3) Optional: Check that columns with similar signatures are really similar

Hashing Columns (Signatures)

- Key idea: "hash" each column C to a small *signature* $h(C)$, such that:
  - (1) $h(C)$ is small enough that the signature fits in RAM
  - (2) $sim(C_1, C_2)$ is the same as the "similarity" of signatures $h(C_1)$ and $h(C_2)$

- Goal: Find a hash function $h(\cdot)$ such that:
  - If $sim(C_1,C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
  - If $sim(C_1,C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$

Min-Hashing

- Goal: Find a hash function $h(\cdot)$ such that:
  - if $sim(C_1,C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
  - if $sim(C_1,C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$

- Clearly, the hash function depends on the similarity metric:
  - Not all similarity metrics have a suitable hash function
- There is a suitable hash function for the Jaccard similarity: It is called Min-Hashing

Min-Hashing

- Imagine the rows of the boolean matrix permuted under random permutation π

- Define a "hash" function $h_\pi(C)$ = the index of the first (in the permuted order π) row in which column C has value 1:

$$h_\pi(C) = min_\pi\, \pi(C)$$

- Use several (e.g., 100) independent hash functions (that is, permutations) to create a signature of a column

Min-Hashing Example

Note: Another (equivalent) way is to store row indexes:

1	5	1	5
2	3	1	3
6	4	6	4

2nd element of the permutation is the first to map to a 1

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

4th element of the permutation is the first to map to a 1

Four Types of Rows

- Given cols C_1 and C_2, rows may be classified as:

	C_1	C_2
A	1	1
B	1	0
C	0	1
D	0	0

  - a = # rows of type A, etc.
- Note: $sim(C_1, C_2) = a/(a + b + c)$
- Then: $\Pr[h(C_1) = h(C_2)] = Sim(C_1, C_2)$
  - Look down the cols C_1 and C_2 until we see a 1
  - If it's a type-*A* row, then $h(C_1) = h(C_2)$
    If a type-*B* or type-*C* row, then not

The Min-Hash Property

0	0
0	0
1	1
0	0
0	1
1	0

- Choose a random permutation π
- Claim: $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
- Why?
  - Let X be a doc (set of shingles), $y \in X$ is a shingle
  - Then: $\Pr[\pi(y) = \min(\pi(X))] = 1/|X|$
    - It is equally likely that any $y \in X$ is mapped to the *min* element
  - Let y be s.t. $\pi(y) = \min(\pi(C_1 \cup C_2))$
  - Then either: $\pi(y) = \min(\pi(C_1))$ if $y \in C_1$, or
    $\pi(y) = \min(\pi(C_2))$ if $y \in C_2$
  - So the prob. that both are true is the prob. $y \in C_1 \cap C_2$
  - $\Pr[\min(\pi(C_1))=\min(\pi(C_2))]=|C_1 \cap C_2|/|C_1 \cup C_2| = sim(C_1, C_2)$

One of the two cols had to have 1 at position *y*

Similarity for Signatures

- We know: $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
- Now generalize to multiple hash functions
- The *similarity of two signatures* is the fraction of the hash functions in which they agree
- Note: Because of the Min-Hash property, the similarity of columns is the same as the expected similarity of their signatures

Min-Hashing Example

Permutation π | Input matrix (Shingles x Documents) | Signature matrix M

2	4	3	1	0	1	0
3	2	4	1	0	0	1
7	1	7	0	1	0	1
6	3	2	0	1	0	1
1	6	6	0	1	0	1
5	7	1	1	0	1	0
4	5	5	1	0	1	0

Signature matrix M

2	1	2	1
2	1	4	1
1	2	1	2

Similarities:

	1-3	2-4	1-2	3-4
Col/Col	0.75	0.75	0	0
Sig/Sig	0.67	1.00	0	0

Min-Hash Signatures

- Pick K=100 random permutations of the rows
- Think of *sig*(C) as a column vector
- *sig*(C)[i] = according to the *i*-th permutation, the index of the first row that has a 1 in column *C*

$$sig(C)[i] = \min(\pi_i(C))$$

- Note: The sketch (signature) of document *C* is small ~100 bytes!

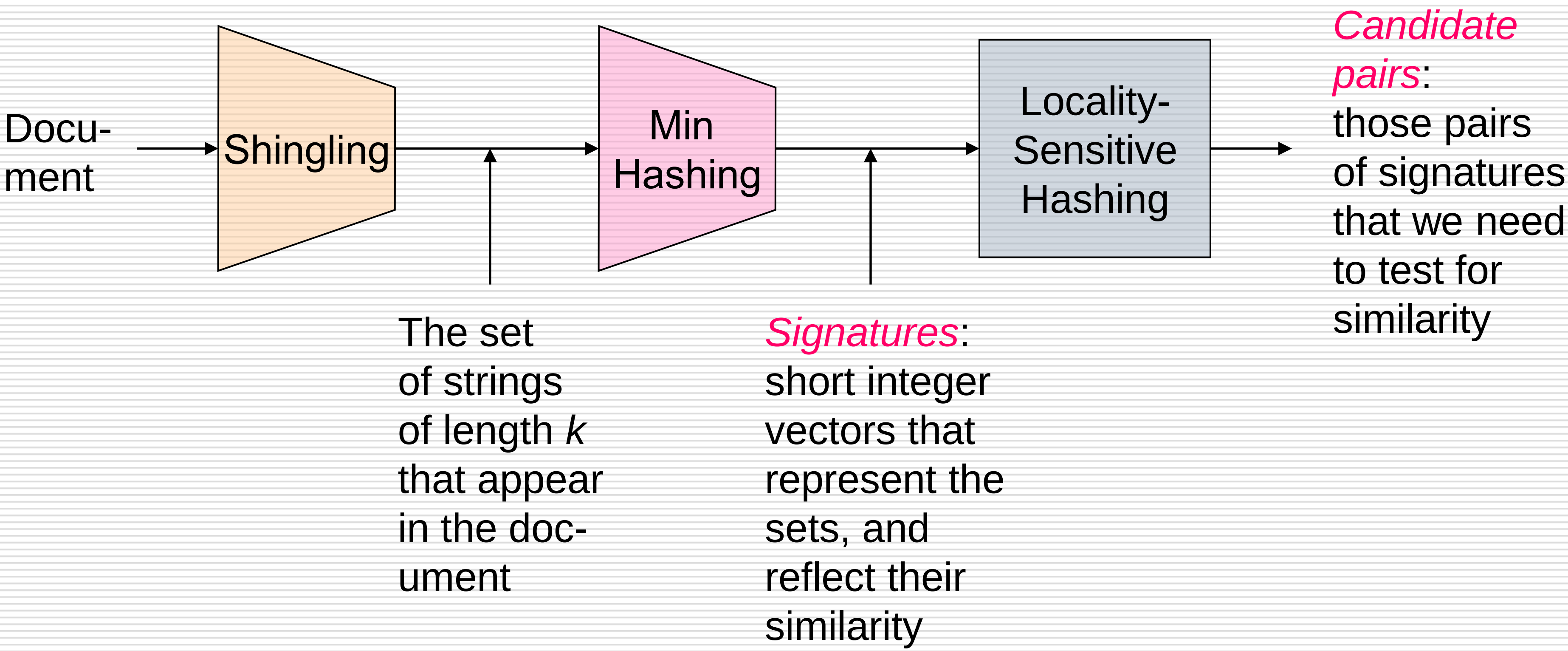
- We achieved our goal! We "compressed" long bit vectors into short signatures

Implementation Trick

- Permuting rows even once is prohibitive
- Row hashing!
  - Pick K = 100 hash functions k_i
  - Ordering under k_i gives a random row permutation!
- One-pass implementation
  - For each column C and hash-func. k_i keep a "slot" for the min-hash value
  - Initialize all $sig(C)[i] = \infty$
  - Scan rows looking for 1s
    - Suppose row j has 1 in column C
    - Then for each k_i :
      - If $k_i(j) < sig(C)[i]$, then $sig(C)[i] \leftarrow k_i(j)$

How to pick a random hash function h(x)?
Universal hashing:
$h_{a,b}(x)=((a \cdot x+b) \bmod p) \bmod N$
where:
a,b … random integers
p … prime number (p > N)

The Big Picture



- Step 3: *Locality-Sensitive Hashing:* Focus on pairs of signatures likely to be from similar documents

Motivation for LSH

- Suppose we need to find near-duplicate documents among $N = 1$ million documents

- Naïvely, we would have to compute pairwise Jaccard similarities for every pair of docs
  - $N(N-1)/2 \approx 5*10^{11}$ comparisons
  - At 10^5 secs/day and 10^6 comparisons/sec, it would take 5 days

- For $N = 10$ million, it takes more than a year…

Problem for Today's Lecture

precision? Shingling

n

high time complexity: O(n^2) ⇨ O(n)

LSH

doc1	doc2		docN
1	1		1
0	1		1
1	1		1
0	1		1
0	0	......	0
1	1		1
0	1		1
1	1		1
0	0		0
......	......		......

i

large space requirement: MinHash

LSH: First Cut

2	1	4	1
1	2	1	2
2	1	2	1

- Goal: Find documents with Jaccard similarity at least *s* (for some similarity threshold, e.g., s=0.8)

- LSH – General idea: Use a function *f(x,y)* that tells whether *x* and *y* is a *candidate pair:* a pair of elements whose similarity must be evaluated

- For Min-Hash matrices:
  - Hash columns of signature matrix *M* to many buckets
  - Each pair of documents that hashes into the same bucket is a candidate pair

Candidates from Min-Hash

2	1	4	1
1	2	1	2
2	1	2	1

- Pick a similarity threshold s ($0 < s < 1$)

- Columns x and y of M are a candidate pair if their signatures agree on at least fraction s of their rows:
  $M(i, x) = M(i, y)$ for at least frac. s values of i
  - We expect documents x and y to have the same (Jaccard) similarity as their signatures

LSH for Min-Hash

2	1	4	1
1	2	1	2
2	1	2	1

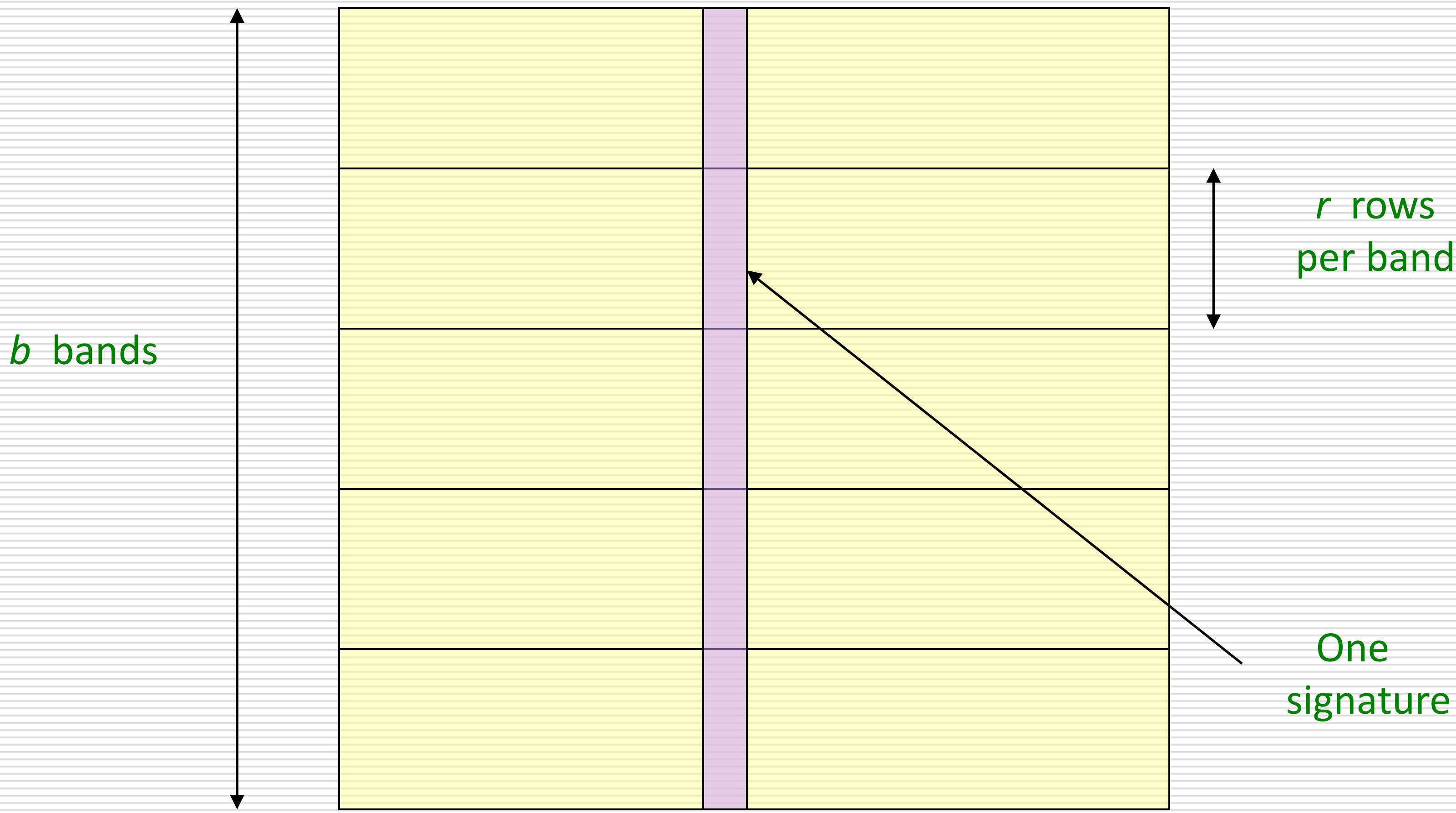
- Big idea: Hash columns of signature matrix *M* several times

- Arrange that (only) similar columns are likely to hash to the same bucket, with high probability

- Candidate pairs are those that hash to the same bucket

Partition *M* into *b* Bands

2	1	4	1
1	2	1	2
2	1	2	1

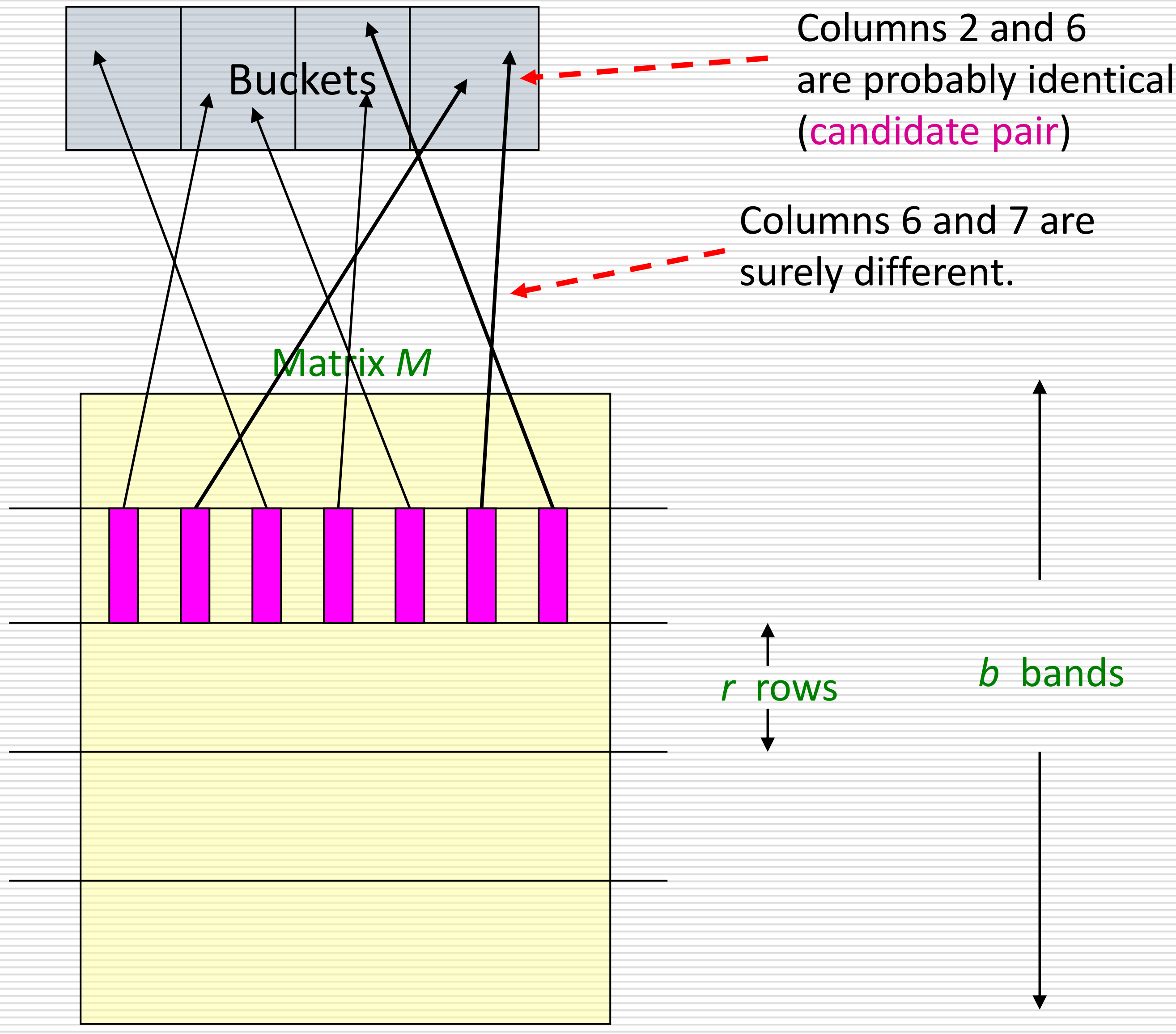


Signature matrix M

Partition M into Bands

- Divide matrix ***M*** into ***b*** bands of ***r*** rows
- For each band, hash its portion of each column to a hash table with ***k*** buckets
  - Make ***k*** as large as possible
- ***Candidate*** column pairs are those that hash to the same bucket for **≥ 1** band
- Tune ***b*** and ***r*** to catch most similar pairs, but few non-similar pairs

Hashing Bands



Simplifying Assumption

- There are enough buckets that columns are unlikely to hash to the same bucket unless they are identical in a particular band

- Hereafter, we assume that “same bucket” means “identical in that band”

- Assumption needed only to simplify analysis, not for correctness of algorithm

Example of Bands

2	1	4	1
1	2	1	2
2	1	2	1

Assume the following case:

- Suppose 100,000 columns of *M* (100k docs)
- Signatures of 100 integers (rows)
- Therefore, signatures take 40Mb
- Choose b = 20 bands of r = 5 integers/band

- Goal: Find pairs of documents that are at least $s = 0.8$ similar

C_1, C_2 are 80% Similar

2	1	4	1
1	2	1	2
2	1	2	1

- Find pairs of $\geq s$=0.8 similarity, set b=20, r=5
- Assume: sim(C_1, C_2) = 0.8
  - Since sim(C_1, C_2) $\geq$ s, we want C_1, C_2 to be a candidate pair: We want them to hash to at least 1 common bucket (at least one band is identical)
- Probability C_1, C_2 identical in one particular band: $(0.8)^5 = 0.328$
- Probability C_1, C_2 are *not* similar in all of the 20 bands: $(1-0.328)^{20} = 0.00035$
  - i.e., about 1/3000th of the 80%-similar column pairs are false negatives (we miss them)
  - We would find 99.965% pairs of truly similar documents

C_1, C_2 are 30% Similar

2	1	4	1
1	2	1	2
2	1	2	1

- Find pairs of $\geq s$=0.8 similarity, set b=20, r=5
- Assume: sim(C_1, C_2) = 0.3
  - Since sim(C_1, C_2) < s we want C_1, C_2 to hash to NO common buckets (all bands should be different)
- Probability C_1, C_2 identical in one particular band: $(0.3)^5 = 0.00243$
- Probability C_1, C_2 identical in at least 1 of 20 bands: $1 - (1 - 0.00243)^{20} = 0.0474$
  - In other words, approximately 4.74% pairs of docs with similarity 0.3% end up becoming candidate pairs
    - They are false positives since we will have to examine them (they are candidate pairs) but then it will turn out their similarity is below threshold s

LSH Involves a Tradeoff

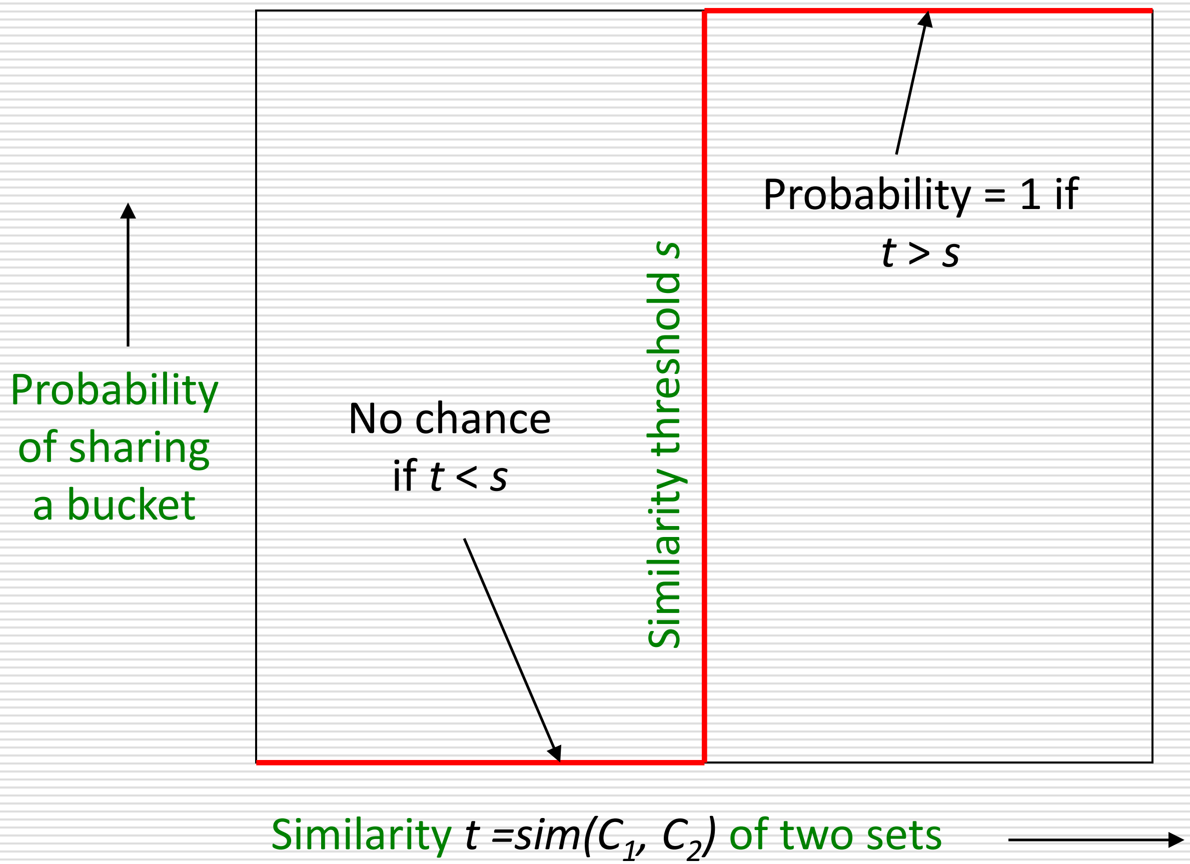
2	1	4	1
1	2	1	2
2	1	2	1

- Pick:
  - The number of Min-Hashes (rows of *M*)
  - The number of bands *b*, and
  - The number of rows *r* per band

  to balance false positives/negatives

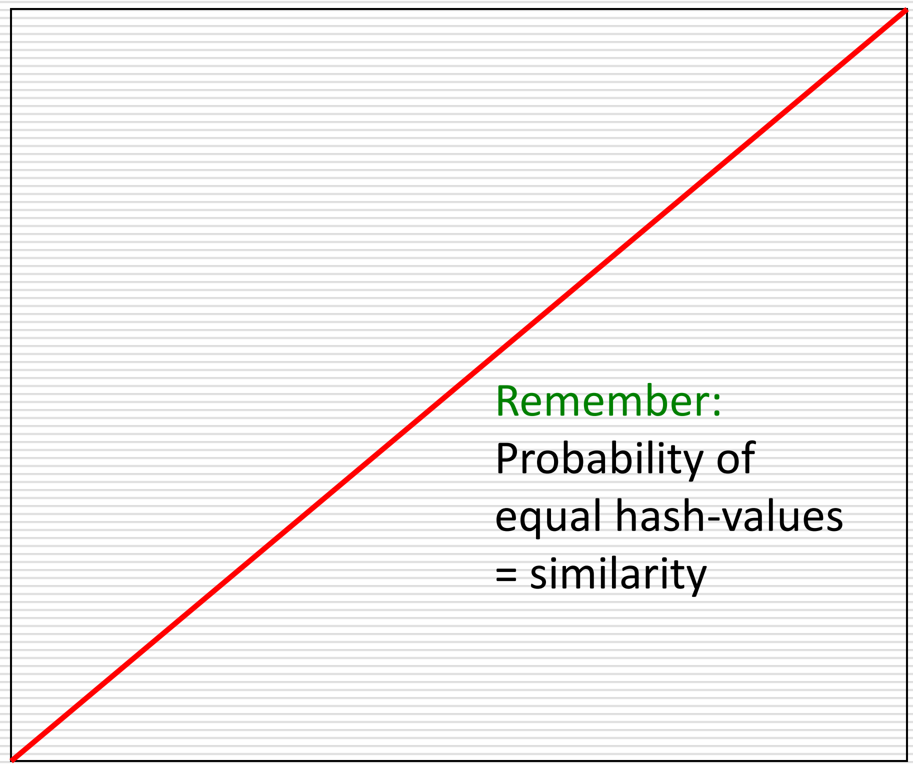
- Example: If we had only 15 bands of 5 rows, the number of false positives would go down, but the number of false negatives would go up

Analysis of LSH – What We Want



What 1 Band of 1 Row Gives You

Probability
of sharing
a bucket



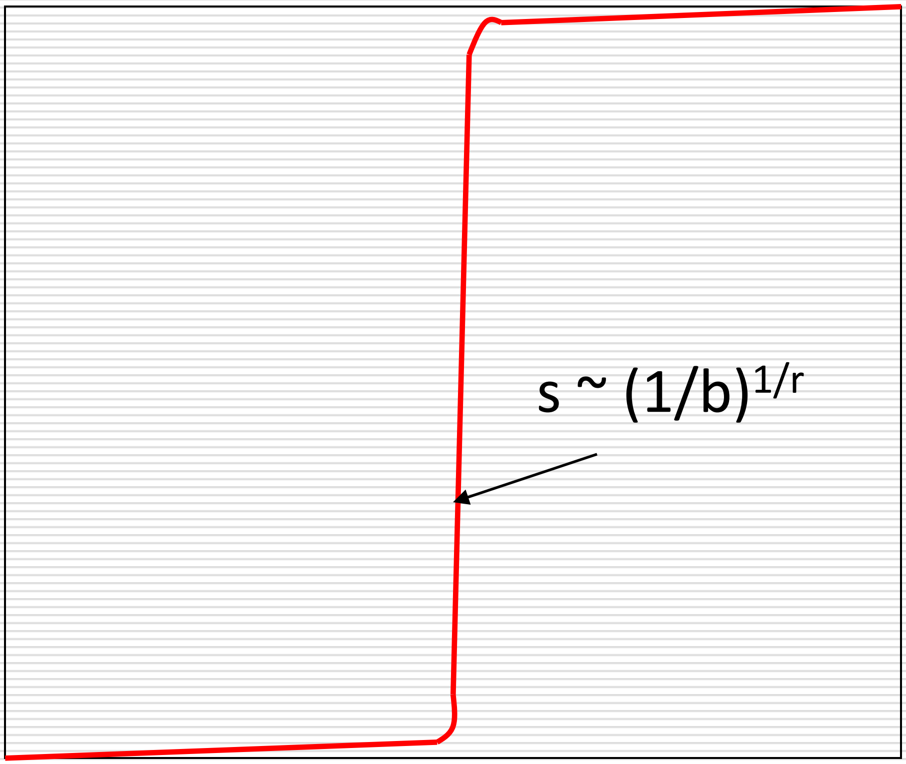
Similarity $t = sim(C_1, C_2)$ of two sets

b bands, r rows/band

- Columns C_1 and C_2 have similarity t
- Pick any band (r rows)
  - Prob. that all rows in band equal = t^r
  - Prob. that some row in band unequal = $1 - t^r$
- Prob. that no band identical = $(1 - t^r)^b$
- Prob. that at least 1 band identical = $1 - (1 - t^r)^b$

What b Bands of r Rows Gives You

Probability of sharing a bucket



Similarity $t=sim(C_1, C_2)$ of two sets

At least one band identical

No bands identical

$$1 - (1 - t^r)^b$$

Some row of a band unequal

All rows of a band are equal

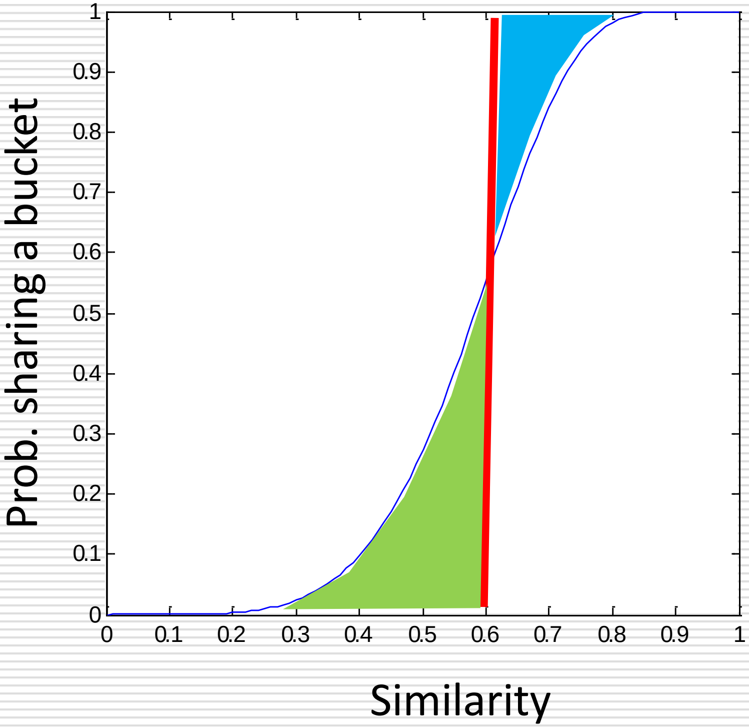
Example: $b = 20$; $r = 5$

- Similarity threshold s
- Prob. that at least 1 band is identical:

s	$1-(1-s^r)^b$
.2	.006
.3	.047
.4	.186
.5	.470
.6	.802
.7	.975
.8	.9996

Picking *r* and *b*: The S-curve

- Picking *r* and *b* to get the best S-curve
  - 50 hash-functions (r=5, b=10)



Blue area: False Negative rate
Green area: False Positive rate

LSH Summary

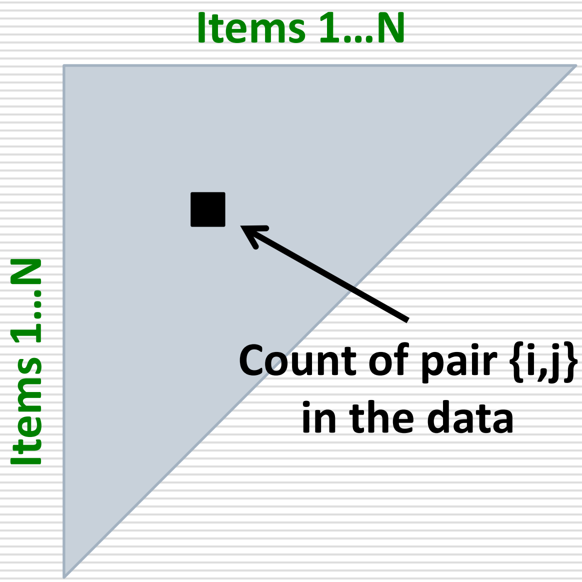
- Tune ***M, b, r*** to get almost all pairs with similar signatures, but eliminate most pairs that do not have similar signatures
- Check in main memory that **candidate pairs** really do have **similar signatures**
- **Optional:** In another pass through data, check that the remaining candidate pairs really represent similar documents

Summary: 3 Steps

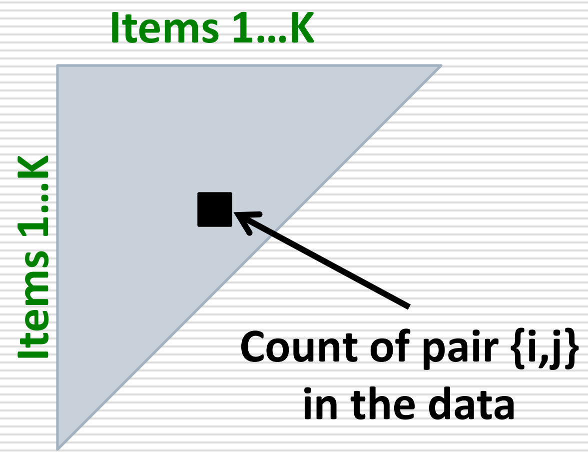
- **Shingling:** Convert documents to sets
  - We used hashing to assign each shingle an ID
- **Min-Hashing:** Convert large sets to short signatures, while preserving similarity
  - We used **similarity preserving hashing** to generate signatures with property $\mathbf{Pr[}h_\pi\mathbf{(C_1) = }h_\pi\mathbf{(C_2)] = }sim\mathbf{(C_1, C_2)}$
  - We used hashing to get around generating random permutations
- **Locality-Sensitive Hashing:** Focus on pairs of signatures likely to be from similar documents
  - We used hashing to find **candidate pairs** of similarity $\geq$ **s**

Relation to Previous Lecture

- Last time: Finding frequent pairs



Naïve solution:
Single pass but requires space quadratic in the number of items

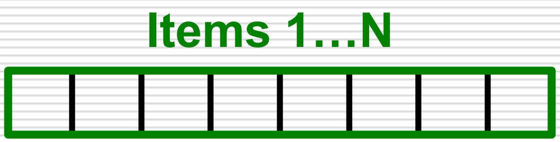


A-Priori:
<u>First pass:</u> Find frequent singletons
For a pair to be a frequent pair candidate, its singletons have to be frequent!
<u>Second pass:</u>
Count only candidate pairs!

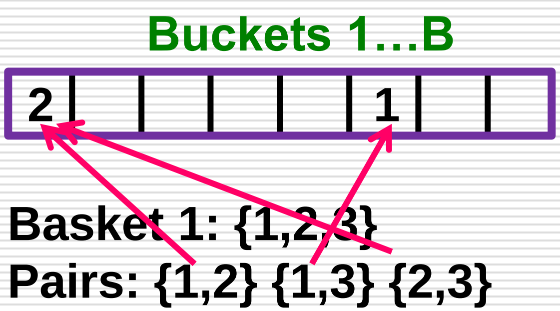
N … number of distinct items
K … number of items with support $\geq s$

Relation to Previous Lecture

- Last time: Finding frequent pairs
- Further improvement: PCY
  - Pass 1:
    - Count exact frequency of each item:

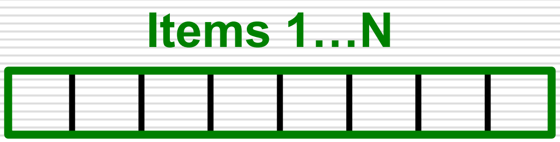


    - Take pairs of items {i,j}, hash them into *B* buckets and count of the number of pairs that hashed to each bucket:

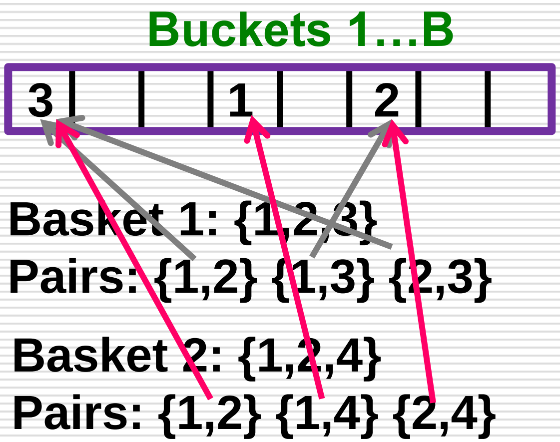


Relation to Previous Lecture

- Last time: Finding frequent pairs
- Further improvement: PCY
  - Pass 1:
    - Count exact frequency of each item:



    - Take pairs of items {i,j}, hash them into *B* buckets and count of the number of pairs that hashed to each bucket:



  - Pass 2:
    - For a pair {i,j} to be a candidate for a frequent pair, its singletons {i}, {j} have to be frequent and the pair has to hash to a frequent bucket!

Relation to Previous Lecture

- Last time: Finding frequent pairs

Previous lecture: A-Priori

Main idea: Candidates

Instead of keeping a count of each pair, only keep a count of candidate pairs!

Today's lecture: Find pairs of similar docs

Main idea: Candidates

-- Pass 1: Take documents and hash them to buckets such that <u>documents that are similar hash to the same bucket</u>

-- Pass 2: Only compare documents that are candidates (i.e., they hashed to a same bucket)

Benefits: Instead of $O(N^2)$ comparisons, we need $O(N)$ comparisons to find similar documents

Acknowledgement

- Slides are adapted from:
  - Prof. Jeffrey D. Ullman
  - Dr. Anand Rajaraman
  - Dr. Jure Leskovec