

# 大数据计算及应用(二)

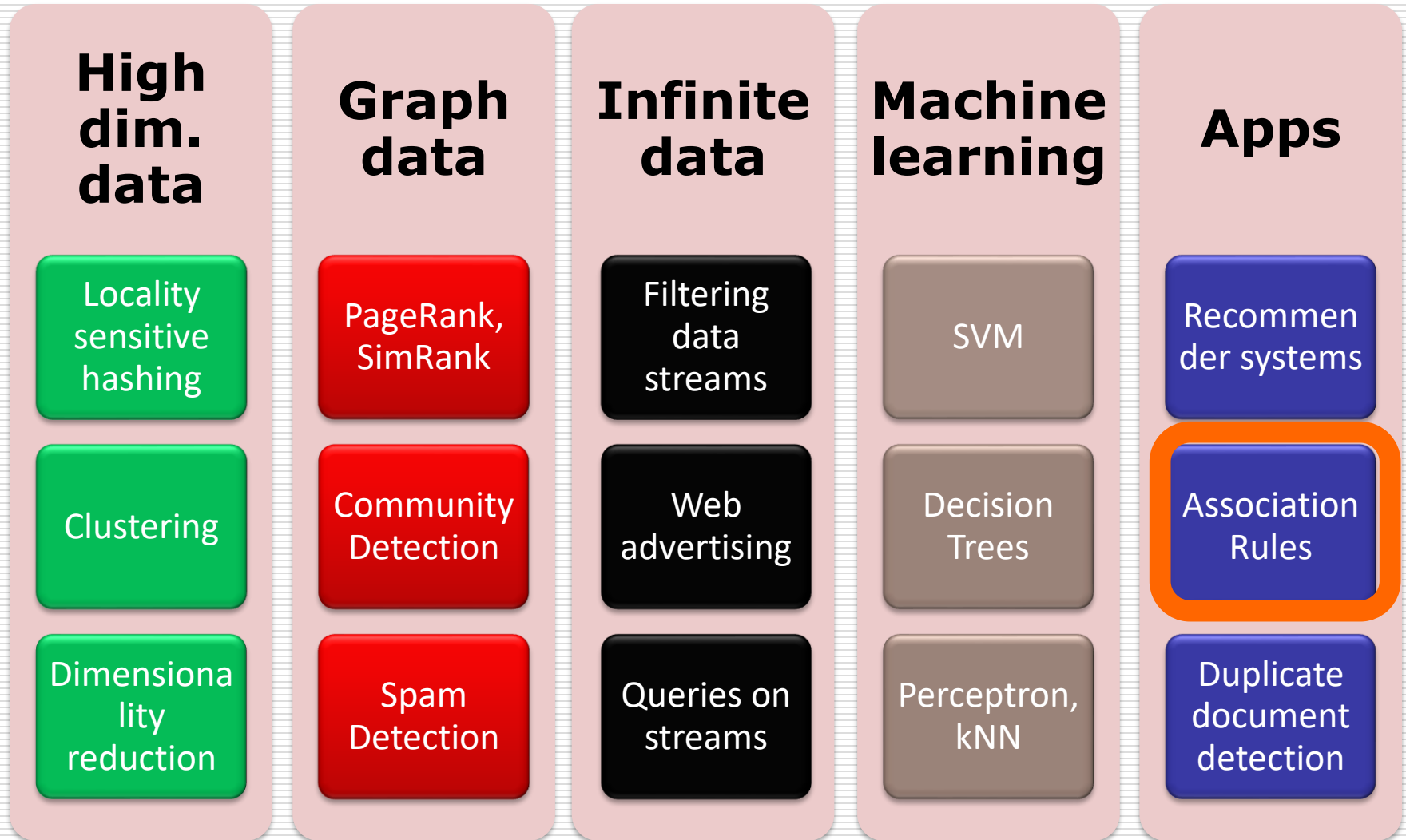
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## Association Rules and Frequent Pattern Mining

Slides adapted from <http://www.mmds.org>

# Agenda

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# Association Rule

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- Items frequently purchased together:

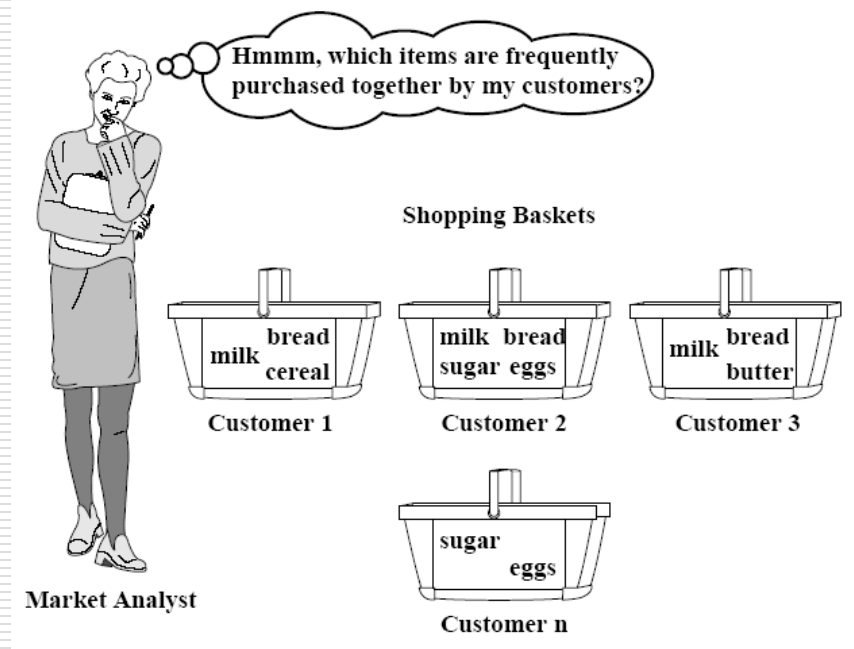
beer → diaper



- Uses:

- Placement
- Advertising
- Sales
- Coupons

- Objective: increase sales and reduce costs



# The Market-Basket Model

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- A large set of *items*, e.g., things sold in a supermarket
- A large set of *baskets*, each of which is a small set of the items, e.g., the things one customer buys on one day

<i>TID</i>	<i>Items</i>
1	Beer, Diaper, Milk
2	Coke, Diaper, Eggs
3	Beer, Coke, Diaper, Eggs
4	Coke, Eggs

# The Market-Basket Model

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- A general many-many mapping (association) between two kinds of things
  - But we ask about connections among “items” not “baskets”
- The technology focuses on **common events**, not rare events (“long tail”)

# Applications – (1)

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- **Items** = products; **baskets** = sets of products someone bought in one trip to the store
- **Example application**: given that many people buy beer and diapers together
  - Run a sale on diapers; raise price of beer
- Only useful if many buy diapers & beer

# Applications – (2)

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- **Items** = words; **Baskets** = Web pages;
- Unusual words appearing together in a large number of documents, e.g., “Brad” and “Angelina” may indicate an interesting relationship

# Applications – (3)

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- ❑ **Items** = sentences; **baskets** = documents containing those sentences
- ❑ Items that appear together too often could represent plagiarism



# Association Rule Mining Applications

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- ☐ Basket Data Analysis
- ☐ Genomic Data
- ☐ Telecommunication
- ☐ Credit Cards/ Banking Services
- ☐ Medical Treatments
- ☐ Web Personalization
- ☐ etc.

# Scale of the Problem

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- ❑ WalMart sells 100,000 items and can store billions of baskets
- ❑ The Web has billions of words and many billions of pages

# Some Definition - Support

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An itemset is **supported** by a basket (transaction) if it is included in the basket

## Market-Basket transactions

<i>TID</i>	<i>Items</i>
1	<b>Beer</b> , <b>Diaper</b> , Milk
2	Coke, Diaper, Eggs
3	<b>Beer</b> , Coke, <b>Diaper</b> , Eggs
4	Coke, Eggs

<Beer, Diaper> is **supported** by basket 1, and 3, and its support is  $2/4=50\%$ .

# Some Definition – Frequent Itemset

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If the support of an itemset exceeds user specified *min\_support* (threshold), this itemset is called a **frequent itemset (pattern)**.

## Market-Basket transactions

<i>TID</i>	<i>Items</i>
1	<b>Beer</b> , <b>Diaper</b> , Milk
2	Coke, Diaper, Eggs
3	<b>Beer</b> , Coke, <b>Diaper</b> , Eggs
4	Coke, Eggs

min\_support=50%

<Beer, Diaper> is a frequent itemset

<Beer, Milk> is not a frequent itemset

# Association Rules

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## □ Association Rules:

If-then rules about the contents of baskets

□  $\{i_1, i_2, \dots, i_k\} \rightarrow j$  means: “if a basket contains all of  $i_1, \dots, i_k$  then it is *likely* to contain  $j$ ”

□ In practice there are many rules, want to find significant/interesting ones!

□ *Confidence* of this association rule is the probability of  $j$  given  $I = \{i_1, \dots, i_k\}$

$$\text{conf}(I \rightarrow j) = \frac{\text{support}(I \cup j)}{\text{support}(I)}$$

# Example: Confidence

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$$T_1 = \{m, c, b\}$$

$$T_2 = \{m, p, j\}$$

$$T_3 = \{m, b\}$$

$$T_4 = \{c, j\}$$

$$T_5 = \{m, p, b\}$$

$$T_6 = \{m, c, b, j\}$$

$$T_7 = \{c, b, j\}$$

$$T_8 = \{b, c\}$$

□ Association rule:  $\{m, b\} \rightarrow c$

■  $\text{Support}(m, b) = 4/8$ ,  $\text{Support}(m, b, c) = 2/8$

■ **Confidence** =  $2/4 = 0.5$

# Association Rules Mining

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- Question: "find all association rules with support  $\geq s$  and confidence  $\geq c$ "
- Hard part: finding the frequent itemsets

# Computation Model

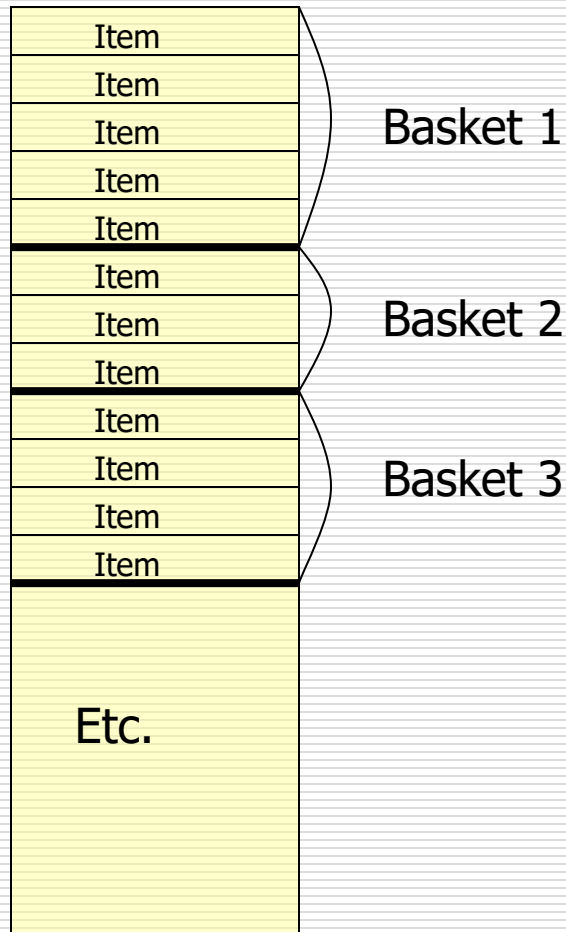
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- Typically, data is kept in flat files rather than in a database system
  - Stored on disk
  - Stored basket-by-basket
  - Expand baskets into pairs, triples, etc. as you read baskets



# File Organization

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**Example:** items are positive integers, and boundaries between baskets are  $-1$

# Computation Model – (2)

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- ❑ The true cost of mining disk-resident data is usually the **number of disk I/O's**
- ❑ In practice, association-rule algorithms read the data in *passes* – all baskets read in turn
- ❑ Thus, we measure the cost by the **number of passes** an algorithm takes

# Main-Memory Bottleneck

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- For many frequent-itemset algorithms, main memory is the critical resource
  - As we read baskets, we need to count something, e.g., occurrences of pairs
  - The number of different things we can count is limited by main memory
  - Swapping counts in/out is a disaster

# Finding Frequent Pairs

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- The hardest problem often turns out to be finding the frequent pairs
  - Why? Often frequent pairs are common, frequent triples are rare
    - Why? Probability of being frequent drops exponentially with size; number of sets grows more slowly with size
- We'll concentrate on pairs, then extend to larger sets

# Naïve Algorithm

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- ❑ Read file once, counting in main memory the occurrences of each pair
  - From each basket of  $n$  items, generate its  $n(n-1)/2$  pairs by two nested loops
- ❑ Fails if  $(\text{\#items})^2$  exceeds main memory
  - **Remember:** #items can be 100K (Wal-Mart) or 10B (Web pages)

## Example: Counting Pairs

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- ❑ Suppose  $10^5$  items
- ❑ Suppose counts are 4-byte integers
- ❑ Number of pairs of items:  $10^5(10^5-1)/2 = 5*10^9$  (approximately)
- ❑ Therefore,  $2*10^{10}$  (20 gigabytes) of main memory needed

# Details of Main-Memory Counting

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## □ Two approaches:

(1) Count all pairs, using a triangular matrix

- requires only 4 bytes/pair

always assume integers are 4 bytes

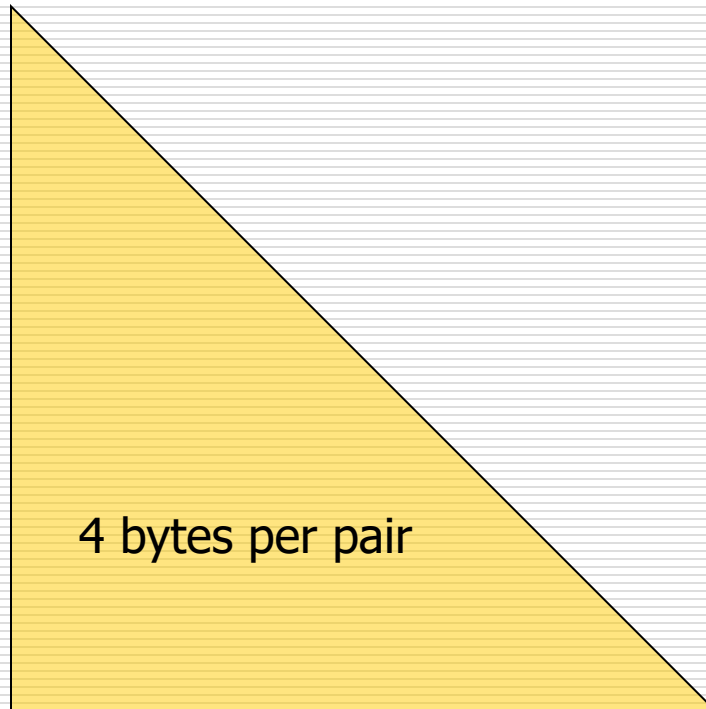
(2) Keep a table of triples  $[i, j, c]$  = "the count of the pair of items  $\{i, j\}$  is  $c$ "

- requires 12 bytes

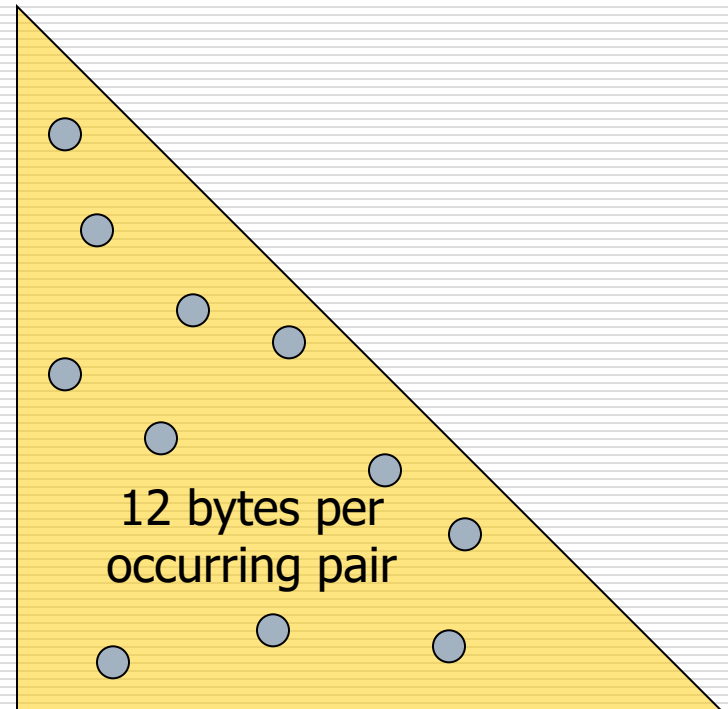
but only for those pairs with count  $> 0$

# Details of Main-Memory Counting

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Method (1)



Method (2)



# Comparing the Two Approaches

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## □ Approach 1: Triangular Matrix

- $n$  = total number of items
- Count pair of items  $\{i, j\}$  only if  $i < j$
- Keep pair counts in lexicographic order:
  - $\{1,2\}, \{1,3\}, \dots, \{1,n\}, \{2,3\}, \{2,4\}, \dots, \{2,n\}, \{3,4\}, \dots, \{n-1,n\}$
- Pair  $\{i, j\}$  is at position  $(i-1)(n-i/2) + j - i$
- Total number of pairs  $n(n-1)/2$ ; total bytes =  $2n^2$
- **Triangular Matrix** requires 4 bytes per pair

## □ Approach 2 uses **12 bytes** per occurring pair (*but only for pairs with count > 0*)

- Beats Approach 1 if less than **1/3** of possible pairs actually occur

# Comparing the Two Approaches

---

## □ Approach 1: Triangular Matrix

- $n$  = total number items
- Count pair of items  $\{i, j\}$  only if  $i < j$

**Problem is if we  
have too many  
items so the pairs  
do not fit into  
memory.**

□ Approach 2:  
(b)

- **Can we do better?**  
possible pairs actually occur

$s = 2n^2$

pair

# Outline

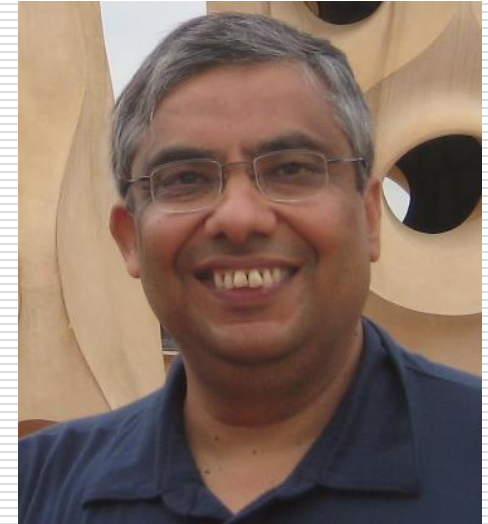
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- Association Rules
- Frequent Itemset Mining Algorithms
  - Apriori
  - FP-growth
- Sequential Pattern Mining Algorithms

# Apriori Algorithm

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- Proposed by Rakesh Agrawal [VLDB'94]
- Key idea:
  - Candidate generation-and-test
  - Anti-monotone property



<http://www.vldb.org> › conf PDF ⋮

## Fast Algorithms for Mining Association Rules - VLDB ...

by R Agrawal · Cited by 28692 — We consider the problem of discovering **association rules** between items in a large database of sales transactions. We present two new **algorithms** for...  
13 pages

# Apriori Algorithm – (1)

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- A two-pass approach called *Apriori* limits the need for main memory
- *Monotonicity* : if a set of items appears at least  $s$  times, so does every subset
  - *Contrapositive for pairs*: if item  $i$  does not appear in  $s$  baskets, then no pair including  $i$  can appear in  $s$  baskets

# Apriori Algorithm – (2)

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- **Pass 1:** Read baskets and count in main memory the occurrences of each item
  - Requires only memory proportional to #items
- Items that appear at least  $s$  times are the *frequent items*

# Apriori Algorithm – (3)

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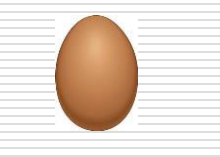
- **Pass 2:** Read baskets again and count in main memory only those pairs both of which were found in Pass 1 to be frequent
- Requires memory proportional to square of *frequent* items only (for counts), plus a list of the frequent items (so you know what must be counted)

# Apriori Algorithm

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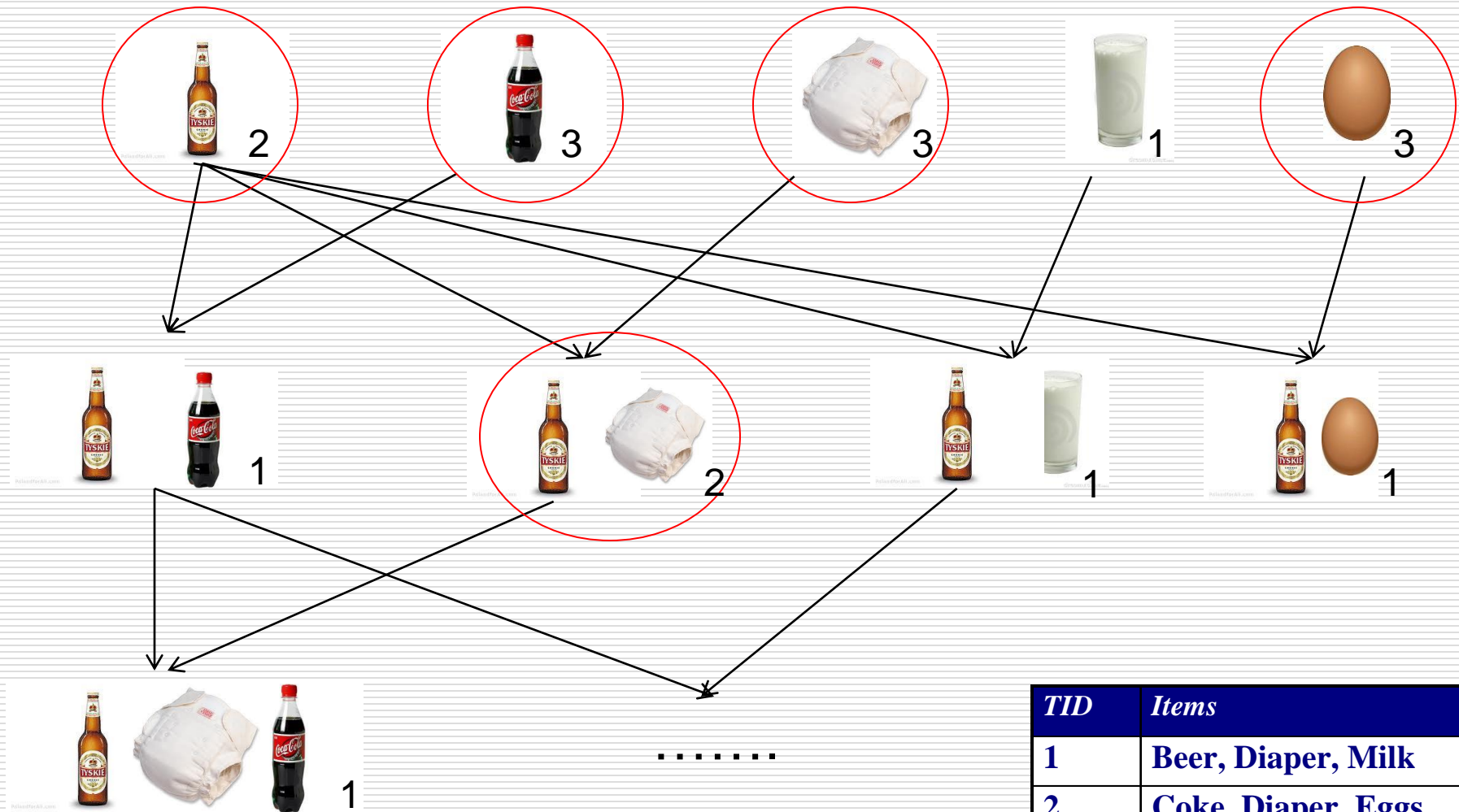
## Market-Basket transactions

<i>TID</i>	<i>Items</i>
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# Naive Algorithm

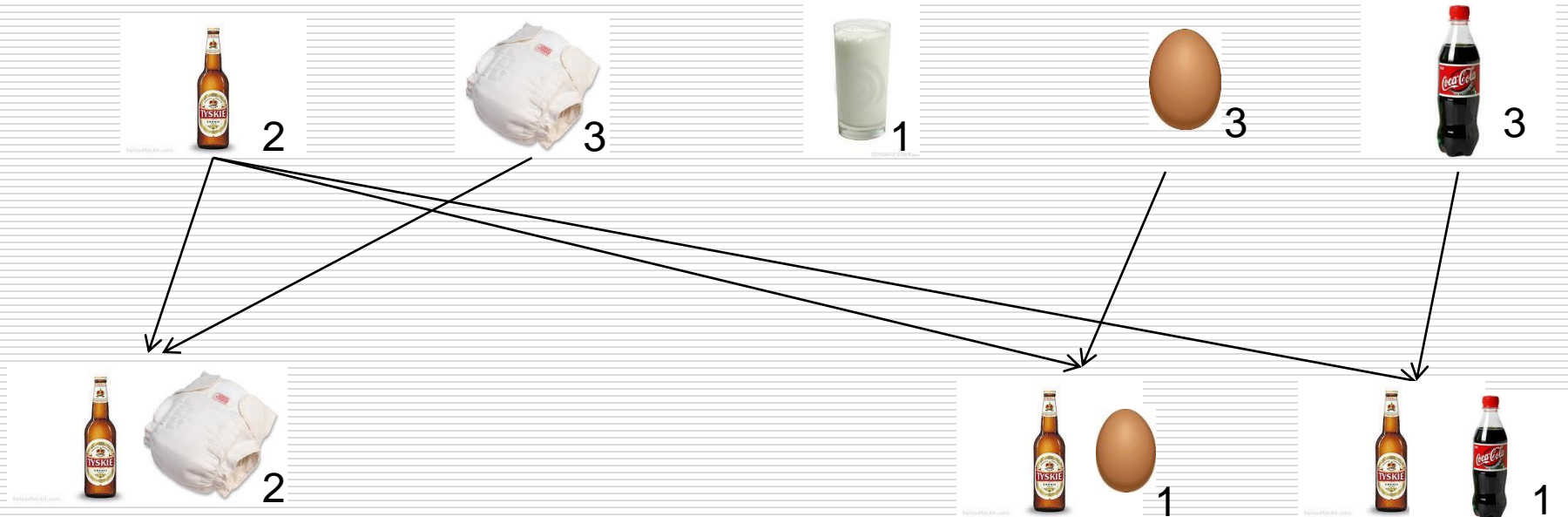


Sup<sub>min</sub>=2

TID	Items
1	Beer, Diaper, Milk
2	Coke, Diaper, Eggs
3	Beer, Coke, Diaper, Eggs
4	Coke, Eggs

# Apriori Algorithm

- Anti-monotone property: If an itemset is not frequent, then any of its superset is not frequent



$\text{Sup}_{\min}=2$

<i>TID</i>	<i>Items</i>
1	Beer, Diaper, Milk
2	Coke, Diaper, Eggs
3	Beer, Coke, Diaper, Eggs
4	Coke, Eggs

# Apriori Algorithm

$\text{Sup}_{\min} = 2$

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

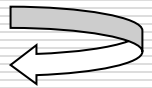
1<sup>st</sup> scan

$C_1$

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

$L_1$

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3



$C_2$

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

2<sup>nd</sup> scan

$C_2$

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

$L_2$

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2



$C_3$

Itemset
{B, C, E}

3<sup>rd</sup> scan

$L_3$

Itemset	sup
{B, C, E}	2

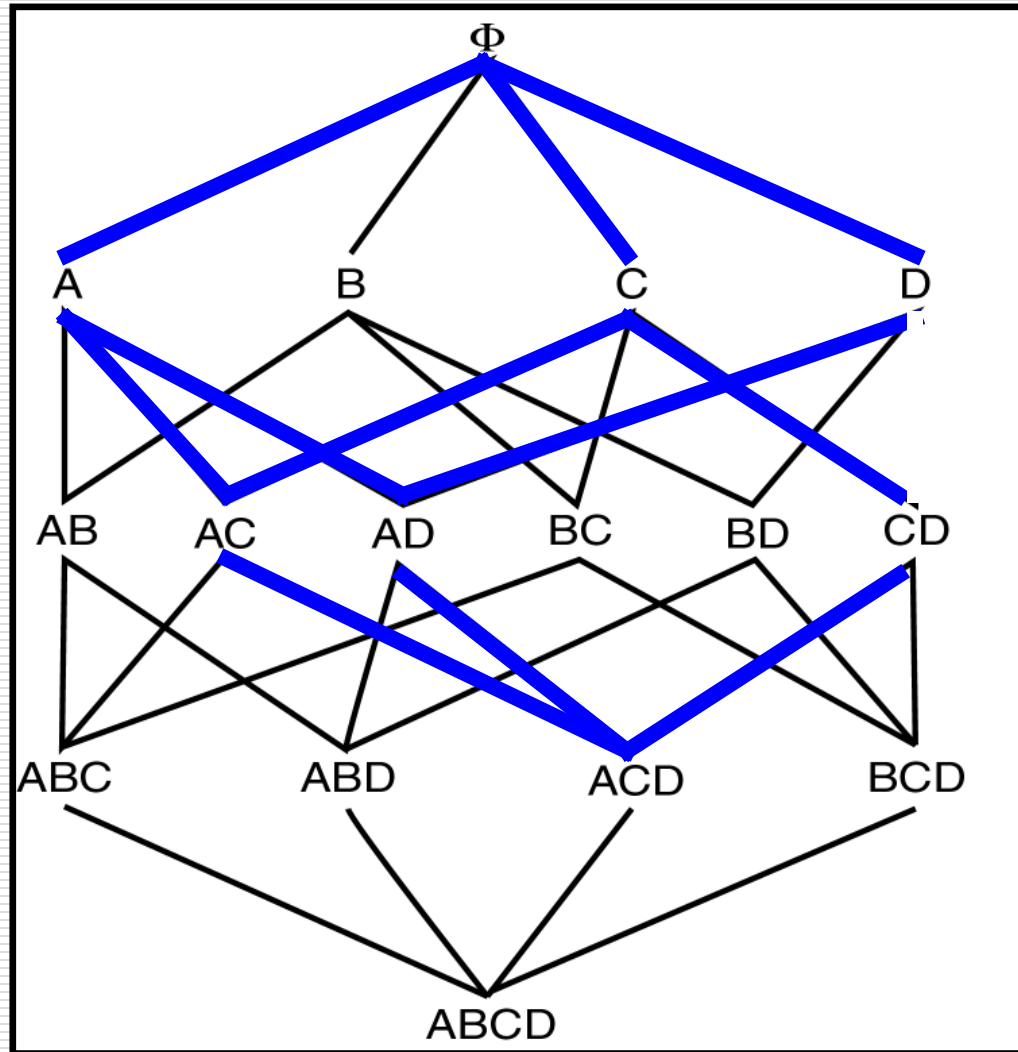
# Apriori Algorithm

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1.  $C_1$  = Itemsets of size one in  $I$ ;
2. Determine all large itemsets of size 1,  $L_1$ ;
3.  $i = 1$ ;
4. Repeat
5.      $i = i + 1$ ;
6.      $C_i$  = Apriori-Gen( $L_{i-1}$ );
7.     Count  $C_i$  to determine  $L_i$ ;
8. until no more large itemsets found;

# Frequent Itemset Property

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# Drawbacks of Apriori

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- Multiple scans of transaction database
  - Multiple database scans are **costly**
- Huge number of candidates
  - To find frequent itemset  $i_1 i_2 \dots i_{100}$ 
    - # of scans: **100**
    - # of Candidates:  $2^{100}-1 = 1.27*10^{30}$

# Improving Apriori: General Ideas

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- ❑ Reduce passes of transaction database scans
- ❑ Shrink number of candidates
- ❑ Facilitate support counting of candidates

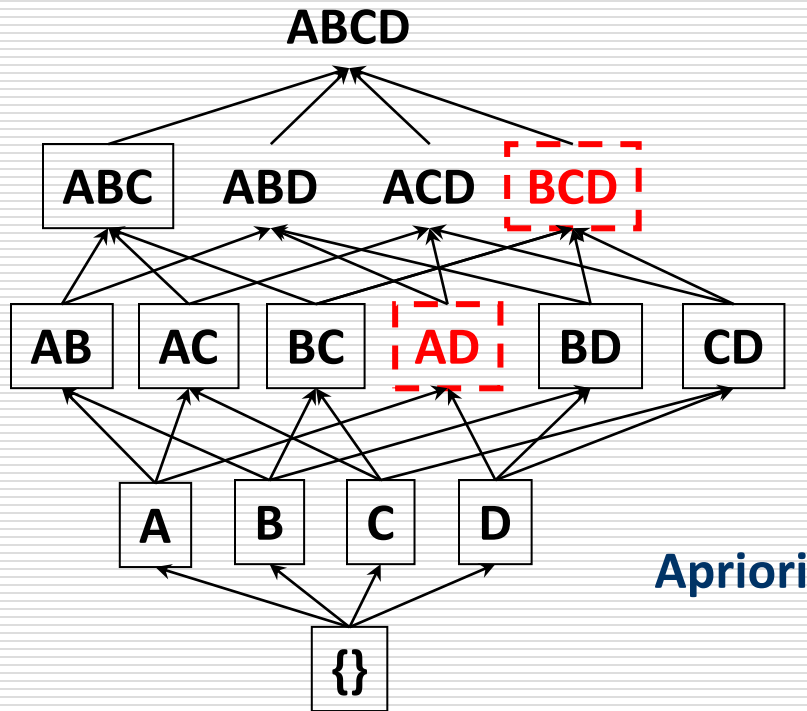
# Improving Apriori's Efficiency

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- ❑ **Hash-based itemset counting**: A  $k$ -itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
- ❑ **Transaction reduction**: A transaction that does not contain any frequent  $k$ -itemset is useless in subsequent scans
- ❑ **Partitioning**: Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB
- ❑ **Sampling**: mining on a subset of given data, need a lower support threshold + a method to determine the completeness
- ❑ **Dynamic itemset counting**: add new candidate itemsets immediately (unlike Apriori) when all of their subsets are estimated to be frequent



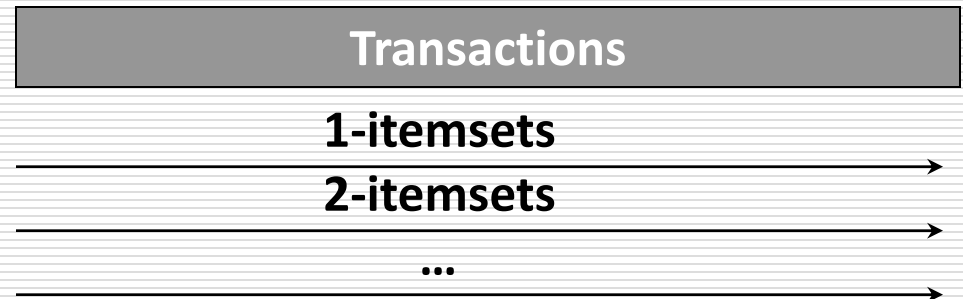
# DIC: Reduce Number of Scans



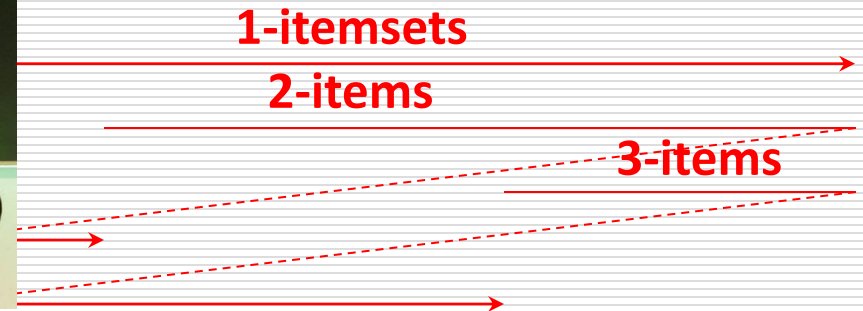
Itemset lattice

Apriori

- Once both A and D are determined frequent, the counting of AD begins
- Once all length-2 subsets of BCD are determined frequent, the counting of BCD begins



S. Brin, R. Motwani, J. Ullman, and S. Tsur. Dynamic itemset counting and implication rules for market basket data. In *SIGMOD'97*



# FP-growth Algorithm

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# FP-Growth

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- Proposed by Jiawei Han [*SIGMOD'00*]
- Uses the Apriori pruning principle
- Scan DB only twice
  - Once to find frequent 1-itemset (single item pattern)
  - Once to construct FP-tree (prefix tree, Trie), the data structure of FP-growth



# Mining Frequent Patterns without Candidate Generation

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- Compress a large database into a compact, Frequent-Pattern tree (FP-tree) structure
  - highly condensed, but complete for frequent pattern mining
  - avoid costly database scans
- Develop an efficient, FP-tree-based frequent pattern mining method
  - A divide-and-conquer methodology: decompose mining tasks into smaller ones
  - Avoid candidate generation: sub-database test only!

# FP-Growth

$\text{Sup}_{\min} = 2$

<i>TID</i>	<i>Items bought</i>
10	{f, a, c, d, g, i, m, p}
20	{a, b, c, f, l, m, o}
30	{b, f, h, j, o, w}
40	{b, c, k, s, p}
50	{a, f, c, e, l, p, m, n}

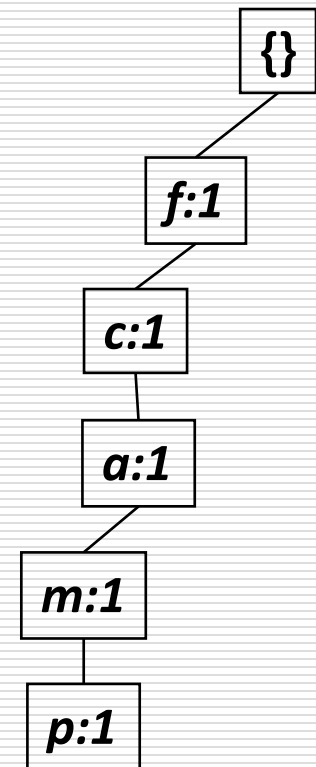
## Header Table

### *Item frequency*

<i>f</i>	4
<i>c</i>	4
<i>a</i>	3
<i>b</i>	3
<i>m</i>	3
<i>p</i>	3



<i>TID</i>	<i>(ordered) frequent items</i>
10	{f, c, a, m, p}
20	{f, c, a, b, m}
30	{f, b}
40	{c, b, p}
50	{f, c, a, m, p}



# FP-Growth

$\text{Sup}_{\min} = 2$

<i>TID</i>	<i>Items bought</i>
10	{f, a, c, d, g, i, m, p}
20	{a, b, c, f, l, m, o}
30	{b, f, h, j, o, w}
40	{b, c, k, s, p}
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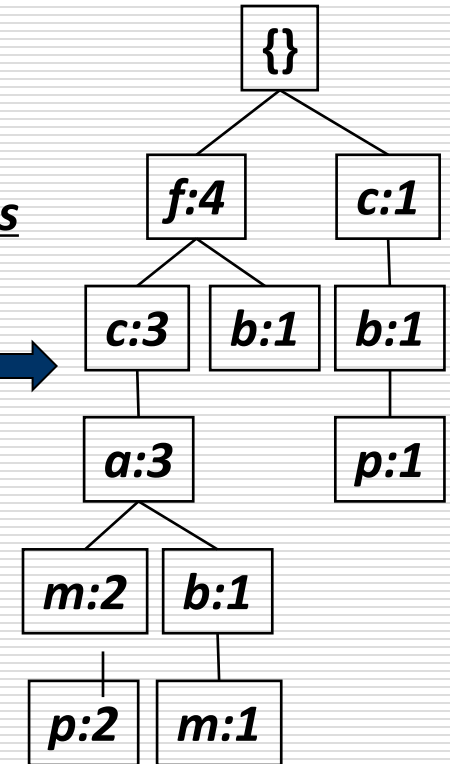
Header Table

*Item frequency*

<i>f</i>	4
<i>c</i>	4
<i>a</i>	3
<i>b</i>	3
<i>m</i>	3
<i>p</i>	3



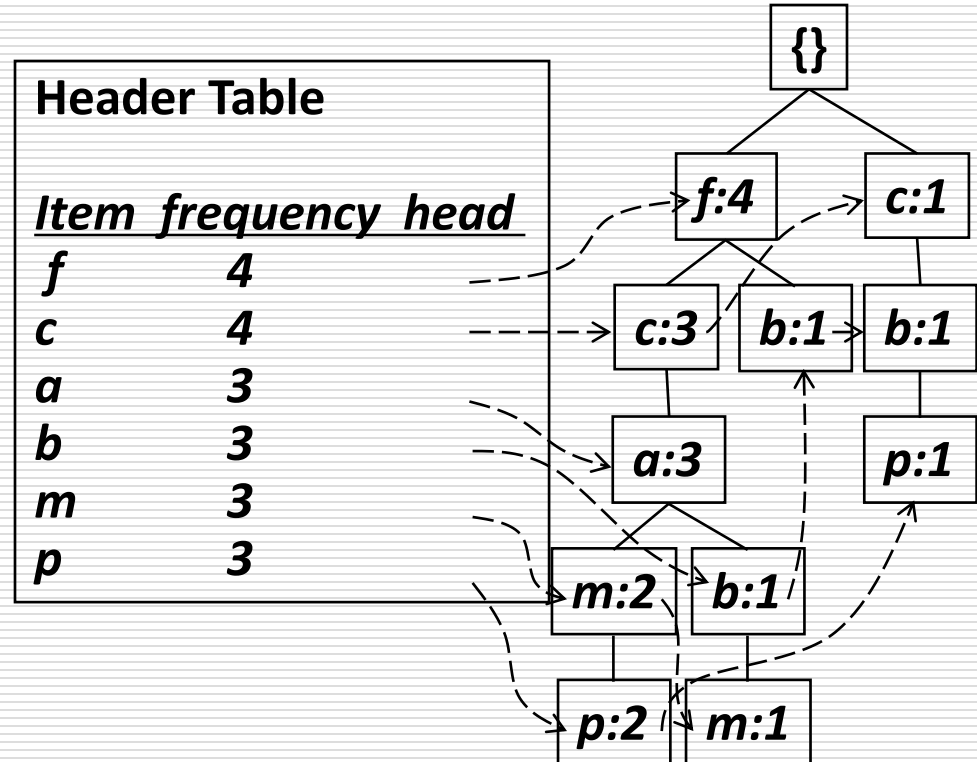
<i>TID</i>	<i>(ordered) frequent items</i>
10	{f, c, a, m, p}
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# FP-Growth

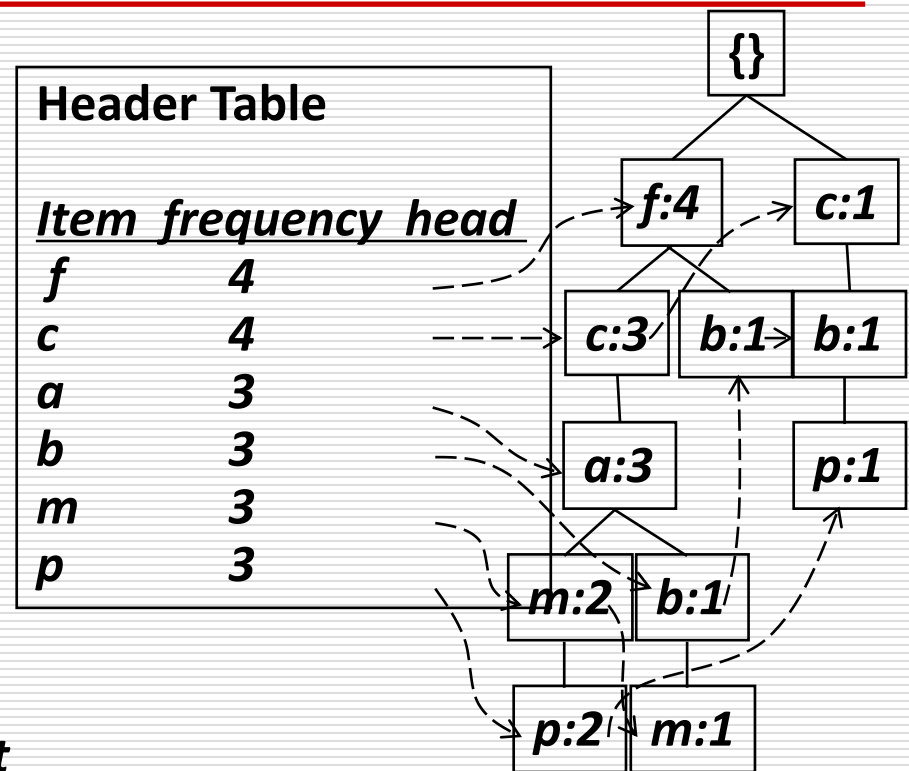
$$\text{Sup}_{\min} = 2$$

<i>TID</i>	<i>(ordered) frequent items</i>
10	{ <i>f</i> , <i>c</i> , <i>a</i> , <i>m</i> , <i>p</i> }
20	{ <i>f</i> , <i>c</i> , <i>a</i> , <i>b</i> , <i>m</i> }
30	{ <i>f</i> , <i>b</i> }
40	{ <i>c</i> , <i>b</i> , <i>p</i> }
50	{ <i>f</i> , <i>c</i> , <i>a</i> , <i>m</i> , <i>p</i> }



# FP-Growth

$$\text{Sup}_{\min} = 2$$



## Conditional pattern bases

Item   cond. pattern base   freq. itemset

<i>p</i>	<i>fcam:2, cb:1</i>	<i>fp, cp, ap, mp, fcp, fap, fmp, cap, cmp, amp, camp, facp, fcmp, famp, fcamp</i>
<i>m</i>	<i>fca:2, fcab:1</i>	<i>fm, cm, am, fcm, fam, cam, fcam</i>
<i>b</i>	<i>fca:1, f:1, c:1</i>	...
<i>a</i>	<i>fc:3</i>	...
<i>c</i>	<i>f:3</i>	...



# Why Is Frequent Pattern Growth Fast?

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- The performance study shows
  - FP-growth is faster than Apriori (in most cases), and is also faster than tree-projection (an order of magnitude on some datasets)
- Reasoning
  - No candidate generation (claimed by the authors)
  - Use compact data structure
  - Eliminate repeated database scan
  - Basic operation is counting and FP-tree building

# Extension of Association Rule Mining

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- ❑ Association rule mining has been extensively studied in the data mining community.
- ❑ There are many efficient algorithms and model variations.
- ❑ Other related work includes
  - Multi-level or generalized rule mining
  - Sequential pattern mining
  - Constrained rule mining
  - Incremental rule mining
  - Maximal and closed frequent itemset mining
  - Numeric association rule mining
  - Rule interestingness and visualization
  - Parallel algorithms
  - ...

# Extension of Association Rule Mining

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  - Numeric association rule mining
  - Rule interestingness and visualization
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  - ...

# Applications

Web Images Maps News Shopping Gmail more ▾

Google™ **sequence mining** Search Advanced Search Preferences

Web

**Sequence mining** - Wikipedia, the free encyclopedia  
Sequence mining is concerned with finding statistically relevant patterns between data examples where the values are delivered in a sequence. ...  
[en.wikipedia.org/wiki/Sequence\\_mining](http://en.wikipedia.org/wiki/Sequence_mining) - 15k - [Cached](#) - [Similar pages](#)

**Publications by Mohammed Javeed Zaki**  
Mohammed J. Zaki, **Sequence Mining** in Categorical Domains: Algorithms and ...  
Mohammed J. Zaki, **Parallel Sequence Mining** on Shared-Memory Machines, ...  
[www.cs.rpi.edu/~zaki/papers.html](http://www.cs.rpi.edu/~zaki/papers.html) - 66k - [Cached](#) - [Similar pages](#)

**[PDF] 7. Sequence Mining** ← **Clicked page**  
File Format: Microsoft Powerpoint - [View as HTML](#)  
**Sequence Mining**. Sequences and Strings. Recognition with Strings. MM & HMM.  
**Sequence Association Rules**. 7/03. **Data Mining** – Sequences ...  
[www.cs.wright.edu/~gdong/mining03/course\\_slides/C7Seq.ppt](http://www.cs.wright.edu/~gdong/mining03/course_slides/C7Seq.ppt) - [Similar pages](#)

**Incremental and Interactive Sequence Mining** - Parthasarathy ...  
Interactive **Sequence Discovery** by Incremental **Mining** - Ming-Yen Lin And (Correct) ... 0.3:  
**Parallel Sequence Mining** on Shared-Memory Machines - Zaki (2000) ...  
[citeseer.ist.psu.edu/235156.html](http://citeseer.ist.psu.edu/235156.html) - 25k - [Cached](#) - [Similar pages](#)

**Sequence Mining** in Categorical Domains: Incorporating Constraints ...  
We present cSPADE, an efficient algorithm for **mining** frequent sequences considering a variety of syntactic constraints.  
[citeseer.ist.psu.edu/616916.html](http://citeseer.ist.psu.edu/616916.html) - 24k - [Cached](#) - [Similar pages](#)  
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Google™ **sequential pattern mining** Search Advanced Search Preferences

Web

**SPAM: Sequential Pattern Mining** ← **Re-query**  
SPAM: **Sequential Pattern Mining**. SPAM is a new algorithm for finding all frequent sequences within a transactional database. The algorithm is especially ...  
[himalaya-tools.sourceforge.net/Spam/](http://himalaya-tools.sourceforge.net/Spam/) - 7k - [Cached](#) - [Similar pages](#)

**[PDF] Multi-dimensional Sequential Pattern Mining**  
File Format: PDF/Adobe Acrobat - [View as HTML](#)  
Multi-dimensional **Sequential Pattern Mining**. Helen Pinto. Jiawei Han. Jian Pei .....  
**SEQUENTIAL PATTERN MINING**. "m1Z@h"kn"r4j&o1l(mb f({Hm1r,h"ka(m1AkEnAo ...  
[www.sal.cs.uiuc.edu/~hanj/pdf/mdseq01.pdf](http://www.sal.cs.uiuc.edu/~hanj/pdf/mdseq01.pdf) - [Similar pages](#)

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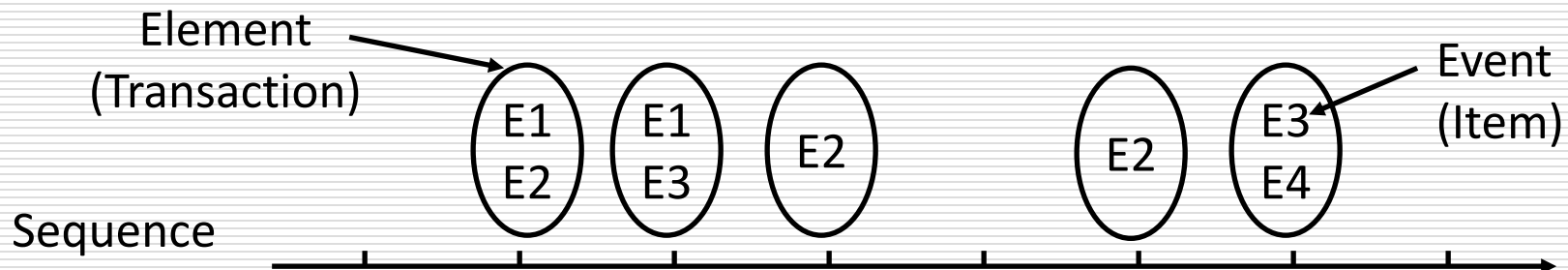
**Mining Sequential Patterns by Pattern-Growth: The PrefixSpan ...**  
**Sequential pattern mining** is an important data **mining** problem with broad applications. However, it is also a difficult problem since the **mining** may have to ...  
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**Mining Sequential Patterns** - Agrawal, Srikant (ResearchIndex)  
We introduce the problem of **mining sequential patterns** over such databases. We present three algorithms to solve this problem, and empirically evaluate ...  
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# Examples of Sequence Data

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Sequence Database	Sequence	Element (Transaction)	Event (Item)
Customer	Purchase history of a given customer	A set of items bought by a customer at time t	Books, diary products, CDs, etc
Web Data	Browsing activity of a particular Web visitor	A collection of files viewed by a Web visitor after a single mouse click	Home page, index page, contact info, etc
Event data	History of events generated by a given sensor	Events triggered by a sensor at time t	Types of alarms generated by sensors
Genome sequences	DNA sequence of a particular species	An element of the DNA sequence	Bases A,T,G,C



# Formal Definition of a Sequence

---

- A sequence is an ordered list of elements (transactions)

$$S = \langle e_1 e_2 e_3 \dots \rangle$$

- Each element contains a collection of items

$$e_i = \{i_1, i_2, \dots, i_k\}$$

- Each element is attributed to a specific time

- A k-sequence is a sequence that contains k items

# Formal Definition of a Subsequence

---

- A sequence  $\langle a_1 a_2 \dots a_n \rangle$  is contained in another sequence  $\langle b_1 b_2 \dots b_m \rangle$  ( $m \geq n$ ) if there exist integers  $i_1 < i_2 < \dots < i_n$  such that  $a_1 \subseteq b_{i_1}, a_2 \subseteq b_{i_2}, \dots, a_n \subseteq b_{i_n}$

Data sequence	Subsequence	Contain?
$\langle \{2,4\} \{3,5,6\} \{8\} \rangle$	$\langle \{2\} \{3,5\} \rangle$	Yes
$\langle \{1,2\} \{3,4\} \rangle$	$\langle \{1\} \{2\} \rangle$	No
$\langle \{2,4\} \{2,4\} \{2,5\} \rangle$	$\langle \{2\} \{4\} \rangle$	Yes

- The support of a subsequence  $w$  is defined as the fraction of data sequences that contain  $w$
- A *sequential pattern* is a frequent subsequence (i.e., a subsequence whose support is  $\geq \text{minsup}$ )

# Sequential Pattern Mining: Definition

---

## □ Given:

- a database of sequences
- a user-specified minimum support threshold, *minsup*

## □ Task:

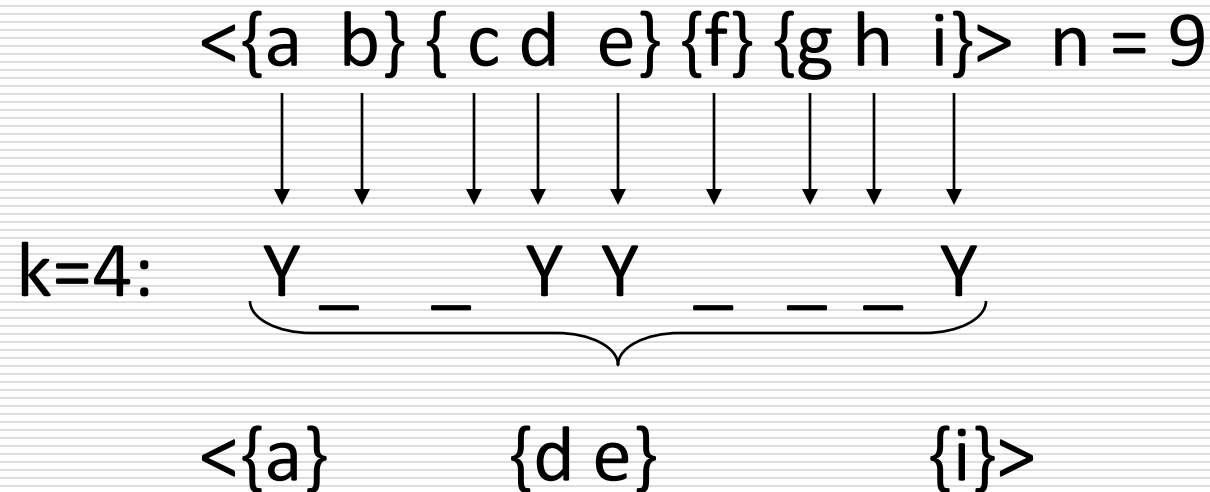
- Find all subsequences with support  $\geq \textit{minsup}$



# Sequential Pattern Mining: Challenge

---

- How many k-subsequences can be extracted from a given n-sequence?



Answer :

$$\binom{n}{k} = \binom{9}{4} = 126$$

# Outline

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- Association Rules
- Frequent Itemset Mining Algorithms
- Sequential Pattern Mining Algorithms
  - GSP
  - SPADE
  - SPAM

# GSP (Generalized Sequential Pattern Mining)

---

- ❑ Proposed by Srikant and Agrawal [*EDBT'96*]
- ❑ Uses the Apriori pruning principle

# Finding Length-1 Sequential Patterns

---

## □ Initial candidates:

■  $\langle a \rangle, \langle b \rangle, \langle c \rangle, \langle d \rangle, \langle e \rangle, \langle f \rangle, \langle g \rangle, \langle h \rangle$

## □ Scan database once, count support for candidates

$min\_sup = 2$

Seq. ID	Sequence
10	$\langle (bd)cb(ac) \rangle$
20	$\langle (bf)(ce)b(fg) \rangle$
30	$\langle (ah)(bf)abf \rangle$
40	$\langle (be)(ce)d \rangle$
50	$\langle a(bd)bcb(ade) \rangle$

Cand	Sup
$\langle a \rangle$	3
$\langle b \rangle$	5
$\langle c \rangle$	4
$\langle d \rangle$	3
$\langle e \rangle$	3
$\langle f \rangle$	2
<del><math>\langle g \rangle</math></del>	1
<del><math>\langle h \rangle</math></del>	1

# Generating Length-2 Candidates

---

51 length-2  
Candidates

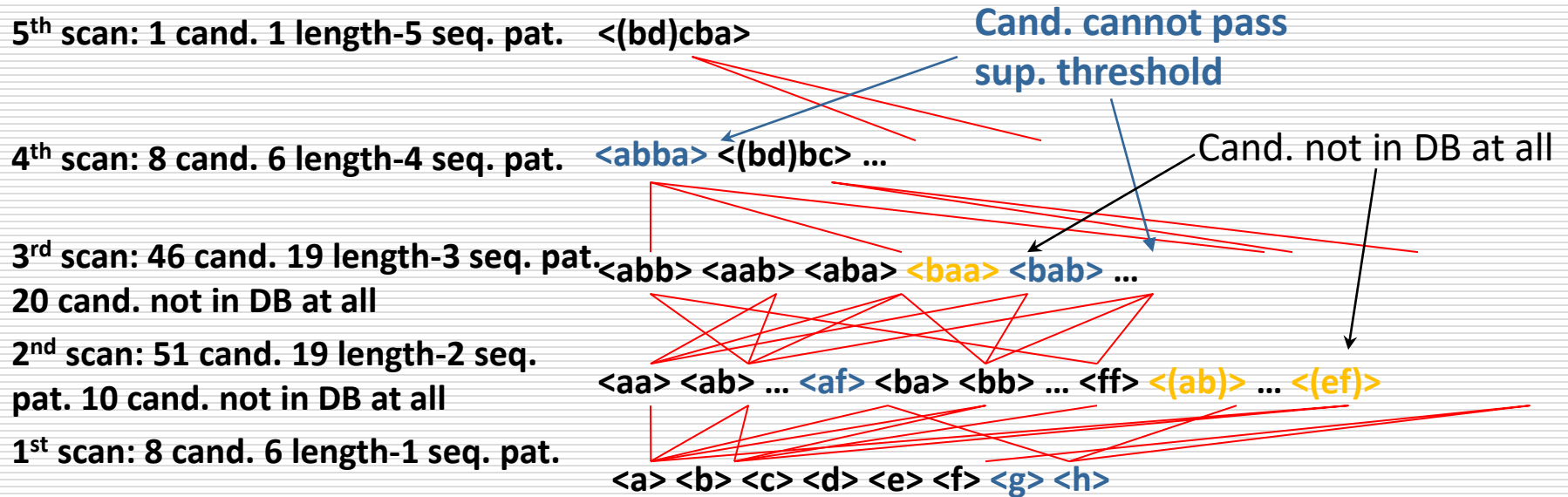
	<a>	<b>	<c>	<d>	<e>	<f>
<a>	<aa>	<ab>	<ac>	<ad>	<ae>	<af>
<b>	<ba>	<bb>	<bc>	<bd>	<be>	<bf>
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	<a>	<b>	<c>	<d>	<e>	<f>
<a>		<(ab)>	<(ac)>	<(ad)>	<(ae)>	<(af)>
<b>			<(bc)>	<(bd)>	<(be)>	<(bf)>
<c>				<(cd)>	<(ce)>	<(cf)>
<d>					<(de)>	<(df)>
<e>						<(ef)>
<f>						

Without Apriori  
property,  
 $8*8+8*7/2=92$   
candidates

Apriori prunes  
44.57%  
candidates

# GSP Mining Process



*min\_sup* = 2

Seq. ID	Sequence
10	<(bd)cb(ac)>
20	<(bf)(ce)b(fg)>
30	<(ah)(bf)abf>
40	<(be)(ce)d>
50	<a(bd)bcb(ade)>

# GSP Algorithm

---

- ❑ Take sequences in form of  $\langle x \rangle$  as length-1 candidates
- ❑ Scan database once, find  $F_1$ , the set of length-1 sequential patterns
- ❑ Let  $k=1$ ; while  $F_k$  is not empty do
  - Form  $C_{k+1}$ , the set of length- $(k+1)$  candidates from  $F_k$ ;
  - If  $C_{k+1}$  is not empty, scan database once, find  $F_{k+1}$ , the set of length- $(k+1)$  sequential patterns
  - Let  $k=k+1$ ;

# GSP Algorithm

---

- Benefits from the Apriori pruning
  - Reduces search space
- Bottlenecks
  - Scans the database multiple times
  - Generates a huge set of candidate sequences



# SPADE Algorithm

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# SPADE Algorithm

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- ❑ Proposed by Zaki *et al.* [*MLJ'01*]
- ❑ Candidate generation-and-test
- ❑ Vertical ID-list data representation based on Lattice-theory
- ❑ Counting support through temporal joins
- ❑ Reduced I/O costs (three DB scans)

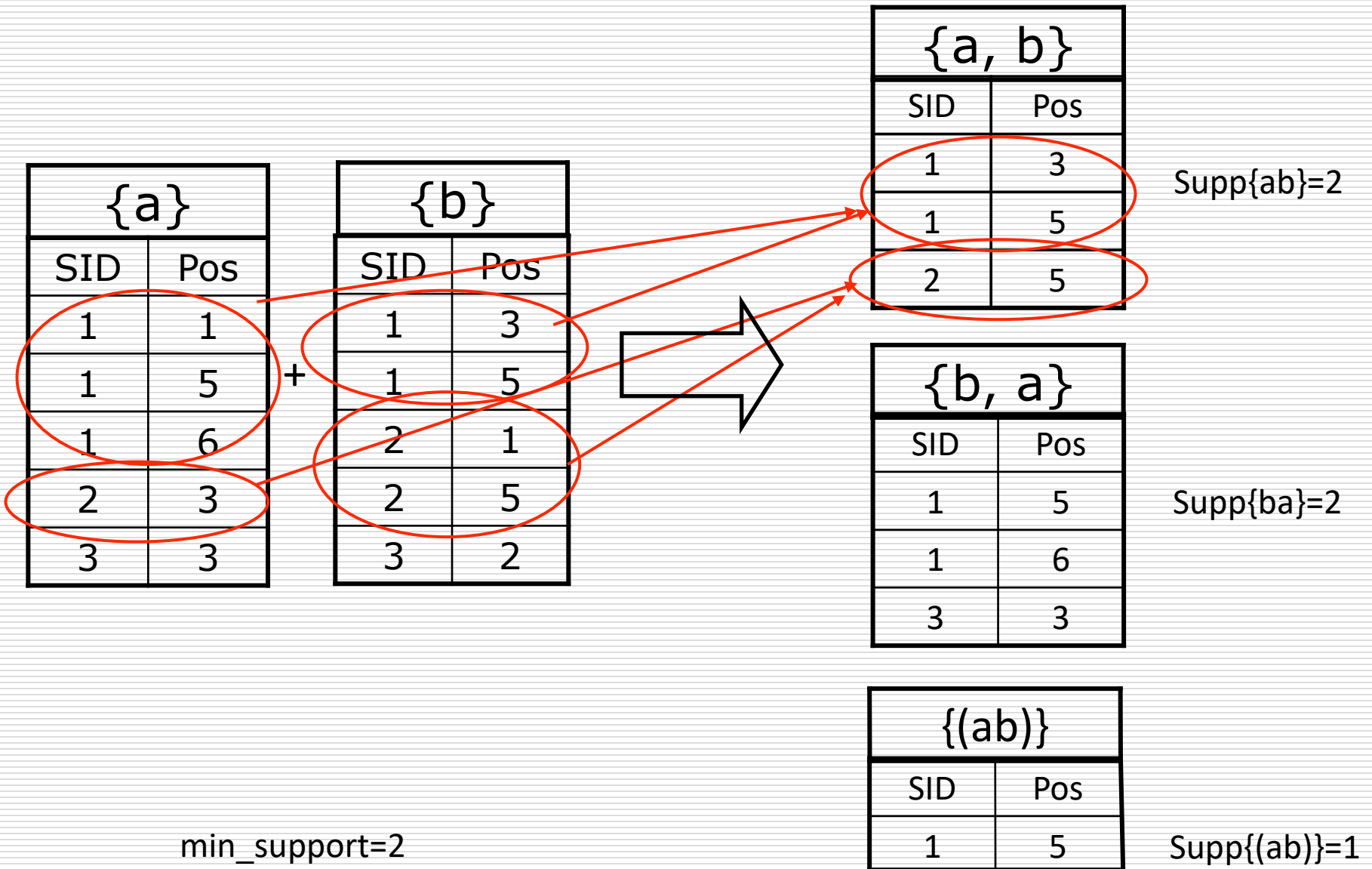
# SPADE Algorithm

ID	Data Sequence
1	<a c (b c) d (a b c) a d >
2	<b (c d) a c (b d) >
3	<d (b c) (a c) (c d) >

ID-List DB

a		b		c		d	
SID	Pos	SID	Pos	SID	Pos	SID	Pos
1	1	1	3	1	2	1	4
1	5	1	5	1	3	1	7
1	6	2	1	1	5	2	2
2	3	2	5	2	2	2	5
3	3	3	2	2	4	3	1
				3	2	3	4
				3	3		
				3	4		

# Temporal Joins



# SPAM Algorithm

---

# SPAM Algorithm

---

- ❑ Proposed by Ayres *et al.* [*KDD'02*]
- ❑ Key idea based on SPADE
- ❑ Using bitmap data representation
- ❑ Faster than SPADE yet space consuming

# SPAM Algorithm

---

ID	Data Sequence
10	<a c (b c) d (a b c) a d>
20	<b (c d) a c (b d)>
30	<d (b c) (a c) (c d)>

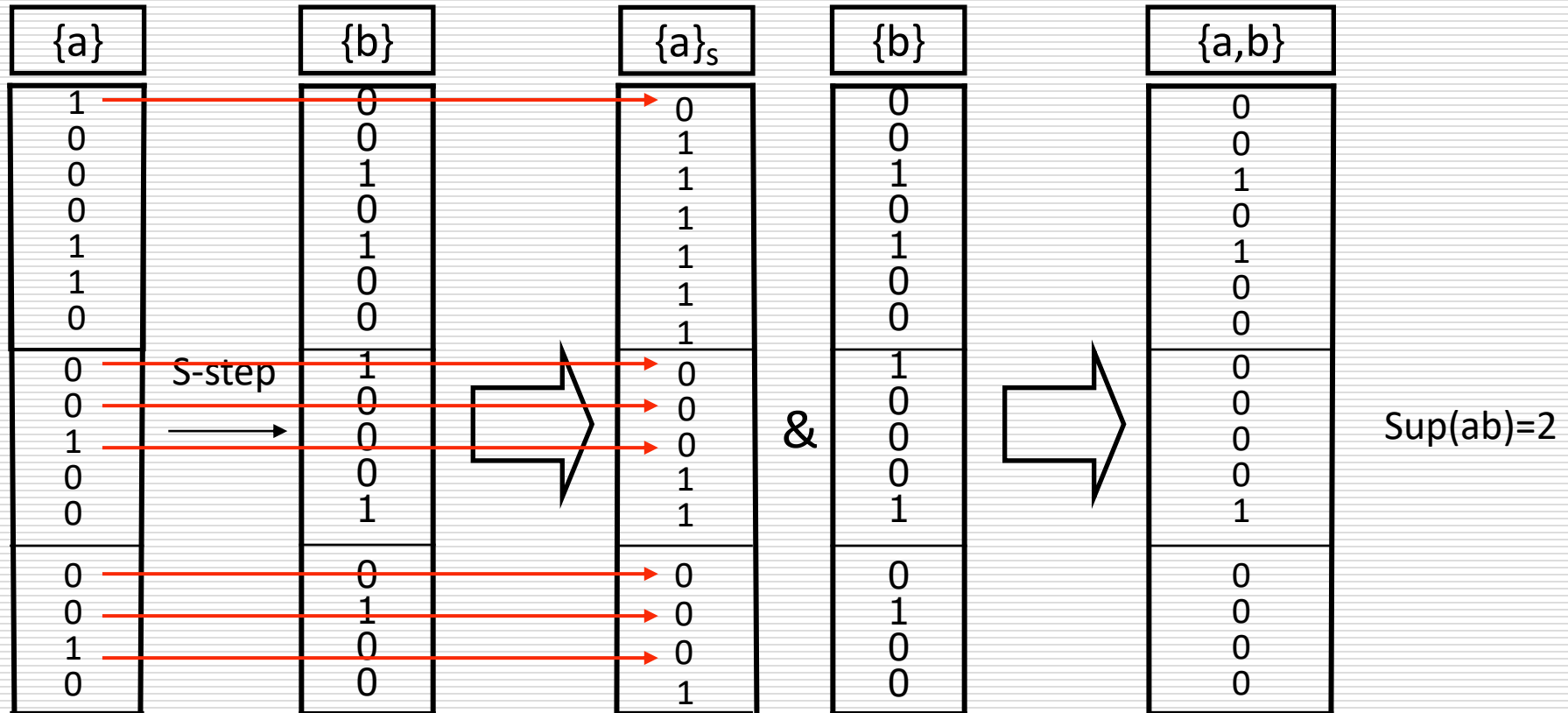
SID EID

10 1  
10 2  
10 3  
10 4  
10 5  
10 6  
10 7  
20 1  
20 2  
20 3  
20 4  
20 5  
30 1  
30 2  
30 3  
30 4

{a}	{b}	{c}	{d}
1	0	0	0
0	0	1	0
0	1	1	0
0	0	0	1
1	1	1	0
1	0	0	0
0	0	0	1
0	1	0	0
0	0	1	1
0	0	0	0
1	0	1	1
0	1	1	0
0	0	1	0
0	0	1	1

# SPAM Temporal Joins

Sequence extended step:



min\_support=2



# Problem of SPAM

---

- Bitmap representation is space consuming  
i.e., data is commonly very sparse

# Acknowledgement

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- Slides are adapted from:
  - Prof. Jeffrey D. Ullman
  - Dr. Jure Leskovec
  - Dr. Wujun Li

# Quiz

---

TID	Items
10	{a, d, e}
20	{a, b, c, e}
30	{a, b, d, e}
40	{a, c, d, e}
50	{b, c, e}
60	{b, d, e}
70	{c, d}
80	{a, b, c}
90	{a, d, e}
100	{a, b, e}

Confidence( $\{bd\} \rightarrow \{e\}$ ) = ?

Confidence( $\{e\} \rightarrow \{bd\}$ ) = ?

答案请发送到邮箱: [bigdatacomputing@163.com](mailto:bigdatacomputing@163.com)

截至时间: 今天(2023年2月24日) 晚上24点

需要注明: (1) 姓名; (2) 学号; (3) 专业