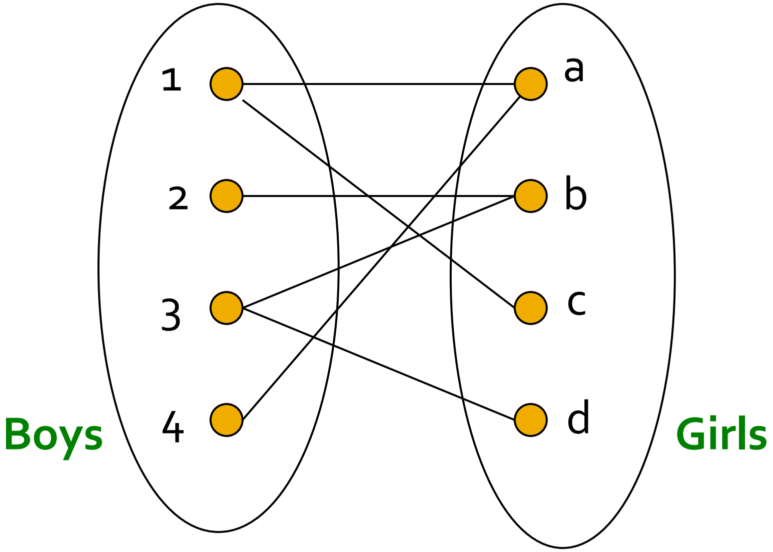
Advertising on the Web
互联网广告

Online Algorithms

- **Classic model of algorithms**
  - You get to see the entire input, then compute some function of it
  - In this context, “offline algorithm”
- **Online Algorithms**
  - You get to see the input one piece at a time, and need to make irrevocable decisions along the way
  - **Similar to the data stream model**

Online Bipartite Matching

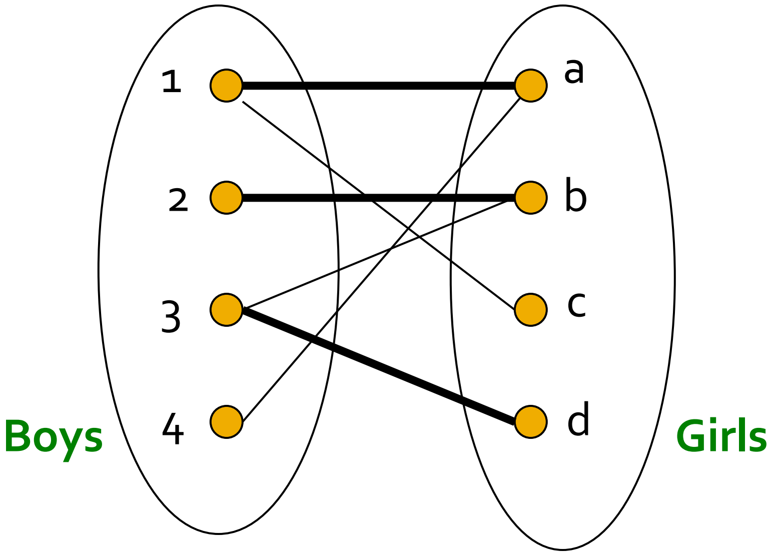
Example: Bipartite Matching



Nodes: Boys and Girls; Edges: Preferences

Goal: Match boys to girls so that maximum number of preferences is satisfied

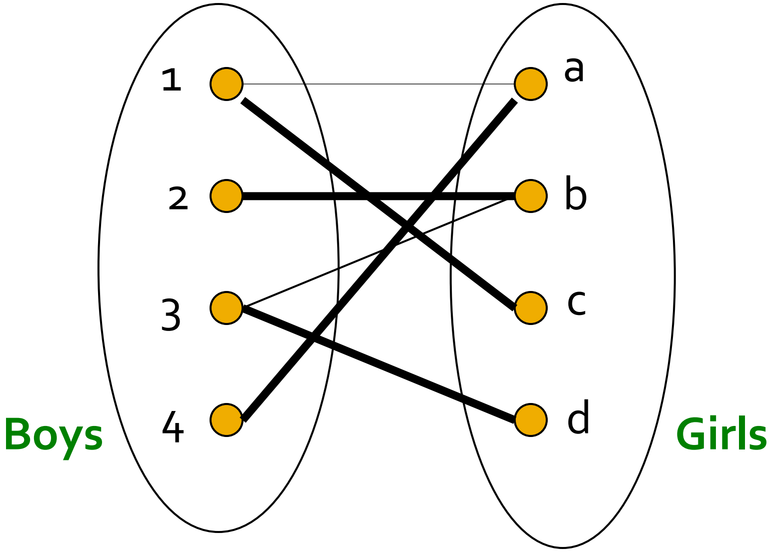
Example: Bipartite Matching



M = {(1,a),(2,b),(3,d)} is a **matching**

Cardinality of matching = |M| = 3

Example: Bipartite Matching



M = {(1,c),(2,b),(3,d),(4,a)} is a

perfect matching

Perfect matching … all vertices of the graph are matched

Maximum matching … a matching that contains the largest possible number of matches

Matching Algorithm

- Problem: **Find a maximum matching for a given bipartite graph**
    - A perfect one if it exists
- There is a polynomial-time offline algorithm based on augmenting paths (Hopcroft & Karp 1973, see http://en.wikipedia.org/wiki/Hopcroft-Karp_algorithm)
- **But what if we do not know the entire graph upfront?**

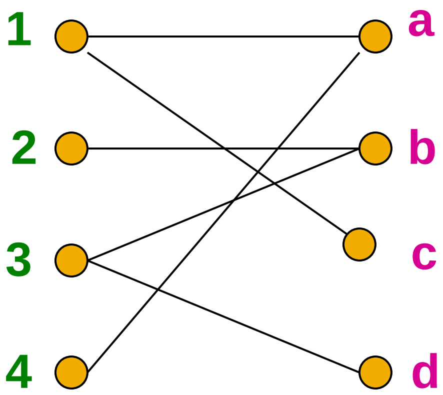
Online Graph Matching Problem

- Initially, we are given the set **boys**
- In each **round**, **one girl's choices are revealed**
    - That is, girl's **edges** are revealed
- **At that time, we have to decide to either:**
    - Pair the **girl** with a **boy**
    - Do not pair the **girl** with any **boy**

- **Example of application:**

  Assigning tasks to servers

Online Graph Matching: Example



(1,a)
(2,b)
(3,d)

Greedy Algorithm

- **Greedy algorithm for the online graph matching problem:**
  - Pair the new girl with **any** eligible boy
    - If there is none, do not pair girl

- **How good is the algorithm?**

Competitive Ratio

- For input ***I***, suppose greedy produces matching $\boldsymbol{M_{greedy}}$ while an optimal matching is $\boldsymbol{M_{opt}}$

Competitive ratio =

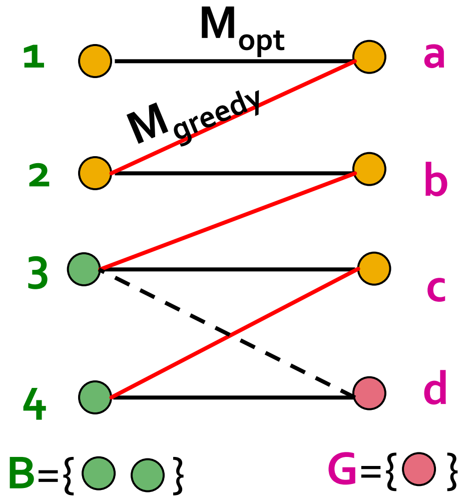
$$\min_{\text{all possible inputs } I} \left(|M_{greedy}| / |M_{opt}|\right)$$

(what is greedy's worst performance over all possible inputs *I*)

Analyzing the Greedy Algorithm

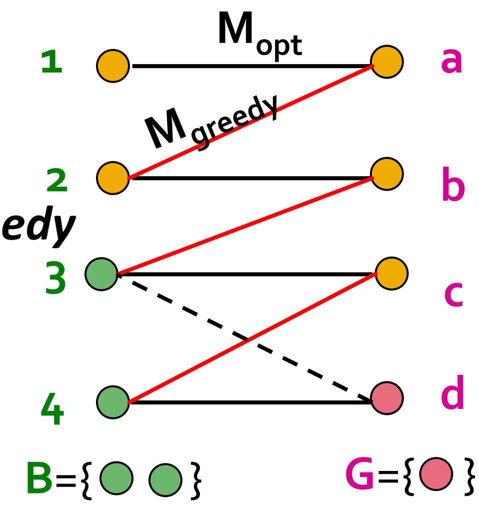
- Consider a case: $M_{greedy} \neq M_{opt}$
- Consider the set *G* of girls matched in M_{opt} but not in M_{greedy}
- Then every boy *B* adjacent to girls in *G* is already matched in M_{greedy}:
  - If there would exist such non-matched (by M_{greedy}) boy adjacent to a non-matched girl then greedy would have matched them
- Since boys *B* are already matched in M_{greedy} then

(1) $|M_{greedy}| \geq |B|$

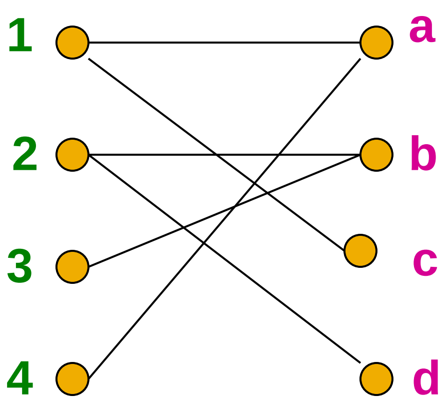


Analyzing the Greedy Algorithm

- **Summary so far:**
  - Girls ***G*** matched in M_{opt} but not in M_{greedy}
  - **(1)** $|M_{greedy}| \geq |B|$
- There are at least $|G|$ such boys ($|G| \leq |B|$) otherwise the optimal algorithm couldn't have matched all girls in ***G***
  - **So:** $|G| \leq |B| \leq |M_{greedy}|$
- By definition of ***G*** also: $|M_{opt}| \leq |M_{greedy}| + |G|$
  - Worst case is when $|G| = |B| = |M_{greedy}|$
- $|M_{opt}| \leq 2|M_{greedy}|$ **then** $|M_{greedy}|/|M_{opt}| \geq 1/2$



Worst-case Scenario

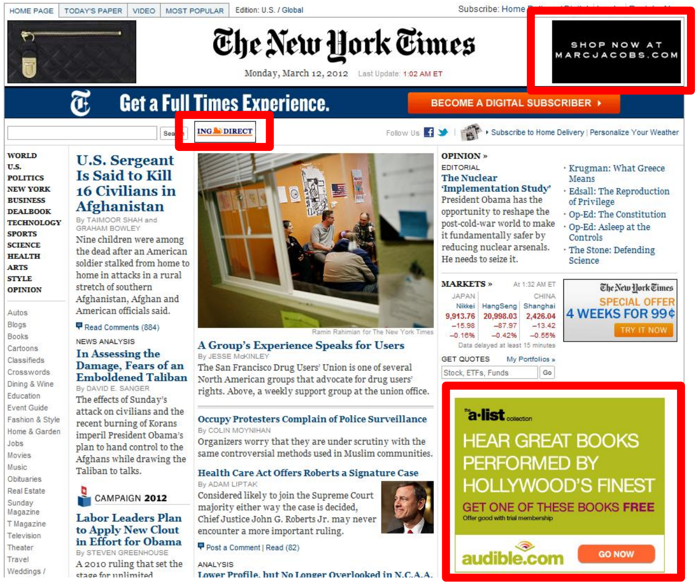


(1,a)
(2,b)

Web Advertising

History of Web Advertising

- **Banner ads (1995-2001)**
  - Initial form of web advertising
  - Popular websites charged *X*\$ for every 1,000 “impressions” of the ad
    - Called “**CPM**” rate (Cost per thousand impressions)
    - Modeled similar to TV, magazine ads
  - From **untargeted** to **demographically targeted**
  - **Low click-through rates**
    - Low ROI for advertisers



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U.S. Sergeant Is Said to Kill 16 Civilians in Afghanistan

By TAIMOOR SHAH and GRAHAM BOWLEY

Nine children were among the dead after an American soldier stalked from home to home in attacks in a rural stretch of southern Afghanistan, Afghan and American officials said.

Read Comments (884)

NEWS ANALYSIS

In Assessing the Damage, Fears of an Emboldened Taliban

By DAVID E. SANGER

The effects of Sunday’s attack on civilians and the recent burning of Korans imperil President Obama’s plan to hand control to the Afghans while drawing the Taliban to talks.

CAMPAIGN 2012

Labor Leaders Plan to Apply New Clout in Effort for Obama

By STEVEN GREENHOUSE

A 2010 ruling that set the stage for unlimited

Ramin Rahimian for The New York Times

A Group’s Experience Speaks for Users

By JESSE McKINLEY

The San Francisco Drug Users’ Union is one of several North American groups that advocate for drug users’ rights. Above, a weekly support group at the union office.

Occupy Protesters Complain of Police Surveillance

By COLIN MOYNIHAN

Organizers worry that they are under scrutiny with the same controversial methods used in Muslim communities.

Health Care Act Offers Roberts a Signature Case

By ADAM LIPTAK

Considered likely to join the Supreme Court majority either way the case is decided, Chief Justice John G. Roberts Jr. may never encounter a more important ruling.

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ANALYSIS

Lower Profile, but No Longer Overlooked in N.C.A.A.

OPINION »

EDITORIAL

The Nuclear ‘Implementation Study’

President Obama has the opportunity to reshape the post-cold-war world to make it fundamentally safer by reducing nuclear arsenals. He needs to seize it.

- Krugman: What Greece Means
- Edsall: The Reproduction of Privilege
- Op-Ed: The Constitution
- Op-Ed: Asleep at the Controls
- The Stone: Defending Science

MARKETS » At 1:32 AM ET

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Nikkei	HangSeng	Shanghai
9,913.76	20,998.03	2,426.04
–15.98	–87.97	–13.42
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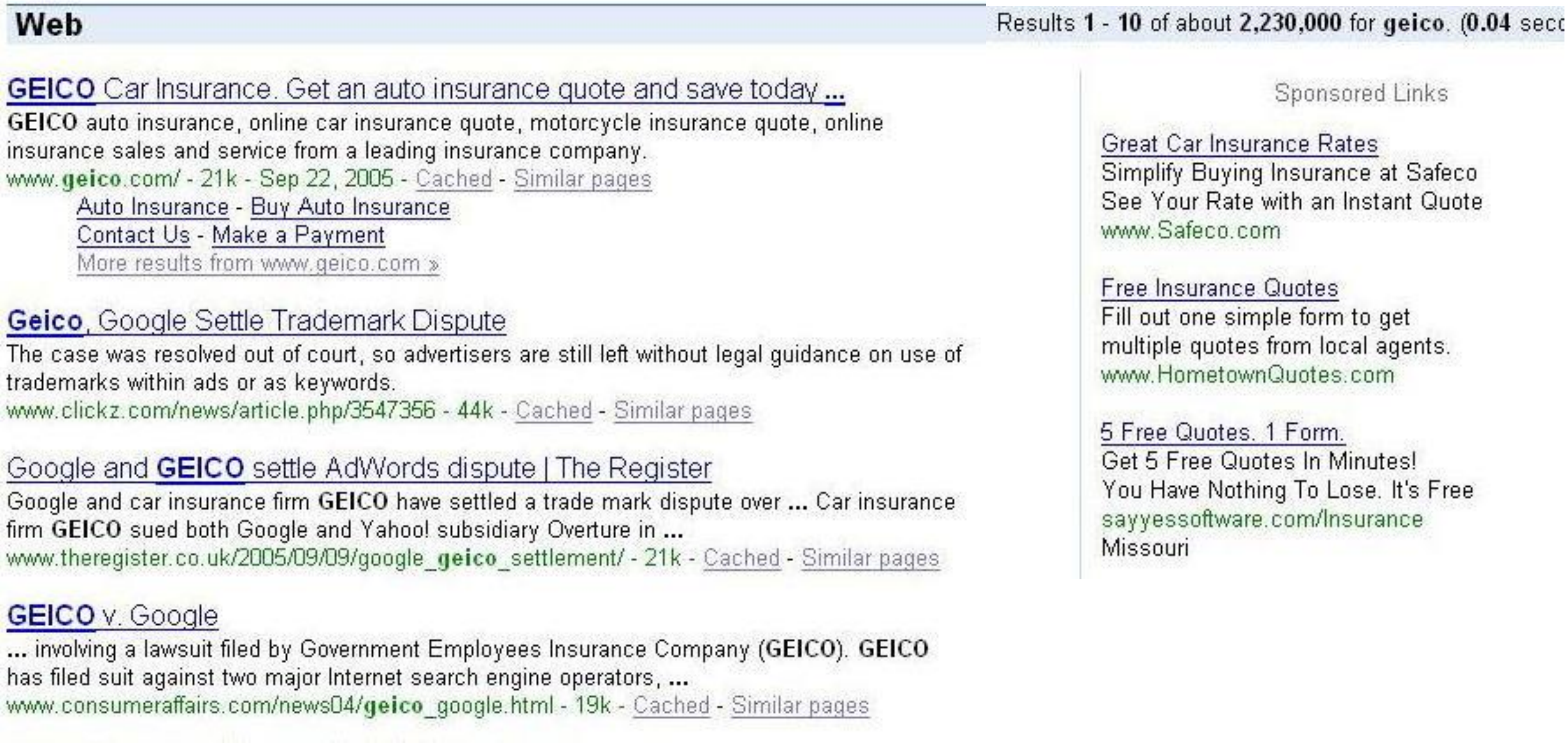
CPM…cost per *mille*

Mille…thousand in Latin

Performance-based Advertising

- **Introduced by Overture around 2000**
  - Advertisers **bid** on **search keywords**
  - When someone searches for that keyword, the **highest bidder's ad is shown**
  - Advertiser is charged only if the ad is clicked on
- Similar model adopted by Google with some changes around 2002
  - Called **Adwords**

Ads vs. Search Results



Web Results **1** - **10** of about **2,230,000** for **geico**. (**0.04** secc

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Geico, Google Settle Trademark Dispute
The case was resolved out of court, so advertisers are still left without legal guidance on use of trademarks within ads or as keywords.
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Web 2.0

- **Performance-based advertising works!**
  - Multi-billion-dollar industry

- **Interesting problem:**
  What ads to show for a given query?
  - (Today’s lecture)

- **If I am an advertiser, which search terms should I bid on and how much should I bid?**
  - (Not focus of today’s lecture)

Adwords Problem

- **Given:**
  - **1.** A set of bids by advertisers for search queries
  - **2.** A click-through rate for each advertiser-query pair
  - **3.** A budget for each advertiser (say for 1 month)
  - **4.** A limit on the number of ads to be displayed with each search query
- **Respond to each search query with a set of advertisers such that:**
  - **1.** The size of the set is no larger than the limit on the number of ads per query
  - **2.** Each advertiser has bid on the search query
  - **3.** Each advertiser has enough budget left to pay for the ad if it is clicked upon

Adwords Problem

- A stream of queries arrives at the search engine: $\boldsymbol{q_1, q_2}$, …
- Several advertisers bid on each query
- When query $\boldsymbol{q_i}$ arrives, search engine must pick a subset of advertisers whose ads are shown

- Goal: **Maximize search engine's revenues**
  - Simple solution: Instead of raw bids, use the "**expected revenue per click**" (i.e., **Bid*CTR**)
- **Clearly we need an online algorithm!**

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
A	$1.00	1%	1 cent
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents
		Click through rate	Expected revenue

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents
A	$1.00	1%	1 cent

Complications: Budget

- Two complications:
  - Budget
  - CTR of an ad is unknown

- Each advertiser has a limited budget
  - Search engine guarantees that the advertiser will not be charged more than their daily budget

Complications: CTR

- **CTR: Each ad has a different likelihood of being clicked**
  - **Advertiser 1** bids $2, click probability = 0.1
  - **Advertiser 2** bids $1, click probability = 0.5
  - **Clickthrough rate (CTR)** is measured **historically**
    - **Very hard problem: Exploration vs. exploitation**

      Exploit: Should we keep showing an ad for which we have good estimates of click-through rate

      or

      Explore: Shall we show a brand new ad to get a better sense of its click-through rate

Greedy Algorithm

- **Our setting: Simplified environment**
  - There is **1** ad shown for each query
  - All advertisers have the same budget ***B***
  - All ads are equally likely to be clicked
  - Value of each ad is the same (=**1**)
- **Simplest algorithm is greedy:**
  - For a query pick any advertiser who has bid **1** for that query
  - **Competitive ratio of greedy is 1/2**

Bad Scenario for Greedy

- Two advertisers A and B
  - ***A*** bids on query ***x***, ***B*** bids on ***x*** and ***y***
  - Both have budgets of **$4**
- Query stream: ***x x x x y y y y***
  - Worst case greedy choice: ***B B B B*** **_ _ _ _**
  - Optimal: **A A A A B B B B**
  - **Competitive ratio = ½**
- This is the worst case!
  - **Note:** Greedy algorithm is deterministic – it always resolves draws in the same way

BALANCE Algorithm [MSVV]

- **BALANCE** Algorithm by Mehta, Saberi, Vazirani, and Vazirani
  - **For each query, pick the advertiser with the largest unspent budget**
    - Break ties arbitrarily (**but in a deterministic way**)

Example: BALANCE

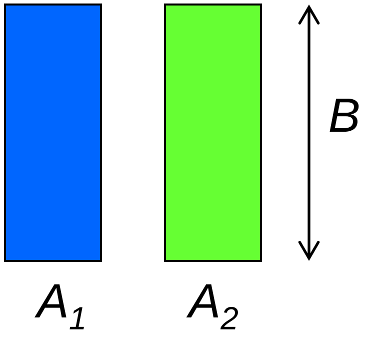
- **Two advertisers A and B**
  - **A** bids on query ***x***, **B** bids on ***x*** and ***y***
  - Both have budgets of **$4**
- **Query stream:** ***x x x x y y y y***
- **BALANCE choice: A B A B B B _ _**
  - Optimal: **A A A A B B B B**
- **In general:** For **BALANCE** on **2** advertisers
  Competitive ratio = ¾

Analyzing BALANCE

- **Consider simple case (w.l.o.g.):**
  - **2** advertisers, **A_1** and **A_2**, each with budget **B** ($\geq$1)
  - Optimal solution exhausts both advertisers' budgets

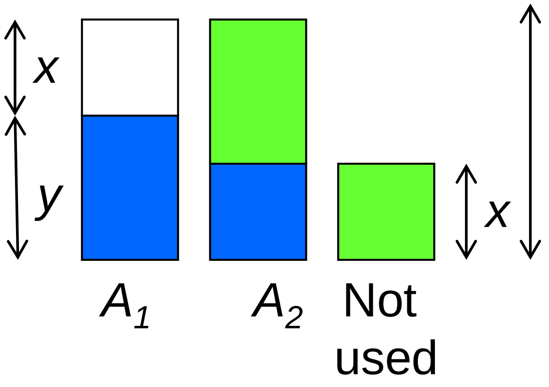
- **BALANCE must exhaust at least one advertiser's budget:**
  - **If not, we can allocate more queries**
    - Whenever BALANCE makes a mistake (both advertisers bid on the query), advertiser's unspent budget only decreases
    - Since optimal exhausts both budgets, one will for sure get exhausted
  - Assume BALANCE exhausts ***A_2***'s budget, but allocates ***x*** queries fewer than the optimal
  - **Revenue: *BAL = 2B - x***

Analyzing Balance



🟦 Queries allocated to A_1 in the optimal solution

🟩 Queries allocated to A_2 in the optimal solution



Optimal revenue = **2B**
Assume Balance gives revenue **= 2B-x = B+y**

Unassigned queries should be assigned to A_2
(if we could assign to A_1 we would since we still have the budget)

Goal: Show we have **$y \geq x$**

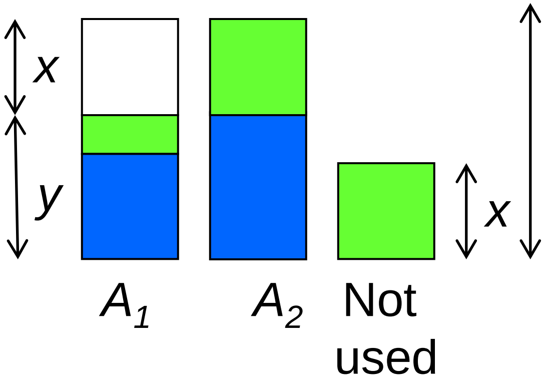
Case 1) ≤ ½ of A_1's queries got assigned to A_2
then $y \geq B/2$

Case 2) > ½ of A_1's queries got assigned to A_2
then $x \leq B/2$ **and** $x + y = B$

Balance revenue is minimum for $x = y = B/2$

Minimum Balance revenue = $3B/2$

Competitive Ratio = 3/4



BALANCE exhausts A_2's budget

BALANCE: General Result

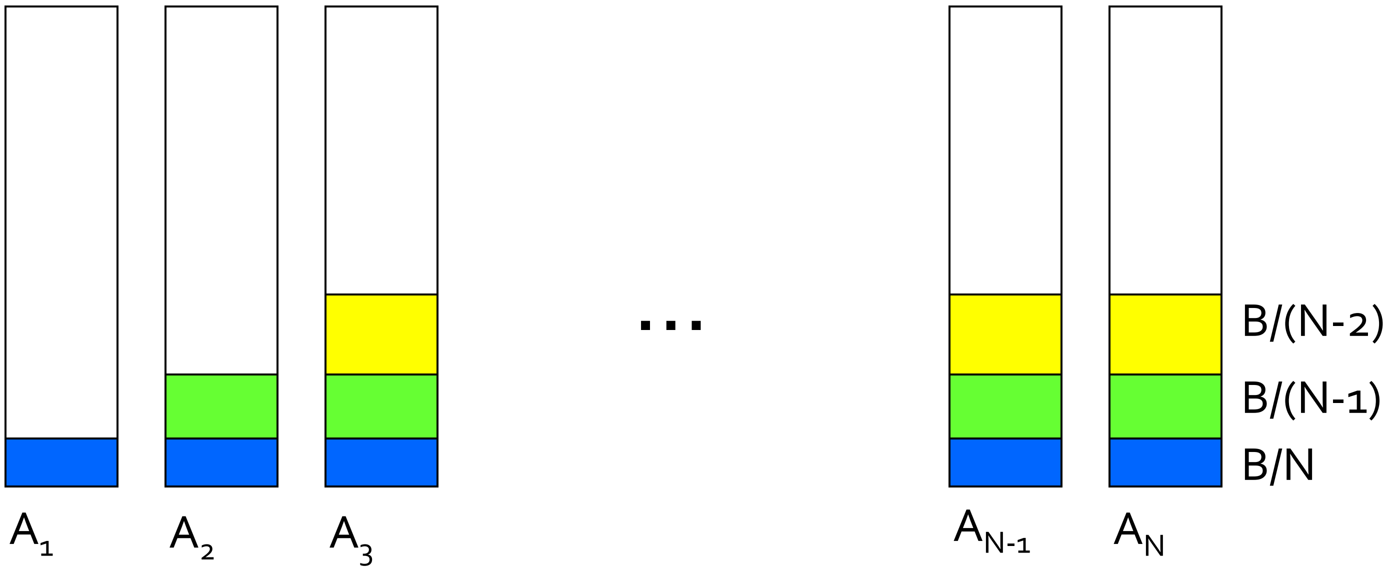
- **In the general case, worst competitive ratio of BALANCE is 1–1/e = approx. 0.63**
  - Interestingly, no online algorithm has a better competitive ratio!

- **Let’s see the worst case example that gives this ratio**

Worst case for BALANCE

- ***N* advertisers:** $A_1, A_2, \dots A_N$
  - Each with budget $B > N$
- **Queries:**
  - $N \cdot B$ queries appear in N rounds of B queries each
- **Bidding:**
  - Round **1** queries: bidders $A_1, A_2, \dots, A_N$
  - Round **2** queries: bidders $A_2, A_3, \dots, A_N$
  - Round ***i*** queries: bidders $A_i, \dots, A_N$
- **Optimum allocation:**
  Allocate round ***i*** queries to A_i
  - Optimum revenue $N \cdot B$

BALANCE Allocation



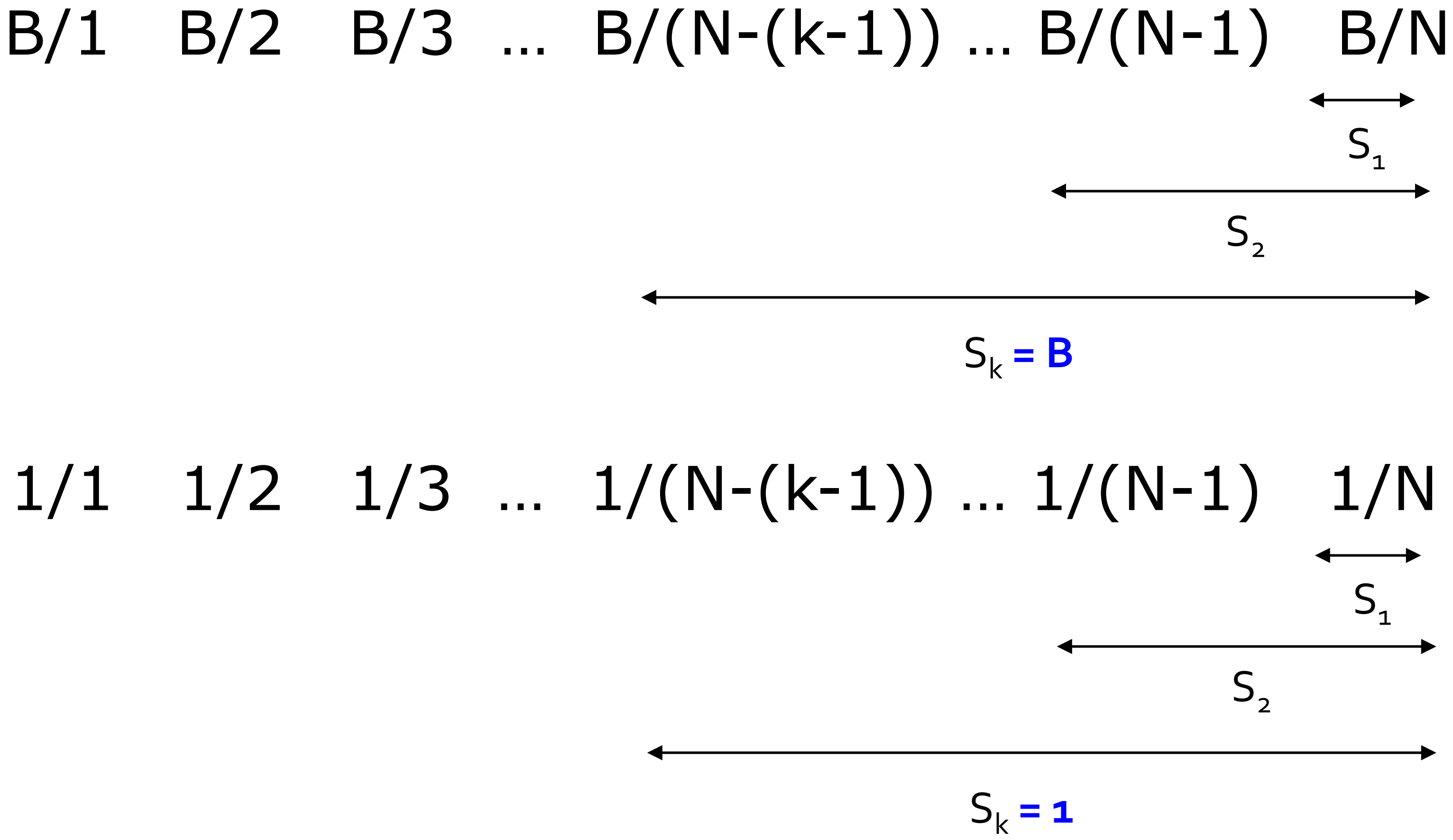
BALANCE assigns each of the queries in round 1 to **N** advertisers.

After ***k*** rounds, sum of allocations to each of advertisers $\boldsymbol{A_k,\ldots,A_N}$ is

$$\boldsymbol{S_k = S_{k+1} = \cdots = S_N = \sum_{i=1}^{k-1} \frac{B}{N-(i-1)}}$$

If we find the smallest *k* such that $S_k \geq B$, then after *k* rounds we cannot allocate any queries to any advertiser

BALANCE: Analysis



$B/1 \quad B/2 \quad B/3 \quad \ldots \quad B/(N-(k-1)) \ldots B/(N-1) \quad B/N$

S_1

S_2

S_k **= B**

$1/1 \quad 1/2 \quad 1/3 \quad \ldots \quad 1/(N-(k-1)) \ldots 1/(N-1) \quad 1/N$

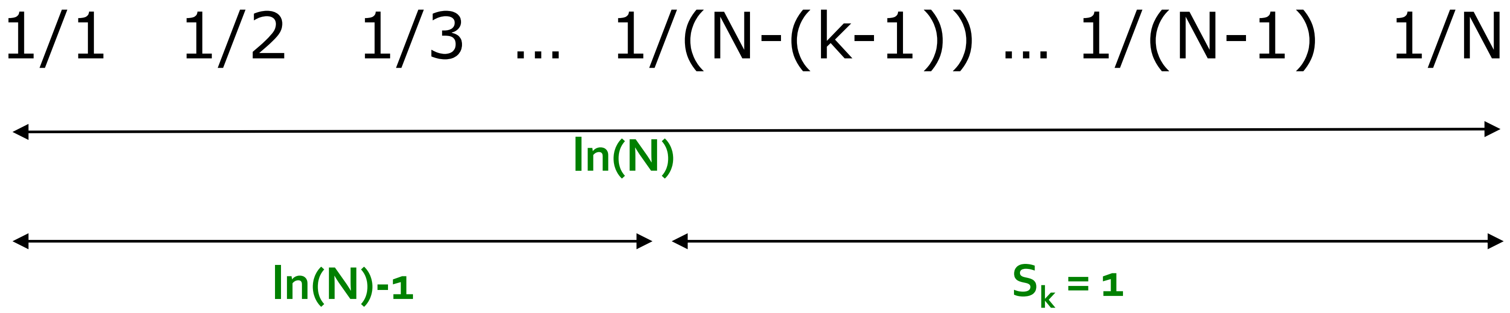
S_1

S_2

S_k **= 1**

BALANCE: Analysis

- **Fact:** $H_n = \sum_{i=1}^{n} 1/i \approx \ln(n)$ for large n
  - Result due to Euler



1/1 1/2 1/3 … 1/(N-(k-1)) … 1/(N-1) 1/N

ln(N)

ln(N)-1

$S_k = 1$

- $S_k = 1$ implies: $H_{N-k} = ln(N) - 1 = ln(\frac{N}{e})$
- We also know: $H_{N-k} = ln(N-k)$
- **So:** $N - k = \frac{N}{e}$
- **Then:** $k = N(1 - \frac{1}{e})$

N terms sum to ln(*N*).
Last *k* terms sum to 1.
First *N*-*k* terms sum
to ln(*N*-*k*) but also to ln(*N*)-1

BALANCE: Analysis

- So after the first **$k=N(1-1/e)$** rounds, we cannot allocate a query to any advertiser
- **Revenue = $B \cdot N\ (1-1/e)$**
- **Competitive ratio = $1-1/e$**

General Version of the Problem

- **Arbitrary bids and arbitrary budgets!**
- Consider we have 1 query ***q***, advertiser ***i***
  - Bid = $\boldsymbol{x_i}$
  - Budget = $\boldsymbol{b_i}$
- **In a general setting BALANCE can be terrible**
  - Consider two advertisers $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$
  - $\boldsymbol{A_1}$**:** $\boldsymbol{x_1}$ **= 1,** $\boldsymbol{b_1}$ **= 110**
  - $\boldsymbol{A_2}$**:** $\boldsymbol{x_2}$ **= 10,** $\boldsymbol{b_2}$ **= 100**
  - Consider we see **10** instances of **q**
  - BALANCE always selects $\boldsymbol{A_1}$ and earns **10**
  - Optimal earns **100**

Generalized BALANCE

- **Arbitrary bids:** consider query ***q***, bidder ***i***
  - Bid = $\boldsymbol{x_i}$
  - Budget = $\boldsymbol{b_i}$
  - Amount spent so far = $\boldsymbol{m_i}$
  - Fraction of budget left over $\boldsymbol{f_i = 1 - m_i/b_i}$
  - Define $\boldsymbol{\psi_i(q) = x_i(1-e^{-f_i})}$
- Allocate query ***q*** to bidder ***i*** with largest value of $\boldsymbol{\psi_i(q)}$
- **Same competitive ratio (1-1/e)**