The finished vehicle routing problem with a heterogeneous transport fleet

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Abstract: A new variant of the vehicle routing problem, the heterogeneous multi-type fleet vehicle routing problem in finished vehicle logistics (HVRP–FVL), is modeled and solved. The HVRP–FVL considers comprehensive transportation costs, including highway toll, labor cost, power cost, time penalty cost, and carbon emissions cost and, as an added complication, highway toll is charged based on the transport vehicle model, loading weight, and travelled distance. The objective is to minimize the total cost of the vehicle fleet. Because of the size and complexity of the search space, a matheuristic is devised using a genetic algorithm (GA) metaheuristic to solve the HVRP–FVL. To improve the computational performance, a dual-chromosome encoding method with adaptive crossover, mutation and climbing operators are incorporated in the GA. A case study from a logistics company is used to evaluate the effectiveness of the proposed algorithm and a series of experiments are conducted. The results demonstrate the proposed approach has good performance and satisfies users in practice. Contributions of the paper are the effective modeling and solution of a real and complex vehicle routing problem in finished vehicle logistics.

Keywords: finished vehicle logistics, truck routing, vehicle fleet logistics, genetic algorithms

1 Introduction

With the rapid expansion of the automotive industry, thirdparty logistics providers have become more popular for finished vehicle transportation. Finished vehicle logistics can be broadly defined as a series of associated processes from automotive factories to final destinations. Among these processes, the transport cost is a very important economic factor. Third-party logistics companies try to minimize their total transportation cost, which consists of highway toll, labor cost, power cost, time penalty cost, carbon emissions cost, and others. In practice, this is normally done in an ad hoc manner using (and depending on) a manager's experience. With the number of cost factors and a heterogeneous fleet, an ad hoc approach dependent on human expertise needs to be improved upon.

In the classic vehicle routing problem (VRP), it is typically assumed that transport cost is based only on distance traveled. However, in practice, there are different highway toll rates for different loading conditions and geographical areas, which make the problem more challenging. By providing an effective and practical solution approach to this complex finished vehicle routing problem, this paper contributes to the automotive logistics field, both to scholars and to practitioners.

The paper is organized as follows. The next section describes the problem. Section 3 carries out a review of the literature. Section 4 devises a math optimization model for the HVRP-FVL. Section 5 presents a customized GA algorithm. Real test instances and algorithm comparisons are presented in Section 6. Finally, conclusions are drawn in Section 7.

2 Problem overview

Automotive factories produce various types of finished vehicles, such as sedan, pickup truck, and SUV. The set of finished vehicles is initially located at a main depot. A third-party logistics company receives orders from the automotive

factories and transports different types of finished vehicles to dealers. The logistics company has its own vehicle fleet consisting of different types of transport vehicles such as 4-axis truck, 5-axis truck, and 6-axis truck. In a typical situation, there are about 100 types of transport vehicles and 600 types of finished vehicles. The dealers can be in different cities and might be far away from each other. Different vehicle transport solutions result in different costs because of the models and numbers of the transport vehicles used. The problem aims to determine a heterogeneous vehicle fleet routing solution to minimize the total cost, which includes toll charge, fuel cost, time penalty cost, and others.

The problem considers a set of ordered items (finished vehicles) characterized by length, width, height, weight, and a set of transport vehicle fleet types characterized by length, width, height, allowable loading weight, and transport cost. The objective is to select the transport vehicles to transport the finished vehicles minimizing the total cost of the transport. We term this problem the Heterogeneous Multitype Fleet Vehicle Routing Problem in Finished Vehicle Logistics (HVRP-FVL). The HVRP-FVL has its own character different from the classic VRP, which are listed next. (1) Both the shape of the finished vehicles and loading space of the transport vehicles in HVRP-FVL are irregular, which leads to more complex multi-dimensional geometry constraints. The classic VRP mostly consider onedimensional constraints. (2) Actual highway toll rates are not only based on travel distance, unlike in the classic VRP. (3) Most VRP models simplify the factors that affect vehicle fuel consumption and carbon emissions. Although the HVRP-FVL is found widely in the real automotive supply chain, there are limited related studied in the literature.

According to the description above, the assumptions and constraints are as follows:

- (1) Any type of the finished vehicle can be loaded in any kind of transport vehicle.
- (2) The finished vehicles to be transported must not exceed the loading limit and geometry constraints of the transport vehicle fleet.

- (3) Each transport vehicle is only allowed to visit each customer once.
- (4) Each transport vehicle needs return to the distribution center (depot) after completing the delivery task.
- (5) Loading and unloading costs are not considered (but could easily be added).

3 Literature review

3.1 Relevant variants of the VRP

Many variants of the VRP have been modeled and applied, such as the capacitated VRP (CVRP) [29], the heterogeneous fleet VRP (HVRP) [26], the VRP with time windows (VRPTW) [8], the Capacitated VRP with Time Windows (CVRPTW) [16], the low-carbon VRP (LCVRP) [22], and the time-dependent VRP (TDVRP) [19]. Among them, the VRPTW is focused on order delivery date. Ghoseiri and Ghannadpour [11] used goal programming and a GA to solve the multi-objective vehicle routing problem with time windows, which aimed to reduce the number of transport vehicles and shorten the total transport distance. Xu et al. [32] studied the VRP with soft time windows considering fuzzy requirements, aiming at reducing costs and improving customer satisfaction.

As for the HVRP, it was proposed by Golden et al. [6]. Later, it was considered by Baldacci et al. [1] and Koç et al. [17], in which one must make additional decisions on fleet composition and customer selection. Kwon et al. [20] considered heterogeneous fixed fleet vehicle routing with carbon emissions to minimizing the sum of variable operation costs and established a mixed integer programming model to assess the value-benefits of purchasing or selling of carbon emissions rights for heterogeneous fleets. Ge et al. [10] designed a quantum GA to solve the HVRP problem, where the optimization goal was to minimize total cost of distribution. Coelho et al. [5] studied a VRP variant inspired by a real case of a large distribution company that could perform multiple trips.

As more attention was paid to environmental protection, fuel consumption became an important economic and environmental factor in the VRP. A model was proposed for calculating total fuel consumption for the time-dependent vehicle routing problem (TDVRP) and solved by an ant colony system (ACS) algorithm hybridized with insertion heuristics [2]. Li and Zhang [21] studied a series of low-carbon VRP problems, which were mainly to study the relationship between carbon emissions and transport costs. Xiao and Konak [31] defined a new mixed integer liner programming model which considered heterogeneous vehicles, time-varying traffic congestion, vehicle time window constraints, the impact of vehicle loads on emissions, and vehicle capacity constraints in the green vehicle routing and scheduling problem.

An exact algorithm was often used to solve the small-scale optimization problems [9], but this approach is not good enough to solve large-scale problems. Heuristic algorithms

have commonly been used to solve large-scale problems. Jiang et al. [14] proposed a method that extended an existing tabu search procedure to solve a problem variant of the VRP with time windows. Wei et al. [30] developed an adaptive variable neighborhood search (AVNS) for the heterogeneous fleet routing problem with three-dimensional loading constraints, and numerical experiments showed that the proposed AVNS outperforms other algorithms for HVRP. Mouthuy et al. [27] solved the VRP with soft time windows (VRPSTW) with a multi-stage large neighborhood search algorithm.

Sbai et al. [28] used an adaptive GA to solve the capacitated vehicle routing problem where customer demand is composed of two-dimensional weighted items with a time window (2L-CVRPTW), and the proposed algorithm performed well. Xu et al. [33] incorporated Simulated Annealing (SA) into GA to solve the VRP, since SA could avoid the drawback of premature convergence compared with GA. Ho et al. [12] developed two hybrid genetic algorithms for the multi-depot VRP, the major difference between two algorithms is whether the initial solutions are generated randomly or not. Computational results showed that a selected, rather than random, initial population could enhance the performance of the GA.

3.2 Finished vehicle logistics (FVL)

Fischer and Gehring [7] and Kim et al. [15] applied information technology to Finished Vehicle Logistics (FVL) operations management. Hu et al. [13] proposed a mixed-integer linear programming model for the finished-vehicle transporter routing problem and designed an evolutionary algorithm to solve the model. Liu et al. [25] defined a simplified geometry model of the finished vehicle and transport vehicle, then introduced a greedy search into a traditional branch and bound algorithm to improve the computational performance for solving heterogeneous multitype fleet vehicle loading problem in FVL. However, these articles ignored the multi-dimensional geometry constraints on both finished vehicles and transport vehicles.

Based on the intrinsic nature of the HVRP-FVL, there are no existing methodologies that can be directly used to solve the HVRP-FVL without substantial modification. To the best of our knowledge, this paper is one of the earliest exploratory research works considering various factors such as multiple types of finished vehicles, multiple types of transport vehicles, dynamical routing cost weight, and so on. Furthermore, the HVRP-FVL is NP-hard as it can be reduced to the general VRP, which is a well-known NP-hard problem [34]. Based on earlier heuristic ideas, we developed a dual-chromosome encoding method that combines adaptive crossover, mutation probability, and a climbing operator to achieve a feasible solution that minimizes the total cost.

4 Optimization model

4.1 Optimization objective

4.1.1 Transport cost F_1 : In the classic VRP model, transport cost optimization is often synonymous with minimizing the travelled distance, ignoring different highway toll rates. Here, a more realistic and comprehensive approach will be presented to calculate total transport cost, which consists of a Fixed cost (C_F) and a Variable cost (C_V).

C_F includes drivers' salaries, depreciation cost, and management expenses, etc.

C_V: toll charge and fuel cost of the transport vehicle.

① In China (and many other countries), the toll charge is a significant part of the transport cost, and depends on some parameters such as unit rate (c), weight (w), and distance (d).

$$\begin{array}{c} C_{toll-charge} \\ = c \times w \times d \end{array} \tag{1}$$

② The calculation of transport vehicle fuel consumption utilizes the integrated mode approach proposed by Bektas and Laporte [3]. Speed (v), distance (d), weight (w), road condition (α), type of transport (β), and oil prices (c_f) were all considered in this method. The model was simplified to the following.

$$C_{fuel-consumption} = c_f \times \phi \times (\alpha \times w + \beta \times v^2) \times d$$
 (2)

4.1.2 Time penalty cost F_2 : It is assumed that the i^{th} transport vehicle is required to service the j^{th} customer in the time window $[e_j, l_j]$. If the time t_i when the transport vehicle arrives at customer j is earlier than the prescribed earliest time e_i , an early penalty is invoked (λ_1). Conversely, if the time t_i is later than l_j , a late penalty is incurred (λ_2).

$$f_2 = \lambda_1 \times \max \left(e_j - t_i, 0 \right) + \lambda_2 \times \max \left(t_i - l_j, 0 \right)$$
(3)

4.1.3 Carbon emissions cost F_3 : Related work on CO_2 emissions reduction can be found in Koc et al. [18]. According to the relationship between carbon emissions I and fuel consumption (F) [4], carbon emissions can be obtained. We monetize the environmental cost in the form of carbon taxes T_c . The δ is the fuel carbon emission conversion factor related to the transport vehicle type used, different horsepower, air resistance caused by windward areas, etc..

$$f_3 = T_c \times F \times \delta$$
 (4)

4.2 The math model

Notation

G = (N, E): Delivery network

N: node set, $N = \{0, 1, \dots, n\}$, 0 indicates the distribution center or depot, others stand for customer points

E: arc set,
$$E = \{(i, j) | i, j \in N, i \neq j\}$$

K: the type of transport vehicles, $K = \{1, 2, \dots, k\}$

P: the type of finished vehicles, $P = \{1, 2, \dots, p\}$ U_k : The number of transport vehicles of type k

 g_p , l_p , w_p , h_p .: The weight, length, width, and height of finished vehicle type p

 G_k , L_k , W_k , H_k ,: The limitation loading weight, length, width, and height of transport vehicle type k

 D_{ip} : The demand of customer i for finished vehicle type

 \mathbb{Q}_k : The self-weight (that is, empty weight) of transport vehicle type k

 $w_{i,ju}^{k}$: The total weight of finished vehicles loaded in the uth transport vehicle of type k on arc (i, j)

 t_j : The time when arriving at j, and

$$t_j = \sum_{k=1}^K \sum_{u=1}^{U_k} \sum_{i=0}^N (t_i + t_{ij} + ct_i) x_{iju}^k \cdot ct_i \text{ is the service time of customer } i, t_{ij} \text{ is travel time on arc } (i, j)$$

 S_p^k : The available status of transport vehicle k for finished vehicle p

Math model

The optimization model below is applied:

$$F = \min \left(\omega_1 f_1 + \omega_2 f_2 + \omega_3 f_3 \right)$$

$$f_{1} = \sum_{u=1}^{U_{k}} \sum_{i=1}^{K} \sum_{j=1}^{N} C_{FK} x_{0}^{k}_{ju} + \sum_{u=1}^{U_{k}} \sum_{k=1}^{K} \sum_{j=0}^{N} \sum_{i=0}^{N} \sum_{ijk}^{K} c_{ij} (Q_{k} + w_{i}^{k}_{ju}) d_{ij} + c_{f} \sum_{u=1}^{U_{k}} \sum_{k=1}^{K} \sum_{i=0}^{N} \sum_{j=0}^{N} x_{iju}^{k} \Phi \left[\alpha_{ij}\right] d_{ij} + \beta_{k} v_{ij}^{2} d_{ij}$$

$$(Q_{k} + w_{ij}^{k}) + \beta_{k} v_{ij}^{2} d_{ij}$$

$$(7)$$

(7)

$$f_2 = \lambda_1 \sum_{i=1}^{N} \max (e_i - t_i, 0) + \lambda_2 \sum_{i=1}^{N} \max (t_i - t_i, 0)$$

(8)

$$f_3 = T_c \times \{ \delta \times \sum_{u=1}^{U_k} \sum_{k=1}^{K} \sum_{i=0}^{N} \sum_{j=0}^{N} x_i j_u \Phi [\alpha_{ij}(Q_k + w_i j_u) + \beta_k v_{ij}^2] d_{ij} \}$$

(9)

Decision variables

$$x_{i\,ju}^{\,k} = \begin{cases} \\ 0, \end{cases}$$
 1, if the u^{th} transport vehicle of
$$y_{i\,u}^{\,k}$$
 (10)

$$= \begin{cases} y_{1u}^{k} \\ = \\ \begin{cases} 1, if the u^{th} transport vehicle of type k \end{cases} \end{cases}$$

(11)

St.

$$\begin{array}{l} x_{i}^{k}_{ju} \left(x_{i}^{k}_{ju} - 1 \right) = 0, \forall i, j \in N; \forall k \in K; \forall u \in U_{K} \\ (12) \\ y_{iu}^{k} \left(y_{iu}^{k} - 1 \right) = 0, \forall i, j \in N; \forall k \in K; \forall u \in U_{K} \\ x_{i}^{k}_{iu} = 0, \forall i \in N; \forall k \in K; \forall u \in U_{K} \\ (14) \\ \sum_{i=0}^{N} x_{i}^{k}_{ju} = \sum_{i=0}^{N} x_{j}^{k}_{iu}, \forall j \in N, \forall k \in K; \forall u \in U_{K} \\ (15) \\ \sum_{u=1}^{U_{k}} \sum_{k=1}^{K} y_{iu}^{k} = 1, \forall i \in N \\ y_{iu}^{k} (S-1) = 0, \forall i \in N; \forall k \in K; \forall u \in U_{K} \\ \sum_{p=1}^{P} \sum_{i=1}^{N} D_{ip} w_{p} y_{iu}^{k} \leq G_{k}, \forall k \in K; \forall u \in U_{K} \\ (18) \\ \sum_{p=1}^{P} \sum_{i=1}^{N} D_{ip} l_{p} y_{iu}^{k} \leq L_{k}, \forall k \in K; \forall u \in U_{K} \\ (19) \\ \sum_{k=1}^{K} \sum_{j=0}^{N} w_{j}^{k}_{iu} - \sum_{k=1}^{K} \sum_{j=0}^{N} w_{i}^{k}_{ju} = \sum_{p=1}^{P} D_{ip} w_{p}, \\ \forall i \in N; \forall u \in U_{K} \\ (20) \\ \sum_{i,j \in S \times S} x_{i}^{k}_{ju} \leq |S-1|, S \subset N \setminus \{0\}; S \neq \emptyset; \forall, k \\ (21) \end{array}$$

Objective Eq. (6) minimizes the total cost. Eq. (7), Eq. (8), and Eq. (9) are the three sub-objective functions that comprise Eq. (6); Eq. (12), Eq. (13), and Eq. (14) limit the decision value range. The conservation of flow during transport is defined by Eq. (15). Any order must not be split and are only be serviced by one transport vehicle as defined by Eq. (16). And Eq. (17) is the width and height limitation of the transport vehicle. The loading weight limitation of transport vehicle is defined by Eq. (18). Similarly, Eq. (19) is the loading length limitation of the transport vehicle. The loading weight on arc (i, j) is defined by Eq. (20). Eq. (21) is the sub-tour constraint.

5 Solution approach

A large amount of literature has shown that the GA is a metaheuristic algorithm with excellent robustness, and it is often combined with other algorithms to solve complicated engineering optimization problems [24]. In this work, a customized GA algorithm is devised for this complex problem.

5.1 Encoding

We used a dual-chromosome encoding method. Firstly, the integer code of the customers in N1 = (1,2,...,n) is used as the first chromosome in which each gene represents a customer and its sequence represents the vehicle routing. The second chromosome also uses an integer code such as N2 = (1, 2,...,m), which represents the selected transport vehicles and their used sequence.

5.2 Initialization

The initial population can have an important effect on the GA search process. A high-quality initial population can improve the efficiency of the search for the global optimal solution. Commonly, transportation $cost(f_I)$ has a large proportion of the total cost(F) and the transport distance (d) is a key factor in determining the cost of transportation. From the cluster analysis point of view, two close customer points are likely to be serviced by the same transport vehicle. Thus, a sweep strategy is applied to generate a better initial population. The implementation procedures for the sweep strategy are as detailed (1/2)bw.

Step 1. Mark all customer points and the distribution center (depot), 0.

Step 2. Generate a ray by connecting the distribution center (depot) to any customer point i, and then begin to sweep all other customer points in a clockwise direction. The swept customer points are recorded one by one in order as an improved initial individual. As shown in Fig.1, there are six customer points, beginning the sweep from 2, and the initial improvement individual obtained is $(2\ 1\ 5\ 4\ 6\ 3)$.

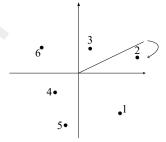


Fig.1 Sweeping Schematic

Step 3. Select different starting points in turn to generate new initial individuals to form the *pop1* number of initial population. As in Fig.1, this results in six improved individuals.

Step 4. if $pop_1 \ge pop$, then terminate; Otherwise, continue to Step 5.

Step 5. Randomly generate the remaining $pop - pop_1$ number of initial individuals. As in Fig.1, if the size of pop is 10, so the remaining four individuals are randomly generated.

Using the above five steps, the first chromosome (N1) is obtained. The second chromosome (N2) is generated uniform randomly to add needed diversity to the population of candidate solutions.

5.3 Fitness function

Calculating fitness is the process of decoding and calculating the final total cost. There exist some constraints of the transport vehicle, such as total loading weight limit and size limit, therefore the transport vehicles must return back to the main depot. Therefore, we need insert '0' into the customer's sequence when the vehicle returns to the main depot.

For example, the customer code is N1 = (1 2 3 4 5 6), and the vehicle code is N2 = (2 1). A corresponding decoding is (0 1 2 3 0 4 5 6 0), which means that No. 2 vehicle starts from the depot, visits customer 1, customer 2, and customer 3 in turn, and then returns to the depot; the No. 1 vehicle transport starts from the depot, serves customer 4, customer 5, customer 6, and then returns to the depot. How many customers to serve and when to return is judged based on the total loading weight limit and the size limit of the vehicles. If the total weight and total length of the cars of the first i customers do not exceed the limit, and the first i+1 customers exceed one (or both) of the limits, the decoding process inserts "0" after the ith position indicating that the vehicle must return to the depot at this juncture.

Then, we calculate the total cost of the route with the objective of minimizing the total cost. So, the total cost is directly taken as the fitness function.

The dual chromosome encoding method accommodates both of situations of insufficient capacity and excess capacity. The only difference is that some chromosomes do not play a role when decoding. For example, when the capacity is insufficient, the last few customer points cannot be served, and when the capacity is excessive, the last vehicles will have no car to transport. When numbering the vehicles, including the depot's information to which it belongs, the problem involving multiple depots can be solved.

5.4 Selection operator

To ensure that superior individuals are selected for crossover, an elitism strategy is adopted. Individuals whose fitness value is top 5% among population are directly selected and the common Roulette method is used to select the remaining individuals.

5.5 Crossover operator

The crossover operation is performed on the two chromosomes separately. The crossover operations are provided as follows:

Randomly select two inspection positions, swap the codes within the two positions, and then delete any duplicate codes. This is a two point crossover.

For example:

Individual1: 1 2 |3 4 5 6 7 8 |9 Individual2: 9 8 |7 6 5 4 3 2 |1

After cross

New individual1: 189765432 New individual2: 921345678

During the execution of the algorithm, the average fitness value is treated as a threshold. For individuals below the threshold, the crossover probability (Pc) is increased so that they can have the opportunity to change their fitness; for

individuals above the threshold, the crossover probability is reduced to prevent the destruction of outstanding individuals. Therefore, an adaptive crossover probability is used as follows:

$$P_c = \begin{cases} P_c(f_{\text{max}} - f_i) / (f_{\text{max}} - f_{\text{avg}}), & f_i > f_{\text{avg}} \\ P_c, & f_i \leq f_{\text{avg}} \end{cases}$$

5.6 Mutation operator

(23)

The mutation operation is designed to enhance the local search ability of the algorithm. The mutation operation is different from the crossover operation. The difference is that the mutation operation only selects one chromosome, while the crossover operation is the crossover of two different chromosomes, and the mutation probability (Pm) is adjusted as the population fitness value changes. Here, the mutation operation is to select randomly two positions on one chromosome, and reverse the order of the middle nodes, an inversion of a randomly selected component of the solution. For example:

Individual: 1 2 | 3 4 5 6 7 8 | 9
After mutation
Individual: 1 2 | 8 7 6 5 4 3 | 9

$$P_{m} = \begin{cases} P_{m}(f_{max} - f_{i})/(f_{max} - f_{avg}), & f_{i} > f_{avg} \\ P_{m}, & f_{i} \leq f_{avg} \end{cases} (23)$$

5.7 Local search operator

To increase the local search capability of the algorithm, a local search operator (climbing) is added. It is realized by the transposition of adjacent nodes. The specific operation is: randomly select a position of a chromosome, exchange it with the subsequent nodes, and judge whether the fitness value after the exchange is better. If so, replace the original individual with the exchanged individual, otherwise, keep the original individual. This is performed once on each child. For example:

Individual: 1 2 | 3 4 5 6 7 8 9 Exchange 2 and 3: 1 3 | 2 4 5 6 7 8 9

5.8 Termination condition

Set the maximum evolutional generation (gen_{max}) as the termination condition of the algorithm. This is dynamic with the scale of the problem.

6 Computational experience

To verify the validity of the improved GA (IGA), it is tested on a PC with an AMD 2.9 GHz processor, 32.00 GB RAM, and the Microsoft Windows 10 operating system. There are no instances publicly available for the FVRP-FVL, so an actual situation from a logistics company is referred to illustrate the approach.

6.1 A case study

Using the data of a logistics company, the proposed algorithm is run. This case considers the situation where the capacity is sufficient and there is one depot. The information of the available transport vehicles and the finished vehicles are listed in Table 1 and 2, respectively. The information related to the distribution center and

customer orders is listed in Table 3. Each order includes the finished vehicles, the customer coordinates, and time windows. Some parameters of model are shown in Table 4. In addition, these parameters are determined by preliminary experimentation:

popsize = 200, $P_c = 0.5$, $P_m = 0.1$, $gen_{max} = 1000$

 Table 1 Information of the transport vehicles

Transport vehicle type	Number of such vehicles	Loading length (m)	Loading width (m)	Loading height (m)	Loading weight (T)	Fixed cost (RMB)	Self- weight (T)
1	5	38	1.90	2.10	20	2000	22
2	5	52	1.95	2.10	30	2400	25
3	5	67	2.20	2.15	40	3000	29

 Table 2 Finished vehicle information

Parameter	Type A	Type B	Type C	Type D	Type E
Length (mm)	3.460E+3	4.135E+3	4.350E+3	4.515E+3	4.865E+3
Width (mm)	1.690E+3	1.828E+3	1.855E+3	1.880E+3	1.895E+3
Height (mm)	1.450E+3	1.485E+3	1.550E+3	1.550E+3	1.730E+3
Weight (kg)	8.700E+2	1.225E+3	1.270E+3	1.320E+3	1.645E+3

Table 3 21 Customer orders information with time windows

Node set	Latitude	Longitude	Earliest time Et(h)	Latest time Lt(h)	Orders
0 (depot)	28.68	115.68	-	-	-
D1	29.27	117.17	2	9	2A+B+2C
D2	28.26	117.07	1	9	1A+3C
D3	28.46	117.95	3	9	3B+2D
D4	27.95	116.36	3	10	4D
D5	30.12	118.16	1	10	3A+B+2D
D6	28.94	118.88	2	8	4E
D7	30.67	117.49	3	5	2A+2D+2E
D8	30.20	115.05	1	5	3A+E
D9	29.08	119.65	1	6	2A+4C
D10	30.39	114.90	3	8	2A+3B+2C
D11	30.60	114.30	2	9	D+4E
D12	30.93	113.90	3	5	A+3D
D13	29.85	114.30	1	6	2A+2C+2D
D14	29.36	113.10	2	10	2B+C+3D
D15	28.56	112.30	1	7	2A+2C+3E
D16	29.70	116.00	2	10	3A+3B+2D
D17	28.23	112.90	3	6	2C+2E
D18	27.83	113.10	3	10	2D+3E
D19	27.63	113.80	2	5	2A+B
D20	27.82	114.90	2	6	4E
D21	27.12	115.00	2	5	3A+B

Table 4 The model parameters					
Parameter	Define	Value	Reference		
c_{ij} (rmb/t/km)	Cost per kilometer per ton	[0.03,0.07]	http://www.chinahighway.com		

v_{ij} (km/h)	Speed	[40,80]	Enterprise data
$c_{ m f}$ (rmb/L)	Oil price	5.8	http://www.sinopecgroup.com
ф	Emission index parameter	0.086	[7]
α _{ij}	Road condition factor	[0.09, 0.15]	[3]
δ (L/kg)	Fuel carbon emission conversion factor	2.7	[22]
$T_{\rm c}({\rm rmb/L})$	Carbon tax	80	[23]
ct(h)	Service time	1	Enterprise data
λ_1/λ_2 (rmb/h	Early/Late penalty factor	10/30	Enterprise data
$w_1/w_2/w_3$	Weight factor	1/1/1	Enterprise data

For instance, with the 21 customer orders information in the table 3, the program runs 100 times independently (that is, different random number seeds), and the best result is obtained. The optimized solution obtained is in table 5. The optimized route is demonstrated in Fig.2. The calculation results are: F=67564, $f_1=35078$, $f_2=95$, and $f_3=32391$.

Table 5 The optimized solutions

Table 5 The optimized solutions					
Route No	Transport vehicle type	Route details	Length loading rate(%)	Weight loading rate(%)	
1	1	0→D20→D21→0	89.41	72.42	
2	3	0→D19→D18→ D15→0	96.86	80.60	
3	2	0→D14→D17→0	85.76	79.11	
4	3	0→D13→D12→ D11→0	97.96	86.45	
5	2	$0 \rightarrow D8 \rightarrow D10 \rightarrow 0$	83.21	76.75	
6	2	0→D16→D1→0	99.17	80.76	
7	1	0→D4→D2→0	90.97	71.33	
8	3	0→D3→D6→D9 →0	97.34	86.54	
9	2	$0 \rightarrow D7 \rightarrow D5 \rightarrow 0$	94.66	80.26	

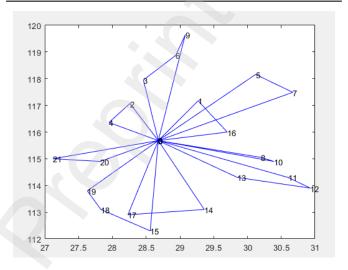


Fig.2 The optimized routes returned by the improved GA

From the analysis of the results, the vehicles are basically fully loaded, especially from the analysis of the length loading rate. Therefore, there is almost no waste of the carrier capacity.

6.2 Improvement analysis

To verify whether the improvement of the GA is effective, a general GA was programmed. In the general GA, the input data was the same, and a single chromosome was used. The initial population was randomly generated, the crossover probability and mutation probability were fixed, and there was no local search operator. To ensure the fairness of the comparison, the general GA also runs 100 times independently, and the best results are compared as follows:

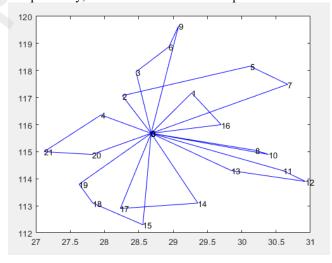


Fig.3 The optimized route returned by the general GA

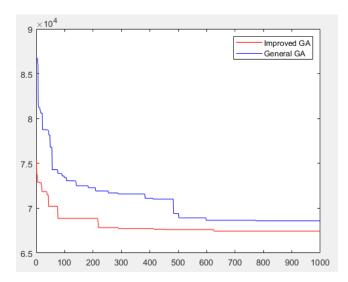


Fig.4 The evolution curve comparison

The experimental results show that the calculation result of the improved algorithm is better than the general algorithm. Taking an average of 100 runs of the algorithm for comparison, the average total cost of the improved algorithm is about 1500 yuan less. The experimental results are shown in Table 6. In table 6, '+' indicates that the improved GA is statistically significantly better at 95% confidence than the general GA according to a Wilcoxon rank sum test in all situations. Furthermore, in all the tables below, "Best", "Mean" and "Std", respectively, represent the best found, the mean, and the standard deviation of the experimental results. From analysis of the experimental results, the general GA

prefers a large transport vehicle, which can result in a waste of capacity, while the use of the dual chromosome encoding facilitates the selection of an appropriate transport vehicle, improving utilization and decreasing cost.

Table 6 Comparison of the two algorithms

_	Method	Best	Mean	Std	Wilcoxon rank sum test
	General GA	6.859E+4	6.982E+4	1.013E+3	
	Improved GA	6.745E+4	6.833E+4	5.310E+2	+

6.3 Extensive computational experience

To further verify the effectiveness of the proposed model and solution approach a series of experiments are conducted. The available transport vehicles and finished vehicles used are as listed in Table 1 and 2, respectively. Population size, crossover probability, and mutation probability are the same as before. Complete customer order information is in Appendix Table 8. The two algorithms are independently run 100 times for each instance. The comparisons of results by the general GA and the improved GA, respectively, are shown in Table 7. The results of the improved GA for the last instance are listed here, that is, the best solution and its path graph for 100 customer orders, which are shown in Appendix Table 9 and in Figure 5. The results show that the improved GA is not only suitable for small cases with 10 customer points, but also effective for large cases such as 100 customer orders, and with the increase of the number of order customers, the improvement effect is more obvious.

Table 7 Comparison of different instances of the two algorithms

Instance No	Number of customer orders	Method	Best	Best improvement ratio	Mean	Mean improvement ratio	Std	Wilcoxon rank sum test
1	10	General GA	3.325E+4		3.325E+4		7.631E-12	
1	10	Improved GA	3.221E+4	3.13%	3.221E+4	3.13%	9.669E-12	+
2	21	General GA	6.859E+4		6.982E+4		1.013E+3	
2	21	Improved GA	6.745E+4	1.66%	6.833E+4	2.13%	5.310E+2	+
3	30	General GA	9.685E+4		9.914E+4		1.929E+3	
3	30	Improved GA	9.582E+4	1.06%	9.689E+4	2.27%	5.891E+2	+
4	40	General GA	1.295E+5		1.335E+5		2.065E+3	
4	40	Improved GA	1.283E+5	0.93%	1.291E+5	3.30%	6.997E+2	+
5	50	General GA	1.601E+5		1.652E+5		2.628E+3	
3	30	Improved GA	1.530E+5	4.43%	1.546E+5	6.42%	8.424E+2	+
6	60	General GA	1.970E+5		2.032E+5		3.769E+3	
U	00	Improved GA	1.854E+5	5.89%	1.870E+5	7.97%	1.187E+3	+
7	70	General GA	2.296E+5		2.354E+5		3.186E+3	
/	70	Improved GA	2.130E+5	7.23%	2.142E+5	9.01%	8.629E+2	+
8	80	General GA	2.760E+5		2.802E+5		2.532E+3	
0	80	Improved GA	2.506E+5	9.20%	2.523E+5	9.96%	1.412E+3	+
9	90	General GA	3.023E+5		3.128E+5		4.857E+3	
9	90	Improved GA	2.725E+5	9.86%	2.750E+5	12.08%	1.733E+3	+
10	100	General GA	3.436E+5		3.522E+5		4.365E+3	
10	100	Improved GA	3.077E+5	10.45%	3.104E+5	11.87%	1.777E+3	+

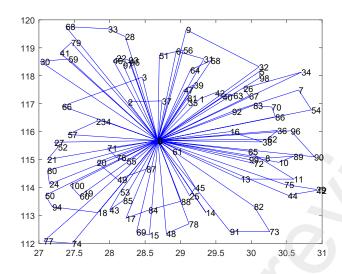


Fig.5 The optimized routes for 100 customer orders returned by the improved GA

7 Conclusion

In this paper, a realistic and complicated vehicle routing optimization problem in finished vehicle logistics is studied. This combines the classic VRP with the characteristics of the finished vehicle logistics. Based on the optimization objectives, the HVRP-FVL model is established and solved by an improved GA. Using real instances provided by a finished vehicle logistics company, a series of experiments are conducted to fully evaluate the proposed algorithm. As the problem size grows, the improved matheuristic performs increasingly better over a standard GA.

As enhancements to this work, the model depends on multiple factors and there are possibly relative relationships among these factors such as multiple depots and weather factors, which are worth exploring in the future. Also, perhaps other strategies or methods could be adopted for encoding, decoding, and operations of the GA algorithm. Moreover, other hybrid optimization algorithms could be explored to solve the HVRP-FVL.

References

- [1] Baldacci, R., Battarra, M., Vigo, D. Routing a heterogeneous fleet of vehicles. *The Vehicle Routing Problem: Latest Advances and New Challenges.*, Springer, Boston, 2008, pp. 3-27
- [2] Balseiro, S.R., Loiseau, I., Ramonet, J. An ant colony algorithm hybridized with insertion heuristics for the time dependent vehicle routing problem with time windows. *Computers & Operations Research.*, 2011, 38, (6), pp. 954-966
- [3] Bektas, T., Laporte, G. The pollution-routing problem.

 *Transportation Research Part B: Methodological., 2011, 45, (8), pp. 1232-1250

- [4] Brandão, J. A tabu search algorithm for the heterogeneous fixed fleet vehicle routing problem. *Computers & Operations Research.*, 2011, 38, (1), pp. 140-151
- [5] Coelho, V.N., Grasas, A., Ramalhinho, H., et al.An ILS-based algorithm to solve a large-scale real heterogeneous fleet VRP with multi-trips and docking constraints. European Journal of Operational Research., 2016, 250, (2), pp. 367-376
- [6] Chris Groër, Bruce Golden, Edward Wasil. A library of local search heuristics for the vehicle routing problem. *Mathematical Programming Computation*, 2010, 2(2), pp.79–101
- [7] Demir, E., Bektaş, T., Laporte, G. A comparative analysis of several vehicle emission models for road freight transportation. *Transportation Research Part D: Transport & Environment.*, 2011, 16, (5), pp. 347-357
- [8] Fischer, T., Gehring, H. Planning vehicle transshipment in a seaport automobile terminal using a multi-agent system. *European Journal* of Operational Research., 2005, 166, (3), pp. 726-740
- [9] Frifita, M., Masmoudi..VNS methods for home care routing and scheduling problem with temporal dependencies, and multiple structures and specialties. *International Transactions in Operational Research.*, 2020, 27, (1), pp. 291-313
- [10] Ge, B.F., Zhang, X.T., Chen, Y.W. Planning model and algorithm for finished vehicle logistic. *Chinese Journal of Management Science.*, 2015, 23, (1), pp. 624-629
- [11] Ge, X.L., Xu, M.Z., Wang, W.X. Study on multi-types vehicle routing problem and its quantum genetic algorithm. *Chinese Journal* of Management Science., 2013, 21, (1), pp. 125-133
- [12] Ghoseiri, K., Ghannadpour, S.F. Multi-objective vehicle routing problem with time windows using goal programming and genetic algorithm. *Applied Soft Computing.*, 2010, 10, (4), pp. 1096-1107
- [13] Ho, W., Ho, G.T., Ji, P., Lau, H.C. A hybrid genetic algorithm for the multi-depot vehicle routing problem. *Engineering Applications* of Artificial Intelligence., 2008, 21, (4), pp. 548-557
- [14] Hu, Z., Zhao, Y., Tao, S., Sheng, Z. Finished-vehicle transporter routing problem solved by loading pattern discovery. *Annals of Operations Research.*, 2015, 234, (1), pp. 37-56

- [15] Jiang, J., Ng, K.M., Poh, K.L., Teo, K.M. Vehicle routing problem with a heterogeneous fleet and time windows. *Expert Systems with Applications.*, 2014, v41, (8), pp. 3748-3760
- [16] Khachay, M. and Y. Ogorodnikov, PTAS for the Euclidean capacitated vehicle routing problem with time windows. *Learning* and *Intelligent Optimization*, LION., 2020.p. 224-230.
- [17] Kim, J., Ok, C.S., Kumara, S., Yee, S.T. A market-based approach for dynamic vehicle deployment planning using radio frequency identification (RFID) information. *International Journal of Production Economics.*, 2010, 128, (1), pp. 235-247
- [18] Koç, Ç, Bektaş, T, Jabali, O, Laporte, G. Thirty years of heterogeneous vehicle routing. *European Journal of Operational Research.*, 2016, 249, (1), pp. 1-21
- [19] Koç, Ç., Bektaş, T., Jabali, O., Laporte, G. The fleet size and mix pollution routing problem. *Transportation Research Part B: Methodological.*, 2014, 70, pp. 239-254
- [20] Kuo,Y. Using simulated annealing to minimize fuel consumption for the time-dependent vehicle routing problem. *Computers & Industrial Engineering.*, 2010, 59, (1), pp. 157-165
- [21] Kwon, Y., Choi, Y., Lee, D. Heterogeneous fixed fleet vehicle routing considering carbon emission. *Transportation Research Part* D: Transport and Environment., 2013, 23, (8), pp. 81-89
- [22] Li, J., Zhang, J.H. Study on the effect of carbon emission trading mechanism on logistics distribution routing decisions. System Engineering-Theory & Practice., 2014, 34, (7), pp. 1779-1787
- [23] Li, J., Zhang, J.H. Vehicle routing problem with time windows based on carbon emissions and speed optimization. System Engineering Theory & Practice., 2014, 34, (12), pp. 3063-3072
- [24] Li, X., Gao, L., Wang, W., et al Particle swarm optimization hybridized with genetic algorithm for uncertain integrated process planning and scheduling with interval processing time. Computers & Industrial Engineering., 2019, 135, (9), pp. 1036-1046
- [25] Liu, J., Smith, A.E., Qian, D. The vehicle loading problem with a heterogeneous transport fleet. *Computers & Industrial Engineering.*, 2016, 97, pp. 137-145

- [26] Meliani, Y., Hani, Y., Elhaq, S.L., et al.. A developed Tabu Search algorithm for heterogeneous fleet vehicle routing problem. IFAC Papers OnLine., 2019, 52, (13), pp. 1051-1056,
- [27] Mouthuy, S., Massen, F., Deville, Y., Van, H.P. A multistage very large-scale neighborhood search for the vehicle routing problem with soft time windows. *Transportation Science.*, 2015, 49, (2), pp. 223-238
- [28] Sbai, I., Limem, O., Krichen, S. An adaptive genetic algorithm for the capacitated vehicle routing problem with time windows and two-dimensional loading constraints. *IEEE/ACS 14th International Conference on Computer Systems & Applications, Hammamet, Tunisia*, 2018, pp. 88-95
- [29] Tao, Y., Wang, F. An effective tabu search approach with improved loading algorithms for the 3L-CVRP. Computers and Operations Research., 2015, 55, pp. 127-140
- [30] Wei, L., Zhang, Z., Lim, A. An adaptive variable neighborhood search for a heterogeneous fleet vehicle routing problem with threedimensional loading constraints. *IEEE Computational Intelligence Magazine.*, 2014, 9, (4), pp. 18-30
- [31] Xiao, Y., Konak, A. The heterogeneous green vehicle routing and scheduling problem with time-varying traffic congestion. *Transportation Research Part E: Logistics and Transportation Review.*, 2016, 88, pp. 146-166
- [32] Xu, J., Yan, F., Li, S. Vehicle routing optimization with soft time windows in a fuzzy random environment. *Transportation Research Part E: Logistics and Transportation Review.*, 2011, 47, (6), pp. 1075-1091
- [33] Xu, Z., Li, H., Wang, Y. An improved genetic algorithm for vehicle routing problem. *IEEE 2011 International Conference on Computational and Information Sciences, Chengdu, China*, 2011, pp.1132-1135
- [34] Yüceer, Ü., Özakça, A. A truck loading problem. *Computers & Industrial Engineering.*, 2010, 58, (4), pp. 766-773

Appendix

Table 8 100 customer orders information with time windows

Node set	Latitude	Longitude	Earliest time Et (h)	Latest time Lt(h)	Orders
D0 (depot)	28.68	115.68	-	-	-
D1	29.27	117.17	2	9	2A+B+2C
D2	28.26	117.07	1	9	1A+3C
D3	28.46	117.95	3	9	3B+2D
D4	27.95	116.36	3	10	4D
D5	30.12	118.16	1	10	3A+B+2D
D6	28.94	118.88	2	8	4E
D7	30.67	117.49	3	5	2A+2D+2E
D8	30.20	115.05	1	5	3A+E
D9	29.08	119.65	1	6	2A+4C
D10	30.39	114.90	3	8	2A+3B+2C
D11	30.60	114.30	2	9	D+4E
D12	30.93	113.90	3	5	A+3D

D13	29.85	114.30	1	6	2A+2C+2D
D14	29.36	113.10	2	10	2B+C+3D
D15	28.56	112.30	1	7	2A+2C+3E
D16	29.70	116.00	2	10	3A+3B+2D
D17	28.23	112.90	3	6	2C+2E
D18	27.83	113.10	3	10	2D+3E
D19	27.63	113.80	2	5	2A+B
D20	27.82	114.90	2	6	4E
D21	27.12	115.00	2	5	3A+B
D22	28.10	118.63	2	9	2A+B+2C
D23	27.81	116.36	1	9	1A+3C
D24	27.15	114.12	3	9	3B+2D
D25	29.13	113.70	3	10	4D
D26	29.89	117.55	1	10	3A+B+2D
D27	27.22	115.62	2	8	4E
D28	28.22	119.40	3	5	2A+2D+2E
D29	30.92	113.93	1	5	3A+E
D30	27.02	118.51	1	6	2A+4C
D31	29.33	118.60	3	8	2A+3B+2C
D32	30.11	118.32	2	9	D+4E
D33	27.98	119.68	3	5	A+3D
D34	30.71	118.13	1	6	2A+2C+2D
D35	29.11	117.04	2	10	2B+C+3D
D36	30.37	116.04	1	7	2A+2C+3E
D37	28.74	117.10	2	10	3A+3B+2D
D38	30.16	115.63	3	6	2C+2E
D39	29.19	117.68	3	10	2D+3E
D40	29.60	117.24	2	5	2A+B
D41	27.30	118.82	2	6	4 E
D42	29.49	117.39	2	5	3A+B
D43	28.00	113.20	2	9	2A+B+2C
D44	30.52	113.72	1	9	1A+3C
D45	29.21	114.00	3	9	3B+2D
D46	28.05	118.55	3	10	4D
D47	29.05	117.49	1	10	3A+B+2D
D48	28.80	112.35	2	8	4E
D49	28.11	114.29	3	5	2A+2D+2E
D50	27.09	113.72	1	5	3A+E
D51	28.70	118.73	1	6	2A+4C
D52	27.27	115.45	3	8	2A+3B+2C
D53	28.15	113.85	2	9	D+4E
D54	30.85	116.78	3	5	A+3D
D55	28.24	114.95	1	6	2A+2C+2D
D56	29.04	118.92	2	10	2B+C+3D
D57	27.41	115.90	1	7	2A+2C+3E
D58	29.43	118.54	2	10	3A+3B+2D
D59	27.42	118.60	3	6	2C+2E
D60	27.58	113.69	3	10	2D+3E
D61	28.89	115.30	2	5	2A+B
D62	30.23	115.73	2	6	4E
D63	29.75	117.30	2	5	3A+B
D64	29.14	118.21	2	9	2A+B+2C

D65	29.96	115.30	1	9	1A+3C
D66	27.33	116.91	3	9	3B+2D
D67	29.97	117.28	3	10	4D
D68	27.38	119.76	1	10	3A+B+2D
D69	28.38	112.41	2	8	4E
D70	30.29	116.88	3	5	2A+2D+2E
D71	27.97	115.41	1	5	3A+E
D72	30.04	114.86	1	6	2A+4C
D73	30.26	112.42	3	8	2A+3B+2C
D74	27.47	112.00	2	9	D+4E
D75	30.47	114.11	3	5	A+3D
D76	28.10	115.07	1	6	2A+2C+2D
D77	27.07	112.08	2	10	2B+C+3D
D78	29.12	112.70	1	7	2A+2C+3E
D79	27.46	119.19	2	10	3A+3B+2D
D80	27.12	114.61	3	6	2C+2E
D81	29.10	117.20	3	10	2D+3E
D82	30.04	113.32	2	5	2A+B
D83	30.03	116.94	2	6	4E
D84	28.55	113.19	2	5	3A+B
D85	28.19	113.53	2	9	2A+B+2C
D86	30.34	116.53	1	9	1A+3C
D87	28.52	114.66	3	9	3B+2D
D88	29.01	113.51	3	10	4D
D89	30.60	115.12	1	10	3A+B+2D
D90	30.89	115.10	2	8	4E
D91	29.70	112.42	3	5	2A+2D+2E
D92	29.73	116.72	1	5	3A+E
D93	28.27	118.56	1	6	2A+4C
D94	27.19	113.31	3	8	2A+3B+2C
D95	28.29	118.50	2	9	D+4E
D96	30.56	116.03	3	5	A+3D
D97	28.19	118.38	1	6	2A+2C+2D
D98	30.12	117.93	2	10	2B+C+3D
D99	29.97	115.00	1	7	2A+2C+3E
D100	27.44	114.08	2	10	3A+3B+2D

Table 9 The optimized solution with 100 customer orders

Ro	ute No	Transport vehicle type	Route Details
	1	3	$D0 \rightarrow D46 \rightarrow D22 \rightarrow D93 \rightarrow D0$
	2	2	$D0 \rightarrow D97 \rightarrow D95 \rightarrow D0$
	3	3	$D0 \rightarrow D1 \rightarrow D42 \rightarrow D63 \rightarrow D40 \rightarrow D0$
	4	3	$D0 \rightarrow D67 \rightarrow D26 \rightarrow D34 \rightarrow D0$
	5	2	$D0 \rightarrow D98 \rightarrow D5 \rightarrow D0$
	6	2	$D0 \rightarrow D32 \rightarrow D9 \rightarrow D0$
	7	3	$D0 \rightarrow D92 \rightarrow D7 \rightarrow D54 \rightarrow D0$
	8	3	$D0 \rightarrow D83 \rightarrow D70 \rightarrow D86 \rightarrow D0$
	9	2	$D0 \rightarrow D35 \rightarrow D81 \rightarrow D0$
	10	2	$D0 \rightarrow D37 \rightarrow D2 \rightarrow D0$
	11	3	$D0 \rightarrow D4 \rightarrow D23 \rightarrow D57 \rightarrow D0$

$12 D0 \rightarrow D27 \rightarrow D52 \rightarrow D21 \rightarrow D0$	
13 $2 D0 \rightarrow D80 \rightarrow D24 \rightarrow D0$	
$14 D0 \rightarrow D100 \rightarrow D60 \rightarrow D19 \rightarrow D0$	
$15 D0 \rightarrow D50 \rightarrow D94 \rightarrow D18 \rightarrow D0$	
$16 D0 \rightarrow D85 \rightarrow D43 \rightarrow D53 \rightarrow D0$	
$17 D0 \rightarrow D87 \rightarrow D49 \rightarrow D20 \rightarrow D0$	
$18 D0 \rightarrow D55 \rightarrow D76 \rightarrow D71 \rightarrow D0$	
$10 1 D0 \rightarrow D96 \rightarrow D90 \rightarrow D0$	
$20 D0 \rightarrow D10 \rightarrow D89 \rightarrow D8 \rightarrow D0$	
$21 D0 \rightarrow D99 \rightarrow D72 \rightarrow D0$	
$22 D0 \rightarrow D65 \rightarrow D11 \rightarrow D13 \rightarrow D0$	
23 $D0 \rightarrow D75 \rightarrow D29 \rightarrow D12 \rightarrow D44 \rightarrow D0$	
$24 D0 \rightarrow D82 \rightarrow D73 \rightarrow D91 \rightarrow D0$	
25 $D0 \rightarrow D78 \rightarrow D48 \rightarrow D0$	
26 $D0 \rightarrow D69 \rightarrow D15 \rightarrow D0$	
27 $D0 \rightarrow D17 \rightarrow D84 \rightarrow D88 \rightarrow D0$	
28 $D0 \rightarrow D61 \rightarrow D45 \rightarrow D25 \rightarrow D0$	
29 $D0 \rightarrow D47 \rightarrow D39 \rightarrow D0$	
$2 \qquad \qquad D0 \rightarrow D64 \rightarrow D58 \rightarrow D0$	
31 $2 D0 \rightarrow D31 \rightarrow D6 \rightarrow D0$	
32 $D0 \rightarrow D16 \rightarrow D38 \rightarrow D0$	
$2 \qquad \qquad D0 \rightarrow D62 \rightarrow D36 \rightarrow D0$	
$1 D0 \rightarrow D14 \rightarrow D0$	
$2 D0 \rightarrow D51 \rightarrow D56 \rightarrow D0$	
$30 D0 \rightarrow D28 \rightarrow D33 \rightarrow D68 \rightarrow D0$	
$2 D0 \rightarrow D41 \rightarrow D79 \rightarrow D0$	
$2 \qquad \qquad D0 \rightarrow D59 \rightarrow D30 \rightarrow D0$	
$2 D0 \rightarrow D66 \rightarrow D3 \rightarrow D0$	
40 $D0 \rightarrow D74 \rightarrow D77 \rightarrow D0$	