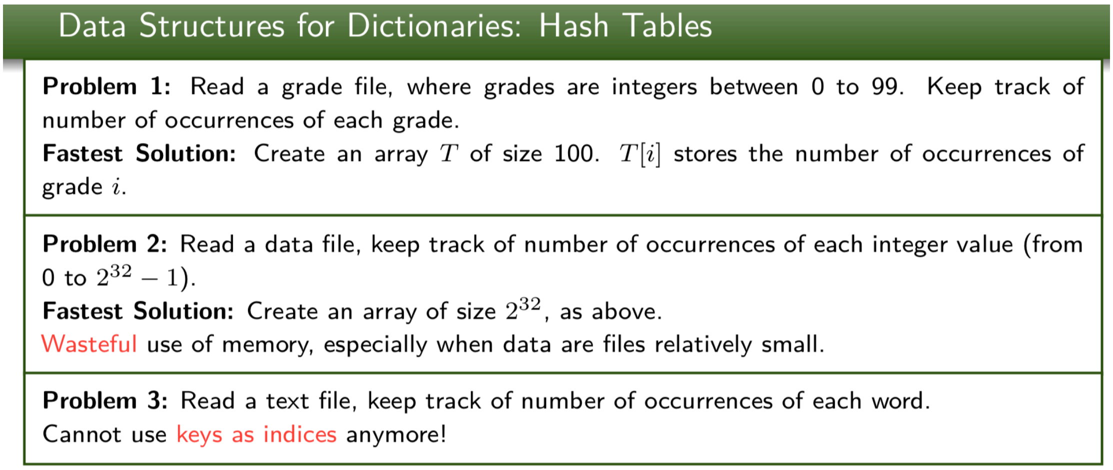
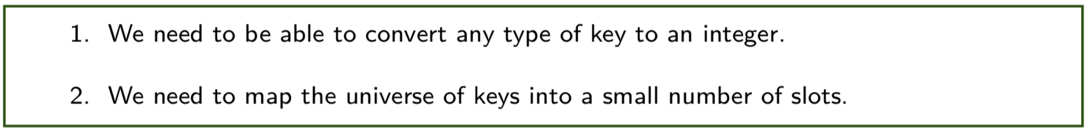


Data Structures for Dictionaries: Hash Tables

Problem 1: Read a grade file, where grades are integers between 0 to 99. Keep track of number of occurrences of each grade.
Fastest Solution: Create an array T of size 100. $T[i]$ stores the number of occurrences of grade i.

Problem 2: Read a data file, keep track of number of occurrences of each integer value (from 0 to $2^{32}-1$).
Fastest Solution: Create an array of size 2^{32}, as above.
Wasteful use of memory, especially when data are files relatively small.

Problem 3: Read a text file, keep track of number of occurrences of each word.
Cannot use keys as indices anymore!



1. We need to be able to convert any type of key to an integer.
2. We need to map the universe of keys into a small number of slots.

csc263 lec4 ppt pg3

研究：

- ➔ 增（插入 insert）
- ➔ 删（delete）
- ➔ 查（寻址/search）

List 是 ADT ADT，其两种基本的实现方式有 array 数组，linkedlist 链表

array[4]=60, 4 是 index 索引，60 是其对应的数据、数值、value
index 索引是提前设置好的

HashTable 基本原理包含了 array，随机顺序访问数据/查询，访问查询速度最快，快速根据 key(index 索引) O(1)
Array 是简单的 HashTable
HashTable: 在直接寻址下，具有关键字 k 的元素被存放在槽 k 中。在散列方式下，该元素存放在 h(k)中；即利用散列函数（hash function）h，由关键字 k 计算出槽的位置。—教材 p143

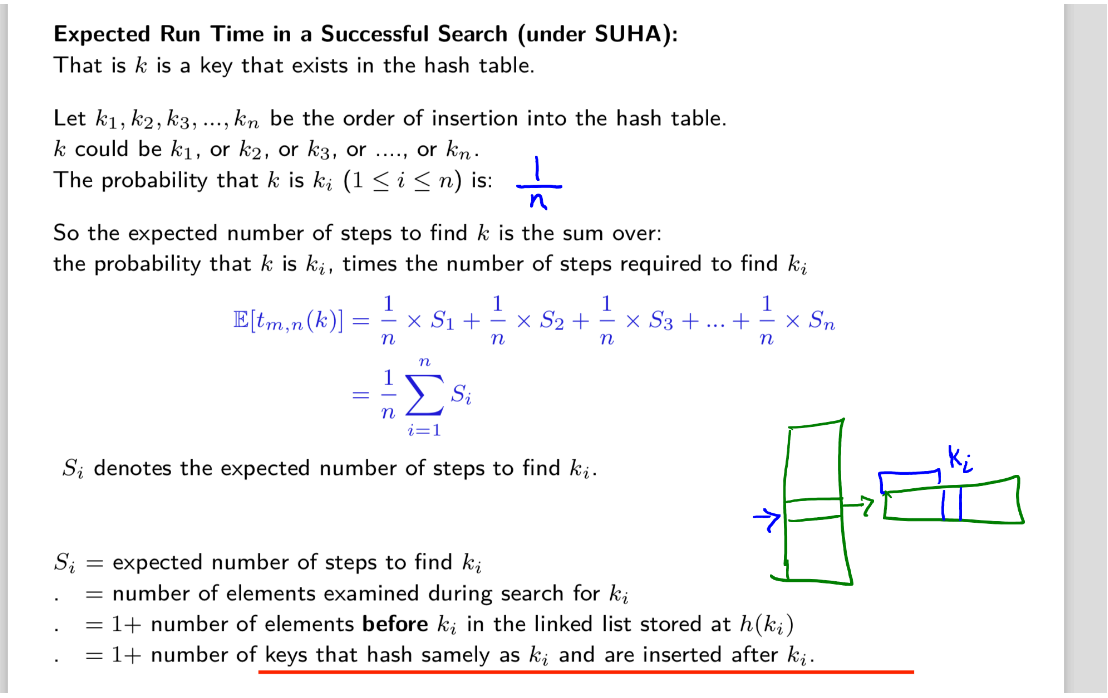
LinkedList 链式 一般是从第一个进行 traverse，O(n)

数据源 (file)– 存储(array)
将数据源里的 type of key 转换成 integer（通过 hash function 转换成具有整数性质的散列值），引出 Hash Function（universe of keys
➔ slots，slot 里面即每个槽位里存的还是原关键字 key）

Hash Table（散列表/哈希表）：元素，关键字（key），散列值（h(k)）

承载因子（load factor）一般是 0.75

目标：使数据尽可能均匀放在 slots 里，ensure uniform



Expected Run Time in a Successful Search (under SUHA):
That is k is a key that exists in the hash table.

Let $k_1, k_2, k_3, ..., k_n$ be the order of insertion into the hash table.
k could be k_1, or k_2, or k_3, or ...., or k_n.
The probability that k is k_i $(1 \leq i \leq n)$ is: $\frac{1}{n}$

So the expected number of steps to find k is the sum over:
the probability that k is k_i, times the number of steps required to find k_i

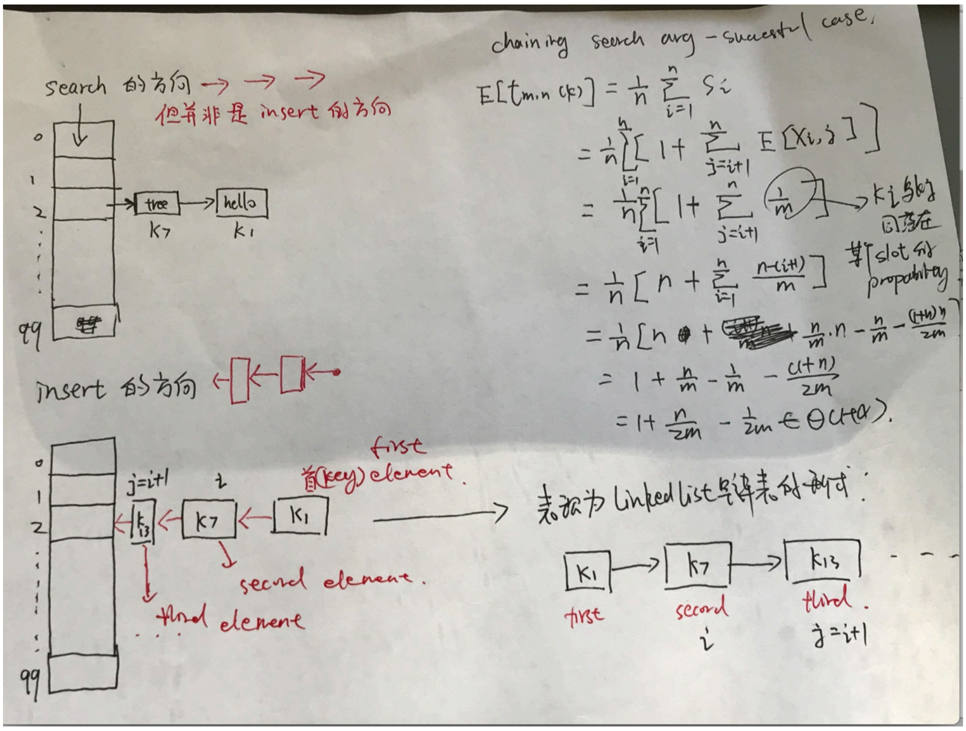
$$\mathbb{E}[t_{m,n}(k)] = \frac{1}{n} \times S_1 + \frac{1}{n} \times S_2 + \frac{1}{n} \times S_3 + ... + \frac{1}{n} \times S_n$$
$$= \frac{1}{n} \sum_{i=1}^{n} S_i$$

S_i denotes the expected number of steps to find k_i.

S_i = expected number of steps to find k_i
. = number of elements examined during search for k_i
. = 1+ number of elements **before** k_i in the linked list stored at $h(k_i)$
. = 1+ number of keys that hash samely as k_i and are inserted after k_i.

csc263 lec4 ppt pg14

注意：链接法（chaining）中的插入和搜索的**顺序**（**不同**）
"在对元素 x 的一次成功查找中，所检查的元素就是 x 所在的链表中 x 前面的元素数多 1。在该链表中因为，新的元素都是在表头插入的，所以出现在 x 之前的元素都是 x 之后插入的" –教材 p146



chaining search avg - successful case,

$$E[t_{m,n}(k)] = \frac{1}{n}\sum_{i=1}^{n} S_i$$
$$= \frac{1}{n}\sum_{i=1}^{n}\Big[1 + \sum_{j=i+1}^{n} E[X_{i,j}]\Big]$$
$$= \frac{1}{n}\sum_{i=1}^{n}\Big[1 + \sum_{j=i+1}^{n} \frac{1}{m}\Big]$$

k_i 与 k_j 同落在某 slot 的 propability

$$= \frac{1}{n}\Big[n + \sum_{i=1}^{n} \frac{n-(i+1)}{m}\Big]$$
$$= \frac{1}{n}\Big[n + \frac{n}{m}\cdot n - \frac{n}{m} - \frac{(1+n)n}{2m}\Big]$$
$$= 1 + \frac{n}{m} - \frac{1}{m} - \frac{(1+n)}{2m}$$
$$= 1 + \frac{n}{2m} - \frac{1}{2m} \in \Theta(1+\alpha).$$

定理 11.1 在 SUHA（简单均匀散列）的假设下，对于用链接法解决冲突的散列表，一次不成功查找的平均时间为 big theta(1+alpha)

定理 11.2 在 SUHA（简单均匀散列）的假设下，对于用链接法解决冲突的散列表，一次成功查找的平均时间为 big theta(1+alpha)