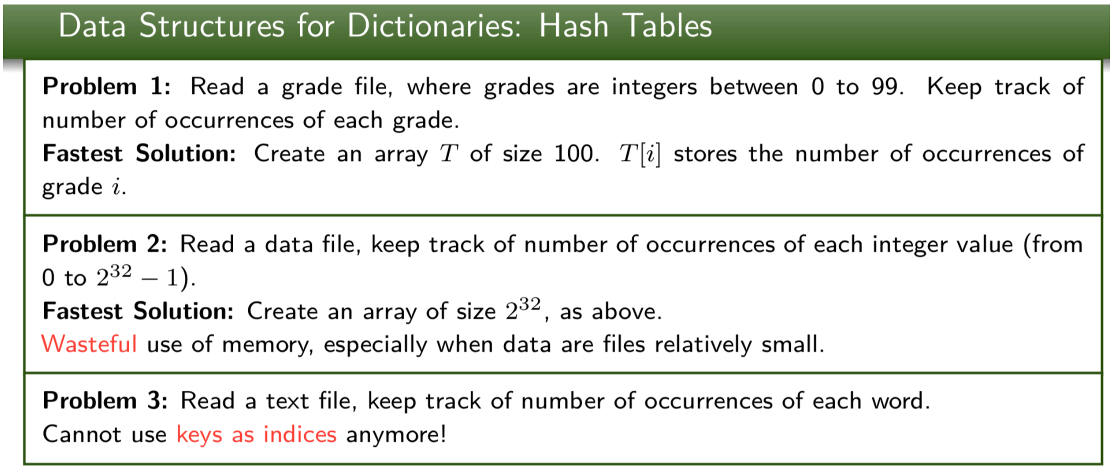
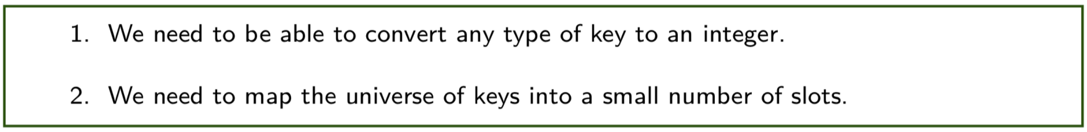


Data Structures for Dictionaries: Hash Tables

**Problem 1:** Read a grade file, where grades are integers between 0 to 99. Keep track of number of occurrences of each grade.
**Fastest Solution:** Create an array $T$ of size 100. $T[i]$ stores the number of occurrences of grade $i$.

**Problem 2:** Read a data file, keep track of number of occurrences of each integer value (from 0 to $2^{32} - 1$).
**Fastest Solution:** Create an array of size $2^{32}$, as above.
Wasteful use of memory, especially when data are files relatively small.

**Problem 3:** Read a text file, keep track of number of occurrences of each word.
Cannot use keys as indices anymore!



1. We need to be able to convert any type of key to an integer.
2. We need to map the universe of keys into a small number of slots.

研究：

➔ 增（插入 insert）
➔ 删（delete）
➔ 查（寻址/search）

List 是 ADT ADT，其两种基本的实现方式有 array 数组，linkedlist 链表

array[4]=60, 4 是 index 索引，60 是其对应的数据、数值、value
index 索引是提前设置好的

HashTable 基本原理包含了 array，随机顺序访问数据/查询，**访问查询速度最快**，快速根据 key(index 索引) O(1)
Array 是简单的 HashTable

LinkedList 链式 一般是从第一个进行 traverse，O(n)

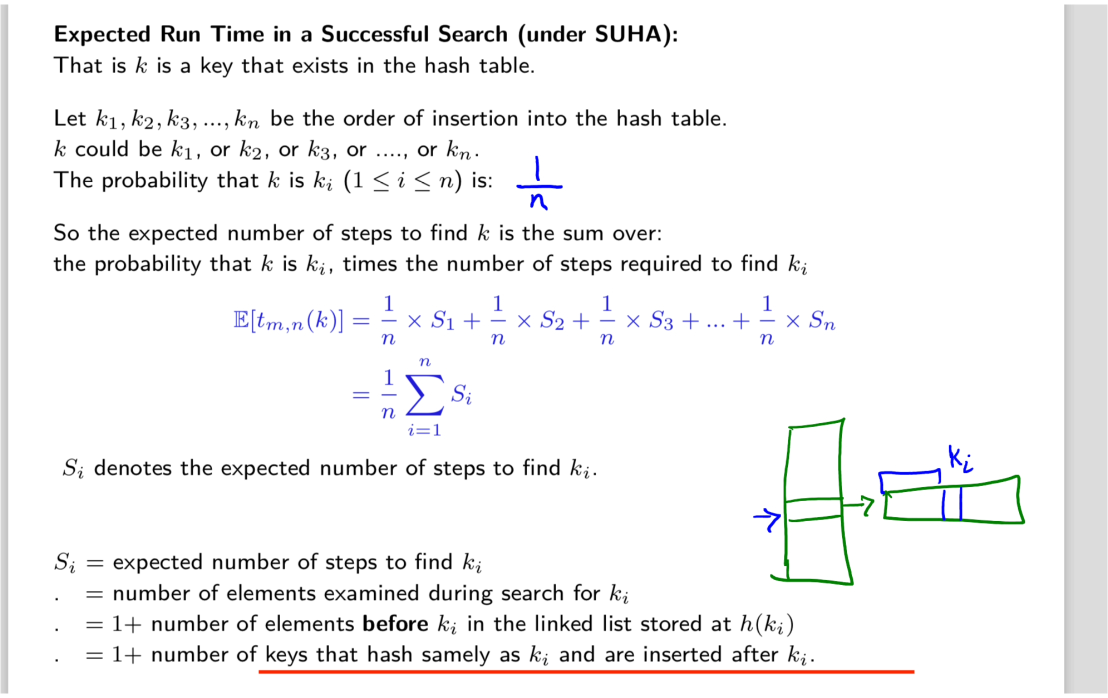
数据源 (file)– 存储(array)
将数据源里的 type of key 转换成 integer（通过 hash function 转换成具有整数性质的散列值），引出 Hash Function（universe of keys

➔ slots，slot 里面即每个槽位里存的还是原关键字 key）

**Hash Table（散列表/哈希表）：元素，关键字（key），散列值（h(k)）**

承载因子（load factor）一般是 0.75

目标：使数据尽可能均匀放在 slots 里，ensure uniform



**Expected Run Time in a Successful Search (under SUHA):**
That is $k$ is a key that exists in the hash table.

Let $k_1, k_2, k_3, ..., k_n$ be the order of insertion into the hash table.
$k$ could be $k_1$, or $k_2$, or $k_3$, or ...., or $k_n$.
The probability that $k$ is $k_i$ $(1 \leq i \leq n)$ is: $\frac{1}{n}$

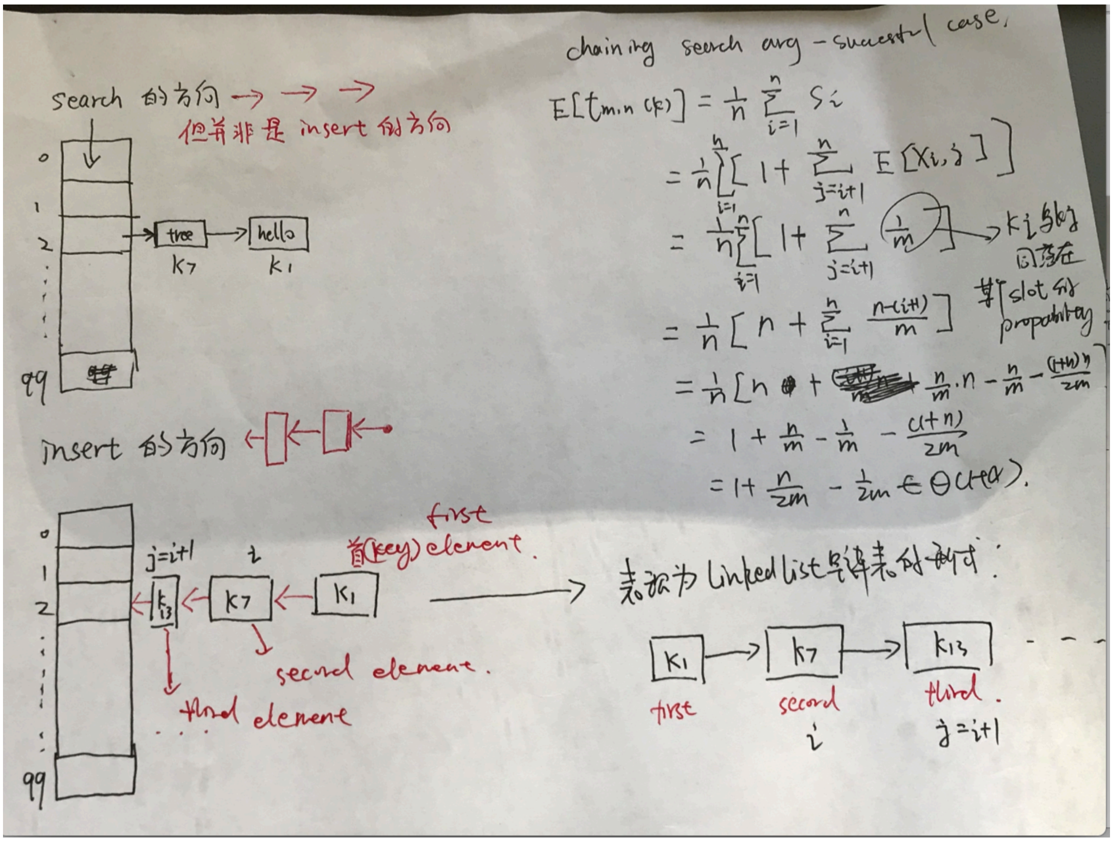
So the expected number of steps to find $k$ is the sum over:
the probability that $k$ is $k_i$, times the number of steps required to find $k_i$

$$\mathbb{E}[t_{m,n}(k)] = \frac{1}{n} \times S_1 + \frac{1}{n} \times S_2 + \frac{1}{n} \times S_3 + ... + \frac{1}{n} \times S_n$$
$$= \frac{1}{n} \sum_{i=1}^{n} S_i$$

$S_i$ denotes the expected number of steps to find $k_i$.

$S_i$ = expected number of steps to find $k_i$
. = number of elements examined during search for $k_i$
. = 1+ number of elements **before** $k_i$ in the linked list stored at $h(k_i)$
. = 1+ number of keys that hash samely as $k_i$ and are inserted after $k_i$.

注意：链式插入中的插入和搜索的**顺序**（不同）
“在对元素 x 的一次成功查找中，所检查的元素就是 x 所在的链表中 x 前面的元素数多 1。在该链表中因为，新的元素都是在表头插入的，所以出现在 x 之前的元素都是 x 之后插入的”



chaining search avg - successful case,

$$E[t_{m,n}(k)] = \frac{1}{n}\sum_{i=1}^{n} S_i$$
$$= \frac{1}{n}\sum_{i=1}^{n}\left[1 + \sum_{j=i+1}^{n} E[X_{i,j}]\right]$$
$$= \frac{1}{n}\sum_{i=1}^{n}\left[1 + \sum_{j=i+1}^{n} \frac{1}{m}\right]$$

→ $k_i$ 与 $k_j$ 同落在某 slot 的 propability

$$= \frac{1}{n}\left[n + \sum_{i=1}^{n} \frac{n-(i+1)}{m}\right]$$
$$= \frac{1}{n}\left[n + \frac{n}{m}\cdot n - \frac{n}{m} - \frac{(1+n)n}{2m}\right]$$
$$= 1 + \frac{n}{m} - \frac{1}{m} - \frac{(1+n)}{2m}$$
$$= 1 + \frac{n}{2m} - \frac{1}{2m} \in \Theta(1+\alpha).$$