Triadic Projection Semantics

Abstract

We refine the Deleuze-aligned difference triad to a mathematically auditable device that integrates with a pullback-based structural core. The projection pipeline

$$\Phi_\kappa = q_\kappa \circ \rho \circ \mathrm{Obs}_\star : X \to I_\kappa$$

induces three regimes: *Displacement* (kernel flow), *Symmetry/Masquerade* (a Φ_κ-preserving étale groupoid action), and *Bifurcation* (transverse discriminant crossings). We separate the discriminant into mechanism-side differential criticality, appearance-side image singularities, and policy-side semantic jumps, and we upgrade the space of forms by a jet-augmented orbit construction. On a regularity hypothesis we prove invariance laws, a short exact sequence

$$0 \to \ker d\Phi_\kappa \to TX \xrightarrow{d\Phi_\kappa} \Phi_\kappa^*(TI_\kappa) \to 0,$$

a Lie correspondence on the identity component, and a dynamic normal form (displacement $\circ$ symmetry $\circ$ displacement). Interfaces to the event-visible layer and to the pullback alignment space are stated explicitly.

1 Motivation and Adjustments

We adopt the standard academic naming and strengthen auditability: (i) rename *Masquerade* as *Symmetry (Masquerade)* to emphasize groupoid actions; (ii) explicitly use an étale groupoid for Φ_κ-preserving local diffeomorphisms, recording isotropy types; (iii) add a jet-augmentation to boundary types to stabilize audits near singular sets; (iv) separate the discriminant into three disjoint sources (mechanism/appearance/policy) to avoid conflation; (v) retain the triad as complete for regular-region classification; singular windows are handled by exception rules only.

Structure of the Section. Section 2 fixes notation and the regularity hypothesis. Section 3 defines the triad and the discriminant calculus. Section 4 constructs forms via groupoid orbits with jet augmentation. Section 5 proves the regular-region results (exactness, Lie correspondence, dynamic normal form). Section 6 states the event-visible and pullback interfaces. A two-line toy example closes the section.

2 Notation and Regularity

Let $X = \Delta_{\mathrm{gen}}$ be the generated difference space. Fix a readable gate $U_\star \subseteq X$ and an observation map $\mathrm{Obs}_\star : U_\star \to \delta_\star$ with a public test family $T_\star$ (calibration, thresholds, time windows, sampling law). Let $\rho : \delta_\star \to \mathcal{R}$ be a least-form selector; let $q_\kappa : \mathcal{R} \to I_\kappa$ be an identity-criterion projection ($\kappa \in \{\text{identity, similarity, analogy, negation}\}$). Set $\Phi_\kappa = q_\kappa \circ \rho \circ \mathrm{Obs}_\star : X \to I_\kappa$. Write $\Sigma \subset I_\kappa$ for appearance singularities and $I_\kappa^{\mathrm{reg}} = I_\kappa \setminus \Sigma$. Let $\nu : I_\kappa^{\mathrm{reg}} \to \mathcal{R}$ be the type map; define the kernel distribution $\mathcal{V} := \ker d\Phi_\kappa \subset TX$.

Regularity hypothesis (H_{reg})**.** On $X \setminus \mathrm{Disc}(\Phi_\kappa)$ the differential $d\Phi_\kappa$ has locally constant rank; ρ is locally constant; ν is locally constant along Φ_κ-fibres.

3 The Triad and the Discriminant Calculus

Definition 3.1 (Difference triad)**.** Displacement *is the flow along* $\mathcal{V} = \ker d\Phi_\kappa$*; if* $\dot{x} \in \mathcal{V}$*, then* $\Phi_\kappa(x(t))$ *is locally constant.* Symmetry (Masquerade) *is the action of the* Φ_κ*-preserving étale groupoid*

$$\mathrm{Aut}_{\Phi_\kappa} := \{\, g : U \to V \ \textit{local diffeomorphism} \mid \Phi_\kappa \circ g = \Phi_\kappa \ \textit{on } U \,\}.$$

Bifurcation *(deterritorialization) is a transverse crossing of the discriminant that changes the type* ν*.*

Definition 3.2 (Discriminant split)**.** *The discriminant decomposes as*

$$\mathrm{Disc}(\Phi_\kappa) = \underbrace{\mathrm{Crit}(\Phi_\kappa)}_{\textit{mechanism: rank drop}} \cup \underbrace{\Phi_\kappa^{-1}(\Sigma)}_{\textit{appearance: image folds}} \cup \underbrace{J(\rho \circ \mathrm{Obs}_\star)}_{\textit{policy: semantic jump}}.$$

On $X \setminus \mathrm{Disc}(\Phi_\kappa)$ *the map* Φ_κ *is a submersion and* ν *is locally constant.*

Remark 3.1 (Boundary semantics)**.** *A pair* (r_i, r_j) *is a semantic boundary when* $\overline{\nu^{-1}(r_i)}$ *and* $\overline{\nu^{-1}(r_j)}$ *meet along* $\mathrm{Disc}(\Phi_\kappa)$*. A boundary witness is a curve* γ *with* $\gamma \pitchfork \mathrm{Disc}(\Phi_\kappa)$ *and a recorded jet-type change.*

4 Forms by Groupoid Orbits and Jet Augmentation

Definition 4.1 (Orbit forms)**.** *Let* $\mathcal{P}$ *be the pseudogroup generated by local flows of vector fields in* $\Gamma(\mathcal{V})$ *and by arrows of* $\mathrm{Aut}_{\Phi_\kappa}$*. For* $z, z' \in \delta_\star$*, write* $z \sim z'$ *if* $z' = h \cdot z$ *for some* $h \in \mathcal{P}$*. The space of forms is* $\mathcal{R} := \delta_\star / \sim$*, stratified by isotropy type. A jet-augmented representative is*

$$r = \Big(\mathrm{nf}(z),\ [\mathrm{Iso}(z)],\ \mathbf{j}_z\Big),$$

where $[\mathrm{Iso}(z)]$ *is the stabilizer class and* $\mathbf{j}_z$ *is a finite germ/jet anchored on* $\mathrm{Disc}(\Phi_\kappa)$ *for auditable boundary types.*

5 Regular-Region Results

Proposition 5.1 (Exactness at regular points)**.** *Under* (H_{reg}) *there is a short exact sequence of vector bundles*

$$0 \longrightarrow \mathcal{V} \longrightarrow TX \xrightarrow{d\Phi_\kappa} \Phi_\kappa^*(TI_\kappa) \longrightarrow 0,$$

and $\mathcal{V}_x = T_x\Big(\Phi_\kappa^{-1}(\Phi_\kappa(x))\Big)$.

Theorem 5.1 (Lie correspondence, identity component)**.** *Let* $\Gamma(\mathcal{V}) = \{\, V \in \mathfrak{X}(X) : d\Phi_\kappa \circ V = 0 \,\}$*. If* V *integrates, its local flow* ψ_t *preserves* Φ_κ*. Conversely, a path* $g_t \in \mathrm{Aut}_{\Phi_\kappa}$ *with* $g_0 = \mathrm{id}$ *has generator* $V \in \Gamma(\mathcal{V})$*. Hence*

$$\mathrm{Lie}\Big(\mathrm{Aut}^0_{\Phi_\kappa}\Big) \cong \Gamma(\mathcal{V}).$$

Theorem 5.2 (Dynamic normal form)**.** *Any local* Φ_κ*-preserving evolution on* I_κ^{reg} *factors as*

$$\textit{displacement} \;\circ\; \textit{symmetry (masquerade)} \;\circ\; \textit{displacement}.$$

Near the singular set, bifurcations must be admitted.

Remark 5.1 (Exception window)**.** *At* $\mathrm{Disc}(\Phi_\kappa)$*, the dimension of* $\mathcal{V}$ *may jump; vertical flows may fail to preserve* Φ_κ *globally. Results above apply on regular open sets; singular cases require stratified or semialgebraic refinements.*

6 Event-Visible and Pullback Interfaces

Event-visible layer. Equip the bridging theory $\mathcal{A}_{\min}$ with time and replayable traces, producing $\mathcal{A}_{\mathrm{evt}}$ and a functor $J : \mathcal{A}_{\min} \to \mathcal{A}_{\mathrm{evt}}$. Define the event-visible alignment space

$$\mathcal{P}_{\mathrm{evt}} := \mathcal{C}_W \times_{\mathcal{A}_{\mathrm{evt}}} \mathcal{D}_W.$$

Non-isolated interleavings that change Φ_κ or ν across executions break alignment; isolation/compensation must be declared.

Pullback interface. Let $F : \mathcal{C}_W \to \mathcal{A}_{\min}$ and $G : \mathcal{D}_W \to \mathcal{A}_{\min}$ be faithful bridging functors; let $S : \mathcal{A}_{\min} \to \delta_\star$ preserve $T_\star$. For $(X, A, \theta) \in \mathcal{P} := \mathcal{C}_W \times_{\mathcal{A}_{\min}} \mathcal{D}_W$, *semantic compatibility* holds when

$$\rho(S \circ F(X)) = \rho(S \circ G(A)), \qquad \Phi_\kappa(S \circ F(X)) = \Phi_\kappa(S \circ G(A))$$

on a regular neighborhood in I_κ^{reg}.

7 Toy Example (two lines)

Let $\Phi(x, y) = (x, y^2)$ and $\rho(u, v) = \mathbf{1}_{\{v \geq 1\}}$. Then

$$\mathrm{Crit}(\Phi) = \{y = 0\}, \quad \Sigma = \{v = 0\} = \Phi(\mathrm{Crit}), \quad J(\rho \circ \Phi) = \{y = \pm 1\}.$$

Displacement flows slide along $\{(x, y) : y \neq 0\}$ preserving $v = y^2$; symmetry adds $(x, y) \mapsto (x, -y)$; crossing $y = \pm 1$ flips the type ν.

Minimal Audit Ledger (this module)

Gate $U_\star$ and tests $T_\star$; ρ, q_κ, Φ_κ; discriminant report $\mathrm{Crit}\cup\Phi_\kappa^{-1}(\Sigma)\cup J(\rho\circ\mathrm{Obs}_\star)$; groupoid data (arrows, isotropy classes); jet-augmented form representatives; regularity domain; boundary witnesses (curves and jet-type transitions).