Pullback-Mediated Context Alignment: A Minimal Structural Device with Identity-Illusion Projection

Abstract

We formalize cross-context alignment as a minimal, auditable structure. Two windowed subcategories embed into a *minimal bridging theory* $\mathcal{A}_{\min}$ carrying four decidable predicates $\mathrm{Pre}, \mathrm{Post}, \mathrm{Err}, \mathrm{Win}$. Structural alignment is the iso-comma 2-pullback $\mathcal{P} = \mathcal{C}_W \times_{\mathcal{A}_{\min}} \mathcal{D}_W$. Semantically, a gate-based observation pipeline

$$\Phi_\kappa = q_\kappa \circ \rho \circ \mathrm{Obs}_\star$$

projects behaviors to an *identity-illusion* space I_κ, generating a difference triad from the kernel flow $\mathcal{V} = \ker d\Phi_\kappa$, the automorphism groupoid $\mathrm{Aut}_{\Phi_\kappa}$, and the discriminant $\mathrm{Disc}(\Phi_\kappa)$. Alignment requires a static witness in $\mathcal{P}$ and *semantic compatibility* under ρ and Φ_κ on a regular neighborhood. We prove preservation under displacements and masquerades, a short exact sequence $0 \to \mathcal{V} \to TX \xrightarrow{d\Phi_\kappa} \Phi_\kappa^*(TI_\kappa) \to 0$, and a dynamic normal form on the regular region. We extend to an event-visible layer for concurrency and a profunctor-based relation layer for soft alignment.

1 Introduction

Large models often produce fluent cross-domain prose while drifting across context boundaries. We replace rhetorical alignment by a *structural* and *semantic* device: (i) a category-theoretic core that makes alignment a 2-pullback, and (ii) a semantics governed by an observation-to-projection pipeline that detects invariance, type stability, and boundary transitions.

Contributions.

- A *minimal bridging theory* $\mathcal{A}_{\min}$ with public predicates Pre/Post/Err/Win and windowed embeddings F, G from context categories.
- A precise *alignment space* as an iso-comma 2-pullback and a static alignment criterion.
- A semantics via $\Phi_\kappa = q_\kappa \circ \rho \circ \mathrm{Obs}_\star$ with a difference triad $(\mathcal{V}, \mathrm{Aut}_{\Phi_\kappa}, \mathrm{Disc}(\Phi_\kappa))$, yielding preservation laws, a short exact sequence, and a dynamic normal form.
- Extensions to event-visible alignment under concurrency and to a relation layer (profunctor halo) enabling soft alignment.

Structure of the Paper. Section 2 fixes notation and the bridging theory. Section 3 develops the 2-pullback core. Section 4 introduces the projection pipeline and proves the main semantic theorems. Section 5 upgrades to event-visible semantics and states concurrency rules. Section 6 gives the relation layer and soft alignment. Section 7 sketches comparisons. Appendices collect auxiliary proofs and a minimal audit ledger.

2 Notation and Conventions

We work in locally small categories. Isomorphisms are written $\xrightarrow{\sim}$. For a smooth map Ψ between manifolds, $d\Psi$ denotes the differential; regular points form the complement of the singular set.

Definition 2.1 (Windows and Bridging)**.** *Let $\mathcal{C}, \mathcal{D}$ be context categories. Windows $\mathcal{C}_W \subseteq \mathcal{C}$ and $\mathcal{D}_W \subseteq \mathcal{D}$ retain public, checkable, composable behaviors. The* minimal bridging theory *$\mathcal{A}_{\min}$ has objects called* manifest functions*, morphisms given by serial composition, and four decidable predicates* $\mathrm{Pre}, \mathrm{Post}, \mathrm{Err}, \mathrm{Win}$*. Faithful functors*

$$F : \mathcal{C}_W \to \mathcal{A}_{\min}, \qquad G : \mathcal{D}_W \to \mathcal{A}_{\min}$$

preserve identities, composition, and these predicates.

Definition 2.2 (Observation-to-Projection Pipeline)**.** *Fix a readable gate $U_\star \subseteq X$ and an observation $\mathrm{Obs}_\star : U_\star \to \delta_\star$ with public tests $T_\star$. Let $\rho : \delta_\star \to \mathcal{R}$ be the least-form selector generated by invariance under the kernel flow and $\mathrm{Aut}_{\Phi_\kappa}$, and by preservation under regular lifts. Let $q_\kappa : \mathcal{R} \to I_\kappa$ be a type projection. Define*

$$\Phi_\kappa := q_\kappa \circ \rho \circ \mathrm{Obs}_\star.$$

Write I_κ^{reg} for the regular region, with type map $\nu : I_\kappa^{\mathrm{reg}} \to \mathcal{R}$. Set the discriminant $\mathrm{Disc}(\Phi_\kappa) = \mathrm{Crit}(\Phi_\kappa) \cup \Phi_\kappa^{-1}(\Sigma) \cup J(\rho \circ \mathrm{Obs}_\star)$.

3 Pullback Core

Definition 3.1 (Alignment Space)**.** *The iso-comma 2-pullback*

$$\mathcal{P} := \mathcal{C}_W \times_{\mathcal{A}_{\min}} \mathcal{D}_W$$

has objects (X, A, θ) with $X \in \mathrm{Ob}(\mathcal{C}_W)$, $A \in \mathrm{Ob}(\mathcal{D}_W)$, and an isomorphism $\theta : F(X) \xrightarrow{\sim} G(A)$ in $\mathcal{A}_{\min}$. A morphism $(f, g) : (X, A, \theta) \to (X', A', \theta')$ satisfies $G(g) \circ \theta = \theta' \circ F(f)$.

Proposition 3.1 (Static Alignment)**.** *Objects X and A are* structurally aligned *iff $(X, A, \theta) \in \mathrm{Ob}(\mathcal{P})$ for some isomorphism θ in $\mathcal{A}_{\min}$.*

Definition 3.2 (Static Boundary)**.** $\partial_{\mathrm{stat}} := \Big(\mathrm{Ob}\,\mathcal{C}_W \times \mathrm{Ob}\,\mathcal{D}_W\Big) \setminus \mathrm{Ob}\,\mathcal{P}$.

4 Semantic Compatibility and Dynamic Invariances

Definition 4.1 (Semantic Compatibility)**.** *Fix a semantics functor* $S : \mathcal{A}_{\min} \to \delta_\star$*. For* $(X, A, \theta) \in \mathcal{P}$*,* semantic compatibility *holds when*

$$\rho\Big(S \circ F(X)\Big) = \rho\Big(S \circ G(A)\Big), \qquad \Phi_\kappa\Big(S \circ F(X)\Big) = \Phi_\kappa\Big(S \circ G(A)\Big)$$

on a regular neighborhood in I_κ^{reg}*.*

Definition 4.2 (Difference Triad)**.** *Let* $\mathcal{V} := \ker d\Phi_\kappa \subset TX$*.* Displacements *are flows of vector fields in* $\Gamma(\mathcal{V})$*.* Masquerades *are paths in the groupoid* $\mathrm{Aut}_{\Phi_\kappa} = \{g : X \to X \mid \Phi_\kappa \circ g = \Phi_\kappa\}$*.* Bifurcations *are transverse crossings of* $\mathrm{Disc}(\Phi_\kappa)$ *that change the type map* ν*.*

Theorem 4.1 (Static + Semantic Alignment)**.** *Let* $(X, A, \theta) \in \mathcal{P}$*. Full alignment holds iff semantic compatibility holds. Moreover, on* I_κ^{reg} *alignment is preserved under displacements and masquerades.*

Theorem 4.2 (Kernel Exactness and Lie Correspondence)**.** *On the identity component,* $\mathrm{Lie}(\mathrm{Aut}^0_{\Phi_\kappa}) \cong \Gamma(\mathcal{V})$*. At regular points there is a short exact sequence of vector bundles*

$$0 \longrightarrow \mathcal{V} \longrightarrow TX \xrightarrow{d\Phi_\kappa} \Phi_\kappa^*(TI_\kappa) \longrightarrow 0,$$

and $\mathcal{V}_x = T_x\Big(\Phi_\kappa^{-1}(\Phi_\kappa(x))\Big)$*.*

Theorem 4.3 (Dynamic Normal Form)**.** *Any local alignment-preserving evolution on* I_κ^{reg} *factors as*

displacement flow $\circ$ *masquerade* $\circ$ *displacement flow.*

Near the singular set, bifurcations must be admitted.

Corollary 4.1 (Semantic Boundary)**.** *If a curve transverse to* $\mathrm{Disc}(\Phi_\kappa)$ *changes* ν *from* r_i *to* r_j*, the pair falls into the semantic boundary* ∂_{sem}*.*

5 Event-Visible Alignment and Concurrency

Definition 5.1 (Event Layer)**.** *Endow* $\mathcal{A}_{\min}$ *with a time object* T *and replayable traces, obtaining* $\mathcal{A}_{\text{evt}}$ *and a functor* $J : \mathcal{A}_{\min} \to \mathcal{A}_{\text{evt}}$*. Set*

$$\mathcal{P}_{\text{evt}} := \mathcal{C}_W \times_{\mathcal{A}_{\text{evt}}} \mathcal{D}_W.$$

A switching law is admissible when layer switches commute with workflow composition.

Proposition 5.1 (Concurrency Discipline)**.** *Non-isolated interleavings that change* Φ_κ *or* ν *across executions break alignment. Isolation or compensation must be declared to restore it.*

6 Relation Layer: Profunctor Halo and Soft Alignment

Definition 6.1 (Profunctor Halo)**.** *Define a profunctor*

$$\mathsf{P}_{F,G}(X, A) := \mathrm{Hom}_{\mathcal{A}_{\min}}(FX, GA)$$

to register alignable but non-equal *pairs. Reflections or kernel pairs carve out the exact-alignment core as a subdevice, while a* type-compatible *halo keeps pairs with ρ-coincident forms.*

7 Comparisons

Decorated or structured cospans can be compared via a manifest-contract functor and the pullback universality. Lenses and optics match bidirectional maintenance with units and idempotents in $\mathcal{A}_{\min}$. Institutions model cross-logic bridges; embeddings and separation boundaries are recorded in $\mathcal{P}$ or ∂_{stat}.

Appendix A: Minimal Audit Ledger

Windows $\mathcal{C}_W, \mathcal{D}_W$ and exceptions; predicates Pre/Post/Err/Win; semantics functor S and projection Φ_κ; structural witness $(X, A, \theta) \in \mathcal{P}$; semantic witness (equalities under ρ and Φ_κ on a regular neighborhood); boundary register (discriminant crossings and concurrency breaks).