Component — Constraints (Guardrails Projection)

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Linear Bands (Invariants)

$$L_{\mu} \le \mu(p) = \sum_{i} r_{i} p_{i} \le U_{\mu}, \qquad L_{h} \le h(p) = \sum_{i} \mathbf{1}[r_{i} > 0] p_{i} \le U_{h}.$$
 (1)

Combined with $p \in \Delta^k$ and caps encoded by r.

Feasibility

The feasible set

$$C = \left\{ p \in \Delta^k \mid L_{\mu} \le \sum_i r_i p_i \le U_{\mu}, \ L_h \le \sum_i \mathbf{1}[r_i > 0] \ p_i \le U_h \right\}$$
 (2)

is nonempty if the bands intersect the convex hull of $\{(r_i, w_i)\}$ under convex combinations; check via a small LP in practice.

KL/Bregman Projection

For a given $q \in \Delta^k$,

$$\min_{x \in \mathcal{C}} D_{\mathrm{KL}}(x \| q) \quad \Rightarrow \quad x_i \propto q_i \exp(-\lambda_0 - \lambda_\mu r_i - \lambda_h w_i). \tag{3}$$

Priority

When both bands cannot be achieved within a δ -small change, prioritize hard legal bounds (e.g., RTP lower bound) and choose the nearest feasible hit-rate.