Component — Simulator (Monte Carlo Confidence)

September 26, 2025

Setup

Draw $X \in \{r_i\}$ with $\mathbb{P}[X = r_i] = p_i$. For N i.i.d. spins $X_1, \ldots, X_N$,

$$\hat{\mu}_N = \frac{1}{N}\sum_{j=1}^{N} X_j, \qquad \widehat{\mathrm{Var}}_N = \frac{1}{N}\sum_{j=1}^{N} X_j^2 - \hat{\mu}_N^2, \qquad \widehat{h}_N = \frac{1}{N}\sum_{j=1}^{N} \mathbf{1}[X_j > 0]. \tag{1}$$

Confidence Intervals (Normal Approx.)

By CLT,

$$\hat{\mu}_N \sim \mathcal{N}\big(\mu,\, \sigma_\mu^2/N\big), \;\; \sigma_\mu^2 = \mathrm{Var}(X), \qquad \widehat{h}_N \sim \mathcal{N}\big(h,\, h(1-h)/N\big). \tag{2}$$

A two-sided $(1-\alpha)$ CI is $\hat{\theta}_N \pm z_{1-\alpha/2}\sqrt{\widehat{\mathrm{Var}}(\hat{\theta}_N)}$.

Sample Size Guidance

To achieve margin ε for μ:

$$N \gtrsim z_{1-\alpha/2}^2\, \sigma_\mu^2/\varepsilon^2, \qquad \sigma_\mu^2 \approx \widehat{\mathrm{Var}}_N \text{ (pilot estimate)}. \tag{3}$$

Goodness-of-Fit (Optional)

Compare simulated frequencies with p via χ^2 or exact multinomial tests.

Reproducibility

Fix random seed; log environment versions; emit raw draws or sufficient statistics for audit.