CS 4104
APPLIED MACHINE LEARNING

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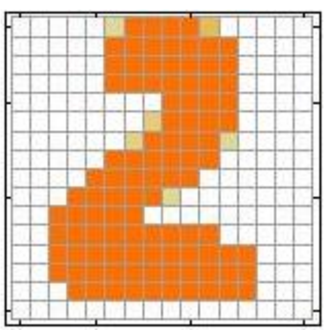
CONVOLUTIONAL NEURAL NETWORK

Why CNN

Image Classification

64x64x3

→ Cat? (0/1)

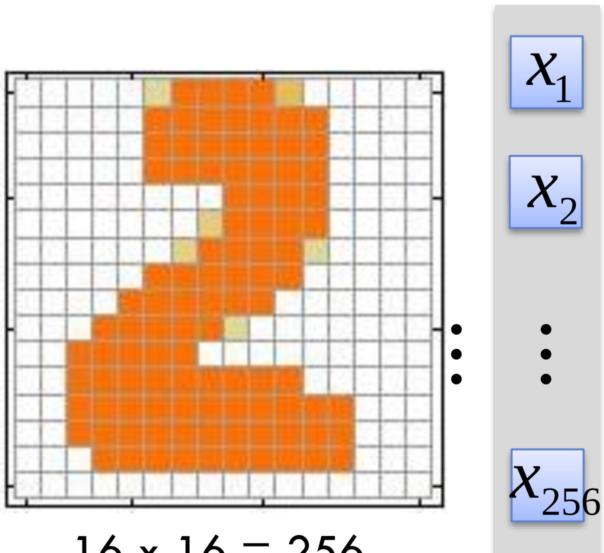


16 x 16 = 256

Text detection

Object detection

Why CNN

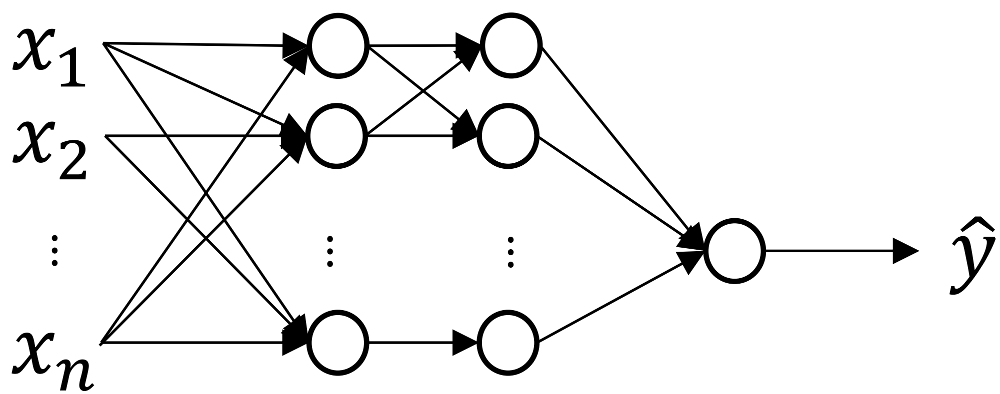


16 x 16 = 256

Ink → 1

No ink → 0

Total weights = ?



1000 1000

Why CNN

Image Classification

64x64x3

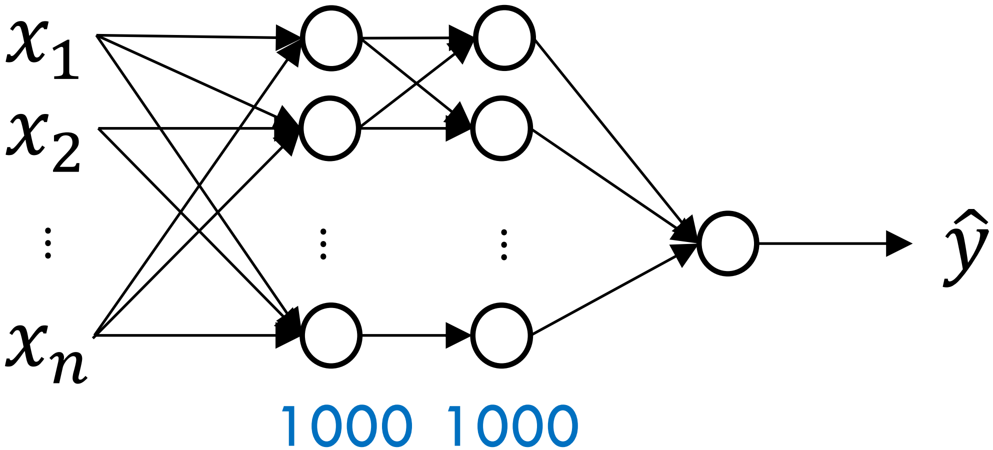
$\longrightarrow$ Cat? (0/1)

1000x1000x3

Why CNN

Image Classification

1000x1000x3



Total weights = ?

CNN

- Neural Networks that use convolution in place of general matrix multiplication in at least one layer

- There are three types of layers in the convolutional network,

  - **Convolution layer (Conv)**
  - **Pooling layer (Pool)**
  - **Fully connected layer (FC)**

Cross-correlation

- Let f be the image,
- w be the kernel of size $m \times n$
  - where $m = 2a + 1$ and $n = 2b + 1$), a and b are the positive integers.
- g be the output image

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

This is called a **cross-correlation** operation:

$$\text{g} = w \otimes f$$

Cross-correlation

$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)\,f(x+s,y+t)$$

At any point (x,y), the response g(x,y) of the filter is the sum of product of filter coefficient and the image pixels

$$\begin{aligned}g(x,y)=\ &w(-1,-1)\,f(x-1,y-1)+\\ &w(-1,0)\,f(x-1,y)+\ldots\\ &w(0,0)\,f(x,y)+\ldots\\ &w(1,1)\,f(x+1,y+1)\end{aligned}$$

Convolution

- Same as cross-correlation, except that the kernel is "*flipped*" (horizontally and vertically)

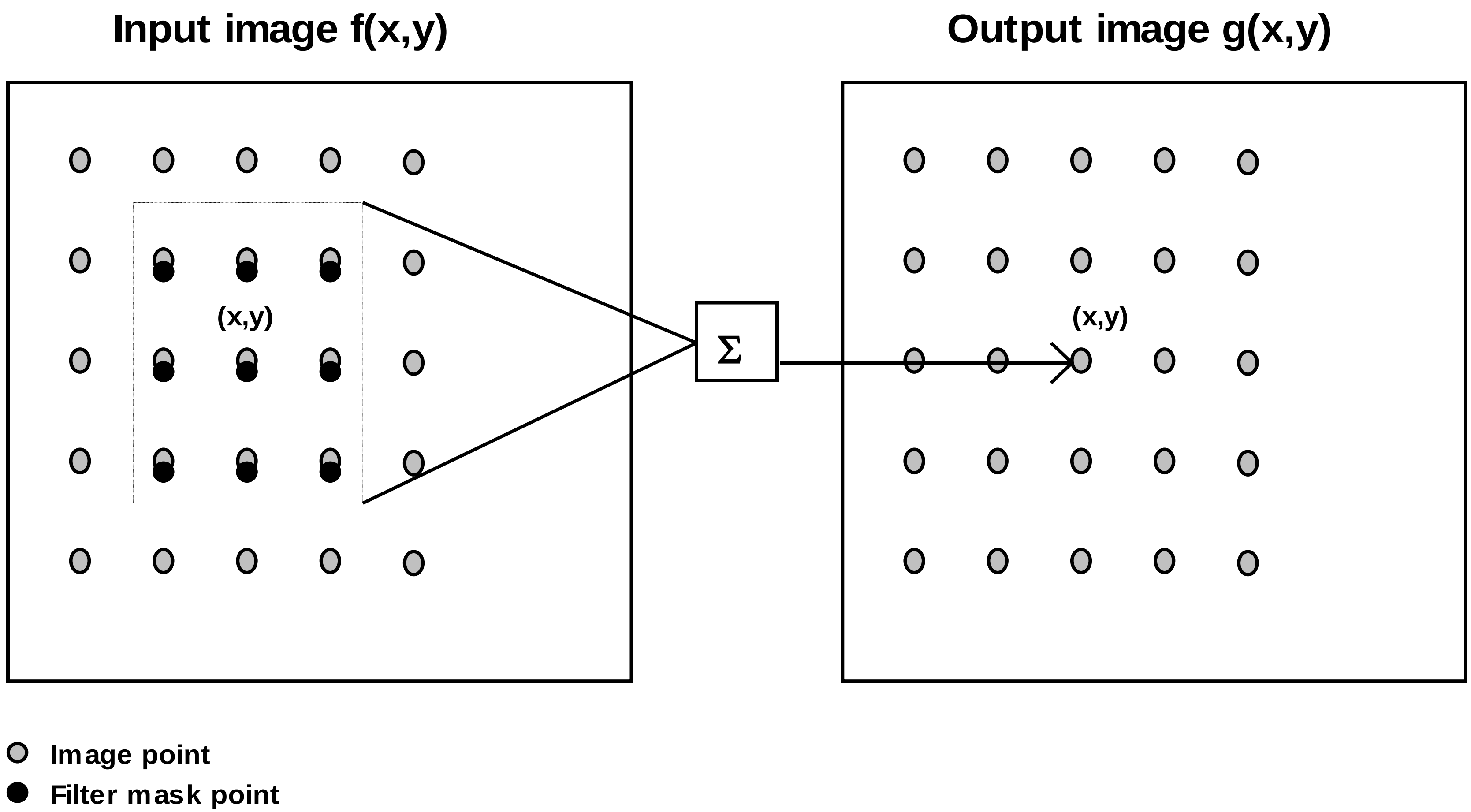
$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)\,f(x-s,y-t)$$

This is called a **convolution** operation:

$$g = w * f$$

- Convolution is **commutative** and **associative**

2D Spatial filtering



2D Spatial filtering

10	11	10	0	0	1
9	10	11	1	0	1
10	9	10	0	2	1
11	10	9	10	9	11
9	10	11	9	99	11
10	9	9	11	10	10

F

H

1/9

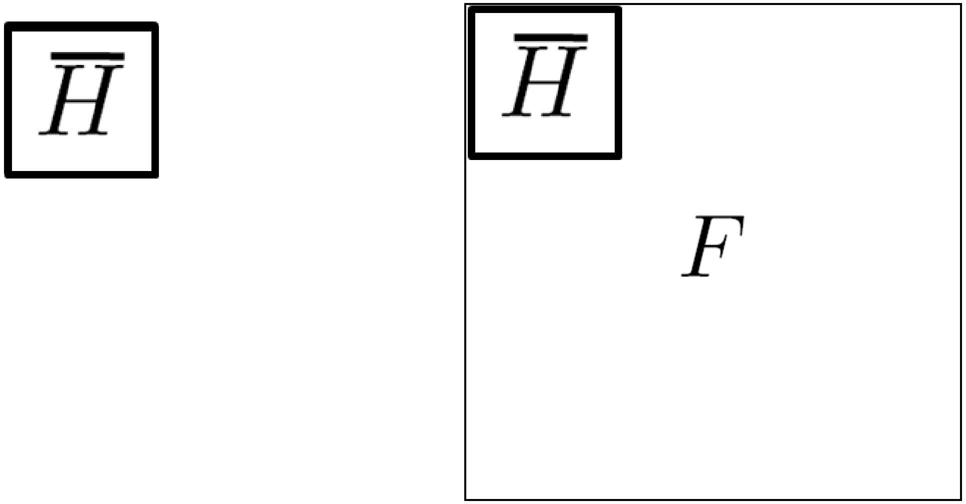
1	1	1
1	1	1
1	1	1

X	X	X	X	X	X
X	10				X
X					X
X					X
X					X
X	X	X	X	X	X

G

$$1/9.(10\times1 + 11\times1 + 10\times1 + 9\times1 + 10\times1 + 11\times1 + 10\times1 + 9\times1 + 10\times1) = 1/9.(90) = 10$$

Convolution



Convolution Examples-Mean filtering

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$*$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$=$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$w \quad * \quad f \quad = \quad g$$

Linear filters: examples

Original

*

0	0	0
0	1	0
0	0	0

=

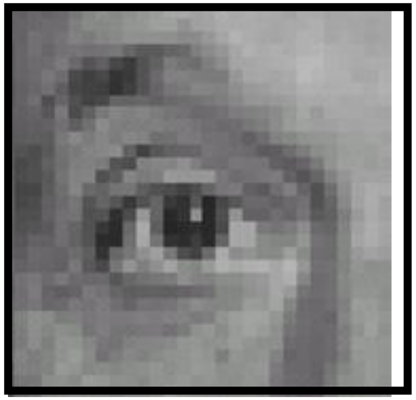
Identical image

Source: D. Lowe

Linear filters: examples

Original

$$* \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} =$$



Shifted left
By 1 pixel

Source: D. Lowe

Smoothing Spatial Filters

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

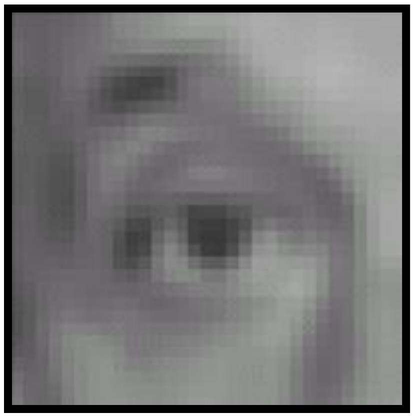
Box Filter **Weighted Average**

the center is the most important and other pixels are inversely weighted as a function of their distance from the center of the mask

Linear filters: examples

Original

$$* \; \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \; =$$

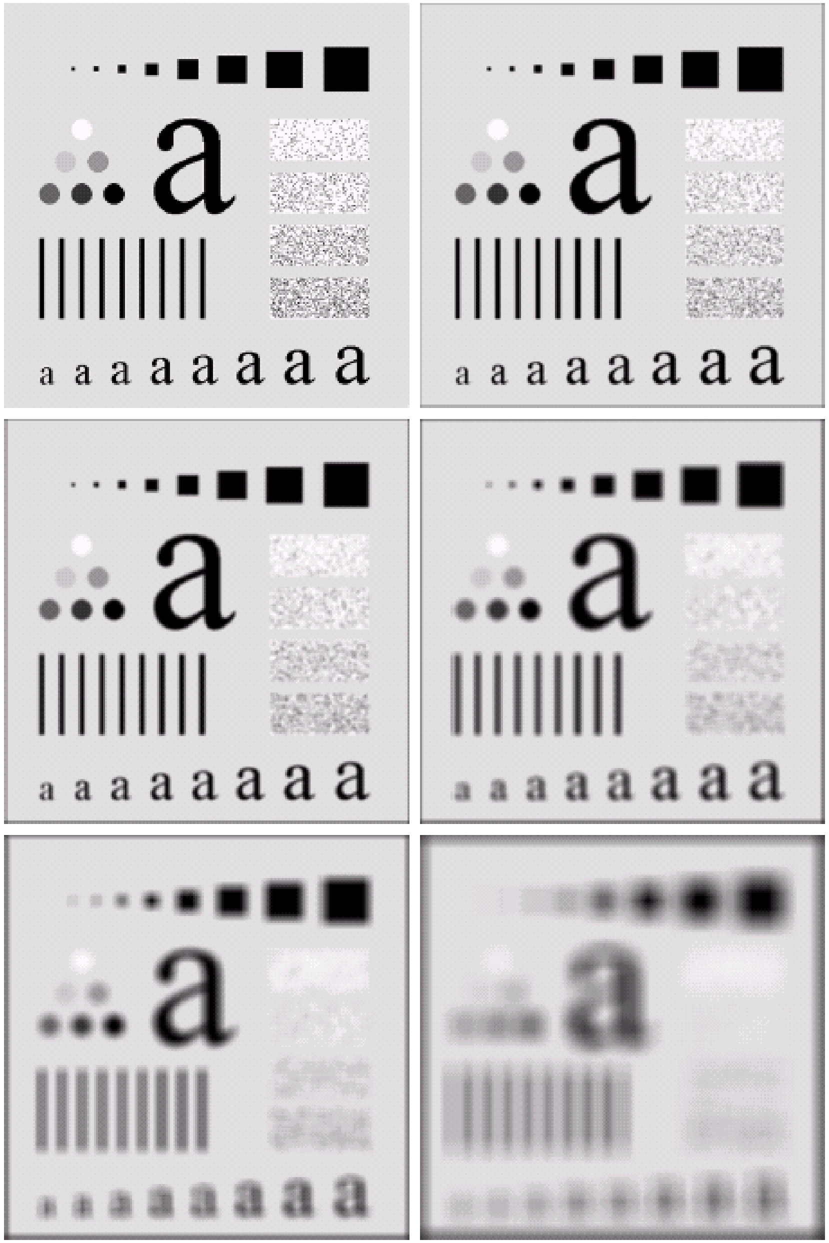


Blur (with a mean filter)

Source: D. Lowe

Smoothing Filters

- (a) is the original image of size 500x500 pixel
- (b)-(f) results of smoothing with square averaging filter masks of size n = 3, 5, 9, 15 and 35, respectively.
- Note:
  - The *big mask is used to eliminate small objects from an image*.
  - The *size of the mask establishes the relative size of the objects* that will be blended with the background.



Gradient Operator

- The first derivatives are implemented using the **magnitude of the gradient**

$$\nabla \mathrm{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

$$M(x, y) = mag(\nabla \mathrm{f}) = [G_x^2 + G_y^2]^{1/2}$$

$$= \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

Gradient Operator

$$M(x,y) = mag(\nabla \mathrm{f}) = [G_x^2 + G_y^2]^{1/2}$$

$$= \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

commonly approx.

the magnitude becomes nonlinear

$$M(x,y) \approx |G_x| + |G_y|$$

Sobel Operators

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

- Sobel operators, 3x3

$$G_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$G_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

$$M(x, y) \approx |G_x| + |G_y|$$

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

the weight value 2 is to achieve smoothing by giving more important to the center point

Sobel Operators

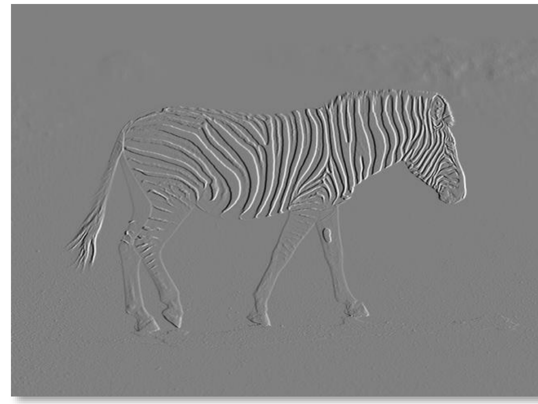
- The *summation of coefficients in all masks equals 0*, indicating that they would give a response of 0 in an area of constant gray level.

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

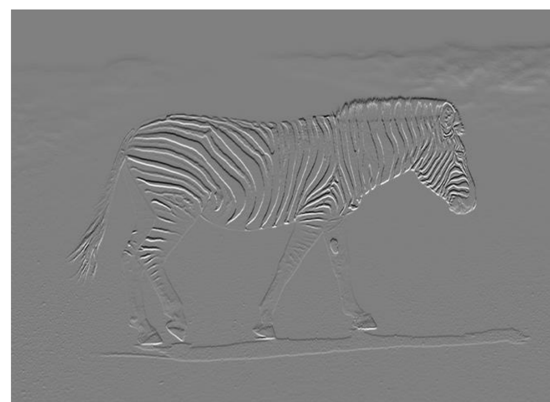
Image Derivatives

$$f$$



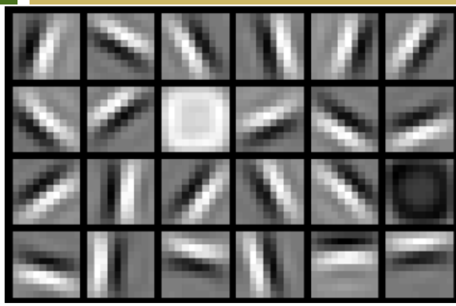
$$\frac{\partial f}{\partial x}$$

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

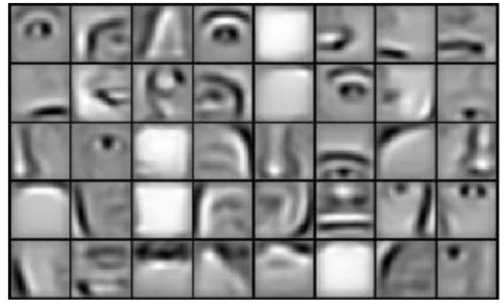


$$\frac{\partial f}{\partial y}$$

CNN … Convolution Layer

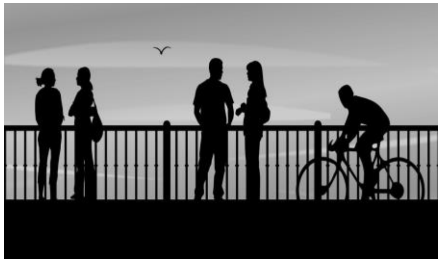


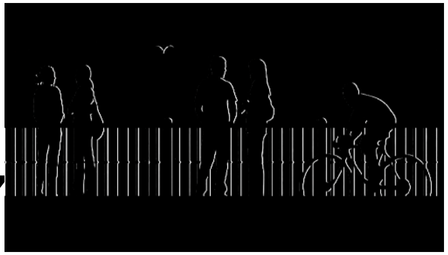
Edge detection in this layer



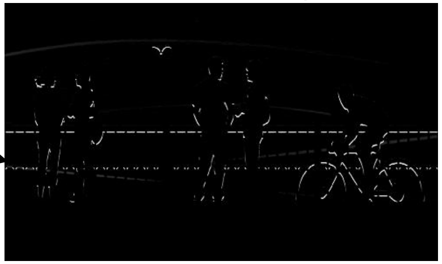
Features detection in this layer

Faces detection in this layer



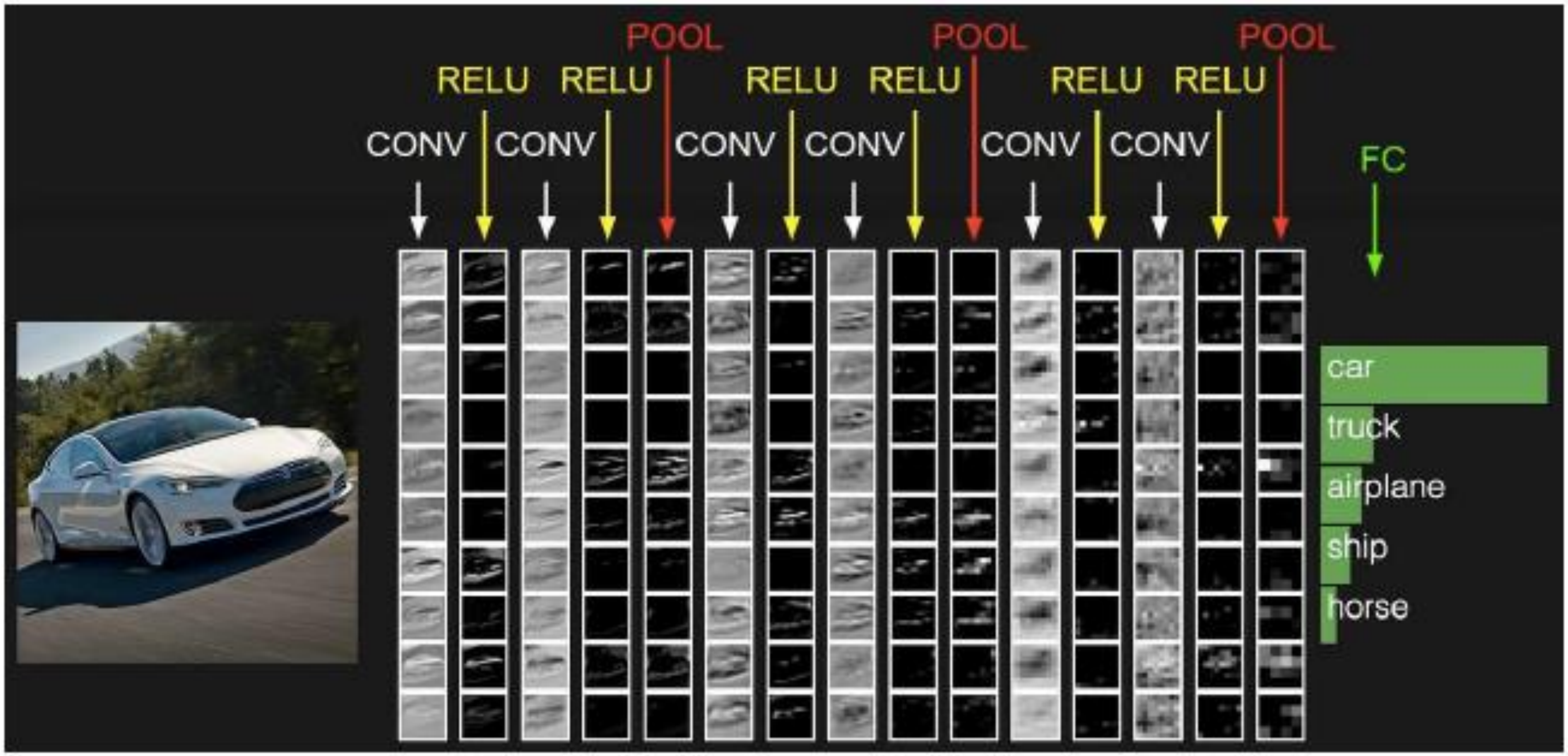


vertical edges



horizontal edges

CNN … Example


POOL
POOL
POOL
RELU
RELU
RELU
RELU
RELU
RELU
CONV
CONV
CONV
CONV
CONV
CONV
FC
car
truck
airplane
ship
horse

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