AI2002 – Artificial Intelligence

Practice Questions

First Order Logic

Instructor:	Mahzaib Younas
Email:	mahzaib.younas@nu.edu.pk
Semester:	Spring 2024
Section:	BSCS-6A

Question 1:

In a kingdom, there are various social hierarchies beyond just being a knight, peasant, or noble. Nobles are not only knights but also have the additional responsibility of being landowners. Peasants, on the other hand, may or may not be serfs, depending on whether they work directly for a noble. Express these intricate relationships using first-order logic.

Q1-

Solution: To represent these relationships in first-order logic, we can introduce additional predicates and functions:

Constants:

Knights, peasants, nobles

Predicates:

isNoble(x): x is a noble
isKnights(x): x is a knight
isPeasant(x): ~~x is a~~ x is a peasant
isLandowner(x): x is a landowner
worksFor(x,y): x works for y (where x is a peasant and y is a noble)

Functions:

ownsLand(x): x owns land (where x is a noble)

~~Now~~
Now, let's represent the given relationships:

Nobles are always knights:

$\forall x\,(isNoble(x) \Rightarrow isKnight(x))$

Nobles are landowners:

$\forall x\,(isNoble(x) \Rightarrow ownsLand(x))$

~~Peasants may or~~

Peasants may or may not be serfs, depending on whether they work directly for a noble:

$\forall x\,(isPeasant(x) \Rightarrow (\exists y(isNoble(y) \wedge worksFor(x,y))))$.

Question 2:

In a diverse society, marriages are not strictly limited to the binary notion of husband and wife. There are instances of same-sex marriages and polyamorous relationships. Additionally, marriages can be temporary or permanent. How would you represent the intricate dynamics of marriage using first-order logic, considering these complexities?

Q2-

Solution:

To represent the relationship between married couples in a diverse society using first-order logic, we need to account for various scenarios:

Constants:

Persons (representing individuals)

Functions:

isMarriedTo(x,y): x is married to y (where x and y are persons)

Predicates:

isHusband(x): x is a husband

isWife(x): x is a wife

isSpouse(x,y): x is a spouse of y

isSameSexMarriage(x,y): x and y are in a same

-sex marriage.

isPolyamorousMarriage(x): x is in a polyamorous marriage.

~~Now, let's repre~~

Now, let's represent the given relationships considering the complexities:

Expressing the relationship between married couples:

$\forall x \forall y\, (\text{isMarriedTo}(x,y) \Leftrightarrow (\text{isHusband}(x) \land \cancel{\text{is wife}}\ \text{isWife}(y)) \lor (\text{isHusband}(y) \land \text{isWife}(x)))$

Accounting for same-sex marriages:

$\forall x \forall y\, (\text{isSameSexMarriage}(x,y) \Rightarrow (\text{isHusband}(x) \land \text{isHusband}(y)) \lor (\text{isWife}(x) \land \text{isWife}(y)))$

Considering polyamorous relationships:

$\forall x\, (\text{isPolyamorousMarriage}(x) \Rightarrow \exists y \exists z\, (\text{isSpouse}(x,y) \land \text{isSpouse}(x,z) \land y \neq z))$

Question 3:

In a futuristic library system, books can belong to multiple genres simultaneously, and new genres can be dynamically created based on user preferences. Additionally, some books might transition from one genre to another over time. How would you represent the dynamic relationship of books belonging to genres using first-order logic, considering these complexities?

Q3-

Solution: To represent the relationship "belongsToGenre" in a futuristic library system using first-order logic, we need to consider the dynamic nature of genre assignments and the possibility of multiple genre memberships:

Constants:

Books (representing individual books)
Genres (representing different genres)

Predicates:

belongstoGenre(x,y): Book x belongs to genre y

Functions:

getGenres(x): Returns the set of genres to which book x belongs

Now, let's represent the given relationship considering the complexities:

Expressing the dynamic relationship of books belonging to genres: $\forall x \forall y$ ~~(belongs to~~ $(belongsToGenre(x,y) \iff y \in getGenres(x))$

Accounting for multiple genre memberships:

$\forall x \forall y\, (belongsToGenre(x,y) \Rightarrow \exists z\, (belongsToGenre(x,z) \wedge z \neq y))$

Incorporating the possibility of new genres based on user preferences:

$\forall x \forall y\, (\exists z\, (belongsToGenre(x,z) \wedge z = y) \Leftrightarrow UserPreference(x,y))$

Question 4:

In the vast ecosystem of a futuristic zoo, animals exhibit complex predator-prey relationships, where some animals may be predators of multiple species while others might be prey to certain predators. Additionally, the relationships between predators and prey evolve dynamically over time due to ecological changes. How would you represent the intricate predator-prey relationships using first-order logic, considering these complexities?

Q4-

Solution: To represent the relationship "isPredatorOf" "isPredatorOf" in a futuristic zoo system using first-order logic, accounting for complex predator-prey relationships and dynamic ecological changes, we can define the following elements:

Constants:

Animals (representing different species of animals)

Predicates:

isPredatorOf(x,y): Animal x is a predator of animal y

isPreyOf(x,y): Animal x is prey to animal y

Functions:

getPreyOf(x): Returns the set of animals that are predators of animal x

getPredatorsOf(x): Returns the set of animals that are predators of animal x.

Now, let's represent the given relationship considering the complexities:

Expressing the dynamic nature of predator-prey relationships: $\forall x \forall y (isPredatorOf(x,y) \iff y \in$ ~~get pre~~ $getPreyOf(x))$

Accounting for ~~anim~~ animals being prey to multiple predators:

$\forall x \forall y (isPreyOf(x,y) \Rightarrow \exists z$
$(isPredatorOf(z,x) \land z \neq y))$

Incorporating ecological changes and evolving relationships:

$\forall x \forall y (\exists z (isPredatorOf \cancel{(x,y)} (z,x) \land z = y) \iff$
$EcologicalChange(x,y))$

Question 5:

Consider a sprawling royal family with intricate lineage and complex familial ties, where siblingship extends beyond direct biological relations to include half-siblings, stepsiblings, and adopted siblings. Additionally, certain cultural and legal nuances influence the recognition of sibling relationships. How would you represent the multifaceted relationship between siblings using first-order logic, considering these complexities?

Q5-

Solution: To represent the relationship "isSiblingOf" in a royal family context with diverse familial ties and cultural intricacies using first-order logic, we can define the following elements:

Constants: Individuals (representing members of the royal family)

Predicates:

isSiblingOf(x,y): Individual x is a sibling of individual y

Functions:

getImmediateSiblings(x): Returns the immediate siblings of individual x

getAllSiblings(x): Returns all siblings (including half-siblings, step-siblings and adopted siblings) of individual x

Now, let's represent the given relationship considering the complexities:

Accounting for direct biological siblingship: $\forall x \forall y$ (isSiblingOf(x,y) $\Leftrightarrow$ isSiblingOf(y,x) $\land$ $x \neq y$)

Including half-siblingship: $\forall x \forall y\,(isSiblingOf(x,y) \Leftrightarrow$
$\exists z\,(hasSameParent(z,x) \land$
$hasSameParent(z,y)) \land x \neq y)$

Incorporating step-siblingship: $\forall x \forall y\,(isSiblingOf(x,y) \Leftrightarrow$
$\exists z$ ~~(has m~~ $(hasMarriageWith(z,x)$
~~$\land$ has children with~~ $\land\, hasChildrenWith(z,y)) \lor$
$(hasMarriageWith(z,y) \land$
~~has C~~ $hasChildrenWith(z,x)) \land x \neq y)$

Recognizing ~~Adopted~~ adopted siblingship:

$\forall x \forall y\,(isSiblingOf(x,y) \Leftrightarrow \exists z$
$(isAdoptedParentOf(z,x) \land$
$isAdoptedParentOf(z,y)) \land x \neq y)$

Considering cultural and legal nuances: $\forall x \forall y\,(isSiblingOf(x,y) \Leftrightarrow$
~~Legal~~ $LegalRecognition(x,y) \land$
$CulturalAcceptance(x,y))$

Question 6:

Consider a dynamic royal court where relationships and attributes evolve over time. Initially, the court comprises King John and Richard the Lionheart, with John wearing a crown and Richard identified as his brother. Further, the left legs of both individuals are distinct. Additionally, it's established that John is the sole king among the mentioned individuals. Now, envision scenarios where the royal family expands, crown ownership shifts, sibling relationships evolve symmetrically, and physical attributes undergo transformations.

1) Extended Royal Siblinghood: In an expanded model, where Richard and John have another sibling named Mary, extend the Brother relation to incorporate Mary's siblinghood with both Richard and John using first-order logic.
2) Regal Heirloom: Introduce a scenario where not only does the queen wear a crown, but also passes it down to her successor. Adjust the existing relations or introduce new ones to represent the inheritance of the crown within the royal lineage using first-order logic.
3) Symmetric Siblinghood Expansion: Expand the model to include a symmetric sibling relationship among all members of the royal family, including extended relatives and adoptees, using first-order logic.
4) Crown Transfer Dynamics: Suppose the crown ownership changes hands frequently among the royal family members due to political intrigues. Update the model dynamically to reflect these changes in crown ownership using first-order logic.
5) Leg Unification Scenario: Imagine a scenario where Richard and John undergo a miraculous transformation, resulting in their left legs becoming identical. Modify the model to represent this transformation

accurately using first-order logic, considering the implications of such a change on the existing facts.

Q6:- Solution:

1) Extended Royal Siblinghood: To ~~am~~ accommodate Mary as another sibling in the model, the Brother relation needs extension. We introduce the atomic sentences:

Brother(Richard, Mary)
Brother(John, Mary)

2) Regal Heirloom: Introduce a new relation, Heir, to signify the inheritance of the crown. After the queen passes away, the heir assumes the crown. The atomic sentences would be:

OnHead(Crown, Queen)
Heir(NextKing, Queen)

3) Symmetric Siblinghood Expansion: Extend the Brother relation to ensure symmetry among all siblings. For every pair of siblings, ensure the relationship holds in both directions:

Brother(Mary, Richard)
Brother(Mary, John)
Brother(Richard, Mary)
Brother(John, Mary)

4) Crown Transfer Dynamics: Suppose the crown ownership shifts from John to Mary. Modify the OnHead relation

and introduce a new relation for the transfer:
OnHead(Crown, Mary)
CrownTransfer(John, Mary)

5) Leg Unification Scenario: If Richard and John's left legs become identical, update the model accordingly:
LeftLegOf(John, LeftLeg)
LeftLegOf(Richard, LeftLeg)

Question 7:

Show progressive unification step-by-step for the sentences "O(F(y), y)" and "O(F(x), J)" to find a unifier.

Q7- Progressive Unification:

Given the sentences:

1) O(F(y),y)
2) O(F(x),J)

Step 1: Initial Unification

- Compare the outermost predicates: O and O. They match.
- Compare the arguments: F(y) and F(x). They do not match.

Step 2: Unifying Inner Terms

- We need to unify F(y) and F(x).
- We apply the substitution {y/x} to F(x).
- F(x) becomes F(y).

Step 3: Updated Unification

- Now both predicates and inner terms match.
- The substitution {y/x} has been applied to F(x).
- We have O(F(y),y) and O(F(y),J).

Step 4: Final Unification:

- Compare the outermost predicates: O and O. They match.
- Compare the arguments: F(y) and F(y). They match.
- Compare the inner terms: y and J. They do not match.

Step 5: Complete Unification:

- We apply the substitution {J/y} to y in the second sentence.
- y becomes J.

Final Result:

- The unifier is {y/x, J/y}.
- Applying this unifier to the second sentence, we get O (F(y), J) which matches the first sentence O(F(y),y).

Solution: The progressive unification steps for the sentences "O(F(y),y)" and "O(F(x),J)" are as follows:

1) Initial Unification:
   - Outer predicates match but inner terms do not.
2) Unifying Inner Terms:
   - Apply the substitution {y/x} to F(x) to get F(y).
3) Updated Unification:
   - Outer predicates and inner terms do not match.
4) Final Unification:
   - Apply the ~~substion~~ substitution {J/y} to match the inner terms.
5) Complete Unification:
   - The unifier is {y/x, J/y}, making the sentences identical.

Question 8:

Given the knowledge that all kings are greedy, and John is a king, infer whether John is greedy using first-order logic inference.

Q8- Solution:

1) Knowledge Base:

- $\forall x$ king (x) $\Rightarrow$ greedy (x) (All kings are ready)
- King(John) (John is a king)

2) Inference:

- We want to infer whether John is greedy.

3) Applying First-Order Logic Inference:

- Given that all kings are greedy and John is a king, we can conclude that John must be greedy based on the knowledge base.
- By applying Modus Ponens, which states that if $P \Rightarrow Q$ and P are true, then, Q is also true, we can infer that since John is a king (P), and all kings are greedy ($P \Rightarrow Q$) therefore, John must be greedy (Q).

4) Conclusion:

- John is greedy based on the inference drawn fr the knowledge ~~basing~~ base using first-order logic inference.

Thank You