# Artificial Intelligence
# AI 2002
# Lecture 14

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# Propositional Logic

- Propositional logic is a **declarative language.**
  - its *semantics* is based on a ***truth relation*** ***between sentences and possible worlds***.
  - Propositional logic **allows partial information using disjunction & negation**
- Propositional logic has a third property that is **compositional**.
- Propositional logic is compositional, i.e., *the meaning of a sentence is a function of the meaning of its parts.*
  - For example: The meaning of $\boldsymbol{B_{1,1}} \wedge \boldsymbol{P_{1,2}}$ is derived from the meaning of $\boldsymbol{B_{1,1}}$ and of $\boldsymbol{P_{1,2}}$

# Propositional Logic

- Meaning in propositional logic is **context-independent** (unlike natural language, where meaning depends on context).
- Propositional logic has very **limited expressive power** (unlike natural language)
  - **For example:** cannot say
    - **“Pits cause Breezes in adjacent squares”.**

we have to write a separate rule about breezes and pits for **EACH** square.

# First Order Logic (FOL)

- The propositional logic assumes the world contains ***facts*** while,
- The **first-order logic** (like natural language) assumes the world contains,

1. ***Objects***
2. ***Relations***
3. ***functions***.

# First Order Logic (FOL)

- The propositional logic assumes the world <u>contains</u> ***facts*** while,
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2. ***Relations***
3. ***functions***.

# First Order Logic (FOL)

## Objects:

- The **nouns and noun phrases** refer to **objects**
  - *In Wumpus-world*, the object examples are (squares, pits, and Wumpus)
  - Some other examples of objects are
    - People
    - Houses
    - Numbers
    - Theories
    - baseball
    - Games
    - centuries, etc.

# First Order Logic (FOL)

## Relations:

- The **verbs and verb phrases** refer to **relations**
  - *In Wumpus-world*, the relation examples are (is breezy, is adjacent to, shoots)
  - Some other examples of relations are
    - Red
    - Round
    - brother of
    - bigger than
    - Inside
    - Part of
    - has colour
    - owns etc.

# First Order Logic (FOL)

**Functions:**

- Some of these relations are **functions**—*relations in which there is only* ***ONE*** *"value" for a given "input".*
  - Some example of relations are
    - father of
    - best friend
    - third inning of
    - one more than
    - end of

# First Order Logic (FOL) Motivation

- The statements that cannot be made in propositional logics but can be expressed with FOL.
- First-order logic can also express facts about ***some* or *all*** of the objects in the universe.

# First Order Logic (FOL) Motivation

- Examples:

1. <u>When you paint a block with green paint, it becomes green.</u>
   - In proposition logic, one would need a statement about **every single block** … for every single aspect of the situation, ***"if this block is black and I paint it, it becomes green"*** and ***"if block # 5 is red and I paint it, it becomes green"***
2. <u>When you sterilize the jar, all the bacteria are dead.</u>
   - In FOL, we can talk about all the bacteria without naming them explicitly.

# Logics

# Logics

Ontological commitment

- what exists in world

- what it assumes about the ***nature of reality (facts)***.

- Mathematically, this commitment is expressed through the ***nature of the formal models*** with respect to which the *truth of sentences is defined*.
  - For example, propositional logic assumes that there are facts that either hold or do not hold in the world.

# Logics

- Logics Commitment/ Language can be expressed in **two types**

1. Ontological Commitment
2. Epistemological Commitment

# Logics

## Ontological Commitment

- what exists in world
- what it assumes about the nature of reality (facts).
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- Example:
  - Propositional logic assumes that there are facts that either **hold or do not hold in the world.**

## Epistemological Commitment

- the possible states of knowledge that it allows with respect to each fact.
- In both propositional and first order logic, a sentence represents a fact and the agent either believes the sentence to be true or false , or has no opinion

.

  - Thus the possible values are: **true/false/unknown**

# Types of Logic

- Different types of logics are

1. Propositional Logic
2. Temporal Logic
3. Fuzzy Logic
4. Probability Theory
5. Temporal Logic

# Types of Logic

- Temporal Logic
  - assumes that facts hold at particular times and
  - those times (which may be points or intervals) are ordered
- Probability Theory/Logic
  - Systems using probability theory can have any degree of belief, ranging from 0 (total disbelief) to 1 (total belief).
- Fuzzy Logic
  - Fuzzy logic has a degree of truth between 0 and 1.
  - For example , the sentence “Vienna is a large city” might be true in our world only to a degree of 0.6 in fuzzy logic

# Summary

| Language/Logic | Ontological (What Exist in the world) | Epistemological (What an agent believes about fact) |
|---|---|---|
| Propositional Logic | Facts | True/false |
| First Order Logic | Facts, Objects, Relations | True/false/unknown |
| Temporal Logic | Facts, Objects, Relations, Times | True/false/unknown |
| Probability theory | Facts | Degree of belief [0,1] |
| Fuzzy Logic | Facts with the degree of truth | Known interval value |

# First Order Logic Models

# Models of First Order Logic

- Models for first-order logic  have objects in them.
- If object exits in the model, so there must be **some domain of the objects**
- The domain of a model is the <u>set of objects or domain elements it contains.</u>
- The domain is required to be nonempty
  - every possible **<u>world must contain at least one object.</u>**

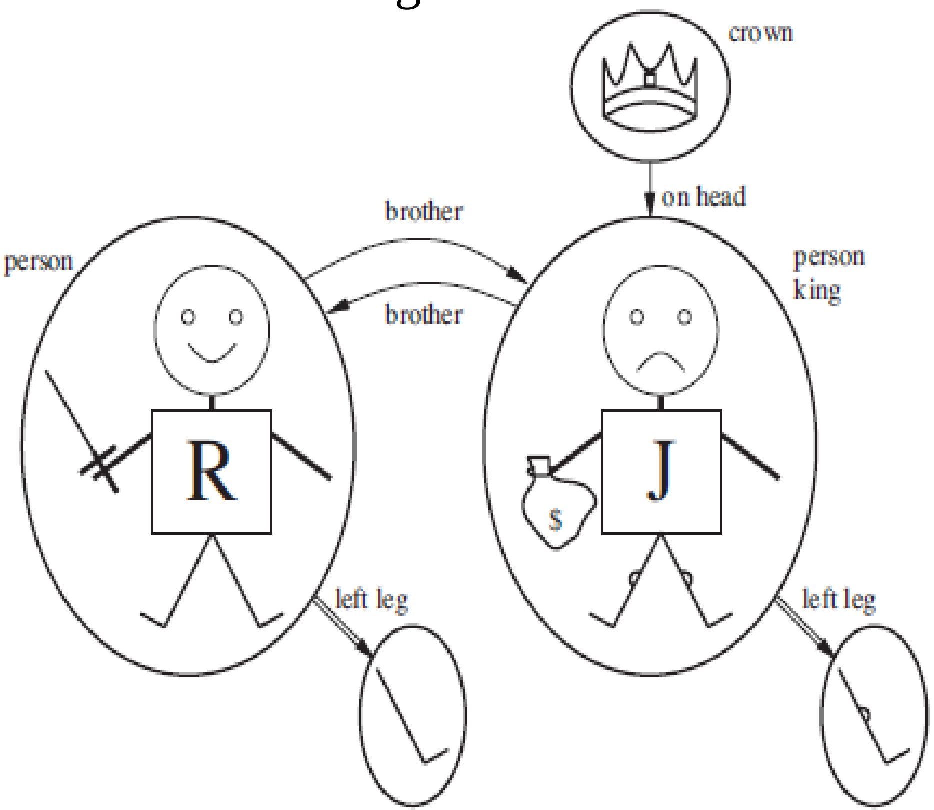
The **<u>first-order logic</u>** assumes the world <u>contains</u>, ***objects***, ***relations*** and ***functions***.

# Models of First Order Logic

- Mathematically speaking, it doesn't matter *what these objects are,*
- ***all that matters is how many there are in each particular model.***

# Models of First Order Logic

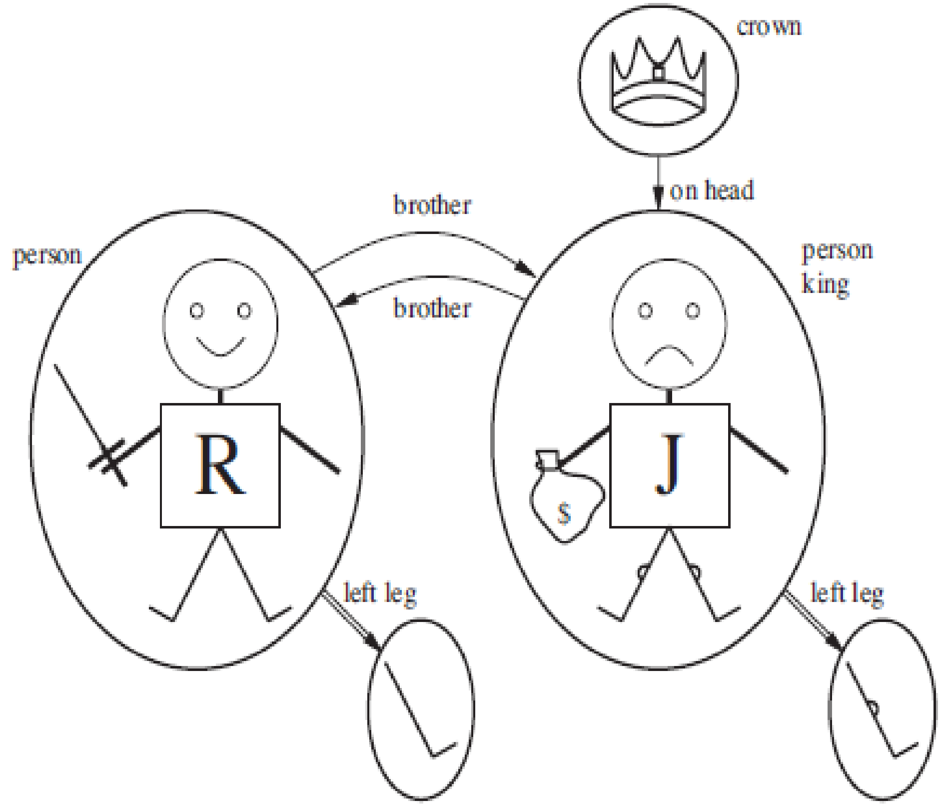
**Richard the Loin heart** was a king of England from 1189 to 1199.
His younger brother was the **evil king John**, who ruled from 1199 to 1215.
The **left legs of Richard and john were different**;
**John had a crown** because he was a king.



# Examples

- The above model Contains

1. Five objects
2. Two Binary Relations
3. Three unary relations
4. One Unary function



# Examples:

**Objects**

Noun and Noun Phrases

We have 5 objects

1. Person King John
2. Person Richard
3. Crown
4. Left Leg of John
5. Left Leg of Richard

**Binary Relation**

verb and Verb Phrases

We have 2 objects

1. Brother
2. On head

**Unary Relations**

Verb and Verb Phrases

We have 3 objects

1. Person
2. King
3. Crown

**One unary function**,

**1. left leg**

# Example:

## Relation

- Onhead<the crown, King john>
- Brother<john, Richard>
- Crown<John>
- Person<Richard>
- King<John>

## Function

- [no other person wear the crown except king]
- <John the King>- onhead(crown)

# Model of First Order Logic

## Tuple

- A tuple is a collection of objects arranged in a fixed order and is written with angle brackets surrounding the objects.

- Tuple Example:

  The "brotherhood" relation in the model" is the set:

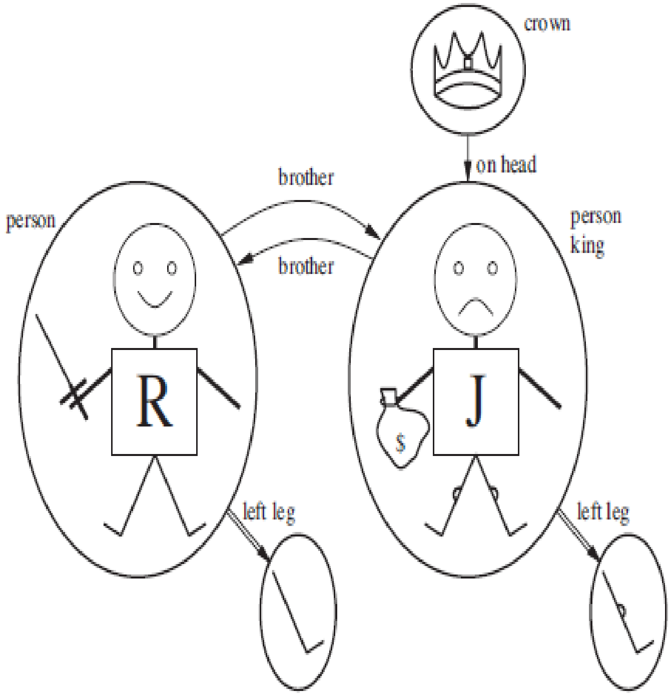
  {<Richard the Lionheart, King John>, <King John, Richard the Lionheart>}.

  The crown is on King John's head, so the "on head" relation contains just one tuple,

  <the crown, King John>.

# Example:

- The "**brother" and "on head"** relations are binary relations.
- The model also contains unary relations, or properties:
  1. The "person" property is true of both Richard and John;
  2. The "king" property is true only of John,
  3. The "crown" property is true only of the crown



# First Order Logics Symbol and Interpretations

# FOL symbol and Interpretations

- Symbol
  - The basic syntactic elements of first-order logic are the  symbols that stand for objects, relations, and functions
  - The symbols will begin with UPPERCASE letters.

- There are three types of symbols in FOL

1. Constant Symbol

   which stands for objects, like Richard  and John

2. Predicate Symbol

   which stands for relations, like  Brother, OnHead, Person, King, and Crown

3. Function Symbol

   which stands for relations, like  Brother, OnHead, Person, King, and Crown

# Syntax of FOL: Basic Elements

| Constants | Predicates | Functions | Variables | Connectives | Quantifiers |
| --- | --- | --- | --- | --- | --- |
| KingJohn | Brother, | Sqrt, | x, y, | $\wedge$ | $\forall$ |
| KingRichard | >, =, ... | LeftLegOf , ... | a, b, ... | $\vee$ | $\exists$ |
| | | | | $\neg$ | |
| | | | | $\Rightarrow$ | |
| | | | | $\Leftrightarrow$ | |

# FOL Symbol and Interpretations

- Interpretation specifies exactly which **objects, relations, and functions** are referred to by the constant, predicate, and function symbols.

- Arity:
  - Each predicate and function symbol comes with an arity that fixes the number of arguments.

# FOL Symbol and Interpretations

- Term
  - A term is a logical expression that refers to an object.

    Function(term1, term2……term n)
- A term may contain:
  - Constant symbol: Fred, Japan, Bacterium 39
  - Variables: a,b, x
  - Functional symbols are applied to one or more terms. F(x), Mother-of(John)
- A term with no variables is called a ground term.

# FOL Symbol and Interpretations

- Sentence
  - A predicate symbol may be applied to terms. On(a, b), Sister(Jane, John), Sister(Mother-of(Jane), Jen)
  - $\boldsymbol{term1} \quad = \quad \boldsymbol{term2}$
  - A functional symbol may be applied to one or more terms. F(x), Mother-of(John).
  - If $\boldsymbol{v}$ is a variable and $\boldsymbol{S}$ is a sentence, then

  - $(\forall \boldsymbol{v}\ \boldsymbol{S})$ and $(\exists \boldsymbol{v}\ \boldsymbol{S})$ are sentences too.

# Atomic Sentence

(or atom for short) is formed from a predicate symbol optionally followed by a parenthesized list of terms

- Syntax:

**Predicate(term1, term 2)**

- Example:

**Brother(King John, Richard the Lion Heart)**

- Atomic sentences can have **complex terms** as arguments.
  - **Married(Father (Richard), Mother (John))**

# Atomic Sentence Example:

$$\underbrace{\underbrace{Brother}_{\text{predicate}}\,(\,\underbrace{\underbrace{KingJohn}_{\text{constant}}}_{\text{term}},\ \underbrace{\underbrace{RichardTheLionheart}_{\text{constant}}}_{\text{term}}\,)}_{\text{atomic sentence}}$$

# Complex Sentence

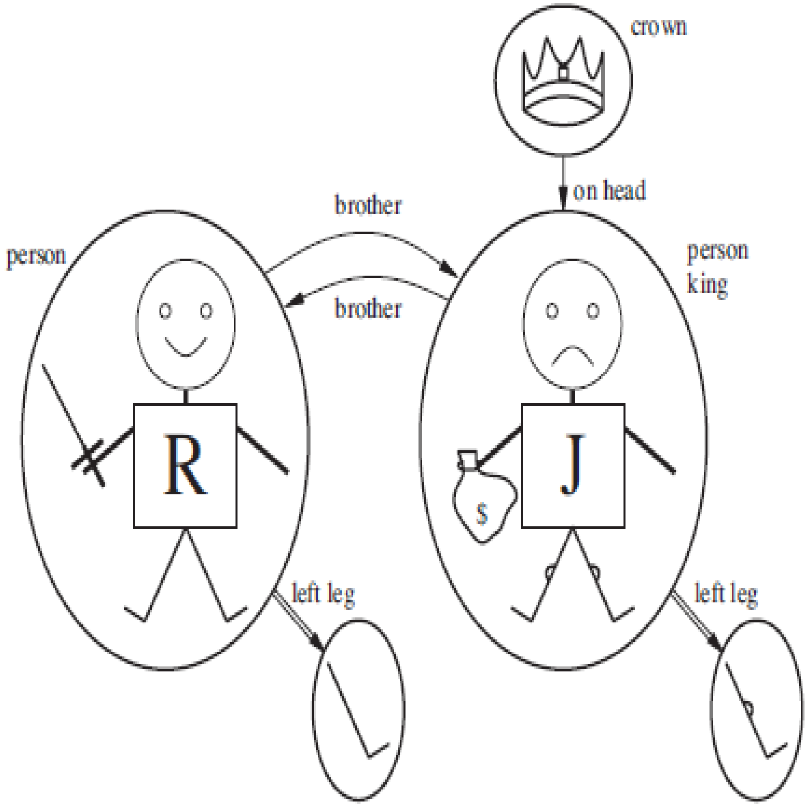
- We can use logical connectives to construct more complex sentences, with the same syntax and semantics as in propositional calculus.

- Examples:

$$\neg Brother(LeftLeg(Richard), John)$$
$$Brother(Richard, John) \wedge Brother(John, Richard)$$
$$King(Richard) \vee King(John)$$
$$\neg King(Richard) \Rightarrow King(John) .$$

# FOL Interpretations

- Constants: KingJohn, Richard
- Predicates: person, king, crown
- Functions: brother, on_head, left_leg



# Quantifiers

# Quantifiers

- Quantifier in the context of AI refers to an element that expresses the quantity of items within a specified range.

- There are two types of quantifiers

1. Universal Quantifiers (For All)
2. Existential Quantifiers (There Exists)

# Universal Quantifiers

- **"All kings are persons"** is written in first-order logic as,

$$\forall x \;\; King(x) \Rightarrow Person(x)$$

- $\forall$ is usually pronounced **"For all . . ."**
- *Intuitively, the sentence* **∀x P***, where* **P** *is any logical expression, says that* **P** *is true for every object* **x***.*
- More precisely, ***∀x P*** *is true in a given model if* ***P*** *is true in* ***ALL*** *possible* ***extended interpretations*** *constructed from the interpretation given in the model,*
  - where each extended interpretation specifies a domain element to which x refers.

# Universal Quantifiers

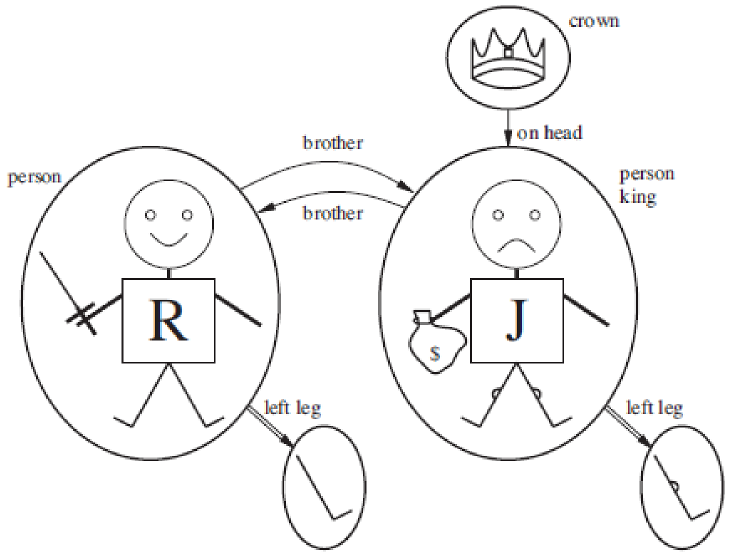
- **"All kings are persons"** is written in first-order logic as,

$$\forall x \;\; King(x) \Rightarrow Person(x)$$

- $\forall$ is usually pronounced **"For all . . ."**
- **Example:** **"For all x, if x is a king, then x is a person."**

We can extend the interpretation in **five** ways:

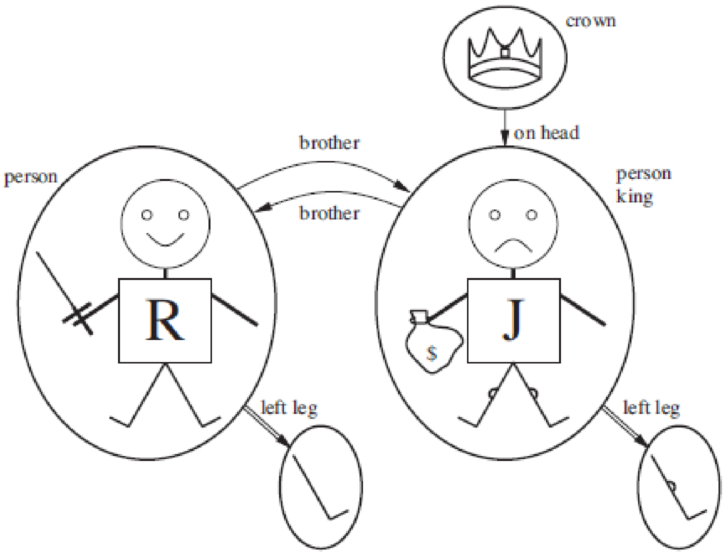
$x \rightarrow$ Richard the Lionheart,
$x \rightarrow$ King John,
$x \rightarrow$ Richard's left leg,
$x \rightarrow$ John's left leg,
$x \rightarrow$ the crown.



# Universal Quantifiers

- The **universally quantified sentence** is equivalent to asserting the following five sentences:

Richard the Lionheart is a king $\Rightarrow$ Richard the Lionheart is a person.
King John is a king $\Rightarrow$ King John is a person.
Richard's left leg is a king $\Rightarrow$ Richard's left leg is a person.
John's left leg is a king $\Rightarrow$ John's left leg is a person.
The crown is a king $\Rightarrow$ the crown is a person.



# Existential Quantifiers

- **"King John has a crown on his head",** we write

$$\exists x \;\; Crown(x) \land OnHead(x, John)$$

- **∃x** is pronounced **"There exists an x such that . . ."** or **"For some x . . .".**
- *Intuitively, the sentence* **∃x P** *says that* **P** *is true for at least one object* **x**.
- More precisely, ***∃x P** is true in a given model if **P** is true in **at least one extended interpretation** that assigns **x** to a domain element*.

# Existential Quantifiers

- That is, at least one of the following is true:

Richard the Lionheart is a crown $\wedge$ Richard the Lionheart is on John's head;
King John is a crown $\wedge$ King John is on John's head;
Richard's left leg is a crown $\wedge$ Richard's left leg is on John's head;
John's left leg is a crown $\wedge$ John's left leg is on John's head;
The crown is a crown $\wedge$ the crown is on John's head.

- The **fifth assertion is true in the model**, so the original existentially quantified sentence is true in the model.

# Existential Quantifiers

- Notice that, by the definition, the sentence would also be true in a model in which **King John was wearing two crowns**.

- **There is a variant of the existential quantifier, usually written $\exists^1$ or $\exists!$, that means**

**"There exists exactly one."**

- Typically, $\Rightarrow$ is the main connective with $\exists$.

# Universal Quantifiers

- Typically **e** is the main connective with ∀.

**Common Mistake:**

- Using A as the main connective with ∀.

Everyone at Berkeley is smart:

$$\forall x \;\; At(x, Berkeley) \;\Rightarrow\; Smart(x)$$

$$\forall x \;\; At(x, Berkeley) \land Smart(x)$$

means "Everyone is at Berkeley and everyone is smart"

# Existential Quantifiers

## Common Mistake:

- Using e as the main connective with ∃

Someone at Stanford is smart:

$\exists x \;\; At(x, Stanford) \wedge Smart(x)$

$$\exists x \;\; At(x, Stanford) \;\Rightarrow\; Smart(x)$$

is true if there is anyone who is not at Stanford!

The **implication is true whenever its premise is false**—*regardless of the truth of the conclusion.*

# Nested Quantifiers

- For example, **"Brothers are siblings"** can be written as

$$\boxed{\forall x \ \forall y} \ Brother(x, y) \Rightarrow Sibling(x, y) .$$

- **<u>Consecutive quantifiers</u>** of the same type can be written as one quantifier with several variables.
- For example, to say that siblinghood is a symmetric relationship, we can write,

$$\boxed{\forall x, y} \ Sibling(x, y) \Leftrightarrow Sibling(y, x)$$

# Nested Quantifiers

- The <u>**order of quantification**</u> **is very important**. For example: **"Everybody loves somebody"** means that for every person, there is someone that person loves:

$$\forall x \ \exists y \ \ Loves(x,y)$$

- On the other hand, to say **"There is someone who is loved by everyone"** we write

$$\exists y \ \forall x \ \ Loves(x,y)$$

# Nested Quantifiers

- *Some confusion may arise when two quantifiers are used with the same variable name.*
- Consider the sentence

$$\forall x \ \ (Crown(x) \lor (\exists x \ \ Brother(Richard, x)))$$

- Here the **x** in **Brother (Richard, x)** is *existentially quantified.*
- ***The*** ***rule*** ***is that the variable belongs to the innermost quantifier that mentions it; then it will not be subject to any other quantification.***

$$\exists z \ Brother(Richard, z).$$

# Reading Material

- **Artificial Intelligence,** A Modern Approach
  **Stuart J. Russell and Peter Norvig**
  - **Chapter 8.**