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BSCS-6F

AI-Assignment : 04



## QNO. 1

(a)

False  $\models$  True

It is a correct, because we know that if 1<sup>st</sup> premise is false in implication, proposition is true.

T.T

| True | False | False $\models$ True |
|------|-------|----------------------|
| F    | F     | T                    |
| T    | F     | T                    |

$\rightarrow$  False  $\rightarrow$  True = True.

(b)

True  $\models$  False

It is a wrong because True  $\rightarrow$  False = False.

T.T

| False | True | True $\models$ False |
|-------|------|----------------------|
| F     | T    | F                    |
| T     | T    | T                    |

$\rightarrow$  T  $\rightarrow$  F = False.

(c)

 $(A \wedge B) \models (A \leftrightarrow B)$ 

It is a correct because  $A \leftrightarrow B$  gives tautology.

T.T

| A | B | $A \wedge B$ | $A \leftrightarrow B$ | $(A \wedge B) \models (A \leftrightarrow B)$ |
|---|---|--------------|-----------------------|----------------------------------------------|
| F | F | F            | T                     | T                                            |
| F | T | F            | F                     | T                                            |
| T | F | F            | F                     | T                                            |
| T | T | T            | T                     | T                                            |

(d)

 $A \leftrightarrow B \models A \vee B$ 

It is wrong, because this is not gives tautology



T.T

(2)

| A | B | $A \vee B$ | $A \leftrightarrow B$ | $(A \leftrightarrow B) \neq A \vee B$ |
|---|---|------------|-----------------------|---------------------------------------|
| F | F | F          | T                     | F                                     |
| F | T | T          | F                     | T                                     |
| T | F | T          | F                     | T                                     |
| T | T | T          | T                     | T                                     |

(C)

$$A \leftrightarrow B \neq \neg A \vee B$$

It is correct, because it gives tautology.

T.T

| A | B | $\neg A$ | $A \leftrightarrow B$ | $\neg A \vee B$ | $(A \leftrightarrow B) \neq \neg A \vee B$ |
|---|---|----------|-----------------------|-----------------|--------------------------------------------|
| T | T | F        | T                     | T               | T                                          |
| T | F | F        | F                     | F               | T                                          |
| F | T | T        | F                     | T               | T                                          |
| F | F | T        | T                     | T               | T                                          |

(F)

$$(A \wedge B) \rightarrow C \neq (A \rightarrow C) \vee (B \rightarrow C)$$

It is a correct, because it gives valid solution.

$$X = (A \wedge B) \rightarrow C \quad Y = (A \rightarrow C) \vee (B \rightarrow C)$$

T.T

| A | B | C | $A \wedge B$ | X | $A \rightarrow C$ | $B \rightarrow C$ | Y | $X \neq Y$ |
|---|---|---|--------------|---|-------------------|-------------------|---|------------|
| T | T | T | T            | T | T                 | T                 | T | T          |
| T | T | F | T            | F | F                 | F                 | F | T          |
| T | F | T | F            | T | T                 | T                 | T | T          |
| T | F | F | F            | T | F                 | T                 | T | T          |
| F | T | T | F            | T | T                 | T                 | T | T          |
| F | T | F | F            | T | T                 | F                 | T | T          |
| F | F | T | F            | T | T                 | T                 | T | T          |
| F | F | F | F            | T | T                 | T                 | T | T          |

(g)

$$(C \vee (\neg A \wedge \neg B)) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$$



It is a correct, because it gives valid solution.

$$X = C \vee (\neg A \wedge \neg B) \quad Y = (A \rightarrow C) \wedge (B \rightarrow C)$$

T.T

| A | B | C | $\neg A$ | $\neg B$ | X | $A \rightarrow C$ | $B \rightarrow C$ | Y | $X = Y$ |
|---|---|---|----------|----------|---|-------------------|-------------------|---|---------|
| T | T | T | F        | F        | T | T                 | T                 | T | T       |
| T | T | F | F        | F        | F | F                 | F                 | F | T       |
| T | F | T | F        | T        | T | T                 | T                 | T | T       |
| T | F | F | F        | T        | F | F                 | F                 | F | T       |
| F | T | T | T        | F        | T | T                 | T                 | T | T       |
| F | T | F | T        | F        | F | F                 | F                 | F | T       |
| F | F | T | T        | T        | T | T                 | T                 | T | T       |
| F | F | F | T        | T        | T | T                 | T                 | T | T       |

(h)

$$(A \vee B) \wedge (\neg C \vee \neg D \vee E) = (A \vee B)$$

It is a correct, because it gives valid solution.

$$X = A \vee B \quad Y = (\neg C \vee \neg D \vee E) \quad Z = X \wedge Y$$

T.T

| A | B | C | D | E | $\neg C$ | $\neg D$ | X | Y | Z | $Z = X$ |
|---|---|---|---|---|----------|----------|---|---|---|---------|
| T | T | T | T | T | F        | F        | T | T | T | T       |
| T | T | T | T | F | F        | T        | T | F | F | F       |
| T | T | T | F | T | F        | F        | T | T | T | T       |
| T | T | T | F | F | F        | T        | T | F | F | F       |
| T | T | F | T | T | T        | F        | T | T | T | T       |
| T | T | F | T | F | T        | T        | T | T | T | T       |
| T | T | F | F | T | T        | T        | T | T | T | T       |
| T | T | F | F | F | T        | T        | T | T | T | T       |
| T | F | T | T | T | F        | F        | T | F | F | F       |
| T | F | T | T | F | F        | T        | T | T | T | T       |
| T | F | T | F | T | F        | F        | T | T | T | T       |
| T | F | T | F | F | F        | T        | T | T | T | T       |
| T | F | F | T | T | T        | F        | T | T | T | T       |
| T | F | F | T | F | T        | T        | T | T | T | T       |
| T | F | F | F | T | T        | T        | T | T | T | T       |
| T | F | F | F | F | T        | T        | T | T | T | T       |
| F | T | T | T | T | F        | F        | T | F | F | F       |
| F | T | T | T | F | F        | T        | T | T | T | T       |
| F | T | T | F | T | F        | F        | T | T | T | T       |
| F | T | T | F | F | F        | T        | T | T | T | T       |
| F | F | T | T | T | T        | F        | T | T | T | T       |
| F | F | T | T | F | T        | T        | T | T | T | T       |
| F | F | T | F | T | T        | T        | T | T | T | T       |
| F | F | T | F | F | T        | T        | T | T | T | T       |
| F | F | F | T | T | F        | F        | T | F | F | F       |
| F | F | F | T | F | F        | T        | T | T | T | T       |
| F | F | F | F | T | F        | F        | T | T | T | T       |
| F | F | F | F | F | T        | T        | T | T | T | T       |



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$$X = (A \vee B), Y = (\neg C \vee \neg D \vee E)$$

TT

[illegible]



(j)  
 $(A \vee B) \wedge \neg(A \rightarrow B)$

It is correct, because it is satisfiable.

T.T

| A | B | $A \vee B$ | $A \rightarrow B$ | $\neg(A \rightarrow B)$ | $(A \vee B) \wedge \neg(A \rightarrow B)$ |
|---|---|------------|-------------------|-------------------------|-------------------------------------------|
| T | T | T          | T                 | F                       | F                                         |
| T | F | T          | F                 | T                       | T                                         |
| F | T | T          | T                 | F                       | F                                         |
| F | F | F          | T                 | F                       | F                                         |

$\left. \begin{matrix} F \\ T \\ F \end{matrix} \right\} \rightarrow$  It is shown that it is satisfy.

(k)

$(A \leftrightarrow B) \wedge (\neg A \vee B)$

It is correct, because it is satisfiable solution.

T.T

| A | B | $\neg A$ | $A \leftrightarrow B$ | $\neg A \vee B$ | $(A \leftrightarrow B) \wedge (\neg A \vee B)$ |
|---|---|----------|-----------------------|-----------------|------------------------------------------------|
| T | T | F        | T                     | T               | T                                              |
| T | F | F        | F                     | F               | F                                              |
| F | T | T        | F                     | T               | F                                              |
| F | F | T        | T                     | T               | T                                              |

(i)

$(A \leftrightarrow B) \leftrightarrow C$

T.T

| A | B | C | $(A \leftrightarrow B)$ | $(A \leftrightarrow B) \leftrightarrow C$ |
|---|---|---|-------------------------|-------------------------------------------|
| T | T | T | T                       | T                                         |
| T | T | F | T                       | F                                         |
| T | F | T | F                       | T                                         |
| T | F | F | F                       | F                                         |
| F | T | T | F                       | T                                         |
| F | T | F | F                       | F                                         |
| F | F | T | T                       | F                                         |
| F | F | F | T                       | T                                         |

4

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Both have same models.



# Q No. 2

Smoke = S

Fire = F

Heat = H

$$S \rightarrow S \quad (a)$$

$$\neg S \vee S$$

valid

(b)

$$S \rightarrow F$$

$$\neg S \vee F$$

satisfiable

(c)

$$(S \rightarrow F) \rightarrow (\neg S \rightarrow \neg F)$$

$$(\neg S \vee F) \rightarrow (\neg \neg S \rightarrow \neg F)$$

$$\neg(\neg S \vee F) \vee (S \vee \neg F)$$

$$(S \wedge \neg F) \vee (S \vee \neg F)$$

$$(S \vee (S \vee \neg F)) \wedge (\neg F \vee (S \vee \neg F))$$

$$(S \vee S) \vee (S \vee \neg F) \wedge (\neg F \vee S) (\neg F \vee \neg F)$$

$$(S \vee \neg F) \wedge (S \vee \neg F)$$

$$(S \vee \neg F)$$

satisfiable

(d)

$$S \vee F \vee \neg F$$

$$S \vee T$$

$$S$$

valid

(e)

$$((S \wedge H) \rightarrow F) \leftrightarrow ((S \rightarrow F) \vee (H \rightarrow F))$$

$$(\neg(S \wedge H) \vee F) \leftrightarrow ((\neg S \vee F) \vee (\neg H \vee F))$$

$$((\neg S \vee \neg H) \vee F) \leftrightarrow ((\neg S \vee F) \vee (\neg H \vee F))$$

$$((\neg S \vee \neg H \vee F) \rightarrow ((\neg S \vee F) \vee (\neg H \vee F))) \wedge (((\neg S \vee F) \vee (\neg H \vee F)) \rightarrow (\neg S \vee \neg H \vee F))$$

(2)

From (1)

$$(S \wedge H \wedge \neg F) \vee ((\neg S \vee F) \vee (\neg H \vee F))$$

$$(S \wedge H \wedge \neg F) \vee (\neg S \vee \neg H \vee F)$$

T

From (2)

$$\neg((\neg S \vee F) \vee (\neg H \vee F)) \vee (\neg S \vee \neg H \vee F)$$

$$\neg(\neg S \vee \neg H \vee F) \vee (\neg S \vee \neg H \vee F)$$

T

Now,

$$(1) \wedge (2)$$

$$T \wedge T$$

$$T$$

Valid



$$(f) \quad (S \rightarrow F) \rightarrow ((S \wedge H) \rightarrow F)$$

$$\begin{aligned} & (\neg S \vee F) \rightarrow (\neg(S \wedge H) \vee F) \\ & \neg(\neg S \vee F) \vee (\neg(S \wedge H) \vee F) \\ & (S \wedge \neg F) \vee (\neg S \vee \neg H \vee F) \\ & (S \vee \neg S) \vee (S \vee \neg H) \vee (S \vee F) \wedge (\neg F \vee \neg S) \vee (\neg F \vee \neg H) \vee (\neg F \vee F) \\ & [(S \vee \neg H) \vee (S \vee F) \vee T] \wedge ((\neg F \vee \neg S) \vee (\neg F \vee \neg H) \vee T) \end{aligned}$$

TAT  
T  
Valid

$$(g) \quad \text{Big} \vee \text{Dumb} \vee (\text{Big} \rightarrow \text{Dumb})$$

$$\begin{aligned} & (\text{Big} \vee \text{Dumb}) \vee (\neg \text{Big} \vee \text{Dumb}) \\ & (\text{Big} \vee \neg \text{Big}) \vee (\text{Big} \vee \text{Dumb}) \vee (\text{Dumb} \vee \neg \text{Big}) \vee (\text{Dumb} \vee \text{Dumb}) \end{aligned}$$

T  
Valid

Qno.3

Food = F  
Party = P  
drinks = D

$$[(F \rightarrow P) \vee (D \rightarrow P)] \rightarrow [(F \wedge D) \rightarrow P]$$

(a)

$$A = F \rightarrow P, B = D \rightarrow P, C = F \wedge D$$

$$X = C \rightarrow P, Y = (A \vee B) \rightarrow X$$



I.T

(8)

| F | P | D | A | B | C | X | A ∨ B | Y |
|---|---|---|---|---|---|---|-------|---|
| T | T | T | T | T | T | T | T     | T |
| T | T | F | T | T | F | T | T     | T |
| T | F | T | F | F | T | F | F     | T |
| T | F | F | F | T | F | T | T     | T |
| F | T | T | T | T | F | T | T     | T |
| F | T | F | T | T | F | T | T     | T |
| F | F | T | T | F | F | T | T     | T |
| F | F | F | T | T | F | T | T     | T |

This is a valid statement.

(b)

Left side

$$(F \rightarrow P) \vee (D \rightarrow P)$$

$$(\neg F \vee P) \vee (\neg D \vee P)$$

$$(\neg F \vee \neg D \vee P)$$

Right side

$$(F \wedge D) \rightarrow P$$

$$\neg (F \wedge D) \vee P$$

$$(\neg F \vee \neg D \vee P)$$

$$L.H.S \equiv R.H.S.$$

So, L.H.S  $\rightarrow$  R.H.S is True.

(c)

$$\begin{aligned} & (F \vee \neg F) \wedge (F \vee \neg D) \wedge (F \vee \neg P) \wedge (\neg P \vee \neg F) \wedge (\neg P \vee \neg D) \wedge \\ & (D \vee \neg D) \wedge (D \vee \neg P) \wedge (D \vee \neg P) \wedge (\neg P \vee \neg F) \wedge (\neg P \vee \neg D) \wedge (\neg P) \end{aligned}$$

Resolution.

$$\begin{aligned} 12 & \rightarrow 2, 8 & (F \vee \neg P) \\ 13 & \rightarrow 3, 7 & (D \vee \neg P) \\ 14 & \rightarrow 4, 1 & (\neg P \vee \neg F) \\ 15 & \rightarrow 5, 6 & (\neg P \vee \neg D) \\ 16 & \rightarrow 4, 12 & (\neg P \vee \neg P) \\ 17 & \rightarrow 10, 13 & (\neg P \vee \neg P) \end{aligned}$$

$$\begin{aligned} 18 & \rightarrow \text{True} \\ \text{i.e. } (P \vee \neg P) &= T \\ \text{Proved } &\rightarrow (c) \end{aligned}$$



# Q.No.4

(a)  
KB

- 1)  $\neg W_{11}$
- 2)  $\neg P_{11}$
- 3)  $B_{21}$
- 4)  $S_{12}$
- 5)  $\neg B_{12}$
- 6)  $S_{12} \rightarrow W_{13}$  or  $\neg S_{12} \vee W_{13}$
- 7)  $S_{12} \rightarrow W_{22}$  or  $\neg S_{12} \vee W_{22}$
- 8)  $B_{21} \rightarrow P_{31}$  or  $\neg B_{21} \vee P_{31}$
- 9)  $B_{21} \rightarrow P_{22}$  or  $B_{21} \vee P_{22}$
- 10)  $\neg B_{12} \rightarrow \neg P_{22}$  or  $B_{12} \vee \neg P_{22}$
- 11)  $\neg B_{12} \rightarrow \neg P_{13}$  or  $B_{12} \vee \neg P_{13}$
- 12)  $\neg B_{12} \rightarrow \neg P_{11}$  or  $B_{12} \vee \neg P_{11}$
- 13)  $\neg S_{21}$
- 14)  $\neg S_{21} \rightarrow \neg W_{22}$  or  $S_{21} \vee \neg W_{22}$

b)

$KB \models \alpha_1 \therefore \alpha_1 = \neg P_{22}$

- |    |               |                     |
|----|---------------|---------------------|
| 15 | $P_{22}$      | Negative conclusion |
| 16 | $\neg P_{22}$ | 10, 15              |
| 17 | .             | 15, 16              |

So,

$KB \models \alpha_1$  is Prove.

c)

$KB \models \alpha_2 \therefore \alpha_2 = W_{13}$

- |    |          |                   |
|----|----------|-------------------|
| 15 | $W_{13}$ | 6, 4 modus Ponens |
|----|----------|-------------------|