AI 2002 – Artificial Intelligence

Practice Questions (Resolution Theorem)

QUESTION 1:

You are engaged in Knowledge Engineering for the Wumpus Cave. You have interviewed an expert on the Wumpus Cave who told you, among other things, "A breeze in square (1.1) is equivalent to a pit in square (1,2) or a pit in square (2,1)." You translated this into propositional logic as,

"(B11 ⇔ P12 V P21),"

and then into Conjunctive Normal Form as

"(BII V P12 V P21) ∧ (P12 V B11) ∧ (P21 V B11)."

Now it is time for the first "live" test of your system. An agent has been lowered down into the Wumpus cave, and reports back by radio, "Square (1,1) has a breeze. Also, I went into square (1.2) and I did not die, so it does not have a pit." You translate this knowledge into propositional logic as "(B11) ∧ (P12)" and add it to your knowledge base.

Next your system is asked to perform inference. The agent asks by radio. "Is it true that square (2,1) has a pit?" You translate this query into propositional logic as the goal sentence "(P21)." You form the negated goal as "(P21)." Your knowledge base plus negated goal is:

(~B11 V P12 V P21)

(~P12 V B11)

(~P21 V B11)

(B11)

(~P12)

(~P21)

Run resolution on this knowledge base until you produce the null clause, "()", thereby proving that the goal sentence is true.

Steps	Formula	Derivations
1	(~B11 V P12 V P21)	**Given**
2	(~P12 V B11)	**Given**
3	(~P21 V B11)	**Given**
4	(B11)	**Given**
5	(~P12)	**Given**
6	(~P21)	**Given**
7	(~B11 V P21)	**1, 5**
8	(P21)	**4, 7**
9	(.)	**6, 8**

QUESTION 2:

Amy, Betty, Cindy, and Diane went out to lunch at a seafood restaurant. Each ordered one fish. Each fish was either a red fish or a blue fish. **Among them they had exactly two red fish and two blue fish.**

You translate this fact into Propositional Logic (in prefix form) as:

/* Ontology: Symbol A/B/C/D means that Amy/Betty/Cindy/Diane had a red fish. */

(or (and A B (~C) (~D)) (and A (~B) C (~D))

(and A (~B) (~C) D) (and (~A) B C (~D))

(and (~A) B (~C) D) (and (~A) (~B) C D)))

Their waiter reported:

"Amy, Betty, and Cindy had exactly one red fish among them; I don't remember who had what. Betty, Cindy, and Diane had exactly one red fish among them; I don't remember who had what."

You translate these facts into Propositional Logic (in prefix form) as:

(or (and A (~B) (~C)) (and (~A) B (~C)) (and (~A) (~B) C))

(or (and B (~C) (~D)) (and (~B) C (~D)) (and (~B) (~C) D))

Betty's daughter asked, "Is it true that my mother had a blue fish?"

You translate this query into Propositional Logic as "(~B)" and form the negated goal as "(B)".

Your resulting knowledge base (KB) plus the negated goal (in CNF clausal form) is:

(A B C) ((~A) (~B) (~C))

(A B D) ((~A) (~B) (~D))

(A C D) ((~A) (~C) (~D))

(B C D) ((~B) (~C) (~D))

((~A) (~B)) ((~A) (~C))

((~B) (~C) ((~B) (~D))

((~C) (~D)) (B)

Write a resolution proof that Betty had a blue fish.

Run resolution on this knowledge base until you produce the null clause, "()", thereby proving that the goal sentence is true. Use as many steps as necessary, ending with the empty clause.

Steps	Formula	Derivations
1	A∧B∧C	**Given**
2	¬A∧¬B∧¬C	**Given**
3	A∧B∧D	**Given**

4	¬A∧¬B∧¬D	**Given**
5	A∧C∧D	**Given**
6	¬A∧¬C∧¬D	**Given**
7	B∧C∧D	**Given**
8	¬A∧¬B	**Given**
9	¬A∧¬C	**Given**
10	¬B∧¬C	**Given**
11	¬B∧¬D	**Given**
12	¬C∧¬D	**Given**
13	B	**Given**
14	¬B (Negated goal)	**Given**
15	B∧¬B	**Resolution: 13, 14, ∅**
16	¬B∧¬C	**Resolution: 10, 11, *¬B***
17	B∧C	**Resolution: 7, 12, *B***
18	C∧D	**Resolution: 5, 6, *C∧D***
19	A∧B	**Resolution: 3, 4, *A∧B***
20	B∧C	**Resolution: 1, 2, *B∧C***
21	¬A∧¬D	**Resolution: 4, 7, *¬A∧¬D***
22	A∧B	**Resolution: 1, 5, *A∧B***
23	¬A	**Resolution: 8, 9, *¬A***
24	¬A∧¬C	**Resolution: 6, 8, *¬A∧¬C***
25	¬B∧¬C	**Resolution: 2, 3, *¬B∧¬C***
26	¬A∧¬C	**Resolution: 9, 10, *¬A∧¬C***
27	∅	**Resolution: 12, 24, ∅**

QUESTION 3:

Use Resolution Theorem to prove the following:

A → [Square (1, 1) = Breeze] = [(Square (1, 2) = Pit) V Square (2,1) = Pit)]

1) Square (1, 2) = Pit = false
2) Square (1, 1) = Breeze = true

By 1 and 2,

A → T = [F V Square (2, 1) = Pit]

[Square (2, 1) = Pit] = T

QUESTION 4:

Convert to CNF:

1) (A ↔ (B V C))

(A → (B V C)) ∧ (B v C) → A
[~A V (B V C)] ∧ [(~(B V C) V A]
[~A V (B V C)] ∧ [~B ∧ ~C) V A]
[~A V B V C] ∧ [(~B V A) ∧ (~C V A)]

2) (C ∧ D → ~E)

~(C ∧ D) V ~E
~C V ¬D V ¬E

3) (A → B) → C

(~A V B) → C

¬ (¬A V B) V C

(A ∧ ¬B) V C

(A V C) ∧ (¬B V C)

4) A → (B ∧ D) ↔ (E V C)

[¬A V (B ∧ D) ↔ (E V C)]
[(¬A V (B ∧ D)) ↔ (E V C)]
[(¬A V (B ∧ D)) → (E V C)] ∧ [(E V C) → (¬A V (B ∧ D))]
[¬(¬A V (B ∧ D)) V (E V C)] ∧ [¬(E V C) V (¬A V (B ∧ D))]
[(A ∧ ¬(B ∧ D)) V (E V C)] ∧ [(¬E ∧ ¬C) V ((¬A V B) ∧ (¬A V D))]
[(A ∧ (¬B V ¬D)) V (E V C)] ∧ [(¬E ∧ ¬C) V ((¬A V B) ∧ (¬A V D))]
(A ∧ ¬B V A ∧ ¬D) V (E V C) ∧ (¬E V ¬C V ¬A V B) ∧ (¬E V ¬C V ¬A V D)
(A V E V C) ∧ (¬B V E V C) ∧ (¬D V E V C)∧(¬E V¬C V ¬A V B) ∧ (¬E V ¬C V ¬A V D)