AI 2002 - Artificial Intelligence (Spring 2023)
Assignment # 4

Topics Covered: Propositional Logic and Reasoning	Submission Deadline: **April 17, 2023, by 16.00 sharp** Only hand-written solutions will be accepted. Submit hand-written solutions to your instructor's office.

Problem # 1: [Propositional logic + model enumeration]

Which of the following are correct? Validate your answers using truth-table.

a. False |= True.
b. True |= False.
c. (A ∧ B) |= (A ⇔ B).
d. A ⇔ B |= A ∨ B.
e. A ⇔ B |= ¬A ∨ B.
f. (A ∧ B) ⇒ C |= (A ⇒ C) ∨ (B ⇒ C).
g. (C ∨ (¬A ∧ ¬B)) ≡ ((A ⇒ C) ∧ (B ⇒ C)).
h. (A ∨ B) ∧ (¬C ∨¬D ∨ E) |= (A ∨ B).
i. (A ∨ B) ∧ (¬C ∨¬D ∨ E) |= (A ∨ B) ∧ (¬D ∨ E).
j. (A ∨ B) ∧ ¬(A ⇒ B) is satisfiable.
k. (A ⇔ B) ∧ (¬A ∨ B) is satisfiable.
l. (A ⇔ B) ⇔ C has the same number of models as (A ⇔ B) for any fixed set of proposition symbols that includes A, B, C.

Problem # 2: [Propositional logic + inference rules]

Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your answers using the equivalence rules.

a. Smoke ⇒ Smoke
b. Smoke ⇒ Fire
c. (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
d. Smoke ∨ Fire ∨ ¬Fire
e. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))
f. (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)
g. Big ∨ Dumb ∨ (Big ⇒ Dumb)

Problem # 3: [Propositional logic + CNF + proof by resolution]

Consider the following sentence:

$$[(\text{Food} \Rightarrow \text{Party}) \vee (\text{Drinks} \Rightarrow \text{Party})] \Rightarrow [(\text{Food} \wedge \text{Drinks}) \Rightarrow \text{Party}]$$

a. Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable.

b. Convert the left-hand and right-hand sides of the main implication into Conjunctive Normal Form (CNF), showing each step, and explain how the results confirm your answer to (a).

c. Prove your answer to (a) using resolution.

Problem # 4: [Wumpus world + entailment]

Suppose the agent has progressed to the point shown in the figure below, having perceived nothing in [1,1], a breeze in [2,1], and a stench in [1,2], and is now concerned with the contents of [1,3], [2,2], and [3,1]. Each of these can contain a pit, and at most one can contain a wumpus.

1,4	2,4	3,4	4,4
1,3 W!	2,3	3,3	4,3
1,2 A S OK	2,2 OK	3,2	4,2
1,1 V OK	2,1 B V OK	3,1 P!	4,1

a. Create a knowledge base to represent the necessary environment rules and agent's observations.

b. Prove that KB |= α1 using resolution theorem where α1 = "There is no pit in [2,2]."

c. Prove that KB |= α2 using inference rules where α2 = "There is a wumpus in [1,3]."