Theory of Automata
Turing Machine

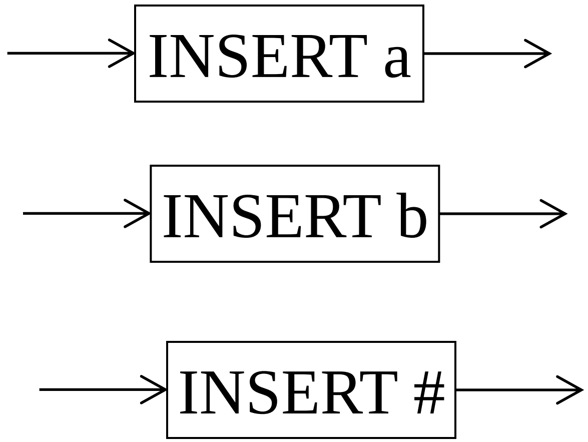
Week 16-Lecture-02

Contents

INSERT subprogram

Sometimes, a character is required to be inserted on the TAPE exactly at the spot where the TAPE Head is pointing, so that the character occupies the required cell and the other characters on the TAPE are moved one cell right. The characters to the left of the pointed cell are also required to remain as such.

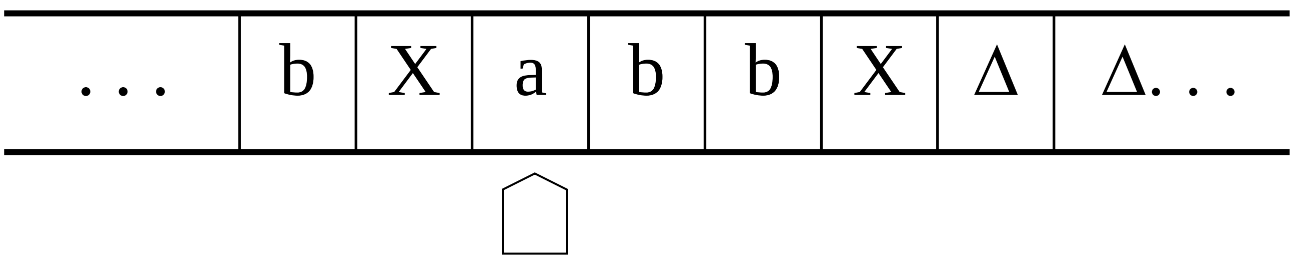
In the situation stated above, the part of TM program that executes the process of insertion does not affect the function that the TM is performing. The subprogram of insertion is independent and can be incorporated at any time with any TM program specifying what character to be inserted at what location. The subprogram of insertion can be expressed as



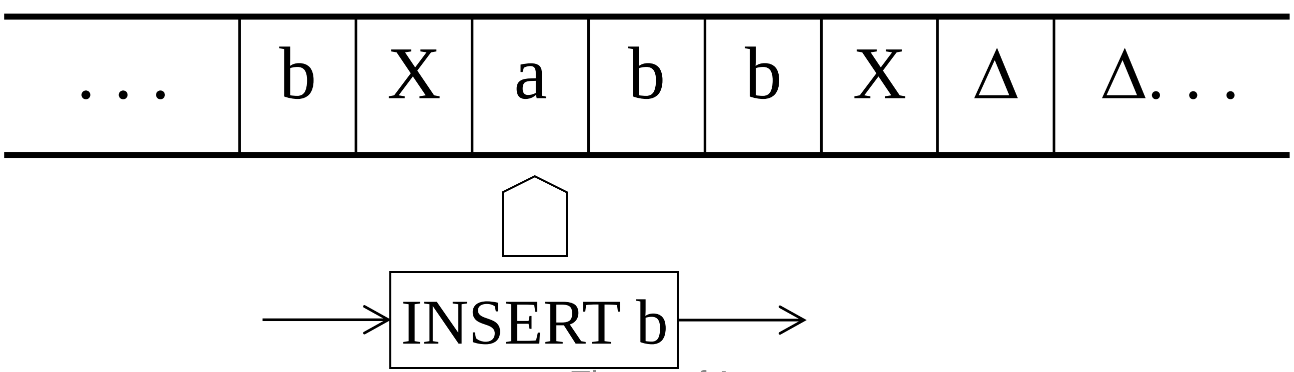
The above diagrams show that the characters a,b and # are to be inserted, respectively. Following is an example showing how does the subprogram INSERT perform its function

Example

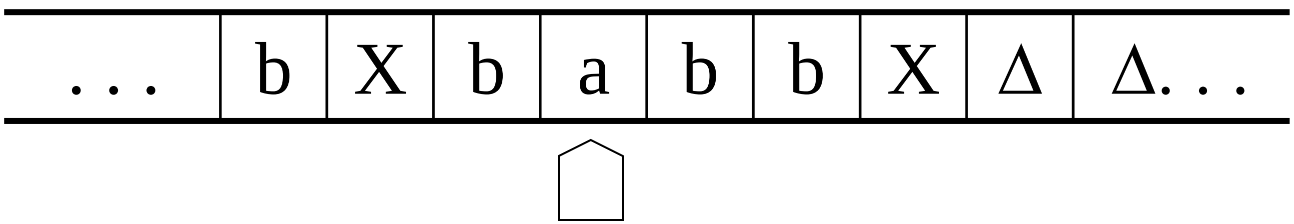
If the letter b is inserted at the cell where the TAPE Head is pointing as shown below



then, it is expressed as



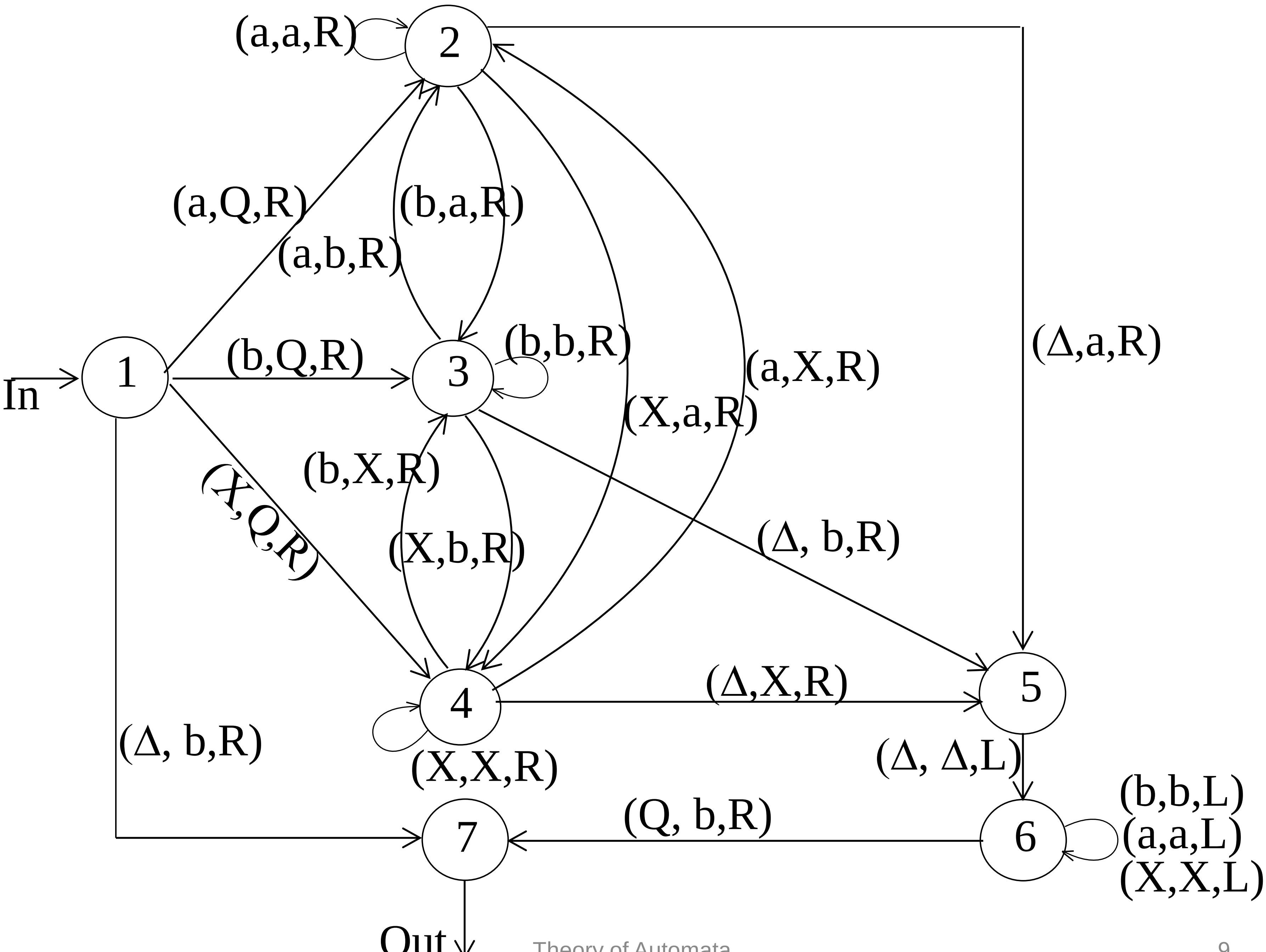
The function of subprogram INSERT b can be observed from the following diagram



Following is the INSERT subprogram

The subprogram INSERT

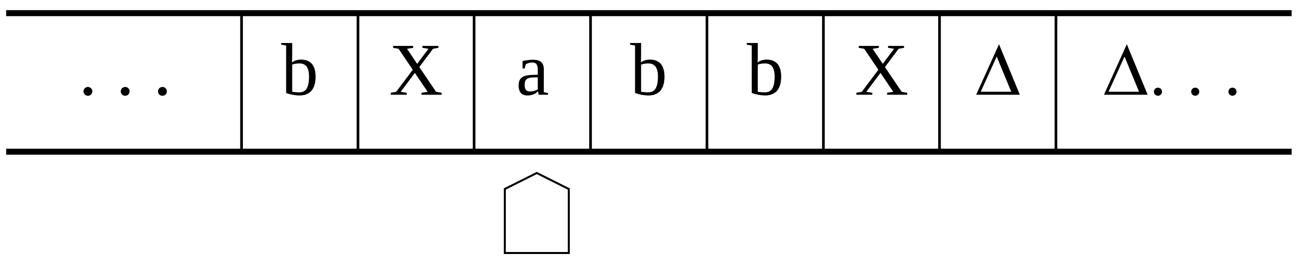
Keeping in view the same example of inserting b at specified location, to determine the required subprogram, first Q will be inserted as marker at the required location, so that the TAPE Head must be able to locate the proper cell to the right of the insertion cell. The whole subprogram INSERT is given as



(a,a,R)
2
(a,Q,R)
(b,a,R)
(a,b,R)
(b,b,R)
(Δ,a,R)
(b,Q,R)
(a,X,R)
In
1
3
(X,a,R)
(b,X,R)
(X,Q,R)
(Δ, b,R)
(X,b,R)
(Δ,X,R)
5
4
(Δ, b,R)
(X,X,R)
(Δ, Δ,L)
(b,b,L)
(Q, b,R)
7
6
(a,a,L)
(X,X,L)
Out

It is supposed that machine is at state 1, when b is to be inserted. All three possibilities of reading a, b or X are considered by introducing the states 2, 3 and 4 respectively. These states remember what letter displaced during the insertion of Q.

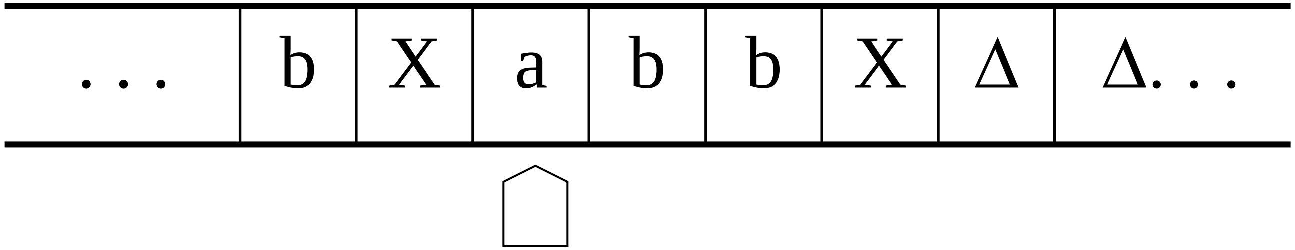
Consider the same location where b is to be inserted



After reading a from the TAPE, the program replaces a by Q and the TAPE Head will be moved one step right. Here the state 2 is entered. Reading b at state 2, b will be replaced by a and state 3 will be entered. At state 3 b is read which is not replaced by any character and the state 3 will not be left.

At state 3, the next letter to be read is X, which will be replaced by b and the state 4 will be entered. At state 4, Δ will be read, which will be replaced by X and state 5 will be entered. At state 5 Δ will be read and without any change state 6 will be entered, while TAPE Head will be moved one step left. The state 6 makes no change whatever (except Q) is read at that state. However at each step, the TAPE Head is moved one step left. Finally, Q is read which is replaced by b and the TAPE Head is moved to one step right.

Hence, the required situation of the TAPE can be shown as



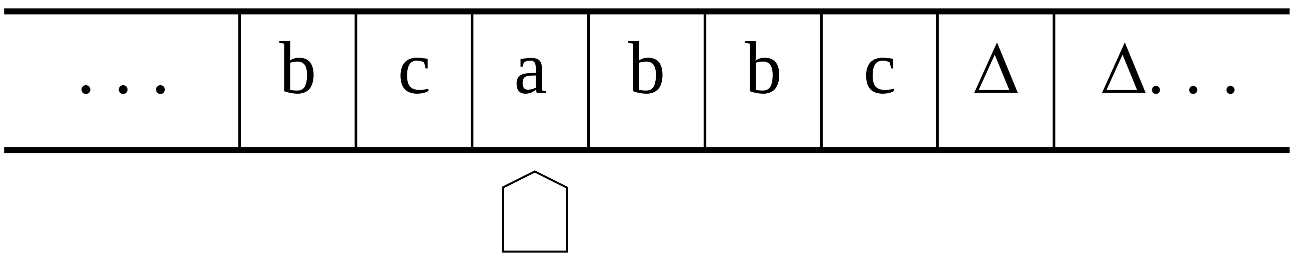
DELETE subprogram

Sometimes, a character is required to be DELETED on the TAPE exactly at the spot where the TAPE Head is pointing, so that the other characters on the right of the TAPE Head are moved one cell left. The characters to the left of the pointed cell are also required to remain as such.

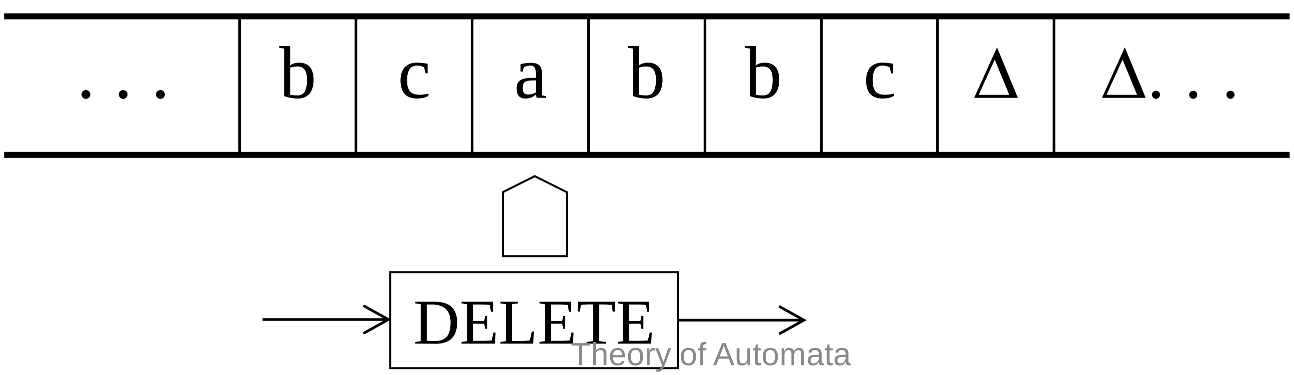
In the situation stated above, the part of TM program that executes the process of deletion does not affect the function that the TM is performing. The subprogram of deletion is independent and can be incorporated at any time with any TM program specifying what character to be deleted at what location. The subprogram of deletion can be expressed as

Example

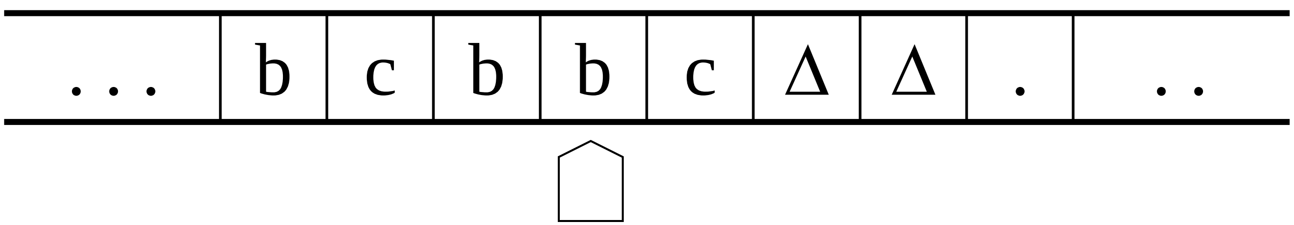
If the letter a is to be deleted from the string bcabbc, shown below



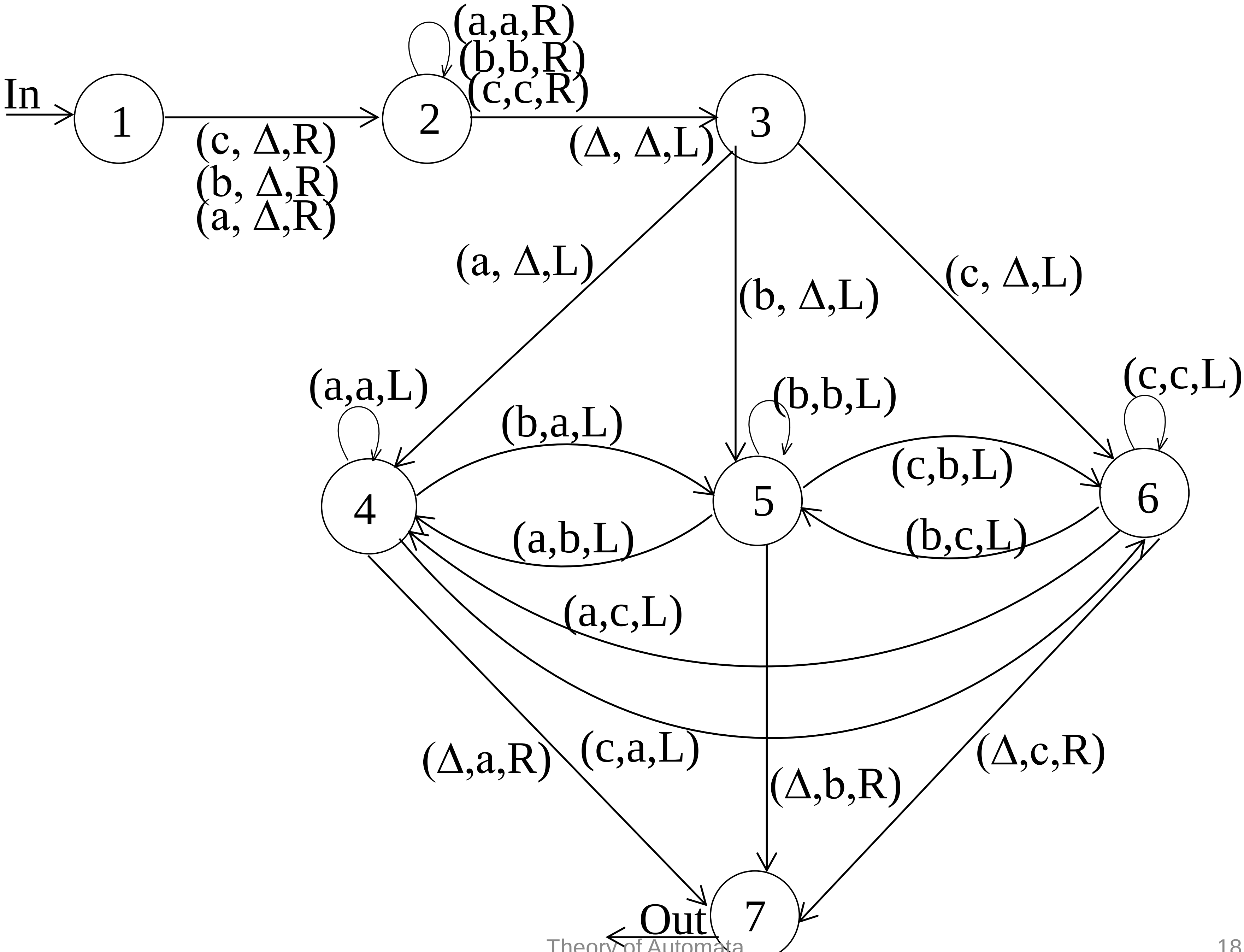
then, it is expressed as



The function of subprogram DELETE can be observed from the following diagram

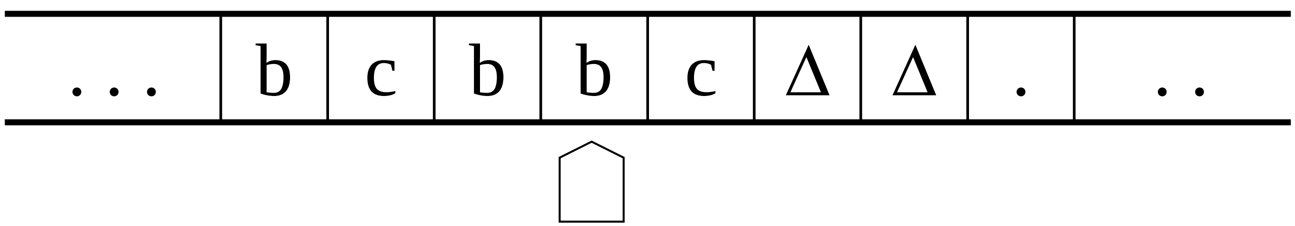


Following is the DELETE subprogram



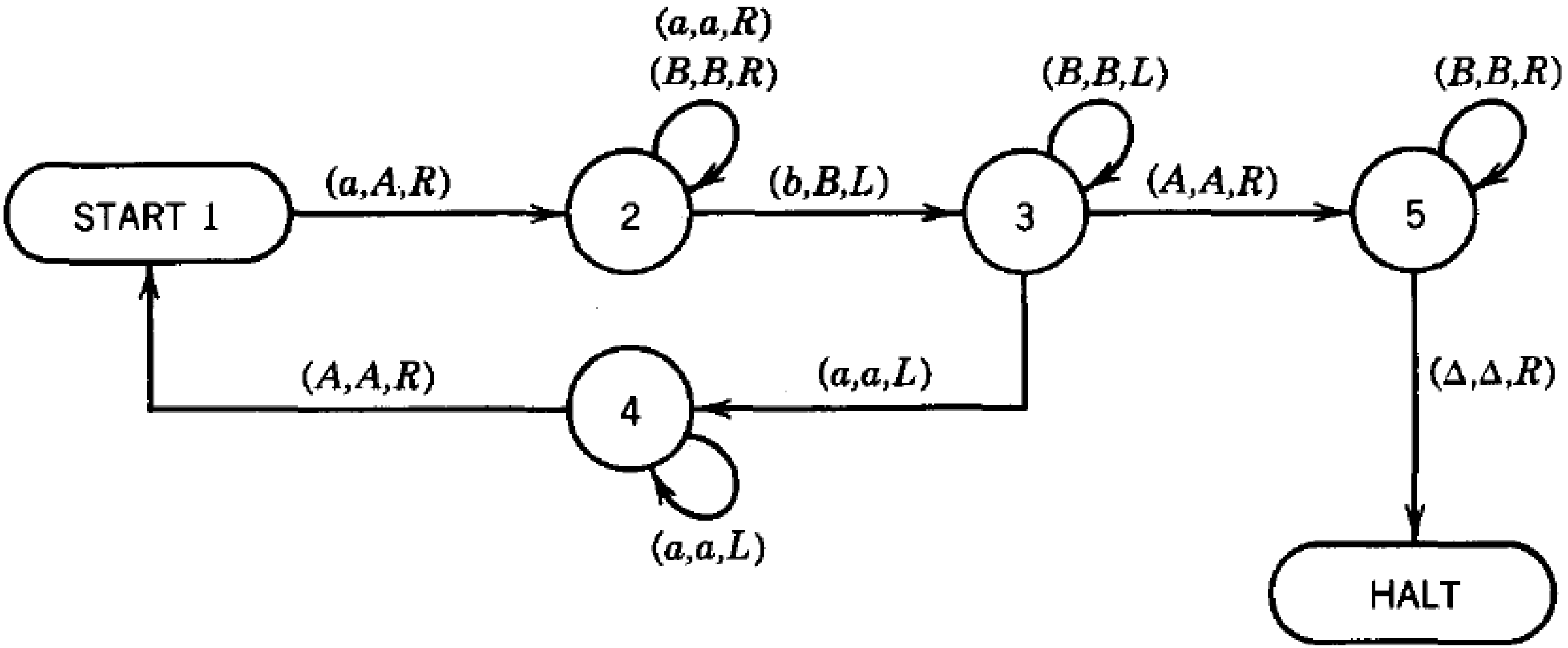
In
1
(c, Δ,R)
(b, Δ,R)
(a, Δ,R)
2
(a,a,R)
(b,b,R)
(c,c,R)
3
(Δ, Δ,L)
(a, Δ,L)
(b, Δ,L)
(c, Δ,L)
(a,a,L)
(b,b,L)
(c,c,L)
(b,a,L)
(c,b,L)
4
5
6
(a,b,L)
(b,c,L)
(a,c,L)
(Δ,a,R)
(c,a,L)
(Δ,b,R)
(Δ,c,R)
Out
7

The process of deletion of letter a from the string bcabbc can easily be checked, giving the TAPE situation as shown below

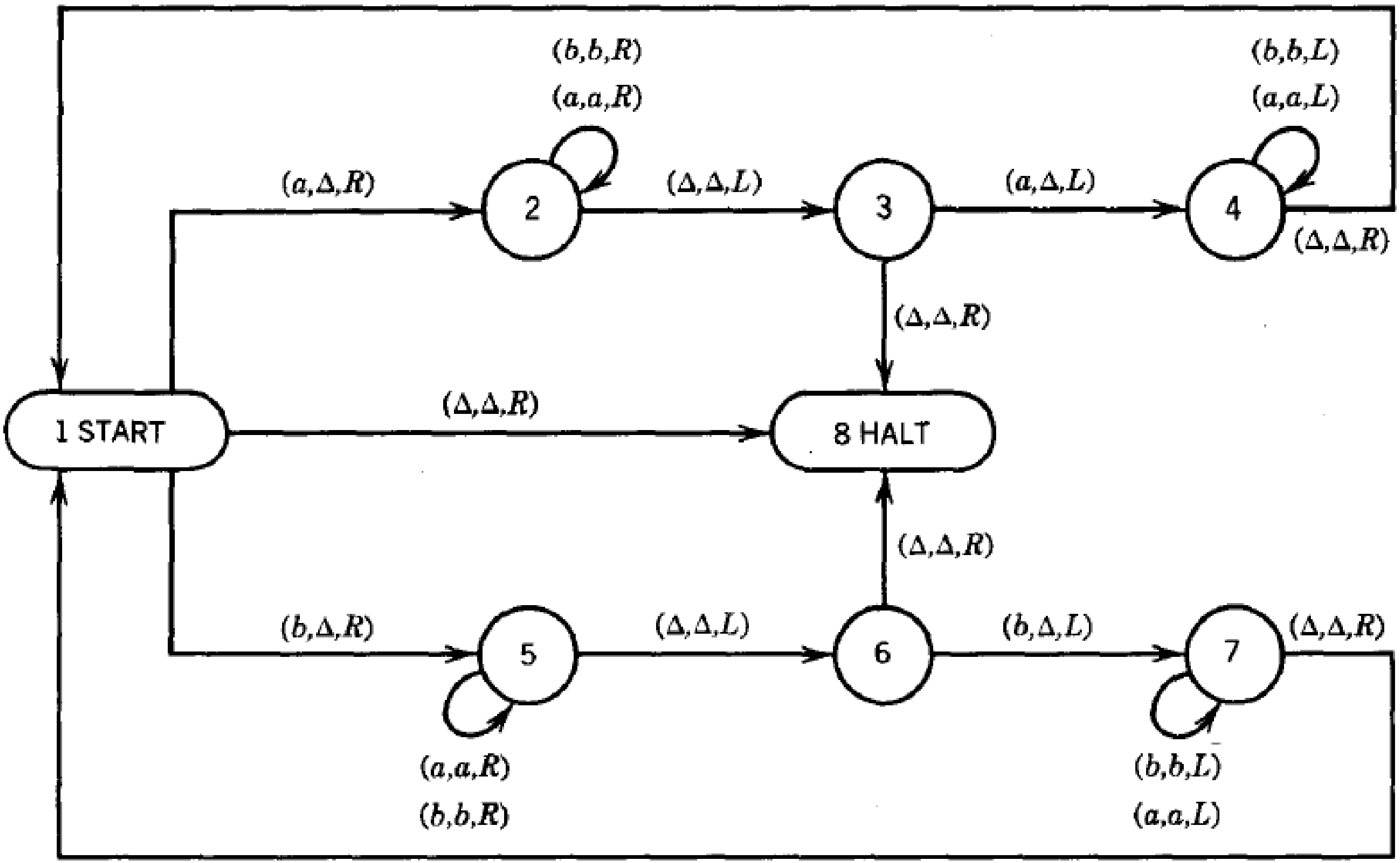


Language Defined by	Corresponding Acceptor	Nondeterminism = determinism?	Language Closed Under	What Can be Decided	Example of Application
Regular expression	Finite automaton Transition graph	Yes	Union, product, Kleene star, intersection, complement	Equivalence, emptiness, finiteness, membership	Text editors, sequential circuits
Context-free grammar	Pushdown automaton	No	Union, product, Kleene star	Emptiness finiteness membership	Programming language statements, compilers
Type 0 grammar	Turing machine, Post machine, 2PDA, *n*PDA	Yes	Union, product, Kleene star	Not much	Computers

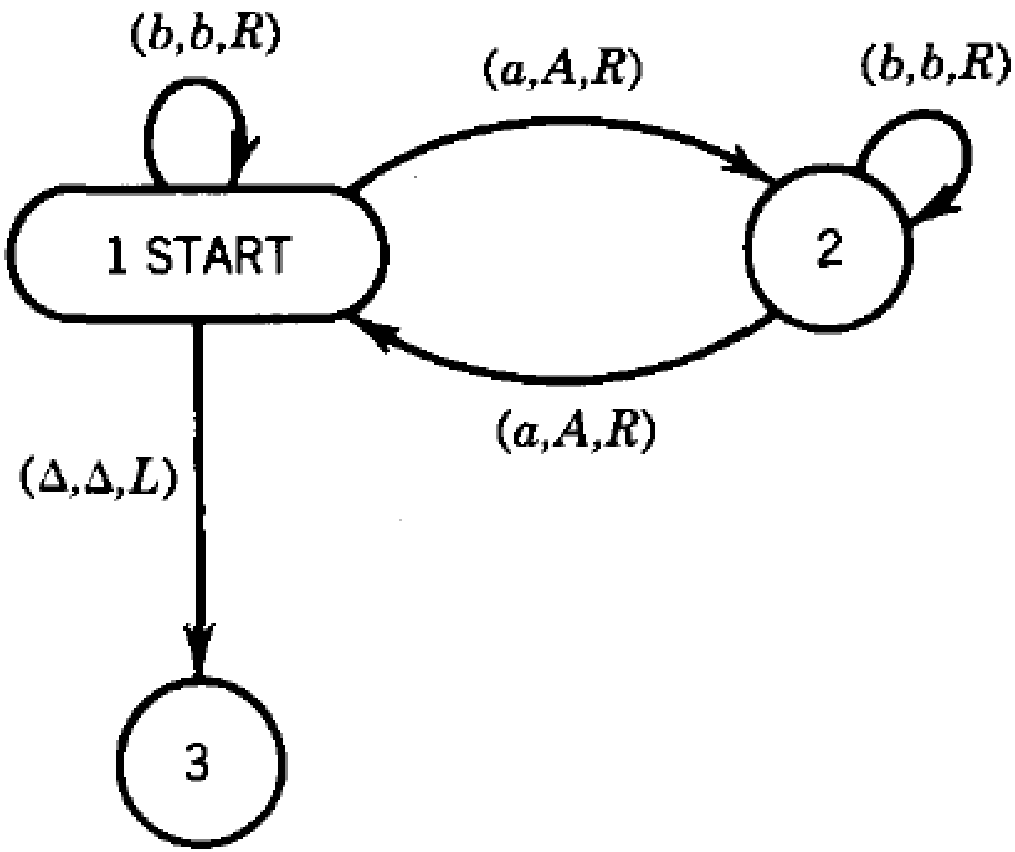
a^nb^n



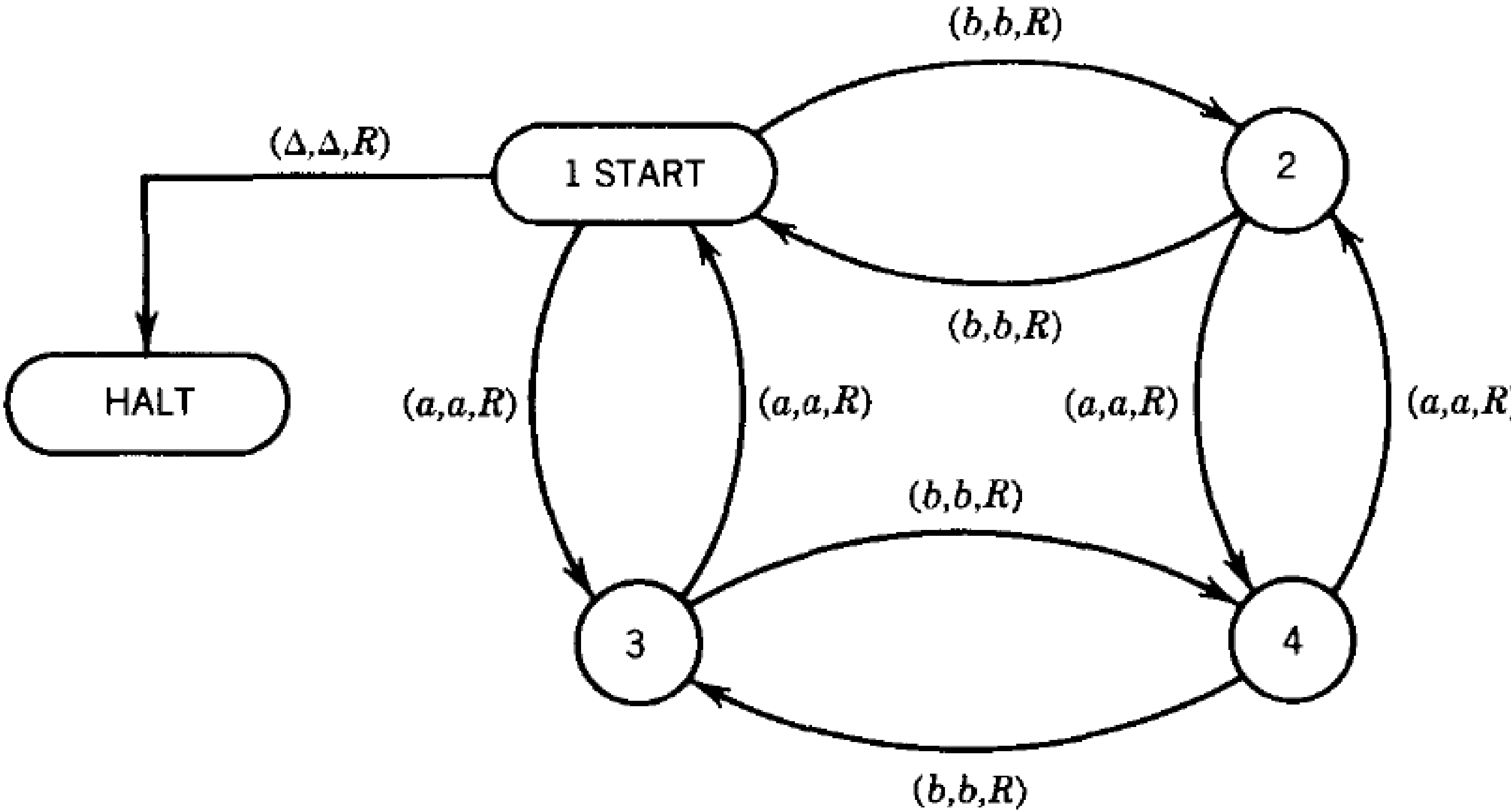
OddPalindrome / EvenPalindrome



Even a’s



Even-Even



Double Word Problem

The language this TM accepts is DOUBLEWORD, the set of all words of the form ww where w is a nonnull string in *{a, b}*.*
DOUBLEWORD **= {aa *bb aaaa abab baba bbbb*** *aaaaaa aabaab . . }*

Sample Problems

16. Build a TM to accept the language $\{a^n b^n a^n\}$ based on the following algorithm:
    - (i) Check that the input is in the form **a*b*a***.
    - (ii) Use DELETE in an intelligent way.

11. Return to the example in the text of a TM to accept EVEN-EVEN based on the algorithm
    1. Move up the string changing a's to A's
    2. Move down the string changing b's to B's

    We can modify this algorithm in the following way: To avoid the problem of crashing on the way down the TAPE change the letter in the first cell to X if it is an a and to Y if it is a b. This way, while charging down the TAPE we can recognize when we are in cell i.

    Draw this TM.

13. Build a TM that accepts the language EVEN-EVEN based on the subroutine DELETE given in this chapter.

Equal-Equal

$S \rightarrow aSbS \mid bSaS \mid \varepsilon$

OR

$S \rightarrow aSb \mid bSa \mid SS \mid \varepsilon$