MEASUREMENT AND INSTRUMENTATION (EEE 326.2)

PART 2 COURSE OUTLINE 09-10-21

1) DC and AC bridges and applications - WK 1
2) General form of AC bridge - universal impedance bridge. - WK 1
3) Electronic instruments for measurement of voltage, current, resistance and other circuit parameters - WK
4) Electronic voltmeters - WK 2
5) AC voltmeters using rectifiers, electronic multimeter, digital voltmeters. - WK 3
6) Oscilloscope: vertical deflection system, horizontal deflection system, probes, sampling CRO. - WK 4

DC AND AC BRIDGES AND APPLICATIONS

DIRECT CURRENT (DC) BRIDGES

Direct current (DC) bridges are resistance measurement instruments which operate using direct current (DC) as their source of power. In other words they are used to measure the resistance of DC circuit elements. DC bridges exist against AC bridges which are instruments used to measure inductances and capacitance of alternating current (AC) circuits. There are several types of DC bridges and the type used for a particular application depends on the range of resistance to be measured. Summarized below are different types of DC bridges and resistance ranges for which they are applicable;

Resistance	Range	DC Bridge Type
Low (resistance of armature, series field windings, armature shunts	$< 1\Omega$	Kelvin double bridge
Medium (resistance of most electrical apparatus)	$1\Omega - 100k\Omega$	Wheatstone bridge Carey-Foster bridge
High (Insulation resistance, high resistance circuit elements)	$> 100k\Omega$	Megohm bridge

1.1 Kelvin Double Bridge

The Wheatstone bridge, to be considered shortly, is considered not suitable for low resistance measurement as result of errors arising from contact resistance, resistance of leads and its lack of sensitivity. Also low resistance measurement requires the four terminal connections such as is provided by the Kelvin Double Bridge. The Kelvin double bridge which is a network of resistors was designed by Lord Kelvin and is illustrated in fig. 1.1.

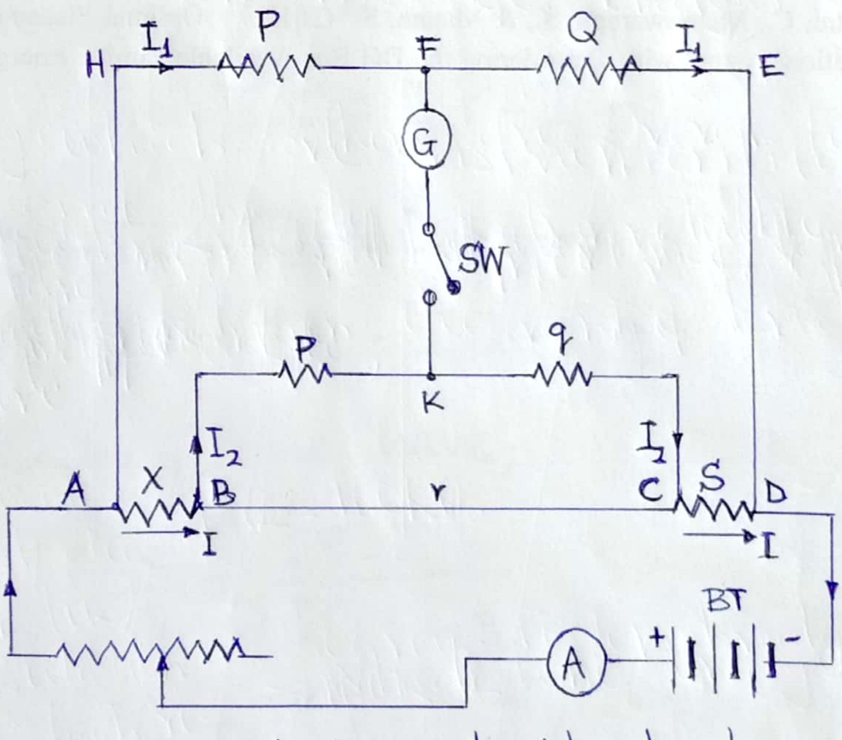


Fig 1.1 Kelvin double bridge

X is the resistance to be measured and S is a known resistance of same order and of same current or higher current rating than the resistance to be measured. Both resistors (X and S) are connected in series with a short link of very low resistance. P, Q, p and q are four known non-inductive resistances one pair of which (either P and p or Q and q) is variable. A current, preferably the rated current of X, is passed through X and S from a low-voltage, high-current battery (BT). A sensitive galvanometer G is connected between the dividing points of PQ and pq. The ratio P/Q and p/q are kept the same by varying the ratios until the galvanometer reads zero. At this point the bridge is said to be balanced.

In the balanced position the currents flowing are as shown in the Fig. 1. Applying KVL to loops AHFKBA and FEDCKF we have;

$$I_1P - I_2p - IX = 0 \quad \text{or}$$

$$IX = I_1P - I_2p \qquad 1.1$$

and

$$I_1Q - IS - I_2q = 0 \quad \text{or}$$

$$IS = I_1Q - I_2q \qquad 1.2$$

Dividing ① by ② we have;

$$\frac{X}{S} = \frac{I_1P - I_2p}{I_1Q - I_2q}$$

$$= \frac{P(I_1 - \frac{p}{P}I_2)}{Q(I_1 - \frac{q}{Q}I_2)} = \frac{P}{Q}, \text{ since } \frac{p}{P} = \frac{q}{Q}$$

Therefore the unknown resistance

$$X = S \times \frac{P}{Q}$$

1.2. Wheatstone Bridge.

The wheatstone bridge presents one of the most widely used methods for the measurement of medium range resistances.

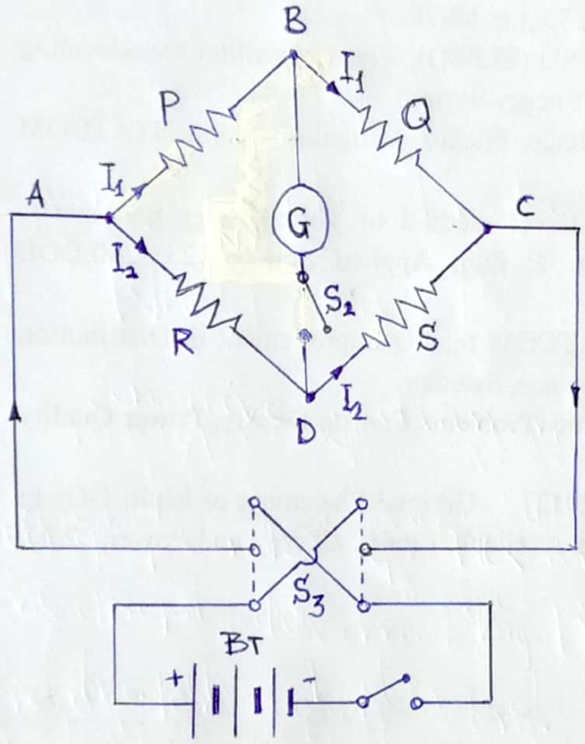


Fig 1.2. Wheatstone Bridge

As seen from fig.1.2 the wheatstone bridge consists of known adjustable resistances P, Q, and S, with an unknown resistance R connected between pts. A and D. When switch S_1 is closed, the arrangement of resistors is such that current can flow through two parallel paths. A sensitive galvanometer G is also connected Between points B and D to provide indication as to when the bridge is balanced.

When S_1 is closed current will flow through both parallel paths. P, Q and S are adjusted such that the deflection of the galvanometer is nil when switch S_2 is closed. When the bridge is balanced the same current I_1 flows through P and Q while same current I_2 flows through R and S. This means the potential at pts B and D must be same. For potential at pts. B and D to be equal means voltage drop from A to B must be equal to voltage drop from A to D and voltage drop from B to C must be equal to voltage drop from D to C.

Hence, $I_1 P = I_2 R$ 1.3

and $I_1 Q = I_2 S$ 1.4

Dividing 1 by 2 we have

$$\frac{P}{Q} = \frac{R}{S}$$

$$\text{or} \quad R = \frac{PS}{Q}$$

Thus the unknown resistance R is computed from the known resistances P, Q and S

<u>Advantages of Wheatstone Bridge</u>

1. The method is mostly analytical, thus eliminating errors of instrument caliberation. The galvanometer is merely to indicate zero current.

2. The balance of the instrument is quite independent of the source emf. Thus, accuracy of measured values is not affected by fluctuation of emf of the source.

Disadvantages of the Wheatstone Bridge

1. ~~Errors~~ arising from descripancies between the true and marked values of the three known resistances, P, Q & S.

2. Personal errors arising from taking the balance pt, taking readings for values of P, Q & S and while making calculation for R.

3. Inaccuracy of balance pt. obtained due to poor sensitivity of galvanometer.

Sensitivity of Wheasfone Bridge

It is often desireble to know the galvanometer response to a small unbalance in the bridge. This response is refered to as the sensitivity of the bridge to unbalance. The approach to calculating the sensitivity to unbalance is by Therenizing the wheatstonebridge shown in fig. 1.3.

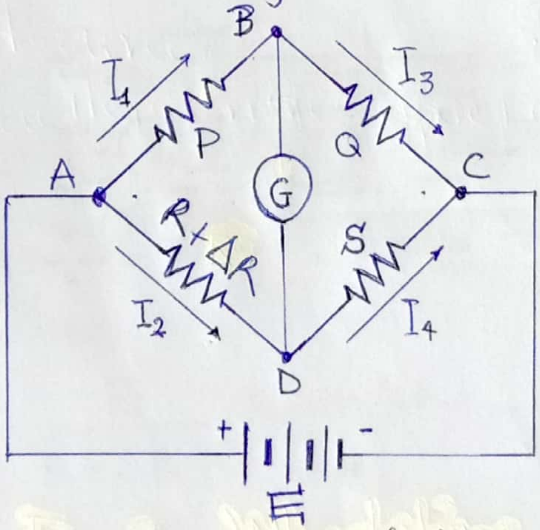


Fig. 1.3. Wheatstone Bridge.

The Thevenin's of the Wheatstone bridge circuit reduces to the circuit of fig. 1.4.

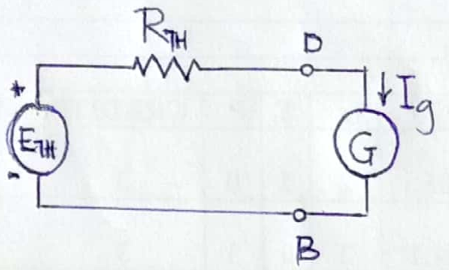


Fig. 1.4 Thevenin's equivalent of Wheatstone bridge circuit.

Since we are interested in getting the current through the galvanometer, I_g, the Thevenin's equivalent is gotten by looking back into the galvanometer terminals B and D of fig. 2. E_{TH} is evaluated as the equivalent voltage across terminals B and D when the galvanometer is replaced with an open circuit. R_{TH} is evaluated as the equivalent resistance looking back into terminals B and D, with the battery replaced by its internal resistance. This is neglected to ease calculations.

The Thevenins or open circuit voltage is evaluated thus;

$$E_{TH} = E_{AD} - E_{AB} \text{ (from fig. 2)}$$
$$= I_2(R + \Delta R) - I_1 P$$

Where ΔR is a small error introduced by galvanometer unbalance.

$$E_{TH} = \frac{E}{R + \Delta R + S}(R + \Delta R) - \frac{E}{(P+Q)} \cdot P$$

$$\therefore E_{TH} = E\left[\frac{R+\Delta R}{R+\Delta R+S} - \frac{P}{P+Q}\right] \quad 1.5$$

$$= E\left[\frac{R+\Delta R}{R+\Delta R+S} - \frac{R}{R+S}\right] \quad \text{since } \frac{P}{P+Q} = \frac{R}{R+S}$$

$$\therefore E_{TH} \simeq \frac{ES\Delta R}{(R+S)^2} \simeq \frac{E\Delta R}{4R} \text{ (for equal ratio bridge)} \quad 1.6$$

The equivalent resistance, R_{TH}, of the Thevenin's equivalent circuit is obtained from fig. which is gotten by looking back into terminals B and D.

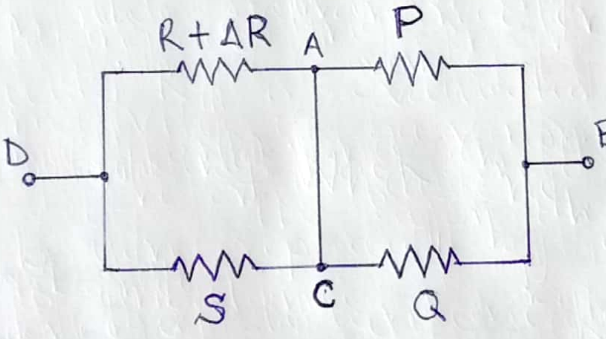


Fig. 1.5 Thevenin's resistance

From fig 1.5, we have;

$$R_{TH} = \frac{(R+\Delta R)S}{R+\Delta R+S} + \frac{PQ}{P+Q}$$

For equal ratio bridge; $(P = Q = R = S)$

$$R_{TH} = \frac{R}{2} + \frac{R}{2} = R$$

Definition of Terms

i) <u>Voltage sensitivity of galvanometer</u>: This is defined as the change in scale units per unit change in voltage of the galvanometer circuit when total resistance seen from galvanometer terminals is the only resistance required for the specified damping.

$$\therefore \quad S_V = \frac{\text{change in scale units}}{\text{change in voltage of galvanometer circuit}}$$

2. Galvanometer Deflection (θ): This is defined as the product of galvanometer voltage sensitivity (S_V) and Thevenin's voltage (E_{TH}) of bridge circuit.

$$\therefore \theta = S_V E_{TH} = S_V E \frac{S \Delta R}{(R+S)^2} \quad 1.7$$

3. Bridge Sensitivity (S_B): This is defined as the change in galvanometer deflection (θ) per unit fractional change in unknown resistance.

$$\therefore S_B = \frac{\theta}{\Delta R / R}$$

$$S_B = \frac{S_V E S R}{(R+S)^2} \quad 1.8$$

From equation 1.8 we see that S_B depends on bridge voltage (E), voltage sensitivity of galvanometer (S_V) and bridge resistances R and S. Rearranging equ. 1.8 we have;

$$S_B = \frac{S_V E}{(R+S)^2 / SR} = \frac{S_V E}{\frac{R}{S} + 2 + \frac{S}{R}} = \frac{S_V E}{\frac{P}{Q} + 2 + \frac{Q}{P}} \quad 1.9$$

From 1.9 we find that when $\frac{R}{S} = \frac{S}{R} = \frac{P}{Q} = \frac{Q}{P} = 1$

$$S_B = \frac{S_V E}{4} \quad 1.10$$

CamScanner

Which represents the condition for maximum sensitivity.

Refering back to the Thevenin's equivalent circuit of the Wheatstone bridge the current passing through the galvanometer is given as;

$$I_g = \frac{E_{TH}}{R_{TH} + R_g} \quad 1.11$$

where R_g = galvanometer current. Therefore for an equal ratio bridge;

$$I_g = \frac{E\Delta R / 4R}{R + R_g} \quad 1.12$$

From 3 galvanometer deflection is given as

$$\theta = \frac{S_v E S \Delta R}{(R+S)^2} \quad 1.13$$

where S_v = voltage sensitivity of galvanometer. In terms of current sensitivity (S_i) we have that;

$$S_v = \frac{S_i}{R_{TH} + R_g} \quad 1.14$$

Putting 1.14 into 1.13 we have that;

$$\theta = \frac{S_i E S \Delta R}{(R+S)^2 (R_{TH} + R_g)} \quad 1.15$$

$$\therefore \theta = \frac{S_i E \Delta R}{4R(R+R_g)} \quad \text{(for equal ratio)} \qquad 1.16$$

Also bridge sensitivity,

$$S_B = \frac{S_v E S R}{(R+S)^2}$$

$$= \frac{S_i E S R}{(R_{TH}+R_g)(R+S)^2} \qquad 1.17$$

$$= \frac{S_i E}{4(R_{TH}+R_g)} \quad \text{(for equal ratio)} \qquad 1.18$$

1.3. Carey-Foster Bridge Method

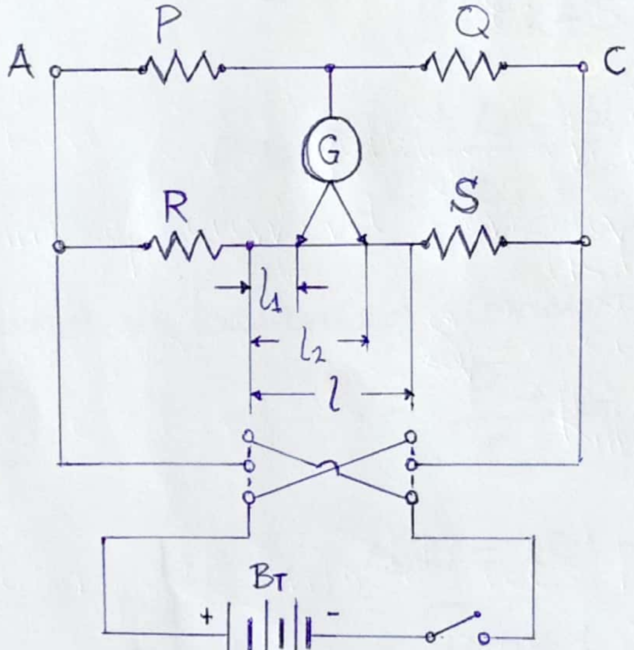


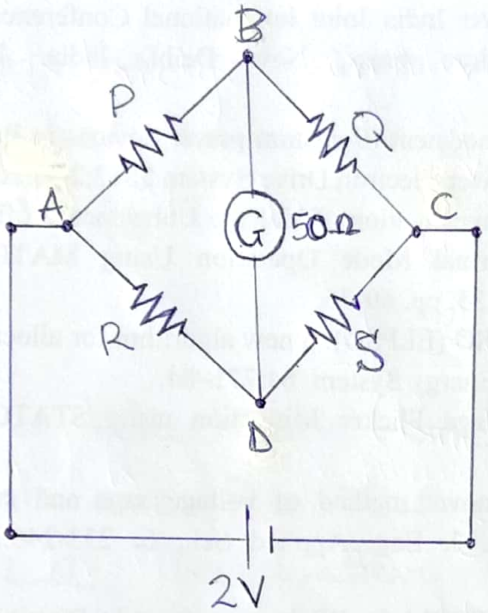
Fig 1·6. Carey-Foster Bridge

The Carey-Foster bridge is a modification of the Wheatstone bridge and is especially suited to comparing of two nearly equal resistances.

As seen in fig 1·6 a slide wire of length l has been included between R and S.

Examples

1. For a Wheatstone bridge as shown $P = Q = 1000\,\Omega$, $S = 100\,\Omega$ and $R = 101\,\Omega$.



Find the magnitude and direction of the current flowing through the galvanometer under the unbalanced condition.

Solution

$$I_g = \frac{E_{TH}}{R_{TH} + R_g} = \frac{E_{TH}}{R_{TH} + 50}$$

Recall, $$E_{TH} = E\left[\frac{R+\Delta R}{R+\Delta R+S} - \frac{P}{P+Q}\right]$$

$$R_{TH} = \frac{(R+\Delta R)S}{R+\Delta R+S} + \frac{PQ}{P+Q}$$

Under balanced conditions

$$R = \frac{PS}{Q} = \frac{1000 \times 100}{1000} = 100\,\Omega$$

$$\therefore \Delta R = 101 - 100 = 1\,\Omega$$

$$\therefore E_{TH} = 2\left[\frac{100+1}{100+1+100} - \frac{1000}{1000+100}\right] = 4.98 \times 10^{-3}\,V$$

$$R_{TH} = \frac{(100+1)\times 100}{100+1+100} + \frac{1000 \times 1000}{1000+1000} = 550.25\,\Omega$$

$$\therefore I_g = \frac{4.98 \times 10^{-3}}{550.25 + 50} = 8.3\,\mu A.$$

From the +Ve sign of E_{TH} pt. D is at a higher potential than pt. B, therefore current will flow from pt. D to B.

2. In a Wheatstone bridge the values of the bridge elements are $P = 100\,\Omega$, $Q = 1{,}000\,\Omega$, $S = 2{,}000\,\Omega$ and $R = 202\,\Omega$. The battery is 5V and has negligible internal resistance. The galvanometer used has a sensitivity of 5 mm/μA and an internal resistance of $200\,\Omega$. Find the deflection of galvanometer caused by the unbalance created by $2\,\Omega$ error in resistance R. Also determine the bridge sensitivity in terms of deflection per unit change in resistance.

Solution

Deflection of galvanometer is given as

θ = Current sensitivity x current flowing in galvanometer

$= S_i \times I_g = 5\,mm/\mu A \times I_g$

Recall, $I_g = \frac{E_{TH}}{R_{TH} + R_g} = \frac{E_{TH}}{R_{TH} + 200}$

$$E_{TH} = E\left[\frac{R + \Delta R}{R + \Delta R + S} - \frac{P}{P + Q}\right]$$

$$E_{TH} = 5\left[\frac{202}{202+2{,}000} - \frac{100}{100+1{,}000}\right]$$

$$= 4.13\,mV$$

$$R_{TH} = \frac{(R+\Delta R)S}{R+\Delta R+S} + \frac{PQ}{P+Q}$$

$$= \frac{(200+2)\times 2{,}000}{200+2+2{,}000} + \frac{1{,}000\times 100}{1{,}000\times 100}$$

$$= 274.4\,\Omega$$

Thus galvanometer current is gotten as

$$I_g = \frac{4.13\times 10^{-3}}{274.4+200}$$

$$= 8.7\,\mu A$$

There fore;

$$\theta = 5\,mm/\mu A \times 8.7\,\mu A$$

$$= 43.5\,mm$$

Sensitivity of bridge is thus evaluated

$$S_B = \frac{\theta}{\Delta R} = \frac{43.5}{2}$$

$$= 21.75\,mm/\Omega$$

3. An equal ratio bridge has the resistance 500 Ω in each arm. The galvanometer in the setup has a resistance of 100 Ω while the voltage across the bridge is 10 V. If the current flowing through the galvanometer is 1 nA determine the error, ΔR in measured resistance responsible for the unbalance.

Solution

$$I_g = \frac{E_{TH}}{R_{TH} + R_g}$$

$R_{TH} = R = 500\,\Omega$ (equal ratio bridge)

$R_g = 100\,\Omega$

$$\therefore I_g = \frac{E_{TH}}{500 + 100} = \frac{E_{TH}}{600}$$

$$\text{or } E_{TH} = 600\, I_g$$

$$= 600 \times 1 \times 10^{-9}\ V \quad \text{——— i}$$

But for an equal ratio bridge

$$E_{TH} = \frac{E\Delta R}{4R} = \frac{10\,\Delta R}{4 \times 500}$$

$$\therefore E_{TH} = 5 \times 10^{-3}\,\Delta R \quad \text{——— ii}$$

Equating i & ii

$$600 \times 10^{-9} = 5 \times 10^{-3}$$

$$\therefore \Delta R = \frac{600 \times 10^{-9}}{5 \times 10^{-3}} = 0.12\, m\Omega$$

4. A Wheatstone bridge has fixed resistances P and Q, with S variable. $Q = 10\,\Omega$, $P = 10{,}000\,\Omega$ and S has a maximum value of $5\,k\Omega$. If the voltage across the bridge is 12V determine (i) The maximum value of the resistance that can be measured with the given arrangement. (ii) How much unbalance would produce a deflection of 2.5mm for the maximum resistance found in (i).

Internal resistance of battery is negligible and the galvanometer has an internal resistance of $100\,\Omega$ and a sensitivity of $100\,mm/\mu A$.

Solution

i) The maximum resistance is gotten as;

$$R = \frac{PS}{Q} = \frac{10000 \times 5000}{10}$$

$$= 5\,M\Omega$$

ii) Recall that galvanometer deflection is given as

$$\theta = \frac{S_i E S \Delta R}{(R+S)^2 (R_{TH} + R_g)}$$

$$\therefore \Delta R = \frac{\theta (R+S)^2 (R_{TH} + R_g)}{S_i E S}$$

But

$$R_{TH} = \frac{RS}{R+S} + \frac{PQ}{P+Q} = \frac{5\times10^6 \times 5\times10^3}{5\times10^6 + 5\times10^3} + \frac{10{,}000 \times 10}{10{,}000 + 10}$$

$$= 5{,}005\,\Omega$$

$$\therefore \Delta R = \frac{2.5(5{,}005+100)(5\times10^{6}+5\times10^{3})^{2}}{100\times10^{6}\times 12\times 5{,}000}$$

$$= 53.28\,k\Omega$$

1.4. Measurement of High Resistance

High resistances, of the order of 100s or 1000s of $M\Omega$ are frequently encountered in electrical equipment. Examples are insulation resistance of cables and machines, leakage resistance of capacitor and resistance in vacuum tube circuits. High resistance may be measured in several ways;

(a) Derect deflection method

(b) Megger method.

(c) Loss of charge method.

(d) Megohm bridge method.

However in this section we focus on the use of the megohm bridge method.

1.4.1 Megohm Bridge Method

The operational arrangement of the megohm bridge is shown in fig. 6. It makes use of a modified Wheatstone bridge which has been fitted with guard electrodes and the galvanometer is replaced by an amplifier and a null detector.

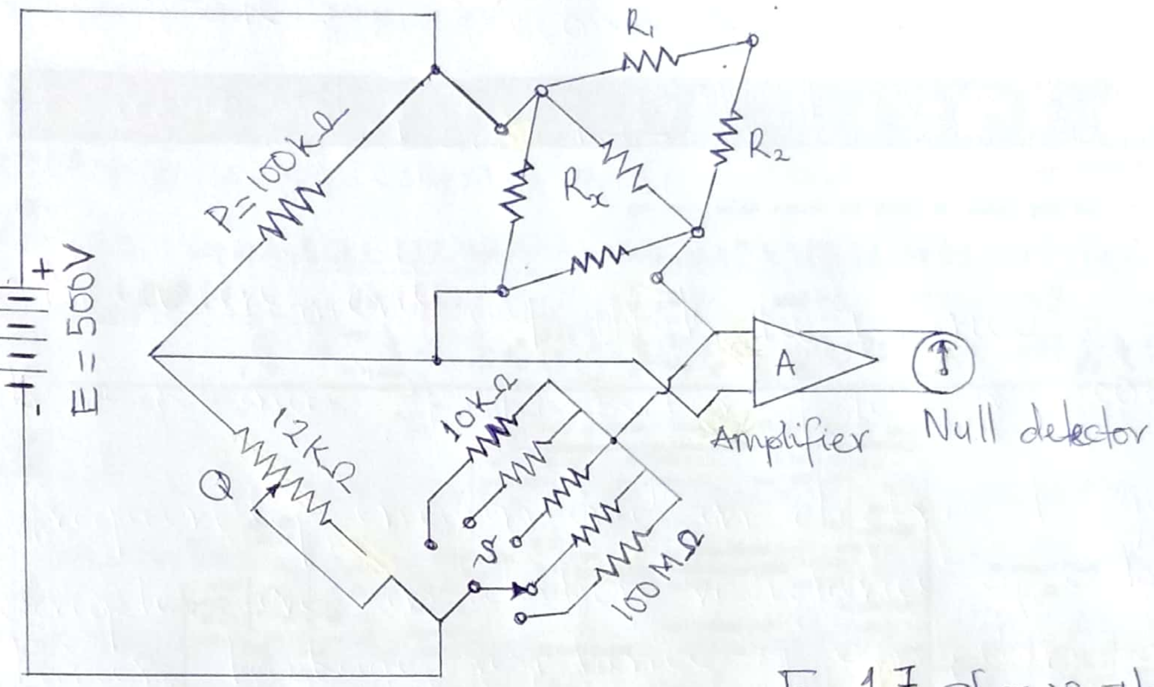


Fig. 1.7 Megohm bridge

Fig. 1.7 shows the complete circuit of a megohm bridge contain, power supply, bridge members, amplifier and a null detector.

ALTERNATING CURRENT (AC) BRIDGES

ALTERNATING CURRENT (AC) BRIDGES

AC bridges present the best and most precise methods for measurement of self and mutual inductance and capacitance. This is because its difficult to get accuracy with deflection methods. Measurement with the AC bridge is done in a similar way as in the DC bridge. The basic AC bridge consist of four arms, a source of power and a null or balance detector. Here the power source is AC and balance detector is sensitive to AC instead of battery and galvanometer as in the case for DC bridges.

2.1 General Form of an AC Bridge

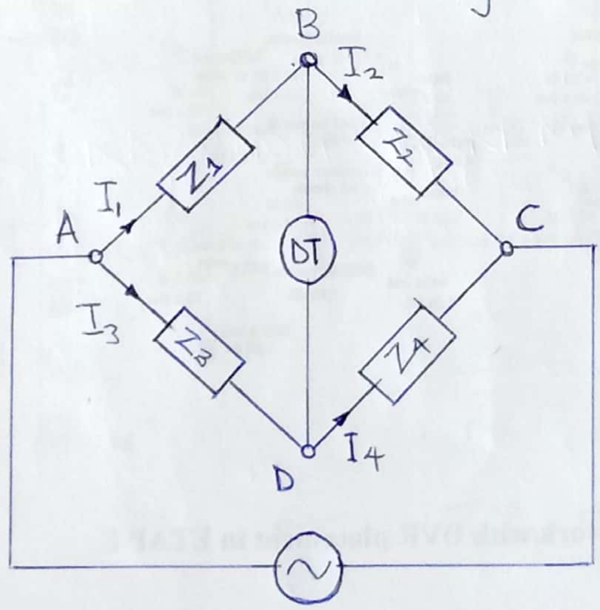


Fig.2.1 Universal impedance bridge

The general form of an AC bridge is given in fig.2.1. It contains four impedances connected in a bridge to an AC source and a null detector DT.

Balance is obtained by adjusting one or more of the impedances and is indicated by a zero response from the detector.

At balance, pts. B and D are at same potential which means voltage drop between pts. A and B is equal to voltage drop between pts A and D

$$\therefore \quad I_1 Z_1 = I_3 Z_3 \qquad 2.1$$

Also the voltage drop between pts. B and C is equal to the voltage drop between pts. D and C.

$$\therefore \quad I_2 Z_2 = I_4 Z_4 \qquad 2.2$$

Dividing 2.1 by 2.2 we have

$$\frac{I_1 Z_1}{I_2 Z_2} = \frac{I_3 Z_3}{I_4 Z_4} \qquad 2.3$$

But at balance current through the detector is zero.

Therefore,

$$I_1 = I_2 \quad \text{and} \quad I_3 = I_4$$

Equation 2.3 then becomes,

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

$$\text{or} \quad Z_1 Z_4 = Z_2 Z_3 \qquad 2.4$$

Equation 2.4 gives the equation for the general form of an AC bridge at balance. This equation is not affected by interchanging the positions of the impedances or supply or indicator.

In polar form equ. 2.4 becomes;

$$(Z_1\angle\theta_1)(Z_4\angle\theta_4) = (Z_2\angle\theta_2)(Z_3\angle\theta_3)$$

or $$Z_1 Z_4 \angle\theta_1+\theta_4 = Z_2 Z_3 \angle\theta_2+\theta_3 \qquad 2.5$$

Equation 2.5 suggests that two conditions must be simultaneously satisfied. The first is the magnitude condition;

$$Z_1 Z_4 = Z_2 Z_3 \qquad 2.6$$

While the second is the phase angle condition;

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3 \qquad 2.7$$

It should be noted that in practice the individual impedance branches could be series or parallel combinations and may include resistance, inductance and capacitance elements in series or in combination

<u>Examples</u>

1. A four arm AC bridge consist of the following impedances:

Arm AB : $Z_1 = 400\angle 60^\circ\ \Omega$ (inductive impedance)

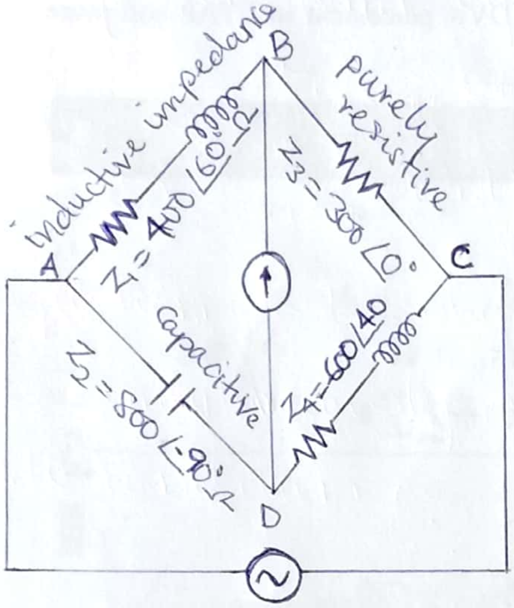
" AD : $Z_2 = 800\angle -90\ \Omega$ (purely capacitive impedance)

" BC : $Z_3 = 300\angle 0^\circ\ \Omega$ (purely resistive)

" DC : $Z_4 = 600\angle 40^\circ\ \Omega$ (inductive impedance)

Determine if the bridge is balanced.

Solution



For bridge to be balanced both magnitude and phase conditions must be met.

$\therefore Z_1 Z_4 = Z_2 Z_3$

$Z_1 Z_4 = 400 \times 600 = 240000$

$Z_2 Z_3 = 800 \times 300 = 240000$

The magnitude condition is satisfied thus. For the phase angle condition we have that

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$$

$$\angle\theta_1 + \angle\theta_4 = 60° + 40° = 100°$$

$$\angle\theta_2 + \angle\theta_3 = -90° + 0° = -90°$$

Therefore the bridge is not balance as the phase angle condition is not satisfied.

2. Three impedances of an AC bridge are;
$Z_1 = 200\angle 60°\ \Omega$; $Z_2 = 400\angle 90°\ \Omega$; $Z_3 = 300\angle 0°\ \Omega$. Determine the value of Z_4 to balance the bridge and what type of impedance.

Solution

For the bridge to balance

$$Z_1 Z_4 = Z_2 Z_3$$

$$\therefore 200 \times Z_4 = 400 \times 300$$

$$Z_4 = \frac{400 \times 300}{200} = 600\,\Omega$$

Also,

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$$

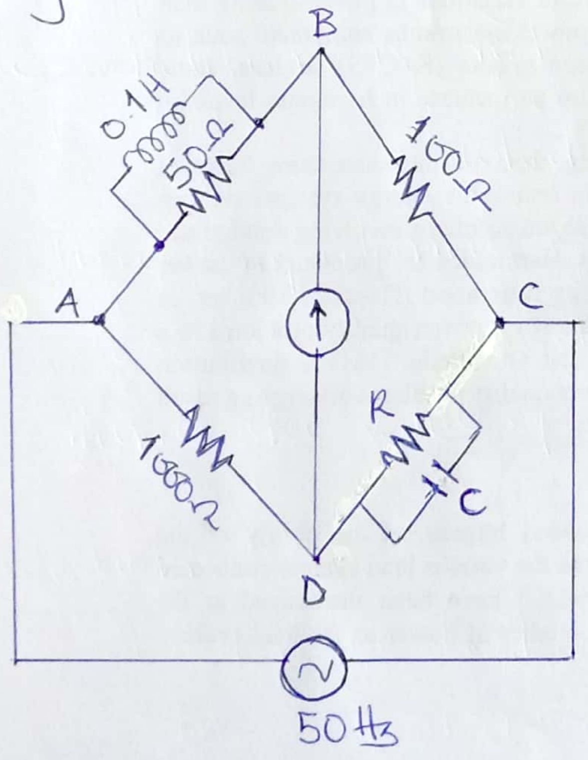
$$\therefore \angle\theta_4 = \angle\theta_2 + \angle\theta_3 - \angle\theta_1$$

$$= 90 + 0 - 60$$

$$= 30^\circ$$

$\therefore Z_4 = 600\angle 30^\circ$ (inductive impedance)

3. The four arms of an AC bridge are composed ~~shown~~. Arm CD is composed of an unknown resistance in parallel with an unknown capacitance. Find the values of R and C that would balance the bridge.



<u>Solution</u>

$$Z_1 = \frac{1}{\frac{1}{50} + \frac{1}{j\omega 0.1}}$$

$$= \frac{1}{0.02 - j0.032}$$

If we rationalize the denominator we have;

$$Z_1 = \frac{0.02 + j0.032}{(0.02 - j0.032)(0.02 + j0.032)}$$

$$= 14.15 + j\,22.52$$

$$Z_4 = \frac{1}{\frac{1}{R} + \frac{1}{-jX_c}} = \frac{1}{\frac{1}{R} + j\omega C} = \frac{R}{1 + j\omega RC}$$

When the bridge is balanced,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\therefore \frac{(14.15 + j22.52)R}{1 + j\omega RC} = 100 \times 1000$$

$$(14.15 + j22.52)R = 100000(1 + j\omega RC)$$

$$14.15R + j22.52R = 100000 + j100000\omega RC$$

Comparing real and imaginary parts we have,

$$14.15R = 100000$$

$$\therefore R = \frac{100000}{14.15} = 7.07\ k\Omega$$

and

$$22.52R = 100000\omega RC$$

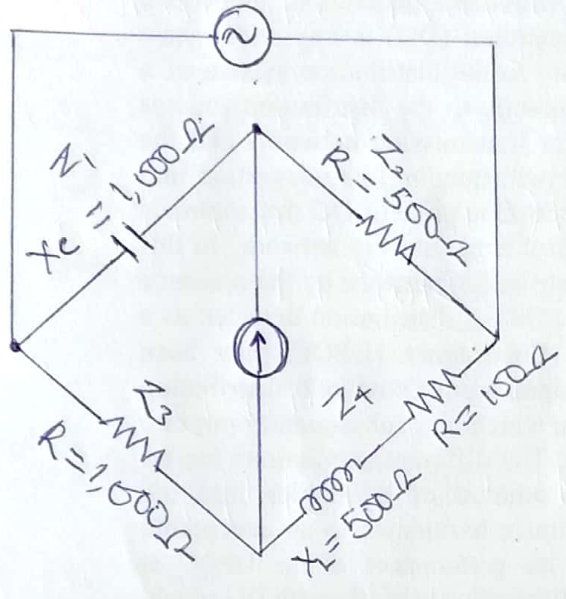
$$\therefore C = \frac{22.52R}{100000\omega R}$$

$$= \frac{22.52}{100000 \times 2 \times 3.142 \times 50}$$

$$= 0.72\ \mu F$$

4. Consider the bridge circuit shown and determine if it is balanced for the circuit elements combination.

Solution



For bridge balance both magnitude and phase conditions must be satisfied.

$Z_1 = 1000 \angle -90^\circ\ \Omega$ (pure capacitive)

$Z_2 = 500 \angle 0^\circ\ \Omega$ (pure resistive)

$Z_3 = 1000 \angle 0^\circ\ \Omega$ (pure resistive)

$Z_4 = R + jX_L = 150 + j500$

$= 509.9 \angle 78.7^\circ\ \Omega$

$\therefore Z_1 Z_4 = 1000 \times 509.9 = 509900$

$Z_2 Z_3 = 500 \times 1000 = 500000$

$\therefore$ Since $Z_1 Z_4 \neq Z_2 Z_3$ condition not satisfied

Also

$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$

$-90 + 78.7 \neq 0 + 0$

$\therefore$ Since $\angle\theta_1 + \angle\theta_4 \neq \angle\theta_2 + \angle\theta_3$ phase angle condition not satisfied.

2.2. Inductance Bridges

Depending on the range of inductance to be measured different inductance bridges exist for measurement of low, medium and high inductance values. Inductance bridges include;

i. Maxwell's bridge ii. Maxwell Wein bridge
iii. Anderson bridge iv. Hay bridge v. Owen bridge

Maxwell's bridge

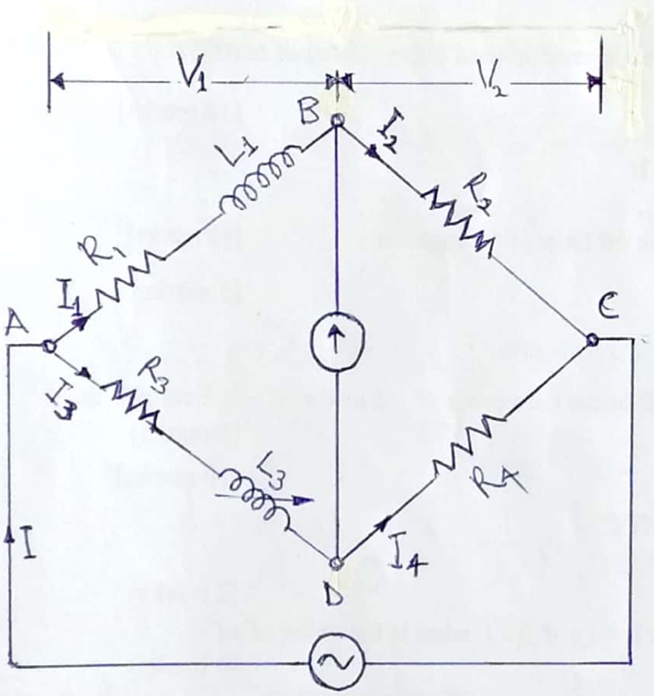


Fig 2.2. Maxwell's bridge

The Maxwell's bridge is shown in fig. 2.2. It provides accurate measurement of medium range inductances ($1 < Q < 10$), where Q is the quality factor of the inductor given as

$$Q = \frac{\omega L}{R}$$

In its operation, the unknown inductance is determined by comparing with a known self-inductance. In fig. 8 L_1 is the unknown inductance while L_3 is a known self-inductance. R_2, R_3 and R_4 are known resistances. The bridge is balanced by varying any one of the resistances R_2 or R_4.

When the bridge is balanced and no current is indicated by the detector we have the following relations

$$I_1 = I_2; \quad I_3 = I_4$$

p.d across AB = p.d across AD = V_1

$$\therefore I_1 Z_1 = I_3 Z_3 = V_1$$

$$Z_1 = R_1 + j\omega L_1 \quad \text{and} \quad Z_3 = R_3 + j\omega L_3$$

$$\therefore I_1(R_1 + j\omega L_1) = I_3(R_3 + j\omega L_3) = V_1 \qquad 2.8$$

Also from fig. 8 we observe that

p.d across BC = p.d across DC = V_2

$$\therefore I_2 Z_2 = I_4 Z_4 = V_2$$

$$Z_2 = R_2 \quad \text{and} \quad Z_4 = R_4$$

$$\therefore I_2 R_2 = I_4 R_4$$

$$\text{or} \quad I_1 R_2 = I_3 R_4 \qquad 2.9$$

Dividing equation 2.8 by equation 2.9

$$\frac{I_1(R_1 + j\omega L_1)}{I_1 R_2} = \frac{I_3(R_3 + j\omega L_3)}{I_3 R_4}$$

$$\frac{R_1}{R_2} + \frac{j\omega L_1}{R_2} = \frac{R_3}{R_4} + \frac{j\omega L_3}{R_4}$$

Comparing real and imaginary parts we have

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\text{or } R_1 = \frac{R_2}{R_4} . R_3 \quad 2.10$$

Also for the imaginary part

$$\frac{\omega L_1}{R_2} = \frac{\omega L_3}{R_4}$$

$$\therefore L_1 = \frac{R_2 L_3}{R_4}$$

Maxwell Wien Bridge

This is also called Maxwell's inductance-capacitance bridge and is shown in fig. 2.3. It is used to measure self-inductance by comparing with a standard variable capacitance. L_1 is an unknown self-inductance, R_1 is an unknown resistance, R_2, R_3, R_4 are known non-inductive resistances and C_4 is a standard variable capacitor. From the bridge arrangement;

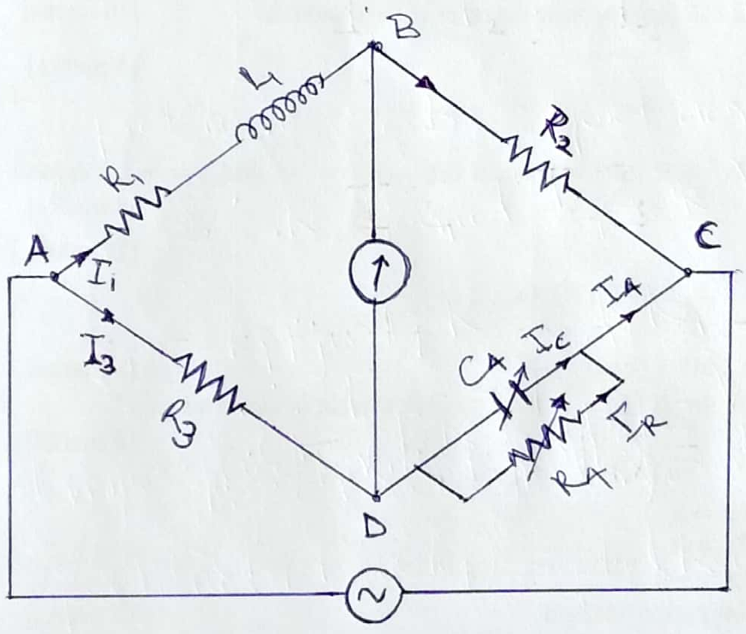


Fig. 2.3. Maxwell Wien bridge.

$Z_1 = R_1 + j\omega L_1$; $Z_2 = R_2$; $Z_3 = R_3$ and

$$Z_4 = \frac{1}{\frac{1}{R_4} + j\omega C_4} = \frac{R_4}{1 + j\omega C_4 R_4}$$

When the bridge is balanced

$$Z_1 Z_4 = Z_2 Z_3$$

$$\therefore \frac{(R_1 + j\omega L_1) R_4}{1 + j\omega C_4 R_4} = R_2 R_3$$

$$\text{or} \quad R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega C_4 R_4 R_2 R_3 \qquad 2.11$$

Comparing real and imaginary parts of eq. 2.11

$$R_1 R_4 = R_2 R_3$$

$$\therefore R_1 = \frac{R_2 R_3}{R_4} \qquad 2.12$$

Also,

$$\omega L_1 R_4 = \omega C_4 R_4 R_2 R_3$$

$$\therefore L_1 = C_4 R_2 R_3 \qquad 2.13$$

Advantages

i. It measures inductance by comparing with a standard capacitor which gives an advantage since a capacitor has no external fields and is more compact

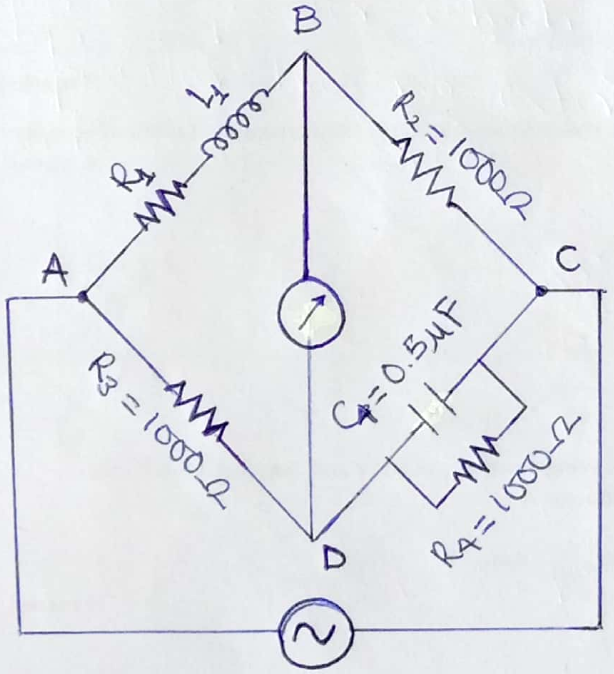
ii. The bridge has simple expressions for both R_1 and L_1.

iii. Balance conditions are independent of frequency.
iv. Q-factor can be measured easily by this bridge

Disadvantages.

i. Although the bridge balance is independent of frequency other circuit elements are not.
ii. This bridge relies on a variable capacitor for its measurement which becomes a disadvantage as the variable standard capacitor is expensive when calibe-rated to high accuracy.
iii. This bridge is only suitable for inductances of medium Q-factor ($1 < Q < 10$) but is not suitable for higher Q-factor.

Example



For the Maxwell's capacitance bridge shown write down the conditions for bridge balance and obtain expressions for R_1 and L_1 in terms of circuit elements. If the circuit combination is as shown obtain values for R_1 and L_1. Also compute the Q-factor of the inductance under test.

Solution

At balance;

$$R_1 = \frac{R_2 R_3}{R_4} = \frac{1000 \times 1000}{1000} = 1000\,\Omega$$

Also,

$$L_1 = C_4 R_2 R_3 = 0.5 \times 10^{6} \times 1000 \times 1000$$
$$= 0.5\,H$$

The Q-factor is gotten as;

$$Q = \frac{\omega L_1}{R_1} = \frac{2\pi \times 50 \times 0.5}{1000} = 0.16$$

Hay Bridge

This is a modification of the Maxwell Wien bridge and is usefull when the phase angle of the inductance is high. The circuit arrangement is represented in fig. 2.4. L_1 and R_1 represent self-inductance and resistance of the coil under test. R_2 R_3 and R_4 are known, non-inductive resistances and C_4 is a standard variable capacitor. Balance is obtained by varying the values of C_4 and R_4

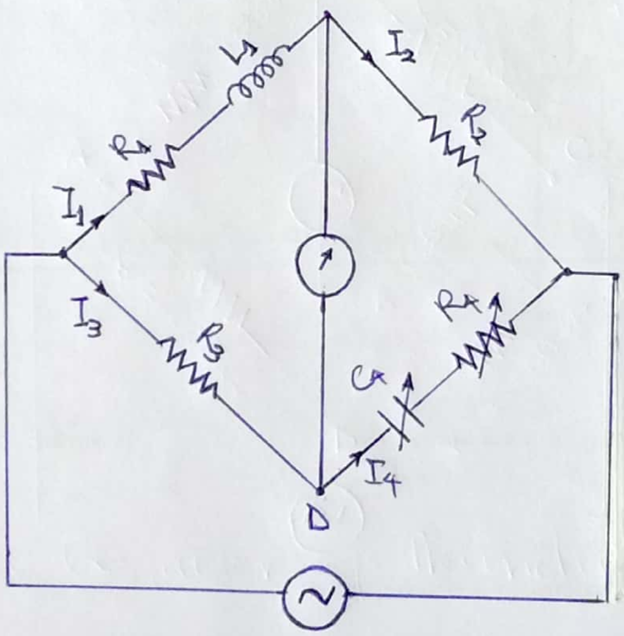


Fig. 2.4 Hay bridge

Following same procedure as for the Maxwell Wien bridge expressions for R_1, L_1 and Q-factor are obtained thus;

$$R_1 = \frac{R_2 R_3 R_4 C_4^2 \omega^2}{1 + \omega^2 C_4^2 R_4^2} \qquad 2.14$$

$$L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 C_4^2 R_4^2} \qquad 2.15$$

$$Q = \frac{\omega L_1}{R_1} = \frac{1}{\omega R_4 C_4} \qquad 2.16$$

2.3 Capacitance Bridge

AC bridge methods present the best and most common methods of measuring capacitance. Some of the more common capacitance bridges are;

i. De Sauty's bridge ii Schering bridge iii High voltage Schering bridge.

De Sauty's Bridge

This bridge is used to determine the value of unknown capacitance by comparing with a known standard capacitance. The circuit is represented in fig. 2.5, where C_1 is the capacitor under test, C_2 is a known standard capacitance, and R_1 and R_2 are known non-inductive resistances.

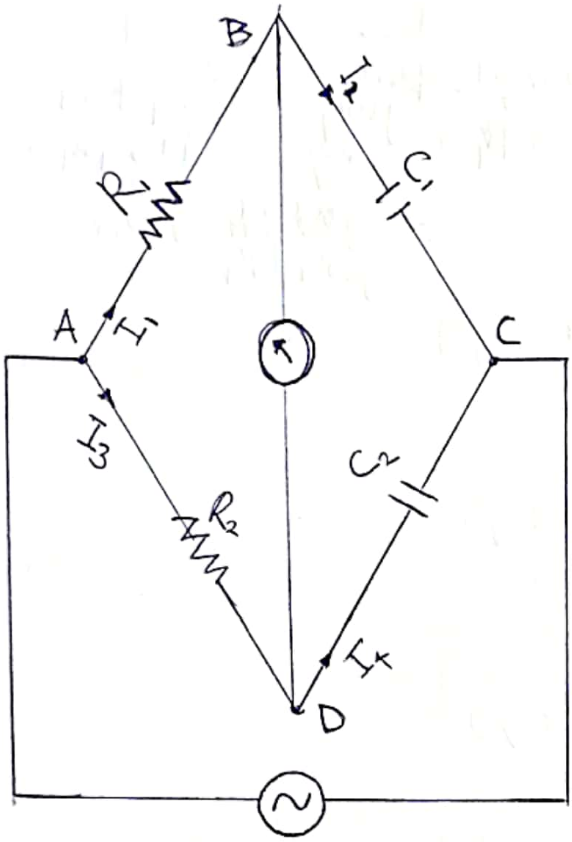


Fig.2.5. De Sauty's bridge

The bridge is balanced by adjusting either R_1 or R_2 :

At balance,

$$I_1Z_1 = I_3Z_3$$

$$I_2Z_2 = I_4Z_4$$

$$Z_1 = R_1 \; ; \; Z_3 = R_2$$

$$Z_2 = \frac{-j}{\omega C_1} \; ; \; Z_4 = \frac{-j}{\omega C_2}$$

$$\therefore I_1R_1 = I_3R_2 \qquad 2.17$$

$$\frac{-I_2 j}{\omega C_1} = \frac{-I_4 j}{\omega C_2} \qquad 2.18$$

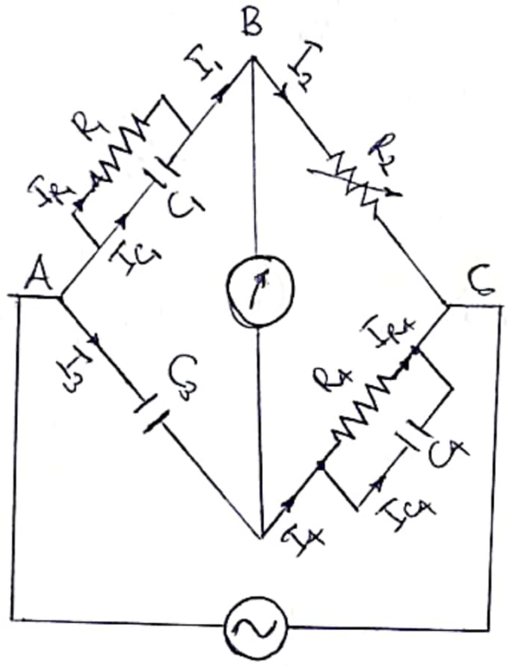
But $I_1 = I_2$ and $I_3 = I_4$, therfore dividing 2.17 by 2.18 yields;

$$\frac{R_1}{-j/\omega C_1} = \frac{R_2}{-j/\omega C_2}$$

$$\therefore C_1 = \frac{C_2R_1}{R_2} \qquad 2.18$$

Schering Bridge

The Schering bridge constitutes one of the most important and useful circuits for measurement of capacitance and dielectric loss. The circuit arrangement is shown in fig. 11.



2.6. Schering bridge

From fig. 2.6 C_1 is the capacitor under test. R_1 the dielectric loss component of C_1. C_3 is a known standard capacitance, C_4 is a variable capacitance, R_2 and R_4 are known non-inductive resistors.

As expected under balance conditions

$$Z_1 Z_4 = Z_2 Z_3$$

From the circuit configuration;

$$Z_1 = \frac{R_1}{1 + j\omega C_1 R_1}; \quad Z_2 = R_2; \quad Z_3 = -j/\omega C_3$$

$$Z_4 = \frac{R_4}{1 + j\omega C_4 R_4}$$

Substituting values of Z_1, Z_2, Z_3 and Z_4 into the balance equation and comparing real and imaginary parts we have

$$C_1 = \frac{C_3 \cos^2\delta}{\omega^2 C_1 C_4 R_1 R_4} \qquad 2.19$$

or

$$C_1 = \frac{C_3 R_4 \cos^2\delta}{R_2}$$

where δ is loss angle of capacitor, $\sin\delta$ is power factor of capacitor and $\tan\delta$ is dissipation factor of capacitor.

Apart from the types of bridge discussed so far other types for the measurement of mutual inductance are

i. Heaviside Campbell bridge.
ii. Carey Foster (Heydweiller) bridge.
iii. Heaviside Campbell equal ratio bridge.
iv. Campbell bridge.

Also there is the transformer ratio bridge (TRB) which can be used to measure;

i. Resistance
ii. Capacitance
iii. Phase angle
iv. Inductance