NMU 21/22-Phy102.2-WK 4: Magnetism(contd.) by H.A Lawal

Mutual Inductance, Self-Inductance, Energy stored in a magnetic field, Magnetic Properties of matter

Properties of magnetic materials: diamagnetism, Paramagnetism, Ferromagnetism and its type, the magnetization curve, Curie Temperature.

Mutual Inductance

While the magnetic flux through a circuit is changing, an EMF is generated in the circuit which is equal to the rate of change of magnetic flux through the circuit.

$$\text{i.e,}\ \epsilon = -\frac{d\varphi_B}{dt} \ldots\ldots\ldots\ldots\ldots (4.1)$$

Inductance is the property that tells us how effectively a conductor induces an emf in another.

Mutual Inductance: When two circuits carrying time-varying currents are close to one another, the magnetic flux through each circuit varies because of the changing current I in the other circuit. Consequently, an emf is induced in each circuit by the changing current in the other. This type of emf is therefore called a mutually induced emf, and the phenomenon that occurs is known as mutual inductance (M). **Mutual inductance occurs when the current in one circuit produces a changing magnetic field that induces an emf in another circuit.** The SI unit for mutual inductance M is called the henry (H) in honor of Joseph Henry (1799–1878), an American scientist who discovered induced emf independently of Faraday.

As an example, let's consider two tightly wound coils (Figure 1). Coils 1 and 2 have N_1 and N_2 turns and carry currents I_1 and I_2 respectively. The flux through a single turn of coil 2 produced by the magnetic field of the current in coil 1 is φ_{21} , whereas the flux through a single turn of coil 1 due to the magnetic field of current in coil 2 is φ_{12}.

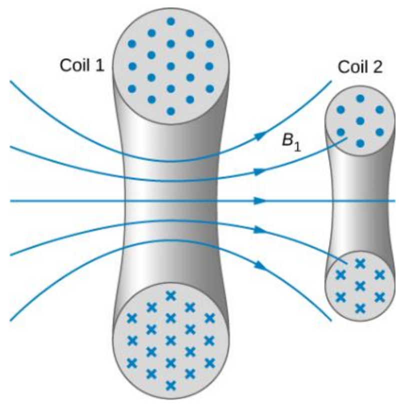


Figure 1 : Some of the magnetic field lines produced by the current in coil 1 pass through coil 2.

The mutual inductance of coil 2 with respect to coil 1 is the ratio of the flux through the turns of coil 2 produced by the magnetic field of the current in coil 1, divided by that current. i.e.

$M_{21} = \frac{N_2\varphi_{21}}{I_1}$, while $M_{12} = \frac{N_1\varphi_{12}}{I_2}$…................(4.2)

In principle, $M_{21} = M_{12} = M$

combining Faraday's law and the definition of mutual inductance,

$$\epsilon_1 = -\frac{d\varphi_{12}}{dt}, \qquad \varphi_{12} = MI_2$$

$$\Rightarrow \epsilon_1 = -\frac{d(MI_2)}{dt}$$

$$\Rightarrow \epsilon_1 = -M\frac{dI_2}{dt} \text{….(4.3)}$$

$$similarly, \epsilon_2 = -M\frac{dI_1}{dt}$$

Self-Inductance

As stated earlier, mutual inductance arises when a current in one circuit produces a changing magnetic field that induces an emf in another circuit. But the magnetic field produced also affect the current in the original circuit that produced the field and this phenomenon called **self-inductance**.

The magnetic field due to a current-carrying wire is proportional to the current, so also is the flux. i.e

$$\varphi_B \propto I$$

$$\Rightarrow \varphi_B = LI \text{………(4.4)}$$

L is known as the self-inductance of the wire.

from Faraday's law, $\epsilon = -\frac{d\varphi_B}{dt} = -\mathrm{L}\frac{dI}{dt}$………………..(4.5)

A circuit element used to provide self-inductance is known as inductor. Its application can be seen in traffic signals, metal detectors, camera flashes and so on.

Example 1: An induced emf of 20.0v is measured across a coil of 50 closely wound turns while the current through it increases uniformly from 0.0 to 5.0A in 0.10s.

(a) what is the self-inductance of the coil? (4×10^{-2} H)

(b) with the current at 5.0A, what is the flux through each turn of the coil? (4×10^{-3} Wb)

Energy Stored in a Magnetic Field

In egn(4.4), we have established that when a conductor carries a current, a magnetic field surrounding the conductor is produced and the resulting magnetic flux is proportional to the current.

i.e $\varphi_B \propto I \Rightarrow \varphi_B = LI$, where L is the inductance of the conductor (inductor).

The energy stored in a magnetic field is equal to the work needed to produce a current through the inductor.

From eqn (4.5) $\epsilon = -\mathrm{L}\frac{di}{dt}$

The power absorbed by the inductor is $P = \epsilon i = L\frac{di}{dt}.i.$

Since $P = \frac{dU}{dt}$ $, \Rightarrow dU = Pdt$. Thus, the total energy stored in the magnetic field when the current increases from 0 to I in a time interval from 0 to t can be determined by integrating this expression

$$U = \int_0^t Pdt = \int_0^t L\frac{di}{dt}idt = L\int_0^I idi = \frac{1}{2}LI^2$$

Hence, the energy stored in the magnetic field of any inductor is equivalent to $U = \frac{1}{2}LI^2$.

where L is the inductance of the material and I is the current passing through the material.

The energy, U, can be expressed as $U = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu N^2 A}{l}.\frac{B^2 l^2}{N^2\mu^2} = \frac{B^2}{2\mu}Al$

The energy density U_B (i.e. energy per unit volume), $U_B = \frac{B^2}{2\mu}$

Example 2: At the instant a current of 0.20 A is flowing through a coil of wire, the energy stored in its magnetic field is 6.0 x 10^{-3}J. What is the self-inductance of the coil? (0.3 H)

Ex: Find the energy stored in the coil in example 1b. (0.5 J)

Magnetic properties of Matter

Magnetic properties of a material refer to the response of the material to an applied magnetic field. Different materials react to the application of magnetic field differently

i. **Intensity of magnetization (I):** The electrons circulating around the nucleus have a magnetic moment. When the material is not magnetized, the magnetic dipole moment sum up to zero. When the material is kept in an external magnetic field, the magnetic moments are aligned in a particular direction and the material gets a net non-zero dipole moment. The net dipole moment per unit volume is defined as magnetization or intensity of magnetization.

ii. **Magnetic Field (H) or Magnetic intensity:** This is the external magnetic field that induces magnetic property in a material. It is produced by the electric current flowing in a solenoid.

iii. **Magnetic permeability (μ):** This is the measure of the **resistance** of a substance against the **formation of a magnetic field**. **Relative permeability** (μ_r) is the ratio of the permeability in a material to the permeability in a vacuum (μ_o). $\mu_r = \frac{\mu}{\mu_0} \Rightarrow \mu = \mu_0 \mu_r$

iv. **Magnetic susceptibility (χ):** When a material is placed in an external magnetic field, the material gets magnetized. For a small magnetizing field, the intensity of magnetization (I) acquired by the material is directly proportional to the magnetic field (H). It indicates the degree of magnetization of a material in response to an applied magnetic field.

$$I \propto H$$

$$I = \chi H, \chi \text{ is the susceptibility of the material.}$$

v. **Retentivity:** The ability of a material to retain or resist magnetization is called retentivity.

vi. **Coercivity:** The coercivity of a material is the ability to withstand the external magnetic field without becoming demagnetized.

Properties of Magnetic Materials

Diamagnetism: This is a phenomenon in which an applied magnetic field creates an **induced magnetic field** in materials in a direction **opposite** to that of the external field. Diamagnetism results from changes in electron orbital motion that are induced by an external field. The effect is extremely small (with magnetic susceptibility on the order of -10^{-5}) and in opposition to the applied field. Diamagnetic materials lose their magnetization when the external magnetic field is removed. Basically, all materials possess diamagnetic tendency but could be hidden in the presence of a greater paramagnetism or ferromagnetism. Diamagnetic materials include water, wood, gold, antimony, most organic compounds etc.

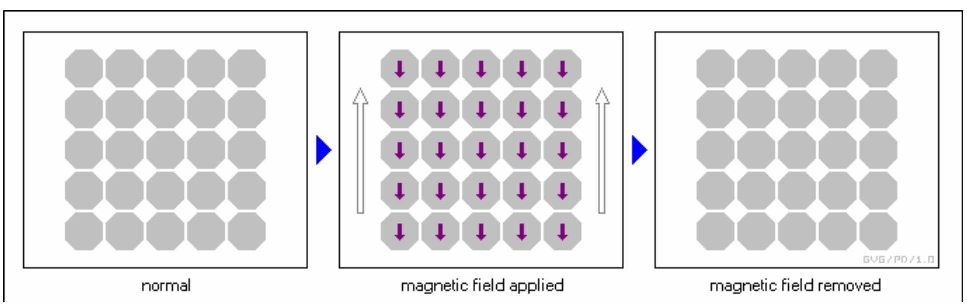


Fig 2: Magnetic moments in diamagnetic materials

Paramagnetism: This is similar to diamagnetism but with magnetic spins aligning **in the direction** of an external (applied) field. In paramagnetism, the magnetic field in the material is strengthened by the induced magnetization. Here, magnetic susceptibility (χ) is small and positive (order of about $+10^{-5}$). Diamagnetic and paramagnetic materials are considered nonmagnetic because the magnetizations are relatively small and persist only while an applied field is present. Examples of paramagnetic materials include most chemical elements like aluminum, platinum, manganese etc and some compounds.

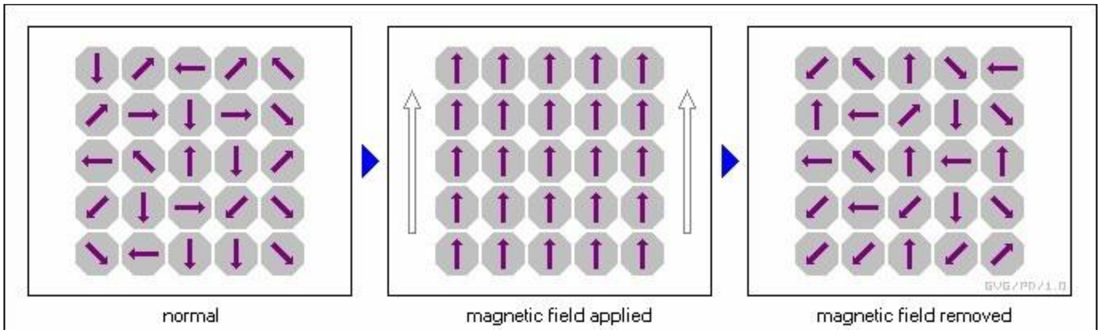


Fig 3: Magnetic moments in paramagnetic materials

Ferromagnetism: Ferromagnetism is the basic mechanism by which a material forms permanent magnet (i.e. materials that can be magnetized by an external magnetic field and remain magnetized after the external field is removed). Ferromagnetism is the strongest type of magnetism with positive magnetic susceptibility (χ) of about 10^3 0r 10^4, even greater. What we normally think of as magnets are made of ferromagnets. However, the ferromagnetic susceptibility is temperature sensitive, above Curie temperature, a ferromagnetic material behaves like a typical paramagnet. Examples of ferromagnets include are iron, cobalt, nickel and some compounds of rare earth metals. Ferromagnetic materials are largely used in storage devices.

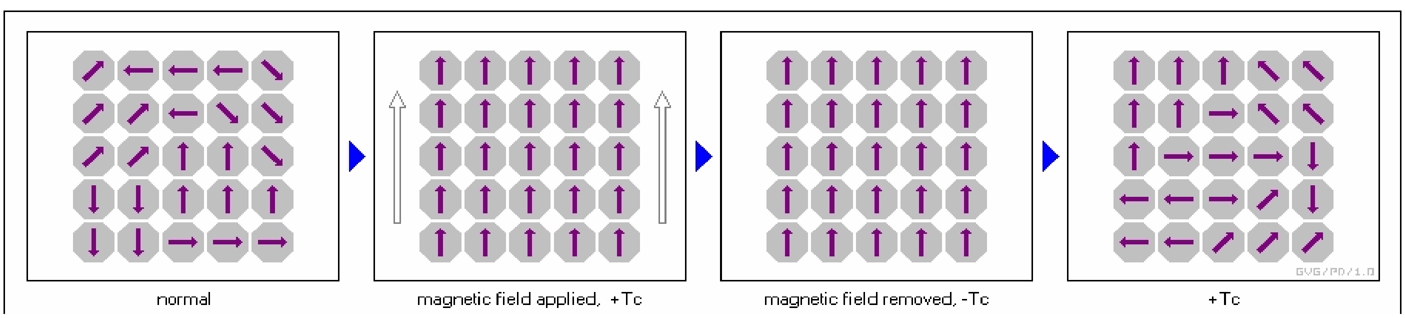
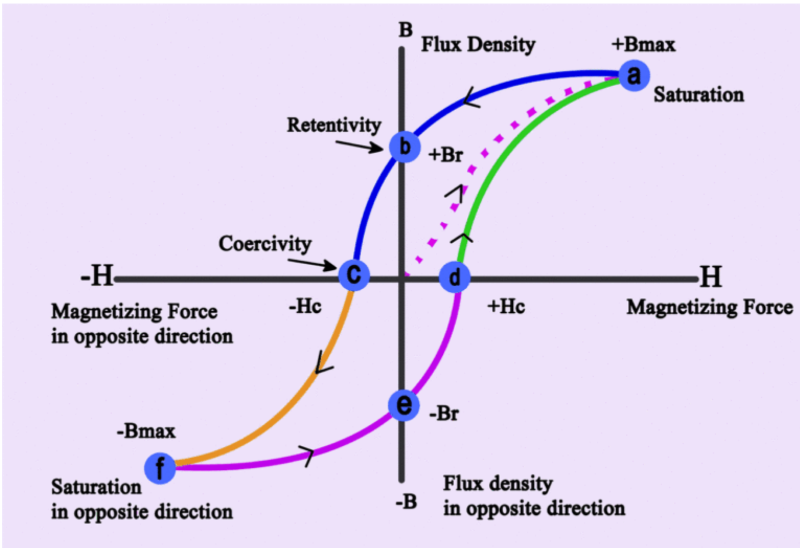


Fig 4: Magnetic moments in ferromagnetic materials

Types of Ferromagnets: Ferromagnetic materials can be divided into magnetically "**soft**" materials which can be magnetized but do not tend to stay magnetized (sometimes called **unmagnetized ferromagnetic materials** e.g., materials like annealed iron) and magnetically "**hard**" materials, which stay magnetized (often called **magnetized ferromagnetic materials** e.g., steel). Permanent magnets are made from "hard" ferromagnetic materials.

In summary, diamagnetism is a property of all materials and opposes applied magnetic fields, but is very weak. Paramagnetism, when present, is stronger than diamagnetism and produces magnetization in the direction of the applied field, and proportional to the applied field. Ferromagnetic effects are very large, producing magnetizations sometimes orders of magnitude greater than the applied field and as such are much larger than either diamagnetic or paramagnetic effects

<u>Magnetization Curve</u>: this is a curve obtained when magnetic flux density (B) is plotted against magnetizing force (H). A magnetization curve showing how the magnetization of a ferromagnetic material varies when subjected to a periodically reversing magnetic field is called **hysteresis loop**.



<u>Fig 5: Hysteresis in ferromagnets</u>

<u>Curie Temperature</u>: this is a temperature at which certain magnetic materials undergo a sharp change in their magnetic properties. The long-range order of a ferromagnet abruptly disappears at Curie temperature hence the material behaves like a paramagnet. At low temperature, magnetic dipoles are aligned. Above Curie temperature, random thermal agitations nudge dipoles out of alignment. Curie temperature varies from one material to another e.g., Fe (1043K), Ni (631K), Co (1394k).

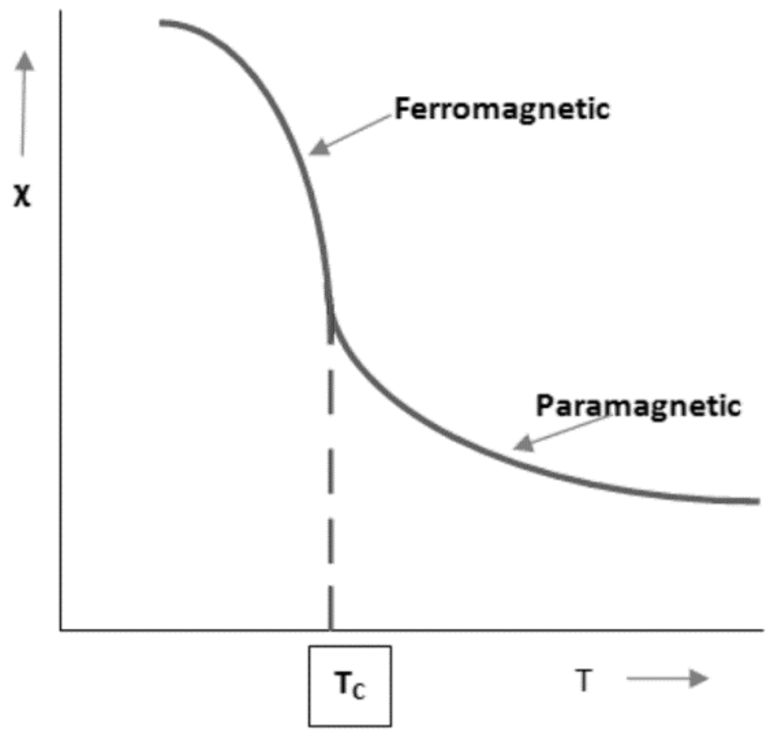


Fig 6: Graphical representation of Curie Temperature

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