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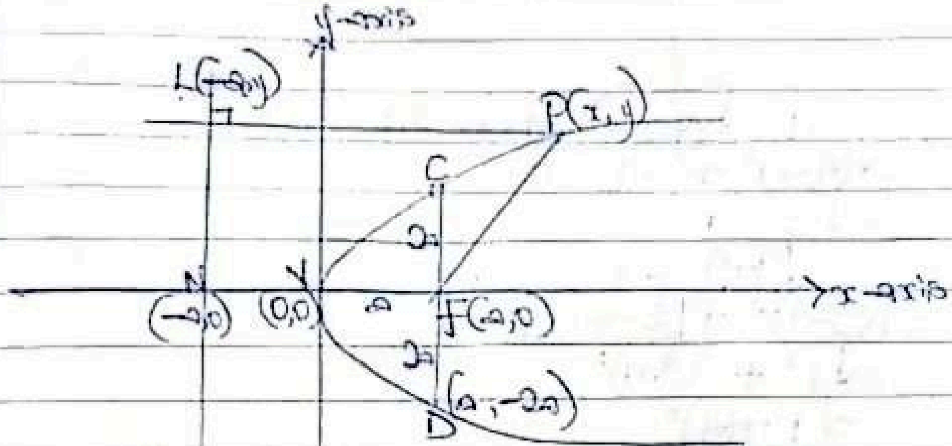
iv) $y^2 = -4ax$.

Note:→ Replace x by $x-h$ has the effect of shifting the graph of the eqn by $|h|$ to the right if h is +ve, to the left if h is −ve.

Detailed Explanation.



N.B Length of latus rectum is $4a$

* $|CD|$ = latus rectum
* Equation of the directrix is $x = -a$
* Coordinates of the ends of latus rectum are $L = (a, 2a)$ and $L' = (a, -2a)$.
* $|LN|$ = Axis of Symmetry.

$$|PF| = |PL|$$

$$\sqrt{(y-0)^2 + (x-a)^2} = \sqrt{(y-y)^2 + [x-(-a)]^2}$$

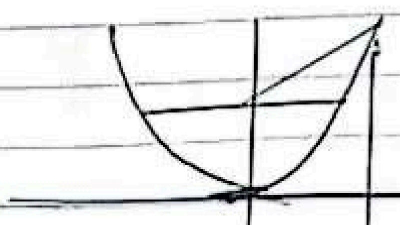
$$y^2 + x^2 - 2ax + a^2 = (x+a)^2$$

$$y^2 + \cancel{x^2} - 2ax + \cancel{a^2} = \cancel{x^2} + 2ax + \cancel{a^2}$$

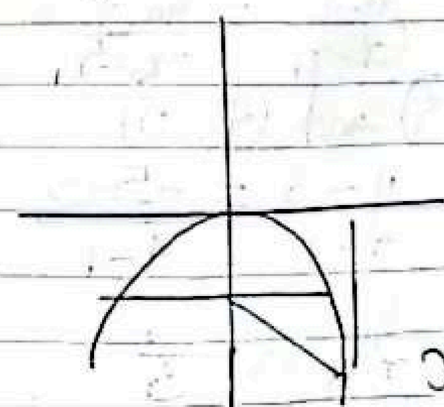
$$y^2 = 2ax + 2ax$$

$$y^2 = 4ax$$

General equation of a parabola.



$x^2 = 4ay$



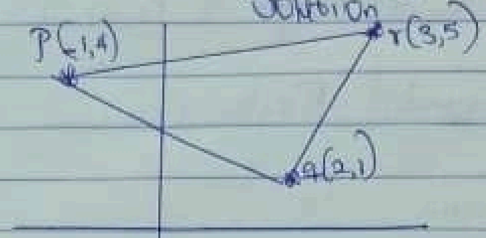
$x^2 = -4ay$

* Study the above Parabola Shape.

Assignment.

1. Show that the point $P(-1,4)$, $q(2,1)$ and $r(3,5)$ are the Vertices Of an isosceles triangle.

Solution



$$d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$$

$$|Pq| = \sqrt{(4-1)^2 + (-1-2)^2}$$
$$= \sqrt{3^2 + (-3)^2}$$
$$= \sqrt{9+9} = \sqrt{18} \text{ Units}$$

$$|Pr| = \sqrt{(5-4)^2 + [3-(-1)]^2}$$
$$= \sqrt{1^2 + (3+1)^2}$$
$$= \sqrt{1+(4)^2}$$
$$= \sqrt{1+16}$$
$$= \sqrt{17} \text{ Units}$$

$$|rq| = \sqrt{(5-1)^2 + (3-2)^2}$$
$$= \sqrt{4^2 + 1^2}$$
$$= \sqrt{16+1} = \sqrt{17} \text{ Units.}$$

Since $|Pr| = |rq|$

$|Pq| \neq |Pr|$

$\therefore$ The point $P(-1,4)$, $q(2,1)$ and $r(3,5)$ are the Vertices Of an isosceles triangle.

5. Find the equation Of the line passing through each pair Of points (i) $(1,3)$ and $(4,-5)$; (ii) $(-2,-6)$ and $(3,1)$.

Solution.

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

(i) $(1,3)$ and $(4,-5)$

$$\frac{y-3}{x-1} = \frac{-5-3}{4-1}$$

$$\frac{y-3}{x-1} = \frac{-8}{3}$$

$$3(y-3) = -8(x-1)$$

3y
3y
3

ii) (-2,-

6. Find
i) parallel
(3,-

$$3y-9=-8x+8$$
$$3y+8x=8+9$$
$$3y+8x=17$$

ii) $(-2,-6)$ and $(3,1)$

$$\frac{y-(-6)}{x-(-2)}=\frac{1-(-6)}{3-(-2)}$$

$$\frac{y+6}{x+2}=\frac{1+6}{3+2}$$

$$\frac{y+6}{x+2}=\frac{7}{5}$$

$$7(x+2)=5(y+6)$$
$$7x+14=5y+30$$
$$5y-7x+30-14=0$$
$$5y-7x+16=0$$

6. Find the equation of the line passing through each point and parallel to the given line:

i) $(3,-2)$ and $5x+4y=-5$ (ii) $(1,6)$ and $3x-7y=4$

Solution

$$5x+4y=-5$$
$$4y=-5x-5$$
$$y=\frac{-5}{4}x-\frac{5}{4}$$
$$m_1=\frac{-5}{4}$$

Parallelism $m_1=m_2$.

$$m=\frac{y-y_1}{x-x_1}$$

$$\frac{-5}{4}=\frac{y-(-2)}{x-3}$$

$$\frac{-5}{4}=\frac{y+2}{x-3}$$

$$4(y+2)=-5(x-3)$$
$$4y+8=-5x+15 \Rightarrow 4y+5x=15-8$$
$$4y+5x=7$$

$= (-7)^2 + 3^2$
$= 49 + 9$
$= 58$ Units

$|AC|^2 + |BC|^2 = |AB|^2$

∴ The points $(6,5)$, $(2,-5)$ and $(-1,2)$ are the Vertices of a right triangle.

$$Area = \frac{1}{2} bh$$
$$= \frac{1}{2} \times \sqrt{58} \times \sqrt{58}$$
$$= \frac{1}{2} \times 58 =$$
$$= 29 \text{ Sq. Units}$$

14. Find the equation of the line i) passing through $(1,-3)$ and parallel to $2x - 3y = 7$
ii) Passing through $(2,-1)$ and perpendicular to $3x + 4y = 6$

Solution

i) $(1,-3)$ $\quad 2x - 3y = 7$
$$2x - 3y = 7$$
$$y = \frac{-2x}{-3} + \frac{7}{-3}$$
$$y = \frac{2}{3}x - \frac{7}{3}$$
$$m_1 = 2/3$$
For Parallelism $m_1 = m_2$
$$\frac{y - y_1}{x - x_1} = m$$
$$\frac{y - (-3)}{x - 1} = \frac{2}{3}$$
$$3(y+3) = 2(x-1)$$
$$3y + 9 = 2x - 2$$
$$3y - 2x + 9 + 2 = 0$$
$$3y - 2x + 11 = 0$$

ii) $3x +$

15 H
th

16.

$\frac{4}{3} = \frac{y-5}{x-2}$
$4(x-2) = 3(y-5)$
$4x-8 = 3y-15$
$3y-4x = -8+15$
$3y-4x = 7$

ii) $2x-4y = 7$
$-4y = -2x+7$
$y = \frac{-2x}{-4} + \frac{7}{-4}$
$y = \frac{1}{2}x - \frac{7}{4}$
$m_1 = 1/2$
For perpendicularity, $m_1m_2 = -1$
$\frac{1}{2}m_2 = -1$
$m_2 = -2$

point

$m = \frac{y-y_1}{x-x_1}$
$-2 = \frac{y-(-1)}{x-3}$
$-2 = \frac{y+1}{x-3}$
$-2(x-3) = y+1$
$-2x+6 = y+1$
$y+2x = 6-1$
$y+2x = 5$

8. Determine the distance d of each line from the Origin
i) $3x-4y = 8$ (ii) $5x+12y = -4$

Solution.

(i) $3x-4y = 8$ $(0,0)$
$d = \frac{|ax_1 + by_1 - c|}{\sqrt{a^2+b^2}} = \frac{|3(0) + (-4)(0) - 8|}{\sqrt{3^2+(-4)^2}}$
$= \frac{|-8|}{\sqrt{9+16}}$

$= \frac{|-8|}{\sqrt{25}} = \frac{8}{5}$ Units

ii $5x + 12y = -4$

$d = \frac{|ax_1 + by_1 - c|}{\sqrt{a^2 + b^2}}$

$= \frac{|5(0) + 12(0) - (-4)|}{\sqrt{(5)^2 + (12)^2}}$

$= \frac{|4|}{\sqrt{25+144}} = \frac{4}{\sqrt{169}}$

$= \frac{4}{13}$ Units

9 Determine the distance 'd' between each point and line;
i) $(5,3)$ and $5x - 12y = 6$ (ii) $(-3,-2)$ and $4x - 5y = -8$

Solution

i) $d = \frac{|ax_1 + by_1 - c|}{\sqrt{a^2 + b^2}}$

$(5,3)$ and $5x - 12y = 6$

$d = \frac{|5(5) + (-12)(3) - 6|}{\sqrt{5^2 + (-12)^2}} = \frac{|25 - 36 - 6|}{\sqrt{25+144}}$

$= \frac{|-17|}{\sqrt{169}} = \frac{17}{13}$ Units

ii) $(-3,-2)$ and $4x - 5y = -8$

$d = \frac{|4(-3) + (-5)(-2) - (-8)|}{\sqrt{4^2 + (-5)^2}}$

$= \frac{|-12 + 10 + 8|}{\sqrt{16+25}} = \frac{6}{\sqrt{41}} \times \frac{\sqrt{41}}{\sqrt{41}}$

$= \frac{6\sqrt{41}}{41}$ Units

10 Determine K so that the distance between $(1,3)$ and $(2K,7)$ is 5.

13 Sho
Vert

Solution:

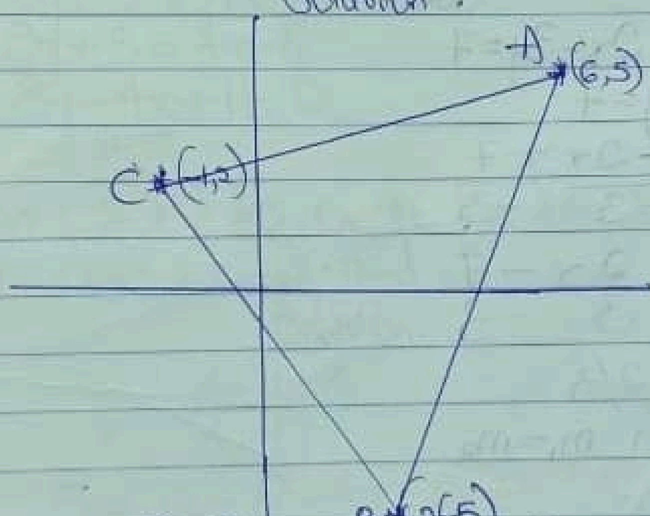
$$(y_2-y_1)^2+(x_2-x_1)^2=d^2$$
$$(7-3)^2+(2k-1)^2=5^2$$
$$\Rightarrow 4^2+(2k)^2-4k+1=25$$
$$\Rightarrow 16+4k^2-4k+1=25$$
$$4k^2-4k=25-17$$
$$4k^2-4k=8$$
$$4k^2-4k-8=0$$
$$4k^2-8k+4k-8=0$$
$$4k(k-2)+4(k-2)=0$$
$$(4k+4)(k-2)=0$$
$$4k+4=0 \quad \text{or} \quad k-2=0$$
$$k=\frac{-4}{4} \quad \text{or} \quad 2$$
$$=-1 \text{ or } 2$$

e:

$y=-8$

13 Show that the points $(6,5)$, $(2,-5)$ and $(-1,2)$ are the vertices of a right triangle and find its area.

Solution.



$$d^2=(y_2-y_1)^2+(x_2-x_1)^2$$
$$|AB|^2=[5-(-5)]^2+(6-2)^2$$
$$=(5+5)^2+4^2$$
$$=(10)^2+16=100+16$$
$$=116 \text{ units}$$

$(2k,7)$

$$|AC|^2=(5-2)^2+[6-(-1)]^2$$
$$=3^2+7^2=9+49$$
$$=58 \text{ units}$$
$$|BC|^2=(-5-2)^2+[2-(-1)]^2$$

ii)
$$3x - 7y = 4$$
$$-7y = -3x + 4$$
$$y = \frac{3}{7}x - \frac{4}{7}$$
$$m_1 = \frac{3}{7}$$

Parallelism $m_1 = m_2$

$$m_2 = \frac{y - y_1}{x - x_1}$$
$$\frac{3}{7} = \frac{y - 6}{x - (+1)}$$
$$\frac{3}{7} = \frac{y - 6}{x - 1}$$
$$3(x-1) = 7(y-6)$$
$$3x - 3 = 7y - 42$$
$$7y - 3x = -3 + 42$$
$$\underline{7y - 3x = 39}$$

7. Find the equation of the line passing through the given point and perpendicular to the given line.
i) (2, 5) and $3x + 4y = 9$ (ii) (3, -1) and $2x - 4y = 7$

Solution

i)
$$3x + 4y = 9$$
$$4y = -3x + 9$$
$$y = \frac{-3}{4}x + \frac{9}{4}$$
$$m_1 = \frac{-3}{4}$$

For perpendicularity ; $m_1 m_2 = -1$

$$\frac{\cancel{-}3}{4}m_2 = \cancel{-}1$$
$$m_2 = \frac{4}{3}$$
$$m = \frac{y - y_1}{x - x_1}$$

ii) $3x+4y=6$

$$y=\frac{-3}{4}x+\frac{6}{4}$$

$$=\frac{-3}{4}x+\frac{3}{2}$$

$$m_1=\frac{-3}{4}$$

For perpendicularity

$$m_1m_2=-1$$

$$\frac{-3}{4}m_2=-1$$

$$m_2=\frac{4}{3}$$

$$m=\frac{y-y_1}{x-x_1}$$

$$\frac{4}{3}=\frac{y-(-1)}{x-2}$$

$$3(y+1)=4(x-2)$$

$$3y+3=4x-8$$

$$\underline{3y-4x+11=0}$$

15. If the point $(2,\mu)$ lies on the line with slope $m=3$ passing through the point $(1,6)$. Find μ.

Solution

$$m=\frac{y-y_1}{x-x_1}$$

$$3=\frac{6-\mu}{1-2}$$

$$3(-1)=6-\mu$$

$$-3=6-\mu$$

$$\mu=6+3$$

$$\mu=\underline{9}$$

Or

$$3=\frac{y-6}{x-1}$$

$$3(x-1)=y-6$$

$(2,\mu)$

$$3(2-1)=\mu-6$$

$$3=\mu-6$$

$$\mu=3+6$$

$$=\underline{9}$$

16. Does the point $(-1,-2)$ lie on the line through the po[ints] $(4,7)$ and $(5,9)$?

Solution

$(4,7)$ and $(5,9)$

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-7}{x-4} = \frac{9-7}{5-4}$$

$$\frac{y-7}{x-4} = \frac{2}{1}$$

$$2(x-4) = y-7$$

$$2x-8 = y-7$$

$$2x-y = -7+8$$

$$2x-y-1 = 0.$$

$(-1,-2)$

$$2(-1)-(-2)-1 = 0$$

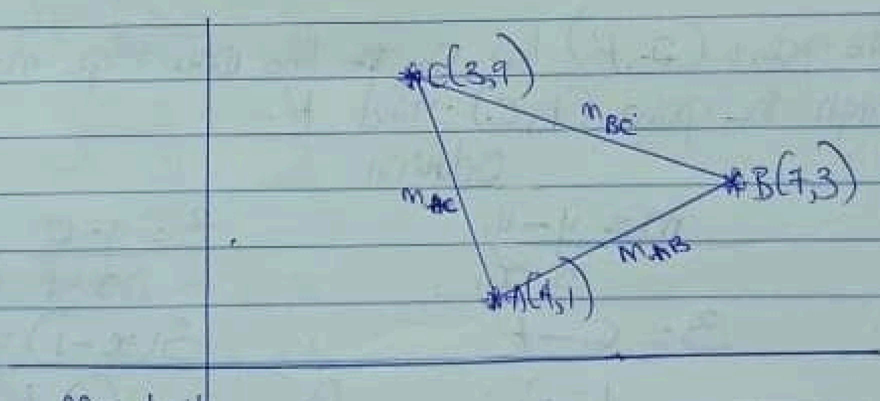
$$-\cancel{2}+\cancel{2}-1 \Rightarrow$$

$$-1 \neq 0$$

$\therefore$ $(-1,-2)$ does not lie on the line.

17 Use slope to determine whether the points $A(4,1)$, $B(7,3)$ and $C(3,9)$ are the vertices of a right triangle.

Solution.



$$m = \frac{y-y_1}{x-x_1}$$

$$m_{AB} = \frac{3-1}{7-4} = \frac{2}{3}$$

$$m_{AC} = \frac{9-1}{3-4} = \frac{8}{-1} = -8$$

$$m_{BC} = \frac{9-3}{3-7} = \frac{6}{-4} = \frac{-3}{2}.$$

∴ Right ...
For Pe...

18 Use Sl...
and D...

$\therefore$ Right angle triangle.

For Perpendicularity $m_1 m_2 = -1$

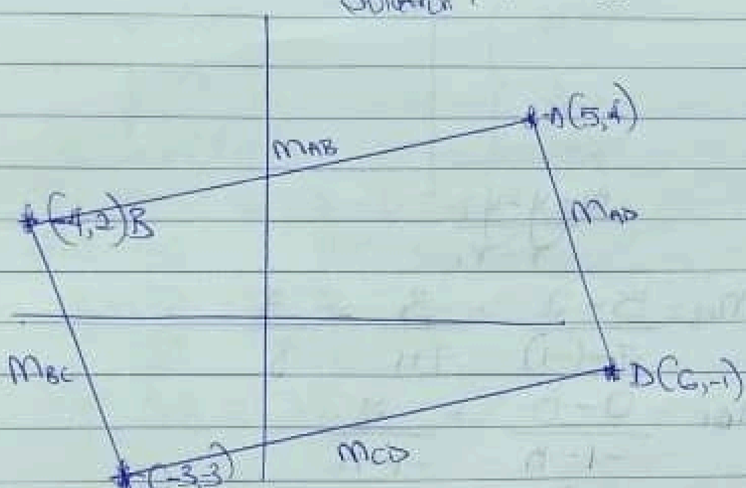
$$m_1 = -\frac{1}{m_2}.$$

$$m_{AB} = \frac{-1}{m_{BC}}$$

$\therefore$ The Points A(4,1), B(7,3) and C(3,9) are On the Vertices Of a right triangle.

18. Use Slope to Show that A(5,4), B(-4,2), C(-3,-3) and D(6,-1) are Vertices Of a parallelogram.

Solution.



$$m = \frac{y - y_1}{x - x_1}$$

$$m_{AB} = \frac{4-2}{5-(-4)} = \frac{2}{9}$$

$$m_{BC} = \frac{2-(-3)}{-4-(-3)} = \frac{2+3}{-4+3} = \frac{5}{-1} = -5$$

$$m_{AD} = \frac{4-(-1)}{5-6} = \frac{4+1}{-1} = -5$$

$$m_{CD} = \frac{-1-(-3)}{6-(-3)} = \frac{-1+3}{6+3} = \frac{2}{9}$$

For Parallelism $m_1 = m_2$.

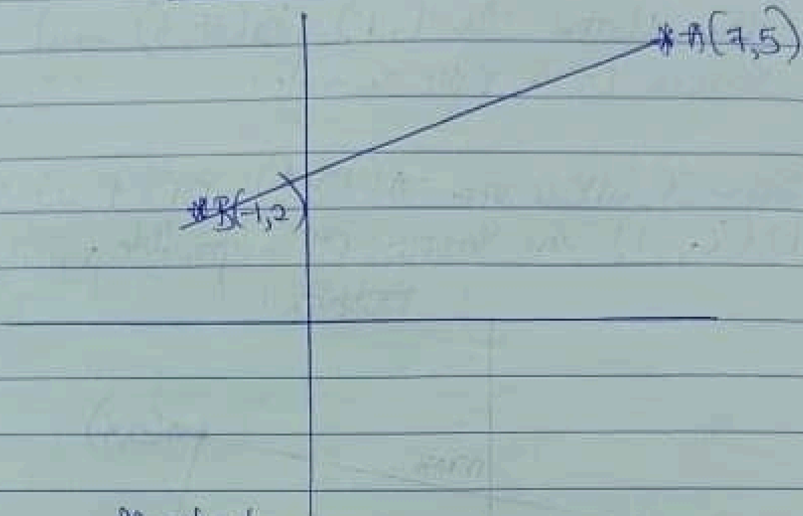
$m_{AB} = m_{CD}$

$m_{BC} = m_{AD}$

$\therefore$ The point A(5,4), B(-4,2), C(-3,-3) and D(6,-1) are Vertices Of a Parallelogram.

20. Determine K so that the points A(7,5), B(-1,2) and C(K,0) are the Vertices Of a right triangle with right angle at B.

Solution.



$$m = \frac{y-y_1}{x-x_1}$$

$$m_{AB} = \frac{5-2}{7-(-1)} = \frac{3}{7+1} = \frac{3}{8}$$

$$m_{BC} = \frac{2-0}{-1-K} = \frac{2}{-1-K}$$

For Perpendicularity

$$m_1 m_2 = -1$$

$$m_1 = \frac{-1}{m_2}$$

$$m_{AB} = \frac{-1}{m_{BC}}$$

$$\frac{3}{8} = \frac{-1}{2/_{-1-K}}$$

$$\frac{3}{8} = \frac{-(-1-K)}{2}$$

$$\frac{3}{8} \times \frac{1+K}{2}$$

$$8(1+K) = 3(2)$$

$$8+8K = 6$$

$$8K = 6-8$$

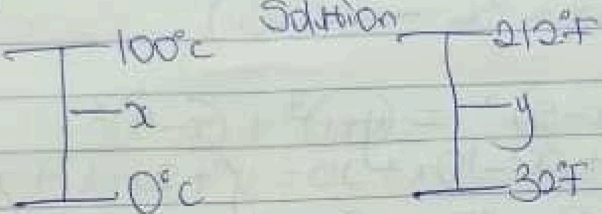
$8h = -2$

$h = \frac{-2}{8} = \frac{-1}{4}$

21 Temperature is usually measured either in Fahrenheit or in Celsius degree. The relation between Fahrenheit and Celsius temperature is given by linear ~~expre~~ equation. The freezing point of water is 0°C or 32°F, and the boiling point of water is 100°C or 212°F. (a) Find an equation giving Fahrenheit temperature y in terms of Celsius temperature x

b) What temperature is the same in both sides?

Solution



a)
$$\frac{y-32}{212-32} = \frac{x-0}{100-0}$$

$$\frac{y-32}{18\cancel{0}} = \frac{x}{10\cancel{0}}$$

$$y-32 = \frac{18x}{10}$$

$$y-32 = \frac{9}{5}x$$

$$y = \frac{9}{5}x + 32$$

b) $y = x$.

$$y - \frac{9}{5}x = 32.$$

$$y - \frac{9}{5}y = 32$$

$$\frac{5y-9y}{5} = 32$$

$$-4y = 160$$

$$y = \underline{-40°C} \text{ or } \underline{-40°F}$$

2. Find the point equidistant from the points $a(-4,3)$, $b(5,6)$ and $c(4,-1)$.

Solution

$P(x,y)$

$ap = bp = cp$

$\sqrt{(y-3)^2 + [x-(-4)]^2} = \sqrt{(y-6)^2 + (x-5)^2} = \sqrt{[y-(-1)]^2 + (x ...}$

$ap = bp$

$(y-3)^2 + (x+4)^2 = (y-6)^2 + (x-5)^2$

$\cancel{y^2} - 6y + 9 + \cancel{x^2} + 8x + 16 = \cancel{y^2} - 12y + 36 + \cancel{x^2} - 10x + 25$

$-6y + 12y + 8x + 10x + 9 + 16 = 36 + 25$

$6y + 18x + \cancel{25} = 36 + \cancel{25}$

$6y + 18x = 36$ ——— (i)

$bp = cp$

$(y-6)^2 + (x-5)^2 = (y+1)^2 + (x-4)^2$

$\cancel{y^2} - 12y + 36 + \cancel{x^2} - 10x + 25 = \cancel{y^2} + 2y + 1 + \cancel{x^2} - 8x + 16$

$-12y - 2y + 8x - 10x + 36 + 25 - 16 - 1 = 0$

$-14y - 2x + 44 = 0$

$-14y - 2x = -44$ ——— (ii) $\times 9$

$6y + 18x = 36$

$-126y - 18x = -396$

$-120y = -360$

$y = \frac{360}{120} = 3$

input $y = 3$ into eqn (i)

$6(3) + 18x = 36$

$18x = 36 - 18$

$18x = 18$

$x = 1$

$P(1,3)$

3. Show that the points $P(1,-2)$, $q(3,2)$ and $r(6,8)$ are collinear i.e lie on the same straight line.

Solution.

$P(1,-2)$, $q(3,2)$ and $r(6,8)$

$m_{pq} = m_{qr} \Rightarrow$ Collinear

4. Find th...

passing ...

and p...

i)

ii)

$m = ...$

m_{pq} ...

m_{qr} ...

$\therefore$ ...

$$m = \frac{y - y_1}{x - x_1}$$

$$m_{pq} = \frac{2-(-2)}{3-1} = \frac{2+2}{2} = \frac{4}{2} = 2$$

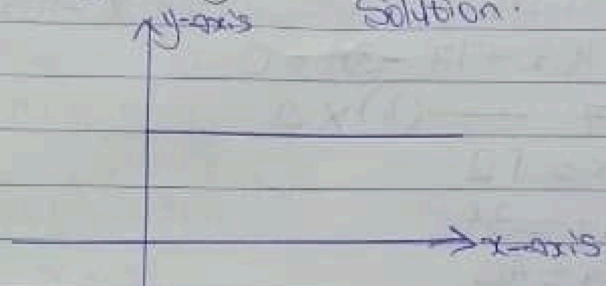
$$m_{qr} = \frac{8-2}{6-3} = \frac{6}{3} = 2.$$

$\therefore$ The points $P(1,-2)$, $q(3,2)$ and $r(6,8)$ are Collinear

4. Find the equation of the line (i) Parallel to the x-axis and passing through the point $(-2,-5)$ (ii) Parallel to the y-axis and passing through the point $(3,4)$.

Solution.

i)



$$m = \frac{y - y_1}{x - x_1}$$

$y = mx + c.$

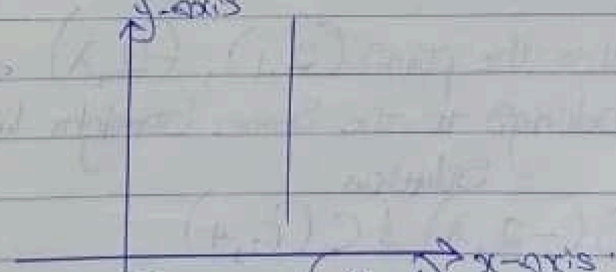
$m_1 = 0$ $\quad (-2,-5)$

$$\cancel{0 =} \frac{y-(-5)}{x-(-2)}$$

$y + 5 = 0$

$y = -5$

ii)



$x = c$ $\quad (3,4)$

$x = 3$

$c = 3$

$x = 3$

$x - 3 = 0$

11. Find the point equidistant from $(1,2)$, $(-1,4)$ and $(5,3)$

Solution

$P(x,y)$ $A(1,2)$, $B(-1,4)$ \$ $C(5,3)$

$AP = BP = CP$

$d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$

$\sqrt{(y-2)^2+(x-1)^2} = \sqrt{(y-4)^2+[x-(-1)]^2} = \sqrt{(y-3)^2+(x-5)^2}$

$(y-2)^2+(x-1)^2 = (y-4)^2+(x+1)^2$.

$\cancel{y^2}-4y+4+\cancel{x^2}-2x+\cancel{1} = \cancel{y^2}-8y+16+\cancel{x^2}+2x+\cancel{1}$

$-4y+8y-2x-2x+4-16=0$

$4y-4x=12$ —— (i)

$(y-4)^2+(x+1)^2 = (y-3)^2+(x-5)^2$

$\cancel{y^2}-8y+16+\cancel{x^2}+2x+1 = \cancel{y^2}-6y+9+\cancel{x^2}-10x+25$

$-8y+6y+2x+10x+17-34=0$

$-2y+12x=17$ —— (ii) $\times 2$

$4y-4x=12$

$-4y+24x=34$

$20x=46$

$x = \frac{46}{20} = 2.3$

$4y-4(2.3)=12$.

$4y = 12+9.2$

$y = \frac{21.2}{4}$

$y = 5.3$

$P(2.3, 5.3)$

12. Determine λ so that the points $(2,1)$, $(-2,\lambda)$ and $(6,4)$ are collinear i.e belongs to the same straight line.

Solution

$A(2,1)$, $B(-2,\lambda)$ \$ $C(6,4)$

$m_{AB} = m_{BC}$

$m_{AB} = \frac{\lambda-1}{-2-2} = \frac{\lambda-1}{-4}$

$m_{BC} = \frac{4-\lambda}{6-(-2)} = \frac{4-\lambda}{6+2} = \frac{4-\lambda}{8}$

$$\frac{\lambda-1}{-4} = \frac{4-\lambda}{8}$$

$$8(\lambda-1) = -4(4-\lambda)$$

$$8\lambda-8 = -16+4\lambda$$

$$8\lambda-4\lambda = -16+8$$

$$4\lambda = -8$$

$$\lambda = \frac{-8}{4} = \underline{\underline{-2}}$$

19 Under what conditions are the points $A(u, v+w)$, $B(v, u+w)$ and $C(w, u+v)$ on the same line?

Solution.

$A(u, v+w)$, $B(v, u+w)$ and $C(w, u+v)$

$$m_{AB} = m_{BC}$$

$$m = \frac{y-y_1}{x-x_1}$$

$$\frac{(u+w)-(v+w)}{v-u} = \frac{(u+v)-(u+w)}{w-v}$$

$$\frac{u+\cancel{w}-v-\cancel{w}}{v-u} = \frac{\cancel{u}+v-\cancel{u}-w}{w-v}$$

$$\frac{u-v}{v-u} = \frac{v-w}{w-v}$$

$$\frac{-(v-u)}{v-u} = \frac{-(w-v)}{w-v}$$

$$-1 = -1$$

$\therefore$ Under the condition where $m_{AB} = m_{BC} = -1$ are the points $A(u, v+w)$, $B(v, u+w)$ and $C(w, u+v)$ on the same line.

Examples

1. Find the focus and the ~~directrix~~ directrix of the parabola $y^2 = 4x$

Solution

(a) Focus $= (1, 0)$

$$y^2 = 4ax$$
$$y^2 = 4x$$
$$4a = 4$$
$$a = 1$$

(b) Directrix $\Rightarrow x = -a$

$$x = -1$$

2. Find the eqn of the parabola whose vertex is the origin and whose focus is the point $(5, 0)$.

Solution

$$y^2 = 4ax$$
$$y^2 = 4(5)x$$
$$y^2 = 20x.$$

3. Write down the eqn of the parabola $y^2 - 4y - 12x + 40 = 0$ in its canonical form and hence find

(i) Vertex (ii) Focus (iii) Directrix.

Solution

$$y^2 - 4y - 12x + 40 = 0$$
$$y^2 - 4y + 40 = 12x$$
$$y^2 - 4y + 4 = 12x - 40 + 4$$
$$(y-2)^2 = 12x - 36$$
$$(y-2)^2 = 12(x-3)$$

(i) Vertex $= (3, 2)$

(ii) Focus $= (h+a, k)$

$$a = 3$$
$$h + a = 3 + 3 = 6$$
$$F = (6, 2)$$

(iii) Directrix $\Rightarrow x = h - a$

$$x = 3 - 3$$
$$x = 0$$

4. Show that $y^2 - 8x + 2y + 9 = 0$ is the eqn of a parabola

Find the focus, directrix and the vertex.

Solution

$y^2 - 8x + 2y + 9 = 0$

$y^2 + 2y + 9 = 8x$

$y^2 + 2y + 1 = 8x - 9 + 1$

$(y+1)^2 = 8x - 8$

$(y+1)^2 = 8(x-1)$

$(y-y_1)^2 = 4a(x-x_1)$

$4a = 8$

$a = 2$

$(x_1, y_1) = (h, k)$

$h = 1, \quad k = -1$

(i) Vertex $= (1, -1)$

(ii) Focus $= (h+a, k)$

$= (1+2, -1)$

$= (3, -1)$

(iii) Directrix $\Rightarrow x = h - a$

$x = 1 - 2$

$x = -1$

* The eqn of the tangent to $y^2 = 4ax$ at the point x_1, y_1

$$\boxed{yy_1 = 2a(x + x_1)}$$

$y^2 = 4ax$

$2y\frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{4a}{2y} = \frac{2a}{y}$

$\frac{dy}{dx} = \frac{y-y_1}{x-x_1}$

$\frac{y-y_1}{x-x_1} = \frac{2a}{y_1}$

$yy_1 - y_1^2 = 2ax - 2ax_1$

$yy_1 = 2ax + y_1^2 - 2ax_1$

$y_1^2 = 4ax_1$ (at point x_1, y_1) ⟵ *

$yy_1 - 2ax = y_1^2 - 2ax_1$

Add $-2ax_1$ to both sides.

$$yy_1 - 2ax - 2ax_1 = y_1^2 - 2ax_1 - 2ax_1$$

$$yy_1 - 2a(x+x_1) = y_1^2 - 4ax_1$$

From eqn *

$$y_1^2 - 4ax_1 = 0$$

$$yy_1 - 2a(x+x_1) = 0$$

$$yy_1 = 2a(x+x_1)$$ —— Eqn of the tangent of a parabola.

Eqn of the normal to $y^2 = 4ax$ at the point x_1, y_1

$$2ay + xy_1 = 2ay_1 + x_1y_1$$

$$y^2 = 4ax$$

$$2y\frac{dy}{dx} = 4a$$

$$\frac{dy}{dx} = \frac{2a}{y}$$

* Normal $\perp$ Tangent.

$$m_1 m_2 = -1$$

$$m_2\left(\frac{2a}{y}\right) = -1$$

$$m_2 = \frac{-y}{2a}$$

$$m_2 = \frac{-y_1}{2a} \text{ at point } (x_1, y_1)$$

$$\frac{y-y_1}{x-x_1} = \frac{-y_1}{2a}$$

$$2ay - 2ay_1 = -xy_1 + x_1y_1$$

$$\boxed{2ay + xy_1 = 2ay_1 + x_1y_1}$$

Examples.

Find the eqn of the tangent to the parabola $y^2 = 12x$ at the point $(3, 6)$

Solution

$$x_1 = 3, \quad y_1 = 6$$

$$y^2 = 4ax$$

$$4ax = 12x$$

$$4a = 12$$

$$a = 3$$

$$yy_1 = 2a(x+x_1)$$

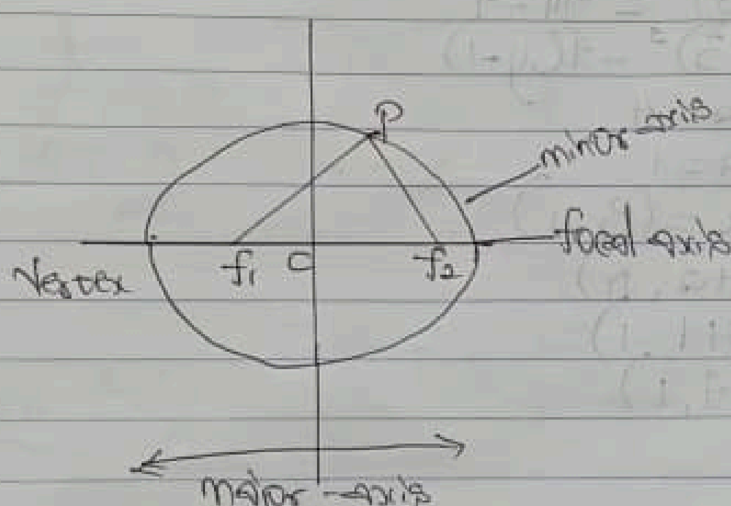
$$y(6) = 2(3)(x+3)$$
$$6y = 6x + 18$$
$$6x - 6y + 18 = 0.$$

Find the eqn of the normal to the parabola $y = 16x$ at the point $(1, -4)$

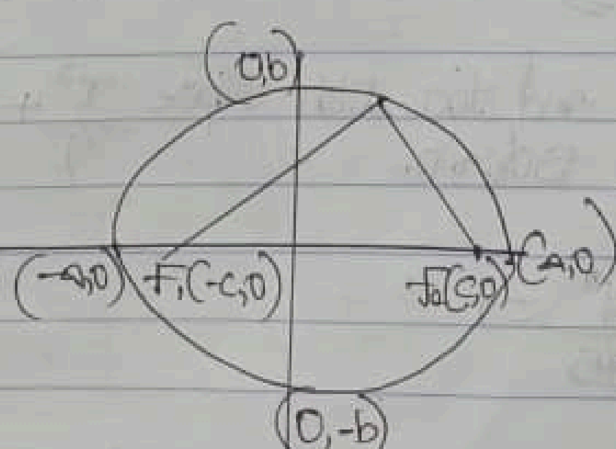
Solution.

$$x_1 = 1, \ y_1 = -4$$
$$4a = 16$$
$$a = 4$$
$$2ay + xy_1 = 2ay_1 + x_1y_1$$
$$2(4)y + x(-4) = 2(4)(-4) + (1)(-4)$$
$$8y - 4x = -32 - 4$$
$$8y - 4x = -36$$
$$2y - x = -9$$
$$x - 2y - 9 = 0$$

Ellipse.



Foci On x-axis.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where $c^2 = b^2 + a^2$

Foci: $F_1(-c, 0)$, $F_2(c, 0)$

Vertices: $(-a, 0)$, $(a, 0)$

Centre: $(0, 0)$

O_8

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$C^2 = a^2 - b^2$

Foci: $F_1(0, -c)$, $F_2(0, c)$

Vertices: $(0, -a)$, $(0, a)$

Centre: $(0, 0)$

** Examples :> $x^2 - 6x - 4y + 13 = 0$. Write it in parabola.

Solution

$x^2 - 6x - 4y + 13 = 0$

$x^2 - 6x = 4y - 13$

$x^2 - 6x + 9 = 4y - 13 + 9$

$(x-3)^2 = 4y - 4$

$(x-3)^2 = 4(y-1)$

$4a = 4$

$a = 1$

Vertex $= (3, 1)$

Focus $= (h+a, k)$

$= (3+1, 1)$

$= (4, 1)$

Directrix

$x = h - a$

$= 3 - 1 = 2$

1. Find the Vertices and foci with ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$

Solution

$C^2 = a^2 - b^2$

Compare: $\frac{x^2}{9} + \frac{y^2}{25} = 1$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$b^2 = 9$ $\quad a^2 = 25$

$b = 3$ $\quad a = 5$

$C^2 = a^2 - b^2$

$= 25 - 9$

$C^2 = 16$

$C = 4$

2. Write the eqn Of the ellipse $25x^2 + 4y^2 - 50x - 16y - 59 = 0$ in the Canonical form. Hence, determine (i) the Coordinate Of the Centre Of the ellipse (ii) the 4 vertices Of the ellipse. (iii) the 2 foci Of the ellipse.

Solution.

$25x^2 + 4y^2 - 50x - 16y = 59$

$25x^2 - 50x + 4y^2 - 16y = 59$

$25(x^2 - 2x) + 4(y^2 - 4y) = 59$

$25[x^2 - 2x + (-1)^2] + 4[y^2 - 4y + (-2)^2] = 59 + 25 + 16$

$$\frac{25(x-1)^2}{100} + \frac{4(y-2)^2}{100} = \frac{100}{100}$$

$$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{25} = 1$$

$a^2 = 25$ $\quad b^2 = 4$

$a = 5$ $\quad b = 2$

ola form

* Write the ellipse $4x^2 + 9y^2 = 36$ in it's Canonical form.

Solution

$$\frac{4x^2}{36} + \frac{9y^2}{36} = \frac{36}{36}$$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$b^2 = 9$ $\quad a^2 = 4$

$b = 3$ $\quad a = 2$

Eqn of the tangent at x_1, y_1 to the ellipse:-

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

Eqn of the normal at $x_1 y_1$ to the ellipse:-

$$a^2 x y_1 - b^2 x_1 y = (a^2 - b^2) x_1 y_1$$

Examples

Find the eqn of the tangent and normal to the ellipse $4x^2 + 25y^2 = 100$ at point $(-3, 8/5)$

Solution

(i) Eqn of the tangent $\quad x_1 = -3, \; y_1 = 8/5$

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

~~Eqn of the normal at x_1, y_1 to the ellipse:-~~

~~$a^2 x y_1 - b^2 x_1 y = (a^2 - b^2) x_1 y_1$~~

~~Find the eqn of the tangent and normal to the ellipse $4x^2 + 25y^2 = 100$~~

$$\frac{4x^2}{100} + \frac{25y^2}{100} = \frac{100}{100}$$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

$$a^2 = 25 \qquad b^2 = 4$$

$$a = 5 \qquad b = 2$$

$$\frac{x(-3)}{25} + \frac{y(8/5)}{4} = 1$$

$$4x(-3) + 5y(8) = 100$$

$$2x(-3) + 5y(4) = 50$$

$$-3x + 10y = 25$$

(ii) Eqn of the normal

$$a^2 x y_1 - b^2 x_1 y = (a^2 - b^2) x_1 y_1$$

$$25x(8/5) - 4(-3)y = (25-4)(-3)(8/5)$$

$$5x(8) + 12y = (21)(-24/5)$$

$$25x(8) + 60y = 21(-24)$$

$$200x + 60y = -504$$

Hyperbola.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$b^2 = c^2 - a^2$

Vertices: $(-a, 0)$, $(a, 0)$

Foci: $F_1(-c, 0)$; $F_2(c, 0)$

Centre: $(0, 0)$

Asymptotes: $y = \pm \frac{b}{a}x$.

A measure of the shape for an ellipse or hyperbola is the quantity $e = c/a$, called the eccentricity. For an ellipse $0 < e < 1$ but for a hyperbola $e > 1$

Example:- Find the vertices and foci of the hyperbola $25x^2 - 16y^2 = 400$.

Solution

$$\frac{25x^2}{400} - \frac{16y^2}{400} = \frac{400}{400}$$

$$\frac{x^2}{16} - \frac{y^2}{25} = 1$$

$a^2 = 16$ $\qquad$ $b^2 = 25$

$a = \pm 4$ $\qquad$ $b = \pm 5$

$c^2 = b^2 + a^2$

$c^2 = 25 + 16$

$c^2 = 41$

$c = \pm \sqrt{41}$

Vertices: $(-4, 0)$, $(4, 0)$

Foci: $(\sqrt{41}, 0)$, $(-\sqrt{41}, 0)$

Eqn of the tangent at x_1, y_1

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

Eqn of the normal at point x_1, y_1

$$a^2xy_1 + b^2x_1y = (a^2 + b^2)x_1y_1$$

Example:-

Find the eqn of the tangent and the normal to the hyperbola $x^2 - 2y^2 = 6$.

= ellipse

lipse

Solution

$$\frac{x^2}{6} - \frac{2y^2}{6} = \frac{6}{6}$$

$$\frac{x^2}{6} - \frac{y^2}{3} = 1$$

$a^2 = 6$ $\quad$ $b^2 = 3$

$a = \pm\sqrt{6}$ $\quad$ $b = \pm\sqrt{3}$

Solution

MTH 104.2 {General mathematics 2}

Course Outline (Calculus)

1. Function Of a real Variable, graphs, limit and idea of continuity.
2. The derivative as limit Of rate Of change. Technique Of differentiation.
3. Application Of derivative On extreme Curve Sketching.
4. Integration as an Inverse Of differentiation. Definite integral
5. Methods Of Integration, application Of integration to areas and Volume.

Textbook :-> College mathematics by Philip. A Schmolt and Frank Ayres Jr (Schaum's Outline Series)

Function.

If a relation between two real Variables Say x and y is Such that when x is given, y is determined then y is Said to be a function Of x. It is denoted by

$$y = f(x)$$

Where x is Called the independent Variable (input) and y is Called the dependent Variable (output).

Also, If y is a function Of u and v i.e $y = f(u, v)$ then y is a function Of two independent Variables u and v.

Let x and y be Set. A function ~~Of~~ from x to y is represented by

$$f: x \to y$$

This is a rule that assigns each element Of x as exactly One element Of y.

A function may be defined as a table Of Values, an equation, a formular Or a graph.

Component Of a function.

Let $y = f(x)$, the Set Of Values Of the Independent Variable x is Called the DOMAIN Of the function while the Set Of Values Of the dependent Variable is Called the RANGE Of the function.

Example :- $y = x^2$ defines a function whose domain consist Of all real numbers, and whose range is all non-negative numbers i.e 0 and positive numbers.

Examples

* $f(x) = \sqrt{x-16}$

Find $f(25)$

Solution

$$f(25) = \sqrt{25-16}$$
$$= \sqrt{9}$$
$$f(25) = 3$$

* $f(x) = \sqrt{x-7}$

find $f(56)$

Solution

$$y = f(x) = \sqrt{x-7}$$
$$f(56) = \sqrt{56-7}$$
$$= \sqrt{49}$$
$$= 7$$

* If $f(x) = x^2 - 5x + 4$

Find (i) $f(0)$ (ii) $f(2)$

Solution

$$f(x) = x^2 - 5x + 4$$

i) $f(0) = 0^2 - 5(0) + 4$

$$f(0) = 4$$

ii) $f(2) = 2^2 - 5(2) + 4$

$$= 4 + 4 - 10$$
$$f(2) = -2$$

* If ~~f(u,v)~~ $f(u,v) = 3u^2 - 2v$, find $f(2,1)$

Solution

$$f(u,v) = 3u^2 - 2v$$
$$f(2,1) = 3(2)^2 - 2(1)$$
$$= 3(4) - 2 = 12 - 2$$

$f(2,1) =$

1. Increasing

the value

$f(x) = 2$

the value

x
$f(x)$

When

i) Strict

Open