THERMODYNAMICS III

ADVANCED THERMODYNAMICS

NOZZLES AND JET PROPULSION

A Nozzle is a duct of smoothly varying cross-sectional area in which a steadily flowing fluid can be made to accelerate by a pressure drop along the duct.

When a fluid flows through a duct with no work or heat transfer, the only factors which can cause a ~~damage~~ change in the fluid properties are

(i) change in the flow area (ii) frictional forces.

For the seek of this course we will assume that frictional force is zero.

The effect of the variation in the cross-sectional area of the duct may be determined if some assumptions are made about the nature of flow.

The simplest type of flow conceivable is one-dimensional steady flow. The flow is said to be one-dimensional if the following conditions are fulfilled.

(i) change in area and curvature of the axis are gradual.

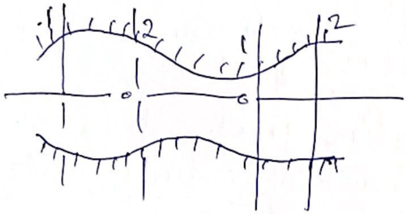
(ii) Thermodynamic and mechanical properties are uniform across planes normal to the axis of the duct.

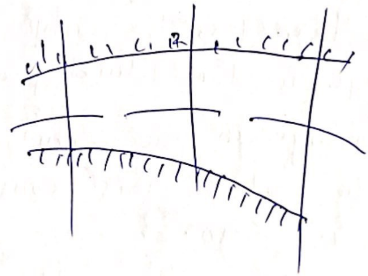
No real flow is truly one-dimensional, but provided that there are no sudden changes in direction or area, and that average values of the properties at any cross-section are used in the analysis, one-dimensional treatment yields results which are sufficiently accurate for many purposes.

NOZZLE SHAPE.

Consider a Stream of fluid at pressure P_1, enthalpy h_1 and a low Velocity C_1. It is required to find the shape of duct which will cause the fluid to accelerate to a high Velocity as the pressure falls along the duct. It can be assumed that $q = 0$ and $W = 0$.

Applying





Applying steady-flow energy equation.

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} \quad \text{------ (i)}$$

Given the Inlet cross-sectional Area A_1, corresponding values of A_2 can be found from the continuity equation

$$\frac{A_2 C_2}{V_2} = \frac{A_1 C_1}{V_1} \quad \text{------ (ii)}$$

where C_2 = Velocity at any point of the nozzle

V = Specific Volume

from equation (i)

$$C_2^2 = 2(h_1 - h_2) + C_1^2$$

$$C_2 = \sqrt{2(h_1 - h_2) + C_1^2} \quad \text{------ (iii)}$$

from equation (ii)

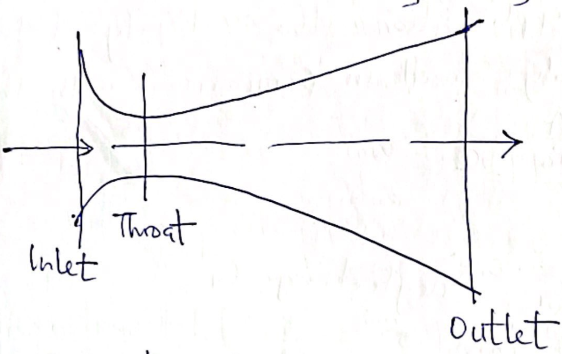
Mass flow rate $\dot{m} = \frac{AC}{V}$

Area per unit mass flow, $\frac{A}{\dot{m}} = \frac{V}{C}$ - - - - - (iv)

Substituting equation (iii) into (iv)

$$\frac{A}{\dot{m}} = \frac{V}{\sqrt{2(h_1-h_2) + C_1^2}}$$

For an Ideal frictionless case, since the flow is adiabatic and reversible, the process undergone is an Isentropic Process. hence S_1 (entropy at any section X–X) = S.



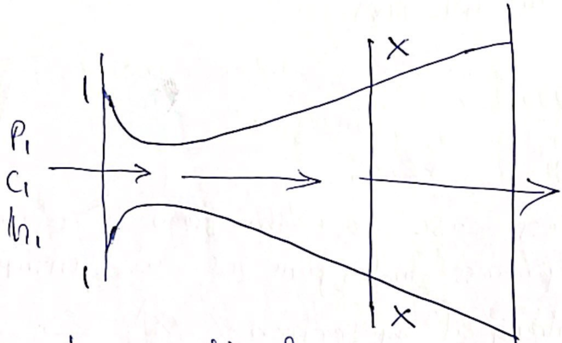
Convergent – divergent nozzle.

The section of minimum area is called the throat of the nozzle. The flow up to the throat is subsonic; the flow after the throat is supersonic. It should be noted that a sonic or a supersonic flow requires a diverging duct to accelerate it.

CRITICAL PRESSURE RATIO

The velocity at the throat of a correctly designed nozzle is the velocity of sound. For a nozzle that is convergent only, then the fluid will attain sonic velocity at exist if the pressure drop across the nozzle is large enough.

The ratio of the pressure at the section where sonic velocity is attained to the inlet pressure of a nozzle is called the critical pressure ratio.



In most practical applications the velocity at the to a nozzle is negligibly small in comparison with the exist velocity. $\frac{A}{\dot{m}} = \frac{V}{C}$

From equation (iii)

$$C = \sqrt{2(h_1-h) + C_1^2} \quad \text{neglecting } C_1$$

$$C = \sqrt{2(h_1-h)} \quad \text{------ (iv)}$$

$$\frac{A}{\dot{m}} = \frac{V}{C} = \frac{V}{\sqrt{2(h_1-h)}} \quad \text{------ (v)}$$

For a perfect gas $h = C_pT$ ------ (vi)

Substitute equation (vi) into (v)

$$\frac{A}{\dot{m}} = \frac{V}{\sqrt{2C_p(T_1-T)}} \quad \text{------ (vii)}$$

③

$$Pv = RT, \quad v = RT/p \quad \text{------} (viii)$$

Substitute equation (viii) into (vii)

$$\frac{A}{\dot{m}} = \frac{RT/P}{\sqrt{2C_pT_1\left(1-\frac{T}{T_1}\right)}} \quad \text{------} (ix)$$

For an Isentropic process let the pressure ratio $\frac{P}{P_1} = x$

$$\frac{T}{T_1} = \left(\frac{P}{P_1}\right)^{(\gamma-1)/\gamma} = x^{(\gamma-1)/\gamma} \quad \text{------} (x)$$

Substitute $P = xP_1$, $T = T_1 x^{(\gamma-1)/\gamma}$ and $\frac{T}{T_1} = x^{(\gamma-1)/\gamma}$ into equation (ix).

$$\frac{A}{\dot{m}} = \frac{RT_1 x^{(\gamma-1)/\gamma}}{P_1 x \sqrt{2C_pT_1\left(1-x^{(\gamma-1)/\gamma}\right)}} \quad \text{------} (xi)$$

For fixed inlet conditions (ie P_1 and T_1 fixed), we have

$$\frac{A}{\dot{m}} = \text{Constant} \times \frac{x^{(\gamma-1)/\gamma}}{x\sqrt{\left(1-x^{(\gamma-1)/\gamma}\right)}} \quad \text{------} (xii)$$

$$\frac{A}{\dot{m}} = \text{Constant} \times \frac{1}{x^{1/\gamma}\sqrt{\left(1-x^{(\gamma-1)/\gamma}\right)}} \quad \text{------} (xiii)$$

$$\frac{A}{\dot{m}} = \frac{\text{Constant}}{\sqrt{x^{2/\gamma} - x^{2/\gamma}\cdot x^{(\gamma-1)/\gamma}}} \quad \text{------} (xiv)$$

Therefore $\frac{A}{\dot{m}} = \frac{\text{Constant}}{\sqrt{(x^{2/\gamma} - x^{(\gamma+1)/\gamma}}}$ ——— (15)

To find the value of the pressure ratio, x, at which the area is a minimum it is necessary to differentiate equation 15 with respect to x and equate to zero

$$\frac{d}{dx}\left[\frac{1}{(x^{2/\gamma} - x^{(\gamma+1)/\gamma})^{1/2}}\right] = 0$$

i.e

$$\frac{\left[\frac{2}{\gamma}x^{(2/\gamma)-1} - \left(\frac{\gamma+1}{\gamma}\right)x^{((\gamma+1)/\gamma)-1}\right)}{2\left(x^{2/\gamma} - x^{(\gamma+1)/\gamma}\right)^{3/2}} = 0 \quad \text{——— 16}$$

Hence the area is a minimum when

$$\frac{2}{\gamma}\cdot x^{(2/\gamma)-1} = \left(\frac{\gamma+1}{\gamma}\right)x^{((\gamma+1)/\gamma)-1} \quad \text{——— (17)}$$

Therefore $x = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}$ ——— (18)

i.e critical pressure ratio, $\frac{P_c}{P_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$ ——— (19)

if you know the critical pressure ratio, you can determine the back pressure if you know the pressure of the air/fluid.

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CRITICAL TEMPERATURE RATIO

The ratio of the temperature at the section of the nozzle where the velocity is sonic to the inlet temperature is called the critical temperature ratio.

Critical temperature ratio, $\frac{T_c}{T_1} = \left(\frac{P_c}{P_1}\right)^{\frac{(\gamma-1)}{\gamma}} = \frac{2}{\gamma+1}$ --- (20)

$\therefore \frac{T_c}{T_1} = \frac{2}{\gamma+1}$ ---- (21)

equation (19) and (21) apply to perfect gases only and not to vapours.

CRITICAL VELOCITY

The critical velocity at the throat of a nozzle can be found for a perfect gas. critical velocity is the velocit of sound at the critical conditions.

from equation (iii)

$$C_c = \sqrt{2(h_1 - h) + C_1^2}$$

where $C_1 = 0$ and $h = C_pT$

$$C_c = \sqrt{2C_p(T_1 - T_c)} = \sqrt{2C_pT_c\left(\frac{T_1}{T_c} - 1\right)} \quad --- (22)$$

from equation (21) $\frac{T_c}{T_1} = \frac{2}{\gamma+1}$

$$C_c = \sqrt{\left[2C_pT_c\left\{\left(\frac{\gamma+1}{2}\right)-1\right\}\right]} = \sqrt{C_pT_c(\gamma-1)} \quad \text{--- (23)}$$

$$C_p = \frac{\gamma R}{\gamma-1} \quad \text{or} \quad C_p(\gamma-1) = \gamma R \quad \text{--- (24)}$$

Substituting equation (24) into (23)

$$C_c = \sqrt{\gamma R T_c} \quad \text{--- (25)}$$

$\therefore$ Critical velocity, $C_c = \sqrt{\gamma R T_c}$

Critical velocity is the velocity of sound at the critical conditions.

Velocity of sound, a is defined by the equation

$a^2 = \frac{dP}{d\rho}$ at constant entropy. --- (26)

where P = pressure and ρ = density

$\rho = \frac{1}{V}$ where V = specific volume --- (27)

$$d\rho = d\left(\frac{1}{V}\right) = -\frac{1}{V^2}dV \quad \text{--- (28)}$$

$$a^2 = -\frac{dP}{dV}V^2 \quad \text{--- (29)}$$

For a perfect gas undergoing an Isentropic process PV^{γ} = Constant

$$PV^{\gamma} = K \quad \text{---} (5) \text{---} \quad (30)$$

$$P = \frac{K}{V^{\gamma}}$$

$$\frac{dP}{dV} = -\frac{\gamma K}{V^{\gamma+1}} \quad \text{------} \quad (31)$$

$$a^2 = \frac{V^2 \gamma K}{V^{\gamma+1}} \quad \text{------} \quad (32)$$

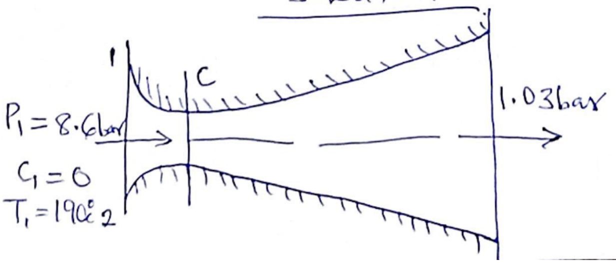
Also $K = PV^{\gamma}$ hence

$$a^2 = \frac{\gamma P V^{\gamma} V^2}{V^{\gamma+1}} = \gamma P V$$

Velocity of Sound $a = \sqrt{\gamma P V}$ ------ (33)

<u>Example 1:</u> Air at 8.6 bar and 190°C expands at the rate of 4.5Kg/s through a convergent-divergent nozzle into a space at 1.03bar. Assuming that the inlet velocity is negligible, calculate the throate and the exist cross-sectional area of the nozzle.

Solution.



Using Critical pressure ratio $\frac{P_c}{P_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$

$P_c = \left(\frac{2}{2.4}\right)^{\frac{1.4}{0.4}} \times 8.6 = 0.5283 \times 8.6 = \underline{4.543}$ bar

Solving for critical Temperature Using $\frac{T_c}{T_1} = \frac{2}{\gamma+1}$

$T_1 = 190°c + 273 = 463°K$

$T_c = \frac{2}{2.4} \times 463 = \frac{926}{2.4} = \underline{\underline{385.8°K}}$

$P_c V_c = RT_c$

$V_c = \frac{RT}{P_c} = \frac{287 \times 385.8}{10^5 \times 4.543} = 0.244 m^3/Kg$

Critical velocity $C_c = \sqrt{\gamma R T_c}$ or $= \sqrt{2C_p(T_1 - T_c)}$

$C_c = \sqrt{1.4 \times 287 \times 385.8} = 393.7 m/s$ or

$C_c = \sqrt{2 \times 1.005 \times 10^3 (463 - 385.8)} = 393.8 m/s$

Solving for area of the throat.

$A_c = \frac{\dot{m} V_c}{C_c} = \frac{4.5 \times 0.244}{393.7} = 0.00279 m^2$

$\therefore$ Area of throat $A = \underline{\underline{2790 mm^2}}$

(6)

Solving for the exist area.

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{(\gamma-1)/\gamma}$$

$$T_2 = \frac{T_1}{\left(\frac{P_1}{P_2}\right)^{(\gamma-1)/\gamma}} = \frac{463}{\left(\frac{8.6}{1.03}\right)^{0.4/1.4}} = \frac{463}{1.834} = 252.5^{\circ}K$$

$$C_2 = \sqrt{2(h_1-h_2)} = \sqrt{2C_p(T_1-T_2)}$$

$$C_2 = \sqrt{2\times 1.005\times 10^3(463-252.5)} = 650.5 \, m/s$$

$$V_2 = \frac{RT_2}{P_2} = \frac{287\times 252.5}{10^5\times 1.03} = 0.7036 \, m^3/Kg$$

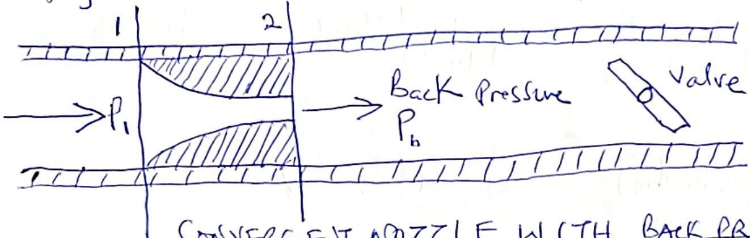
Then to find the exist area

$$A_2 = \frac{\dot{m}V_2}{C_2} = \frac{4.5\times 0.7036}{650.5} = 0.00487 m^2$$

$\therefore$ Exist Area $= \underline{\underline{4870 \, mm^2}}$

MAXIMUM MASS FLOW.

Consider a Convergent nozzle expanding into a space, the pressure of which can be varied, while the inlet pressure remain fixed. The nozzle is shown diagrammatically in the figure below. ~~when the backth~~



CONVERGENT NOZZLE WITH BACK PRESSURE

When the back pressure P_b is equal to P_1, then no fluid can flow through the nozzle. As P_b is reduced the mass flow through the nozzle increases, since the enthalpy drops, ~~and~~ hence the velocity, Increases, However, when the back pressure reaches the critical value, it is found that no further reduction in back pressure can affect the mass flow.

When the back pressure is exactly equal to the critical ~~value, it is found that no further~~ pressure P_c, then the velocity at exit is Sonic and the mass flow through the nozzle is at a maximum. if the back pressure is reduced below the critical value then the mass flow remains at the maximum value, the exit pressure remains at P_c, and the fluid expands violently outside the nozzle down to the back pressure. it can be seen that the maximum mass flow through a convergent nozzle is obtained when the pressure ratio across the nozzle is the critical pressure ratio.

When a nozzle operates (7) with the maximum mass flow it is said to be choked. A correctly designed convergent divergent nozzle is alway choked.

From the diagram below, the pressure waves emanate from point P at the velocity of sound relative to the fluid, a, while the fluid moves with a velocity, C. The absolute velocity of the pressure waves travelling back upstream is therefore given by $(a-C)$. Now when the fluid velocity is subsonic, then $C < a$, and the pressure waves can move back upstream; however, when the flow is sonic or supersonic (i.e $C = a$ or $C > a$), then the pressure waves cannot be transmitted back upstream.

Example 2: A fluid at 6.9 bar and 93°C enters a convergent nozzle with negligible velocity, and expands isentropically into a space at 3.6 bar. Calculate the mass flow per square metre of exit area:
(i) When the fluid is helium ($C_p = 5.19 KJ/KgK$.)
(ii) When the fluid is ethane ($C_p = 1.88 KJ/KgK$).

Assuming that both helium and ethane are perfect gases and take the respective molar masses as 4Kg/Kmol and 30 Kg/Kmol.

Solution.

i) When the fluid is helium.

Solving for specific gas constant of helium $R = \frac{\bar{R}}{M}$

$$R = \frac{8314.5}{4} = 2079 Nm/KgK$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$

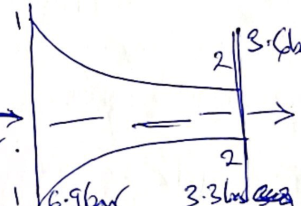
Solving for specific heat capacity ratio of helium (γ)

$$\gamma = \frac{C_p}{C_p - R} = \frac{5.19 \times 10^3}{5.19 \times 10^3 - 2079} = \underline{\underline{1.667}}$$

Solving for critical pressure (P_c)

$$\frac{P_c}{P_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{2.667}\right)^{\frac{1.667}{0.667}} = 0.487$$

$P_c = P_1 \times 0.487 = 3.36$ bar.



$\therefore$ Critical pressure $P_c = 3.36$ bar.

The actual back pressure is 3.6 bar, hence in this case the fluid does not reach the critical conditions and the nozzle is not choked.

Solving for T_2 at the exit.

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{(\gamma-1)/\gamma} = \left(\frac{6.9}{3.6}\right)^{\frac{(1.667-1)}{1.667}} = \left(\frac{6.9}{3.6}\right)^{0.4} = 1.297$$

$$T_2 = \frac{T_1}{1.297} = \frac{93+273}{1.297} = 282.3°K.$$

$$C_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2C_p(T_1 - T_2)}$$

$$C_2 = \sqrt{2 \times 5.19 \times 10^3 (366 - 282.2)} = 932.7 \text{ m/s}$$

$$V_2 = \frac{RT_2}{P_2} = \frac{2079 \times 282.2}{10^5 \times 3.6} = 1.63\, m^3/Kg$$

$$\dot{m} = \frac{A_2 C_2}{V_2} = \frac{1 \times 932.7}{1.63} = 572.3\, Kg/s$$

$\therefore$ mass flow per Square metre of exit area = 572.3 Kg/s

ii) Solving for when fluid is ethane.

Using the Same procedure for ethane.

$$R = \frac{\bar{R}}{m} = \frac{8314.5}{30} = 277.1\, Nm/KgK$$

$$\frac{\gamma - 1}{\gamma} = \frac{R}{C_p}$$

$$\gamma = \frac{1}{1 - 0.147} = 1.172$$

Then

$$\frac{P_c}{P_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{2.172}\right)^{\frac{1.172}{0.172}} = 0.57$$

$$P_c = 0.57 \times 6.9 \text{ bar}$$

$\therefore$ critical pressure, $P_c = 3.93$ bar

The actual back pressure is 3.6 bar, hence in this case the fluid reaches critical conditions at exit and the nozzle is choked. The expansion from the exit pressure 3.93 bar down to the back pressure of 3.6 bar must take

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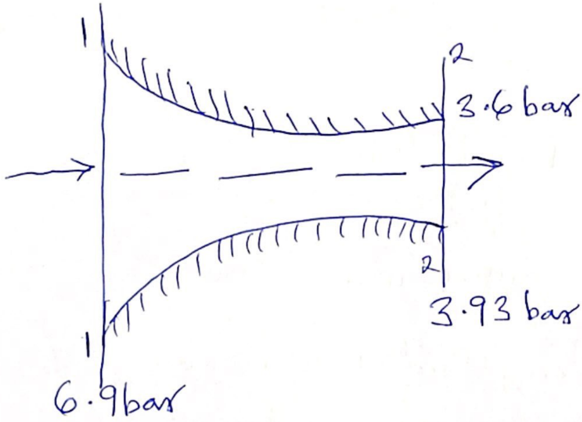
Place outside the nozzle



Since the nozzle is choked

$$\frac{T_c}{T_1} = \frac{2}{\gamma+1} = \frac{2}{2.172}$$

$$T_2 = T_c = \frac{2 \times 366}{2.172} = \underline{\underline{337^\circ K}}$$

$$C_2 = C_c = \sqrt{\gamma R T_c} = \sqrt{1.172 \times 277.1 \times 337}$$

$$\therefore C_2 = C_c = \underline{\underline{331 m/s}}$$

$$V_2 = \frac{RT_2}{P_2} = \frac{277.1 \times 337}{3.93 \times 10^5} = 0.238 m^3/Kg$$

$$\dot{m} = \frac{A_2 C_2}{V_2} = \frac{1 \times 331}{0.238} = 0.238 m^3/Kg.$$

$\therefore$ The mass flow per square metre of exit area = $\underline{1391 kg/s}$

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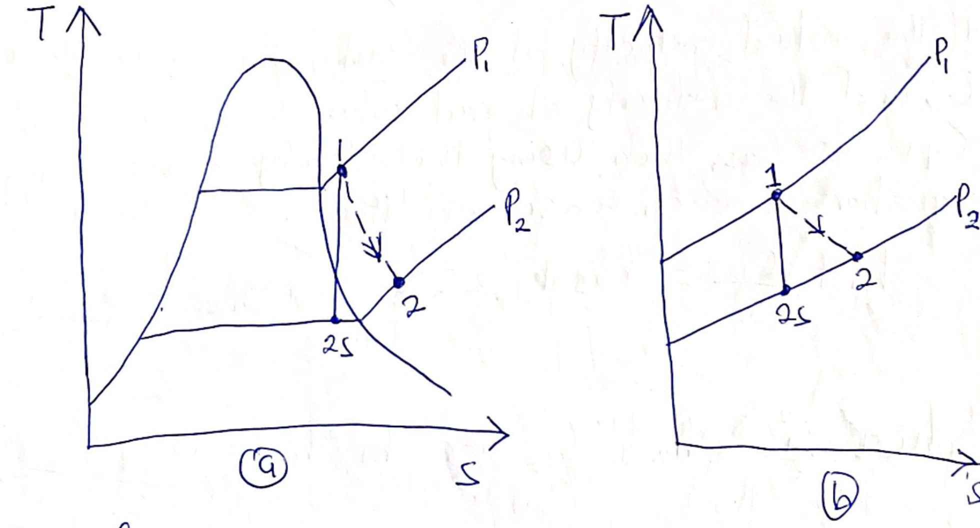
NOZZLE EFFICIENCY.

Due to friction between the fluid and the walls of the nozzle, and to friction within the fluid itself, the expansion process is irreversible, although still approximately adiabatic. In nozzle design it is usual to base all calculations on isentropic flow and then to make an allowance for friction by using a coefficient or an efficiency.



Typical expansions between P_1 and P_2 in a nozzle are shown in the diagram above for vapour in fig (a) and perfect gas in fig (b).

The line 1–2s on each diagram represents the ideal isentropic expansion and the line 1–2 represents the actual irreversible adiabatic expansion.

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Nozzle efficiency is ~~the~~ defined by the ratio of the actual enthalpy drop to the Isentropic enthalpy drop ~~drop~~ between the same pressure.

$$\text{Nozzle efficiency} = \frac{h_1 - h_2}{h_1 - h_{2s}} \quad \text{------(i)}$$

For a perfect gas, this equation (i) is reduced to

$$\text{Nozzle efficiency} = \frac{C_p(T_1 - T_2)}{C_p(T_1 - T_{2s})} = \frac{T_1 - T_2}{T_1 - T_{2s}} \quad \text{---(ii)}$$

if the actual velocity at the exit from the nozzle is C_2, and the velocity at exit when the flow is Isentropic is C_{2s}, then using the steady-flow energy equation in each case we have.

$$h_1 + \frac{C_1^2}{2} = h_{2s} + \frac{C_{2s}^2}{2} \quad \text{or} \quad h_1 - h_{2s} = \frac{C_{2s}^2 - C_1^2}{2} \quad \text{-(iii)}$$

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} \quad \text{or} \quad h_1 - h_2 = \frac{C_2^2 - C_1^2}{2} \quad \text{--(iv)}$$

Substituting equation (iii) & (iv) into equation (i)

$$\text{Nozzle efficiency} = \frac{C_2^2 - C_1^2}{C_{2s}^2 - C_1^2} \quad \text{------(V)}$$

when the Inlet velocity C_1 is negligibly small then

$$\text{Nozzle efficiency} = \frac{C_2^2}{C_{2s}^2} \quad \text{------(vi)}$$

Velocity coefficient: is defined as the ratio of the actual exit velocity to the exit velocity when the flow is Isentropic between the same pressures.

$$\text{Velocity coefficient} = \frac{C_2}{C_{2s}} = \sqrt{\text{Nozzle efficiency}}$$

Coefficient of discharge: ~~its~~ the ratio of the actual mass flow through the nozzle, $\dot{m}$, to the mass flow which would be passed if the flow were Isentropic $\dot{m}_s$; This is called the coefficient of discharge.

$$\text{Coefficient of discharge} = \frac{\dot{m}}{\dot{m}_s}$$

Nozzles in practice are used with a variety of shapes and cross-sections. The cross-section can be either circular or rectangular, and the axis of the nozzle can be straight or curved.

Example 3: Gases expand in a propulsion nozzle from 3.5 bar and 425°C down to a back pressure of 0.97 bar, at the rate of 18 Kg/s. Taking a coefficient of discharge of 0.99 and a nozzle efficiency of 0.94, calculate the required throat and exit areas of the nozzle. for the gases take $\gamma = 1.333$ and $C_p = 1.11$ KJ/KgK. Assume that the Inlet velocity is negligible

Solution

Solving for critical pressure $\frac{P_c}{P_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$

$$\frac{P_c}{P_1} = \left(\frac{2}{2.333}\right)^{\frac{1.333}{0.333}} = 0.54$$

$$P_c = P_1 \times 0.54 = 3.5 \times 0.54 = 1.89 \text{ bar}.$$

$\therefore$ Critical pressure $P_c = \underline{1.89 \text{ bar}}$

The nozzle is therefore choking and a convergent-divergent nozzle is required.

$$\frac{T_c}{T_1} = \frac{2}{\gamma+1} = \frac{1}{1.1665}$$

$$T_c = \frac{425 + 273}{1.1665} = 598.4 \text{ K}$$

Solving for Critical Velocity.

$$C_c = \sqrt{2(h_1 - h_c)} = \sqrt{2(C_p(T_1 - T_c)} \text{ or } C_c = \sqrt{\gamma R T_c}$$

$$C_c = \sqrt{2 \times 1.11 \times 10^3 (698 - 598.4)}$$

$$C_c = \underline{470.3 \text{ m/s}}$$

Solving for Specific Gas Constant.

$$C_p = \frac{\gamma R}{\gamma - 1}$$

$$R = \frac{C_p(\gamma - 1)}{\gamma} = \frac{1.11 \times 10^3 \times 0.33}{1.33} = 277.3 \text{ Nm/KgK}$$

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(11)

$$V_c = \frac{RT_c}{P_c} = \frac{277.3 \times 598.4}{10^5 \times 1.89} = 0.878 m^3/Kg$$

Solving for Isentropic mass flow rate.

$$\text{Discharge Coefficient} = \frac{\dot{m}}{\dot{m}_s}$$

$$0.99 = \frac{18}{\dot{m}_s}$$

$$\dot{m}_s = 18.18 Kg/s$$

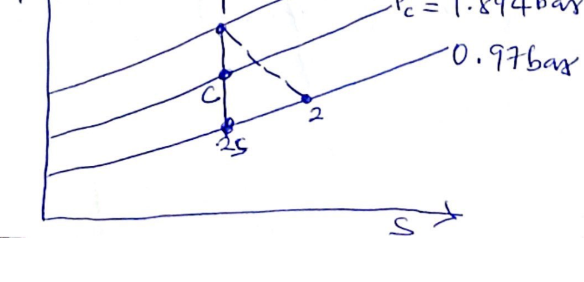
$$\text{Area of throat } A_c = \frac{\dot{m}_s V_c}{C_c} = \frac{18.18 \times 0.878}{470.3}$$

$$A_c = 0.0339 m^2$$

for an Isentropic expansion from the Inlet conditions down to the back pressure, the temperature exit is T_2 given by equation below

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = \frac{698}{T_2} = \left(\frac{3.5}{0.97}\right)^{0.333/1.333} = 1.378$$

$$T_2 = \frac{698}{1.378} = 506.6°K$$



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nozzle efficiency $= 0.94 = \dfrac{T_1 - T_2}{T_1 - T_{2s}} = \dfrac{698 - T_2}{698 - 506.6}$

$T_2 = 698 - 0.94(698 - 506.6)$
$= 698 - 180 = 518.1°K$

$V_2 = \dfrac{RT_2}{P_2} = \dfrac{277.3 \times 518.1}{0.97 \times 10^5} = \underline{1.48 m^3/Kg}$.

$C_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2C_p(T_1 - T_2)}$

$C_2 = \sqrt{2 \times 1.11 \times 10^3 (698 - 518.1)} = 632 m/s$

$A_2 = \dfrac{\dot{m}_2 V_2}{C_2} = \dfrac{18 \times 1.48}{632} = \underline{0.0422 m^2}$

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