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MTH 102.2

25/10/21

Vectors, Geometry and Dynamics.

Vectors

In many Scientific Calculations/applications, Certain quantities Only of magnitude, dis magnitudes, quantities are refer to as Scalars. E.g age, height, temp, Speed, Volume, Mass, density, length etc.

There are Other quant. which of both magnitude and direction. Some examples Of Such are Velocity, acceleration, force, momentum, displacement, electric Or magnetic field intensity. This quant. are Called Vectors.

A complete characterization Of a Scalar quantity requires length and Support i.e a Specified Unit and a number Stating how many times that Unit is Contained in that quantity While the Complete Characterization Of a Vector quantity requires length, Support and Sense. i.e a Specified Unit and a number Stating how many times that Unit is Contained in that quantity and in the Sense Of direction.

Representation Of a Vector.

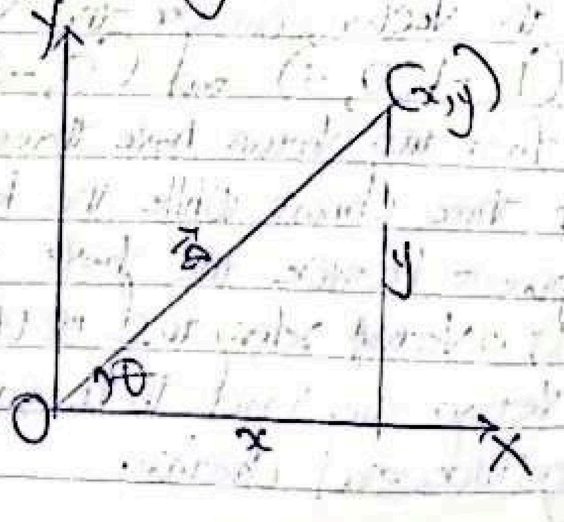
A Vector in the plane is a directed line Segment graphically, a vector is rep. by a line $\overline{OP}$ directed from the initial point O to the final terminal point P and denoted by $\overrightarrow{OP}$.

Vectors in two dimensions.

A Vector in two dimensions may be Written as $\vec{a} = xi + yj$ Or $\vec{a} = (x, y)$ Where x, y are

N.B In One dimension, $\vec{a} = xi$

the Component along the OX and OY axis respectively.



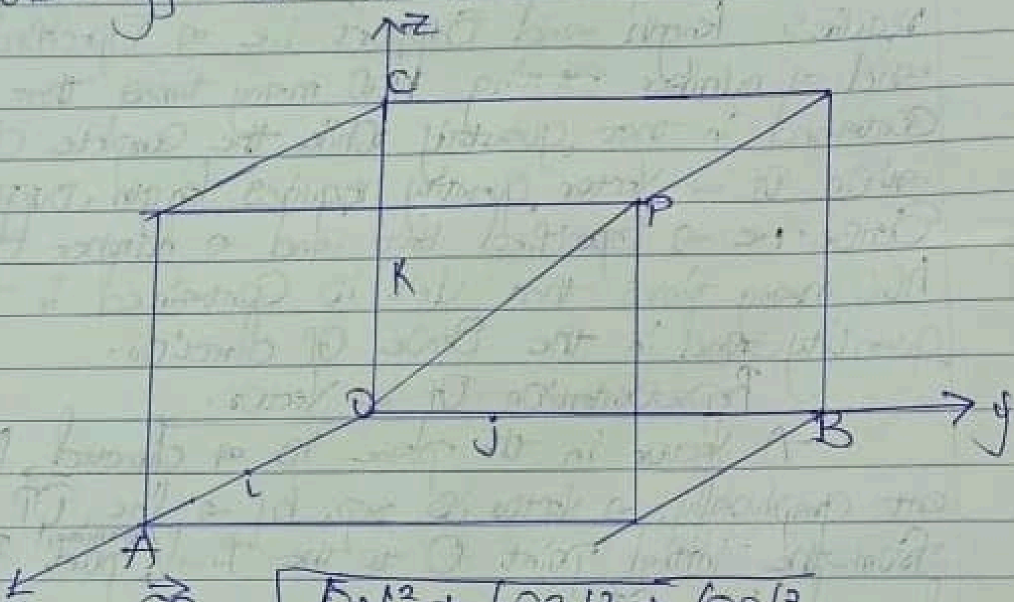
The modulus of $\vec{a}$ is given by $\sqrt{x^2+y^2}$

i.e $|\vec{a}| = \sqrt{x^2+y^2}$.

The angle θ which the $\vec{a}$ makes with the x unit, is given by $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Vectors in three dimensions.

A vector a in three dimensions may be written as $\vec{a} = xi + yj + zk$ or $\vec{a} = (x, y, z)$ where x, y, z are the component of $\vec{a}$ along the directions of i, j and k respectively i.e $\vec{OA} = xi$, $\vec{OB} = yj$ and $\vec{OC} = zk$



$$\vec{OP} = \sqrt{|\vec{OA}|^2 + |OB|^2 + |OC|^2}$$
$$= \sqrt{(xi)^2 + (yj)^2 + (zk)^2}$$
$$= \sqrt{x^2 + y^2 + z^2}$$

Components of a given Vector.

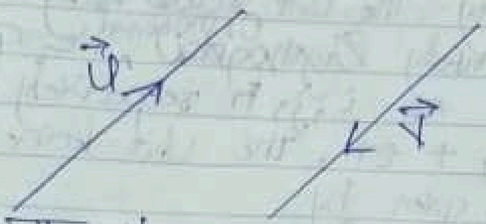
The real numbers V_i are called the component / coordinate of the vector. Consider the vectors; $(3, 1, 2)$, $(-1, 0, 3)$, $(1, -1, 0, 5)$ and $(0, -3, 4, 0)$.

The first two vectors have three component because they consist of three element while the last two consist of four component because the have four element.

N.B The component of a vector refers to d no of element in dat vector.

Two vectors are said to be equal if they have the same magnitude and direction.

If two vectors $\vec{U}$ and $\vec{V}$ are such that $\vec{U} = -\vec{V}$



i.e The vectors are parallel but opposite in sense. Example

If $\vec{A} = (8, 1, 6)$ and $\vec{B} = (a+b, c-1, a-b)$ find the component a, b & c if $\vec{A} = \vec{B}$.

Solution.

$a + b = 8$ —— (i)

$c - 1 = 1$

$c = 1 + 1$

$c = 2$

$a - b = 6$ —— (ii)

From (i)

$a = 8 - b$ —— (iii)

$8 - b - b = 6$

$-2b = -2$

$b = 1$

$a = 8 - 1$

$= 7$

* Find the modulus of $\vec{A} = 3i + 5j$.

Solution

$|A| = \sqrt{3^2 + 5^2}$

$= 9 + 25$

$= \sqrt{34}$ units.

* Find the modulus of $\vec{A} = 5i - 2j - 4\hat{k}$.

Solution

$|A| = \sqrt{5^2 + (-2)^2 + (-4)^2}$

$= \sqrt{25 + 4 + 16}$

$= \sqrt{45} = \sqrt{15 \times 3}$

$= \sqrt{5 \times 3 \times 3} = 3\sqrt{5}$ u.

Unit Vectors

A Unit Vector $\hat{a}$ is defined as a Vector whose modulus is Unity. The Unit Vectors along the x, y, z axis which are mutually Orthogonal i.e perpendicular and normally written as $\hat{i}, \hat{j}, \hat{k}$ respectively. Given a Vector $\vec{a} = xi + yj + zk$, the Unit Vector $\hat{a}$ along the Vector $\vec{a}$ is given by

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

Example :- Find the Unit Vector in the direction of $\vec{a} = 2i + j - 2k$.

Solution.

Let $\hat{a}$ be the Unit Vector in d direction Of $\vec{a}$.

$$|\vec{a}| = \sqrt{2^2 + 1^2 + (-2)^2}$$
$$= \sqrt{4+1+4}$$
$$= \sqrt{9} \text{ units} = 3 \text{ units}$$
$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{2i + j - 2k}{\sqrt{9}}$$
$$= \frac{1}{\sqrt{9}}(2i + j - 2k)$$
$$= \frac{1}{3}(2i + j - 2k)$$

* Find the Unit Vector parallel to the resultant Vectors $\vec{r}_1 = 2i + 4j - 5k$, $\vec{r}_2 = i + 2j + 3k$.

Solution.

let R be the resultant Vectors.

$$R = \vec{r}_1 + \vec{r}_2$$
$$= 2i + 4j - 5k + i + 2j + 3k$$
$$= 3i + 6j - 2k.$$
$$|R| = \sqrt{3^2 + 6^2 + (-2)^2}$$
$$= 9 + 36 + 4 = \sqrt{49}$$
$$= 7 \text{ Units.}$$
$$\hat{r} = \frac{R}{|R|} = \frac{1}{7}(3i + 6j - 2k)$$

Addition Of Vectors in Components.

Vectors may be added algebraicly by adding their Corresponding Scalar Component

$V_1 = a_1 i + b_1 j + c_1 k$

$V_2 = a_2 i + b_2 j + c_2 k$

$V_1 + V_2 = (a_1 i + b_1 j + c_1 k) + (a_2 i + b_2 j + c_2 k)$

$= (a_1 + a_2) i + (b_1 + b_2) j + (c_1 + c_2) k.$

Example

if $\vec{A} = 2i - 4j + 3k$, $\vec{B} = 5i + 3j - 6k$ then $\vec{A} + \vec{B}$

$= (2+5)i + (-4+3)j + (3-6)k$

$= 7i - j - 3k$

Subtraction Of Vectors in Components.

The difference between two vectors is Obtained by Subtracting the Corresponding components. If

$V_1 = a_1 i + b_1 j + c_1 k$

$V_2 = a_2 i + b_2 j + c_2 k$

$V_1 - V_2 = (a_1 i + b_1 j + c_1 k) - (a_2 i + b_2 j + c_2 k)$

$= (a_1 - a_2) i + (b_1 - b_2) j + (c_1 - c_2) k.$

Example.

$\vec{A} = 6i + 2j - 4k$

$\vec{B} = 3i - 5j - 7k.$

$\vec{A} - \vec{B} = (6-3)i + [2-(-5)]j + [-4-(-7)]k.$

$= 3i + (2+5)j + (-4+7)k$

$= 3i + 7j + 3k$

Scalar multiplication Of Vectors

Scalar multiplication Can be accomplished component by Component. If C is a Scalar and $V = ai + bj$ is a Vector. Then

$CV = C(ai + bj)$

$= aci + bcj$

Similarly, if C is a Scalar and $\vec{V}$ is a Vector then $|CV| = |C||V|$ i.e the absolute Value Of CV

Examples:

Given $\vec{a} = 2i + 3j - k$, $\vec{b} = 3i + 4j + 3k$ and $\vec{c} = 5i + 4j + 3k$.

(i) Find $2\vec{a}$

(ii) $-3\vec{c}$

(iii) $6\vec{a} + \vec{b} - \vec{c}$

Solution.

(i) $2(2i + 3j - k) = 4i + 6j - 2k$.

(ii) $-3(5i + 4j + 3k) = -15i - 12j - 9k$.

(iii) $6(2i + 3j - k) + 3i + 4j + 3k - (5i + 4j + 3k)$

$= 12i + 18j - 6k + 3i + \cancel{4j} + \cancel{3k} - 5i - \cancel{4j} - \cancel{3k}$

$= 10i + 18j - 6k$

* If $c = -2$ and $\vec{v} = -3i + 4j$. Show that $|cv| = |c||v|$

Solution

$cv = -2(-3i + 4j)$

$= 6i - 8j$

$|cv| = \sqrt{6^2 + (-8)^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100} = 10$ units.

$|v| = \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9 + 16} = \sqrt{25}$

$= 5$ units.

$|c| = \sqrt{(-2)^2} = \sqrt{4}$

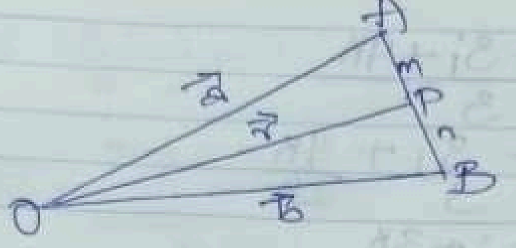
$= 2$

$|c||v| = 2 \times 5 = 10$ units.

N.B The absolute value of a magnitude is the opposite operator of that magnitude.

Position Vector

Consider a point A, B with position vectors $\vec{a}, \vec{b}$



Let P be the point with position vector $\vec{r}$ on AB such that

$AP : BP = m : n$

$$\frac{AP}{BP} = \frac{m}{n}$$

$$AP(n) = BP(m)$$

$$n(\vec{r} - a) = m(b - \vec{r})$$

$$n\vec{r} - na = mb - m\vec{r}$$

$$n\vec{r} + m\vec{r} = mb + na$$

$$\vec{r}(m+n) = mb + na$$

$$\vec{r} = \frac{mb + na}{m+n}$$

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$$\vec{r} = \frac{n\vec{A} + m\vec{B}}{m+n}$$

Example :-> Given that $\vec{A} = 2i + 5j + 5k$ and $\vec{B} = 2i - j + 2k$ are position vectors of point A and B respectively. Find the position vector of a point r which divides AB in the ratio 2:1

Solution

Let the position vector of point r be $\vec{r}$

$$\vec{r} = \frac{n\vec{A} + m\vec{B}}{m+n}$$

2 : 1

m n

$$\vec{r} = \frac{1(2i+5j+5k) + 2(2i - j + 2k)}{2+1}$$

$$= \frac{2i + 5\hat{j} + 5k + 4i - 2j + 4k}{3}$$

$$\vec{r} = \frac{6i + 3j + 9k}{3}$$

$$\vec{r} = \frac{6i}{3} + \frac{3j}{3} + \frac{9k}{3}$$

$$= 2i + j + 3k$$

* Find the position vectors of X, Y and Z which divides AB in the ratio 1:3, 3:1 and 3:-1 respectively. Given that the position vectors of A and B are $5i + 2j + k$ and $i - 2j - 3k$.

Solution.

(i) $\vec{r} = \dfrac{n\vec{A} + m\vec{B}}{m+n}$

$1 : 3$

$m + n = 4$

$m = 1,\ n = 3$

$$\vec{r} = \frac{3(5i + 2j + k) + 1(i - 2j - 3k)}{4}$$

$$= \frac{15i + 6j + \cancel{3k} + i - 2j - \cancel{3k}}{4}$$

$$= \frac{16i + 4j}{4}$$

$$\vec{r}_x = 4i + j$$

(ii) $\vec{Y} = \dfrac{n\vec{A} + m\vec{B}}{m+n}$

$3 : 1$

$m \quad n$

$m + n = 3 + 1 = 4$

$$= \frac{1(5i + 2j + k) + 3(i - 2j - 3k)}{4}$$

$$= \frac{5i + 2j + k + 3i - 6j - 9k}{4}$$

$$= \frac{8i - 4j - 8k}{4}$$

$$\vec{Y} = 2i - j - 2k$$

iii $\vec{Z} = 3$ ...

Is d ...

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iii $\vec{Z} = \underset{m}{3} : \underset{n}{-1}$

$m + n = 3 - 1 = 2$

$$\vec{z} = \frac{n\vec{A} + m\vec{B}}{m+n}$$

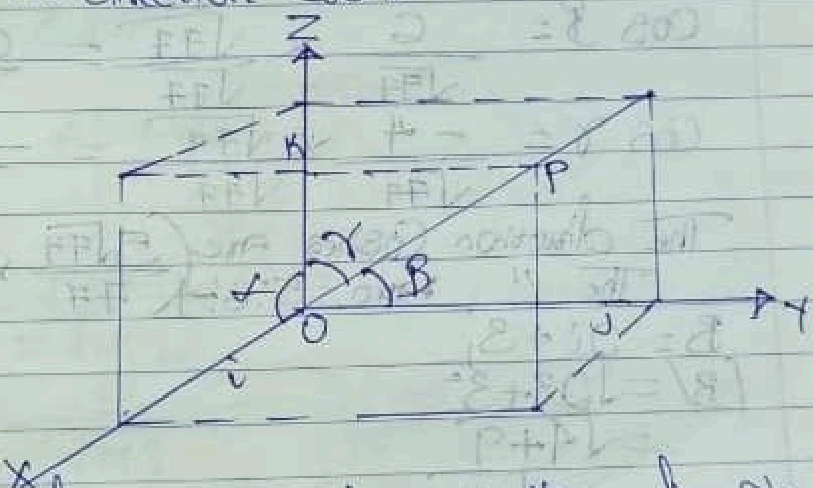
$$= \frac{-1(5i + 2j + k) + 3(i - 2j - 3k)}{2}$$

$$= \frac{-5i - 2j - k + 3i - 6j - 9k}{2}$$

$$= \frac{-2i - 8j - 10k}{2}$$

$$\vec{Z} = -i - 4j - 5k$$

The direction Cosine.



The direction of a vector in three dimension is determined by the angles which the vector makes with the three axes of reference. From the above figure, suppose OP makes angles α, β, γ with OX, OY and OZ respectively. By resolving OP along OX, OY and OZ, we have

$x = OP\cos\alpha$

$y = OP\cos\beta$

$z = OP\cos\gamma$

but $|OP| = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

Hence

$$\cos\alpha = \frac{x}{|\vec{r}|} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos\beta = \frac{y}{|\vec{r}|} = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$\cos\gamma = \frac{z}{|\vec{r}|} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

Examples:-

1 If $A = 5i + 6j - 4k$ and $B = 2i + 3j$. Find the direction Cosines of A and B.

Solution.

$A = 5i + 6j - 4k$

$|A| = \sqrt{5^2 + 6^2 + (-4)^2}$

$= \sqrt{25 + 36 + 16}$

$= \sqrt{77}$ Units

* Direction ratio ⇒ Direction Component / element.

$$\cos\alpha = \frac{5}{\sqrt{77}} \times \frac{\sqrt{77}}{\sqrt{77}} = \frac{5\sqrt{77}}{77}$$

$$\cos\beta = \frac{6}{\sqrt{77}} \times \frac{\sqrt{77}}{\sqrt{77}} = \frac{6\sqrt{77}}{77}$$

$$\cos\gamma = \frac{-4}{\sqrt{77}} \times \frac{\sqrt{77}}{\sqrt{77}} = -\frac{4\sqrt{77}}{77}$$

The direction Cosines are $\left(\frac{5\sqrt{77}}{77}, \frac{6\sqrt{77}}{77}, \frac{-4\sqrt{77}}{77}\right)$

The " ratio $= 5:6:-4$

$B = 2i + 3j$

$|B| = \sqrt{2^2 + 3^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$

$$\cos\alpha = \frac{2}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

$$\cos\beta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\cos\gamma = \frac{0}{\sqrt{13}} = 0$$

The direction Cosines are $\left(\frac{2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13}, 0\right)$

The direction ratio $= 2:3$

2. Find the direction Cosine Of $R = 3i - 9j + 2k$.

Solution

$|R| = \sqrt{94}$

$\cos \alpha = \frac{3}{\sqrt{94}} = \frac{3\sqrt{94}}{94}$

$\cos \beta = \frac{-9}{\sqrt{94}} = \frac{-9\sqrt{94}}{94}$

$\cos \gamma = \frac{2}{\sqrt{94}} = \frac{\sqrt{94}}{47}$

The direction Cosine are $\left(\frac{3\sqrt{94}}{94}, \frac{-9\sqrt{94}}{94}, \frac{\sqrt{94}}{47}\right)$

If $A = 4i + 6j + 2k$ and $B = i + 6j + k$ find

(a) The magnitude and direction Cosine Of $A+B$

(b) " " " Cosine Of $A-B$

Solution

(a) $A+B = \begin{array}{r} 4i + 6j + 2k \\ i + 6j + k \\ \hline 5i + 12j + 3k \end{array}$

$|A+B| = \sqrt{5^2 + 12^2 + 3^2}$
$= \sqrt{178}$

$\cos \alpha = \frac{5}{\sqrt{178}} = \frac{5\sqrt{178}}{178}$

$\cos \beta = \frac{12}{\sqrt{178}} = \frac{6\sqrt{178}}{89}$

$\cos \gamma = \frac{3}{\sqrt{178}} = \frac{3\sqrt{178}}{178}$

The direction Cosine are $\left(\frac{5\sqrt{178}}{178}, \frac{6\sqrt{178}}{89}, \frac{3\sqrt{178}}{178}\right)$

(b) $A-B = \begin{array}{r} 4i + 6j + 2k \\ -\quad i + 6j + k \\ \hline 3i + k \end{array}$

$|A-B| = \sqrt{3^2 + 1^2} = \sqrt{10}$

$\cos \alpha = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$

$$\cos\beta = \frac{0}{\sqrt{10}} = 0$$

$$\cos\gamma = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

The direction Cosines are $\left(\frac{3\sqrt{10}}{10}, 0, \frac{\sqrt{10}}{10}\right)$

Linear independence (Using the concept of determinant)

Given the three Vectors $A = i + 2j - 5k$, $B = i + j + 2k$ and $C = i + 4j - 19k$

$$\begin{vmatrix} 1 & 2 & -5 \\ 1 & 1 & 2 \\ 1 & 4 & -19 \end{vmatrix}$$

$$1\begin{vmatrix} 1 & 2 \\ 4 & -19 \end{vmatrix} - 2\begin{vmatrix} 1 & 2 \\ 1 & -19 \end{vmatrix} + -5\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix}$$

$1(-19-8) - 2(-19-2) - 5(4-1)$

$1(-27) - 2(-21) - 5(3)$

$= -27 + 42 - 15$

$= 42 - 42 = 0$

N.B If the final solution is zero, then it is linearly dependent but if the final solution is not zero, then it is linearly independent.

i.e $|Det| = 0$ (dependent)

$|Det| \neq 0$ (independent)

* Given $A = 2i + j + k$, $B = 3i + 4k$ and $C = i + j + k$. Determine if it is linearly dependent or independent.

Solution

$$\begin{vmatrix} 2 & 1 & 1 \\ 3 & 0 & 4 \\ 1 & 1 & 1 \end{vmatrix}$$

$$2\begin{vmatrix} 0 & 4 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 3 & 4 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 0 \\ 1 & 1 \end{vmatrix}$$

$2(0-4) - 1(3-4) + 1(3-0)$

$2(-4) - 1(-1) + 1(3)$

$-8 + 1 + 3 = -4$

$\therefore$ it is linearly independent.

Given $A = 2i + 2j$; $B = i - j + k$ and $C = -j - 2k$

Solution.

$$\begin{vmatrix} 2 & 2 & 0 \\ 1 & -1 & 1 \\ 0 & -1 & -2 \end{vmatrix}$$

$$2\begin{vmatrix} -1 & 1 \\ -1 & -2 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix} + 0\begin{vmatrix} 1 & -1 \\ 0 & -1 \end{vmatrix}$$

$2(2+1) - 2(-2) + 0$

$2(3) + 4 = 10$

$\therefore$ it is linearly independent.

Vector differentiation. 1/11/21

Let $\vec{r} = xi + yj + zk$

Then, the first Order derivative $\frac{dr}{dt}$ which is also called the differential coefficient of r with respect to t is

$$\frac{dr}{dt} = \frac{dxi}{dt} + \frac{dyj}{dt} + \frac{dzk}{dt}.$$

Hence, the Second Order derivative is $\frac{d^2r}{dt^2}$

Formulae Of Differentiation:

If A, B, C are differentiable Vector functions of u and ϕ is a differentiable Scalar function Of u, then

1. $\frac{d(A+B)}{du} = \frac{dA}{du} + \frac{dB}{du}$

2. $d(A \cdot B) = \frac{A\,dB}{du} + \frac{B\,dA}{du}$

3. $d(A \times B) = \frac{A \times dB}{du} + \frac{B \times dA}{du}$

4. $\frac{d(\phi A)}{du} = \frac{\phi\, dA}{du} \pm \frac{A\, d\phi}{du}$

Example.

If $r = (3t - t^3)i + 3t^2 j + (3t + t^3)k$

Find $\frac{dr}{dt}$ and $\frac{d^2 r}{dt^2}$.

Solution.

$$r = (3t - t^3)i + 3t^2 j + (3t + t^3)k$$

$$\frac{dr}{dt} = (3 - 3t^2)i + 6tj + (3 + 3t^2)k$$

$$\frac{d^2 r}{dt^2} = -6ti + 6j + 6tk$$

2. Given $r = t^2 i - tj + (2t+1)k$. Find the Values Of

(a) $\frac{dr}{dt}$ (b) $\frac{d^2 r}{dt^2}$ (c) $\frac{dr}{dt}\Big|_{t=0}$ (d) $\left|\frac{dr}{dt}\right|$ (e) $\left|\frac{d^2 r}{dt^2}\right|_{t=0}$

Solution.

(a) $\frac{dr}{dt} = 2ti - j + 2k$

(b) $\frac{d^2 r}{dt^2} = 2i$

(c) $\frac{dr}{dt} = 2ti - j + 2k$

$t = 0$

$$\frac{dr}{dt}\Big|_{t=0} = -j + 2k$$

(d) $\frac{dr}{dt} = 2ti - j + 2k$

$$\left|\frac{dr}{dt}\right| = \sqrt{(2t)^2 + (-1)^2 + (2)^2}$$

$$= \sqrt{4t^2 + 1 + 4}$$

$$= \sqrt{4t^2 + 5}$$

(e) $\frac{d^2 r}{dt^2} = 2i$

$$\left|\frac{d^2 r}{dt^2}\right| = \sqrt{2^2} = 2$$

3. $R = \sin t$

(a) $\frac{dR}{dt}$

(a) $\frac{dR}{dt} =$

(b) $\left|\frac{dR}{dt}\right|$

(c) $\frac{d^2 R}{dt^2}$

(d) $\left|\frac{d^2 R}{dt^2}\right|$

4. If

F

(a) $\frac{dR}{dt}$

$\left|\frac{dR}{d}\right.$

3. $R = \sin t\, i - \cos t\, j + k$. Find

(a) $\frac{dR}{dt}$ (b) $\left|\frac{dR}{dt}\right|$ (c) $\frac{d^2R}{dt^2}$ (d) $\left|\frac{d^2R}{dt^2}\right|$

Solution

(a) $\frac{dR}{dt} = \cos t\, i - (-\sin t) j$

$= \cos t\, i + \sin t\, j$

(b) $\left|\frac{dR}{dt}\right| = \sqrt{\cos^2 t + \sin^2 t}$

Recall $\sin^2 t + \cos^2 t = 1$

$= \sqrt{1} = 1$

$\left|\frac{d^2r}{dt^2}\right|$ NOTE

(c) $\frac{d^2R}{dt^2} = -\sin t\, i + \cos t\, j$

$= \cos t\, j - \sin t\, i$

(d) $\left|\frac{d^2R}{dt^2}\right| = \sqrt{(\cos^2 t) + (\sin^2 t)}$

$= \sqrt{\cos^2 t + \sin^2 t}$

$= \sqrt{1} = 1$

4. If $R = 3\cos t\, i + 3\sin t\, j + 4tk$

Find (a) $\frac{dR}{dt}$ (b) $\left|\frac{dR}{dt}\right|$

Solution.

(a) $\frac{dR}{dt} = 3(-\sin t) i + 3(\cos t\, j) + 4k$

$= -3\sin t\, i + 3\cos t\, j + 4k$

$\left|\frac{dR}{dt}\right| = \sqrt{(-3\sin t)^2 + (3\cos t)^2 + (4)^2}$

$= \sqrt{9\sin^2 t + 9\cos^2 t + 16}$

$= \sqrt{9(\sin^2 t + \cos^2 t) + 16}$

$= \sqrt{9(1) + 16} = \sqrt{9 + 16}$

$= \sqrt{25} = \underline{\underline{5}}$

Application to Velocity and acceleration.

Let $\vec{r} = xi + yj + zk$ be the position vector of a particle r. If the particle moves along a curve, then dr/dt gives the velocity of the particle and d^2r/dt^2 gives the acceleration of the particle.

Examples

1 A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$

(a) Determine the velocity and acceleration at any time.

(b) Find the magnitude of the acceleration at $t = 0$

Solution

Since $\vec{r} = xi + yj + zk$

$$\vec{r} = 2t^2 i + (t^2 - 4t)j + (3t - 5)k$$

(a) Velocity $= \dfrac{dr}{dt}$

$$\frac{dr}{dt} = 4ti + (2t - 4)j + 3k$$

(b) Acceleration $= \dfrac{d^2r}{dt^2}$

$$\frac{d^2r}{dt^2} = 4i + 2j$$

$$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{4^2 + 2^2} = \sqrt{16+4}$$

$$= \sqrt{20} = \sqrt{2\times2\times5}$$

$$= 2\sqrt{5}\,m/s^2$$

2. A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$ and $z = 2\sin 3t$, where t is the time.

(a) Determine the velocity and acceleration at any time t.

(b) Find the magnitude of the velocity and acceleration at $t = 0$.

Solution.

Since $\vec{r} = xi + yj + zk$

$$x = e^{-t}, \quad y = 2\cos 3t \quad z = 2\sin 3t.$$

$$\vec{r} = e^{-t}i + 2\cos 3t\, j + 2\sin 3t\, k$$

(a) $$\frac{d\vec{r}}{dt} = -e^{-t}i + 6(-\sin 3t)j + 6\cos 3t\, k$$

$$= -e^{-t}i - 6\sin 3t\, j + 6\cos 3t\, k.$$

$*\ \dfrac{d^2\vec{r}}{dt^2} = e^{-t}i - 18\cos 3t\,j + 18(\cancel{\cos}-\sin 3t)k$

$= e^{-t}i - 18\cos 3t\,j - 18\sin 3t\,k.$

(b) $\left|\dfrac{d\vec{r}}{dt}\right| = \sqrt{(-e^{-t})^2 + (-6\sin 3t)^2 + (6\cos 3t)^2}$

$= \sqrt{(-e^{-t})^2 + 36\sin^2 3t + 36\cos^2 3t}$

$= \sqrt{(-e^{-t})^2 + 36(\sin^2 3t + \cos^2 3t)}$

$\left|\dfrac{d\vec{r}}{dt}\right|_{t=0} = \sqrt{(-e^{0})^2 + 36}$

$= \sqrt{(-1)^2 + 36} = \sqrt{+1+36}$

$= \sqrt{37}\ m/s$

$\left|\dfrac{d^2\vec{r}}{dt^2}\right| = \sqrt{(e^{-t})^2 + (-18\cos 3t)^2 + (-18\sin 3t)^2}$

$= \sqrt{e^{-2t} + 324\cos^2 3t + 324\sin^2 3t}$

$= \sqrt{e^{-2t} + 324(\cos^2 3t + \sin^2 3t)}$

$\left|\dfrac{d^2\vec{r}}{dt^2}\right|_{t=0} = \sqrt{e^{-2t} + 324}$

$= \sqrt{e^{0} + 324} = \sqrt{1 + 324}$

$= \sqrt{325}$

$= \underline{5\sqrt{13}\ m/s^2}$

Or

$\dfrac{d\vec{r}}{dt}_{t=0} = -e^{-t}i - 6\sin 3t\,j + 6\cos 3t\,k$

$= -e^{0}i - 6\sin 0 + 6\cos 0$

Recall $\sin 0 = 0$, $\cos 0 = 1$

$\dfrac{d\vec{r}}{dt}_{t=0} = -i + 6k$

$\left|\dfrac{d\vec{r}}{dt}\right|_{t=0} = \sqrt{(-1)^2 + 6^2} = \sqrt{1+36} = \sqrt{37}\ m/s.$

$\dfrac{d^2\vec{r}}{dt^2}_{t=0} = e^{0}i - 18j$

$= i - 18j$

$\left|\dfrac{d^2\vec{r}}{dt^2}\right|_{t=0} = \sqrt{1 + 324} = \sqrt{325} = \underline{5\sqrt{13}\ m/s^2}.$

Vector - integration

Integration is the reverse of differentiation. We have definite and indefinite integrals.

Indefinite integrals are integrals that contain an arbitrary constant C is $\int f(t)\,dt = f(t) + C$

While definite integrals are those in which limits are applied (i.e upper limit and lower limit) e.g

$$\int_a^b f(t)\,dt = f(b) - f(a)$$

21/1/2

1. If $f(t) = ti + (t^2 - 2t)j + (3t^2 + t^3)k$. Find

(a) $\int f(t)\,dt$ (b) $\int_0^2 f(t)\,dt$

Solution

(a) $\int f(t)\,dt = \int [ti + (t^2 - 2t)j + (3t^2 + t^3)k\,dt]$

$$= \frac{t^2}{2}i + \left(\frac{t^3}{3} - \frac{2t^2}{2}\right)j + \left(\frac{3t^3}{3} + \frac{t^4}{4}\right)k + C$$

$$= \frac{1}{2}t^2 i + \left(\frac{1}{3}t^3 - t^2\right)j + \left(t^3 + \frac{1}{4}t^4\right)k + C.$$

(b) $\int_0^2 f(t)\,dt = \left[\frac{1}{2}t^2 i + \left(\frac{1}{3}t^3 - t^2\right)j + \left(t^3 + \frac{1}{4}t^4\right)k + C\right]_0^2$

$$= \left[\frac{1}{2}(2)^2 i + \left(\frac{1}{3}(2)^3 - 2^2\right)j + \left((2)^3 + \frac{1}{4}(2^4)\right)k\right] -$$

$$\left[\frac{1}{2}(0)^2 i + \left(\frac{1}{3}(0)^3 - 0^2\right)j + \left((0)^3 + \frac{1}{4}(0)^4\right)k\right]$$

$$= \frac{1}{2}(4)i + \left(\frac{1}{3}(8) - 4\right)j + \left(8 + \frac{1}{4}(16)\right)k$$

$$= 2i + \left(\frac{8 - 12}{3}\right)j + (8 + 4)k$$

$$= 2i + \left(\frac{-4}{3}\right)j + 12k$$

$$= 2i - \frac{4}{3}j + 12k$$

2. Evaluate $\int_1^2 f(t)\,dt$ if $f(t) = (t - t^2)i + 2t^3 j - 3k$.

Solution

$$\int_1^2 f(t)\,dt = \left[\left(\frac{t^2}{2} - \frac{t^3}{3}\right)i + \frac{2t^4}{4}j - 3tk\right]_1^2$$

$$= \left[\left(\frac{1}{2}t^2 - \frac{1}{3}t^3\right)i + \frac{1}{2}t^4 j - 3tk\right]_1^2$$

$$\left[\left(\frac{2^2}{2} - \frac{(2)^3}{3}\right)i + \frac{2(2)^4}{4}j - 3(2)k\right] - \left[\left(\frac{1^2}{2} - \frac{(1)^3}{3}\right)i + \frac{2(1)^4}{4}j - 3(1)k\right]$$

$$= \left(2 - \frac{8}{3}\right)i + 8j - 6k - \left[\left(\frac{1}{2} - \frac{1}{3}\right)i + \frac{1}{2}j - 3k\right]$$

$$= \frac{-2}{3}i + 8j - 6k - \frac{1}{6}i - \frac{1}{2}j + 3k.$$

$$= \frac{-2}{3}i - \frac{1}{6}i + 8j - \frac{1}{2}j - 6k + 3k$$

$$= \frac{-4i - i}{6} + \frac{16j - j}{2} - 3k$$

$$= \frac{-5i}{6} + \frac{15j}{2} - 3k$$

Partial differentiation of vectors.

Let f be a vector with more than one variables. F is represented as

$$f = u(x, y, z, t)i + v(x, y, z, t)j + w(x, y, z, t)k$$

then the partial derivation of f with respect to each of x, y, z and t is as below:

$$\frac{df}{dx} = \frac{du}{dx}i + \frac{dv}{dx}j + \frac{dw}{dx}k$$

$$\frac{df}{dy} = \frac{du}{dy}i + \frac{dv}{dy}j + \frac{dw}{dy}k$$

$$\frac{df}{dz} = \frac{du}{dz}i + \frac{dv}{dz}j + \frac{dw}{dz}k$$

$$\frac{df}{dt} = \frac{du}{dt}i + \frac{dv}{dt}j + \frac{dw}{dt}k$$

These are the first order partial derivatives of f with respect to x, y, z, t. Other higher order partial derivatives are $\frac{d^2f}{dx^2}$, $\frac{d^2f}{dy^2}$, $\frac{d^2f}{dxdy}$, $\frac{d^2f}{dydx}$, $\frac{d^2f}{dz^2}$ and $\frac{d^2f}{dt^2}$.

Example

1. If $F = (2x^2 - x^4)i + (e^{xy} - y\sin x)j + x^2\cos y\, k$

Find (a) $\frac{dF}{dx}$ (b) $\frac{dF}{dy}$ (c) $\frac{d^2F}{dx^2}$ (d) $\frac{d^2F}{dy^2}$ (e) $\frac{d^2F}{dxdy}$ (f) $\frac{d^2F}{dydx}$

Solution

$F = (2x^2 - x^4)i + (e^{xy} - y\sin x)j + x^2\cos y\, k$

(a) $\frac{dF}{dx} = (4x - 4x^3)i + (ye^{xy} - y\cos x)j + 2x\cos y\, k$

(b) $\frac{dF}{dy} = 0i + (xe^{xy} - \sin x)j + x^2(-\sin y)k$

$= (xe^{xy} - \sin x)j - x^2\sin y\, k$

(c) $\frac{dF}{dx} = (4x - 4x^3)i + (ye^{xy} - y\cos x)j + 2x\cos y\, k$

$\frac{d^2F}{dx^2} = (4 - 12x^2)i + [y^2e^{xy} - y(-\sin x)]j + 2\cos y\, k$

$= (4 - 12x^2)i + (y^2e^{xy} + y\sin x)j + 2\cos y\, k$

(d) $\frac{dF}{dy} = (xe^{xy} - \sin x)j - x^2\sin y\, k$

$\frac{d^2F}{dy^2} = x^2e^{xy}j - x^2(\cos y)k$

$= x^2e^{xy}j - x^2\cos y\, k$

(e) $\frac{d^2F}{dxdy} = ?$ $F = (2x^2 - x^4)i + (e^{xy} - y\sin x)j + x^2\cos y\, k$

$\frac{dF}{dx} = (4x - 4x^3)i + [ye^{xy} - y(\cos x)]j + 2x\cos y\, k$

$= (4x - 4x^3)i + (ye^{xy} - y\cos x)j + 2x\cos y\, k$

$\frac{d^2F}{dxdy} = (xye^{xy} - \cos x)j + 2x(-\sin y)k$

$= (xye^{xy} - \cos x)j - 2x\sin y\, k$

(f) $\frac{d^2F}{dydx} = ?$

$F = (2x^2 - x^4)i + (e^{xy} - y\sin x)j + x^2\cos y\, k$

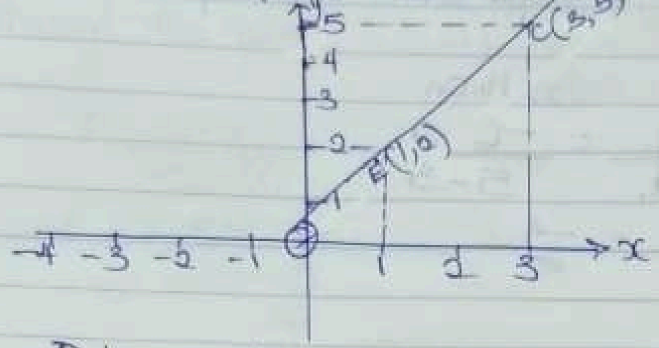
$\frac{dF}{dy} = (xe^{xy} - \sin x)j + x^2(-\sin y)k$

$= (xe^{xy} - \sin x)j - x^2\sin y\, k$

$\frac{d^2F}{dydx} = (xye^{xy} - \cos x)j - 2x\sin y\, k$

Straight lines. 18-11-21

Cartesian Plane $\mathbb{R}^2$



Distance between two points;

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$E(\underset{x_1}{1}, \underset{y_1}{2}) \quad C(\underset{x_2}{3}, \underset{y_2}{5})$

$= \sqrt{(3-1)^2 + (5-2)^2}$

$\sqrt{2^2 + 3^2}$

$= \sqrt{4+9} = \sqrt{13}$ Units.

Example 2 :> Find the distance between point $P(\overset{x_1}{3}, \overset{y_1}{-1})$ and $Q(\underset{x_2}{7}, \underset{y_2}{2})$

Solution

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7-3)^2 + (2+1)^2}$

$= \sqrt{4^2 + 3^2}$

$= \sqrt{16+9}$

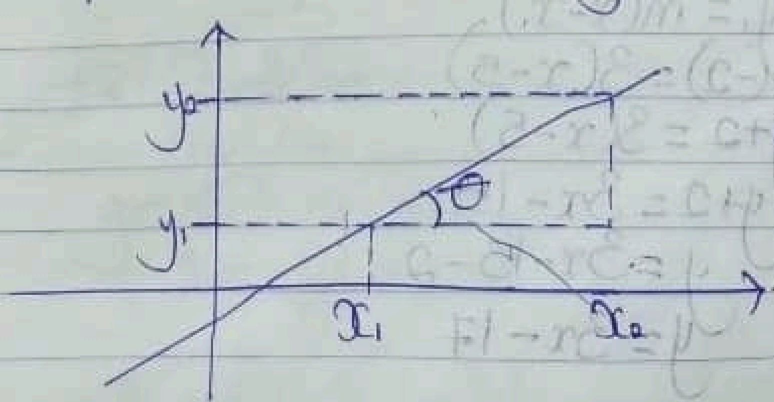
$= \sqrt{25}$

$= \underline{\underline{5}}$ Units

Inclination and Slope of a line

$0 \leq \theta \leq 180°$

Slope can also be called gradient.



Slope $m = \tan\theta$

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Example :→ Find the Slope between the point $\overset{x_1\ y_1}{(3, 2)}$ and $\overset{x_2\ y_2}{(5, -6)}$

Solution

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-2}{5-3}$$

$$= \frac{-8}{2} = \underline{\underline{-4}}$$

Equation Of a line :

$$y = mx + c$$

* $y = 5x + 7 \longrightarrow$ The Slope is 5

* $\frac{7y}{7} = \frac{5x}{7} + \frac{7}{7}$

$y = \frac{5}{7}x + 1$ ∴ The Slope is 5/7

Point Slope form

$(3, 4)$, $m = 3$. Find the equation Of the line :

$$m = \frac{y - y_1}{x - x_1}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 3(x - 3)$$

$$y - 4 = 3x - 9$$

$$y = 3x - 9 + 4$$

$$y = 3x - 5$$

Example :→ Find the equation Of the line having Slope 3 and passing through the point $(5, -2)$.

Solution :

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = 3(x - 5)$$

$$y + 2 = 3(x - 5)$$

$$y + 2 = 3x - 15$$

$$y = 3x - 15 - 2$$

$$y = 3x - 17$$

Example 2:> Find the equation of the line passing through the point $(3,2)$ and $(5,-6)$

Solution

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-6-2}{5-3} = \frac{-8}{2}$$

$$= -4$$

$$y - y_1 = m(x - x_1)$$

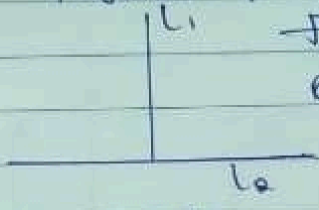
$$y - 2 = -4(x-3)$$

$$y - 2 = -4x + 12$$

$$y = -4x + 12 + 2$$

$$y = -4x + 14$$

Parallel & Perpendicular line:



- For parallel lines, the slope of l_1 is equal to l_2 i.e $m_1 = m_2$
- For perpendicular line $m_1 m_2 = -1$

Example :> Find the line parallel to the line $2x - 5y = 7$ and passing through the point $(3,4)$.

Solution

$$2x - 5y = 7$$

$$-5y = -2x + 7$$

$$y = \frac{-2x}{-5} + \frac{7}{-5}$$

$$y = \frac{2}{5}x - \frac{7}{5}$$

$$m_1 = \frac{2}{5}$$

For Parallelism

$$m_1 = m_2$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{2}{5}(x-3)$$

$$5(y-4) = 2(x-3)$$

$$5y - 20 = 2x - 6$$
$$5y = 2x - 6 + 20$$
$$5y = 2x + 14$$
$$y = \frac{2}{5}x + \frac{14}{5}$$

Example 2 :> Find the line Perpendicular to the line $2x - 5y = 7$ and passing through the point $(3, 4)$.

Solution

$$2x - 5y = 7$$
$$-5y = 7 - 2x$$
$$y = \frac{-2x}{-5} + \frac{7}{-5}$$
$$y = \frac{2}{5}x - \frac{7}{5}$$
$$m_1 = 2/5$$
$$m_1 m_2 = -1$$
$$\frac{2}{5} m_2 = -1$$
$$m_2 = \frac{-5}{2}$$
$$y - 4 = \frac{-5}{2}(x - 3)$$
$$2(y - 4) = -5(x - 3)$$
$$2y - 8 = -5x + 15$$
$$2y = -5x + 15 + 8$$
$$2y = -5x + 23$$

Distance between a point and a line.

$$d = \frac{|ax_1 + by_1 - c|}{\sqrt{a^2 + b^2}}$$

Example :> Given the line $3x + 4y = 2$, find the distance between the points $(2, -6)$ and the line, also find the distance between the points and the Origin.

Solution

$$3x + 4y = 2$$

1) $d = \frac{|ax_1 + by_1 - c|}{\sqrt{a^2+b^2}}$

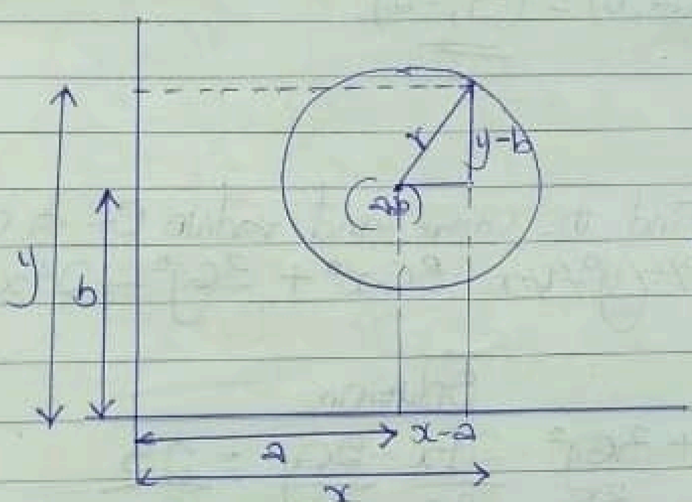
$d = \frac{|(3\times 2)+(4\times -6)-2|}{\sqrt{3^2+4^2}} = \frac{|6-24-2|}{\sqrt{9+16}}$

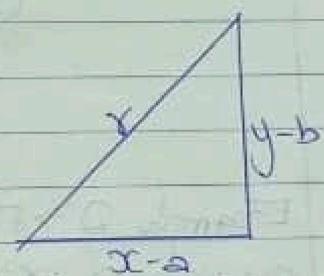
$= \frac{|-20|}{\sqrt{25}} = \frac{20}{5} = 4$

2) $d = \frac{|ax_1 + by_1 - c|}{\sqrt{a^2+b^2}} = \frac{|(3\times 0)+(4\times 0)-2|}{\sqrt{3^2+4^2}} = \frac{|0+0-2|}{\sqrt{25}}$

$= \frac{|-2|}{5} = \frac{2}{5}$

Circle 22/11/21





$r^2 = (x-a)^2 + (y-b)^2$

If the Centre is the Origin, it will be $r^2 = x^2 + y^2$

$r^2 = (x-a)^2 + (y-b)^2$

$r^2 = x^2 - 2ax + a^2 + y^2 - 2by + b^2$

$x^2 + y^2 - 2ax - 2by + a^2 + b^2 - r^2 =$

$x^2 + y^2 + 2gx + 2fy + C = 0$

Where $a = -g$ and $b = -f$ and $C = a^2 + b^2 - r^2$.

How to know the eqn of a Circle.

I It is a Second degree equation in x and y.

II Coefficient of x^2 and y^2 is equal

III Has no xy term.

* Find the equation of the circle centre $(\overset{x}{3}, \overset{y}{-2})$, $r = 2$ units

Solution

$(x-3)^2 + [y-(-2)]^2 = 2^2$

$x^2 - 6x + 9 + y^2 + 4y + \cancel{4} = \cancel{4}$

$x^2 + y^2 - 6x + 4y + 9 = 0.$

* Given the circle $x^2+y^2-6x+4y+9=0$ find the centre and radius of the circle.

Solution

$x^2+y^2-6x+4y+9=0$

$x^2-6x+y^2+4y=-9$

$x^2-6x+(-3)^2+y^2+4y+(2)^2=-9+(-3)^2+2^2$

$(x-3)^2+(y+2)^2=\cancel{-9}+\cancel{9}+2^2$

$(x-3)^2+(y+2)^2=2^2.$

$a=3,\ b=-2;\ r=2.$

$(a,b)=(3,-2)$

Example 2:> Find the centre and radius of a circle whose equation is ~~x^2+y^2+6x~~ $36x^2+36y^2-24x-36y-23=0$.

Solution

$$\frac{36x^2}{36}+\frac{36y^2}{36}-\frac{24x}{36}-\frac{36y}{36}=\frac{23}{36}$$

$$x^2+y^2-\frac{2}{3}x-y=\frac{23}{36}$$

$$x^2-\frac{2}{3}x+y^2-y=\frac{23}{36}$$

$$\left(x-\frac{1}{3}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{23}{36}+\left(\frac{-1}{3}\right)^2+\left(\frac{-1}{2}\right)^2$$

$$\left(x-\frac{1}{3}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{23}{36}+\frac{1}{9}+\frac{1}{4}$$

$$\left(x-\frac{1}{3}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{23+4+9}{36}$$

$$\left(x-\frac{1}{3}\right)^2+\left(y-\frac{1}{2}\right)^2 = \left(\frac{\sqrt{36}}{\sqrt{36}}\right)^2$$

Centre $(1/3, 1/2)$ $\qquad r = \sqrt{\frac{36}{36}} = \sqrt{1} = \underline{\underline{1}}$

Example 3:→ Find the equation of the circle whose centre is $(5,-4)$ and which passes through the point $(-3,2)$.

Solution

$$\text{distance} = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$
$$= \sqrt{[2-(-4)]^2+(-3-5)^2}$$
$$\sqrt{6^2+(-8)^2}$$
$$= \sqrt{36+64}$$
$$= \sqrt{100}$$
$$r = 10 \text{ units}$$
$$(x-a)^2+(y-b)^2 = r^2.$$
$$(x-5)^2+[y-(-4)]^2 = (10)^2$$
$$x^2-10x+25+y^2+8y+16 = 100$$
$$x^2+y^2-10x+8y = 100-(25+16)$$
$$x^2+y^2-10x+8y = 100-41$$
$$x^2+y^2-10x+8y = 59$$
$$\underline{x^2+y^2-10x+8y-59=0}$$

Equation of the tangent at the point (x_1, y_1)

$$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0.$$

Example:→ Show ~~that~~ the point $(2,3)$ lies on the circle $x^2+y^2-3x+4y-19=0$. Hence or otherwise, determine the eqn of the tangent ~~to~~ the ~~solution~~ circle at the point $(2,3)$.

Soln:→ $(2)^2+(3)^2-3(2)+4(3)-19$

$$4+9-6+12-19$$
$$= 4+9+12-6-19$$
$$= 25-25$$
$$= 0. \qquad \text{Q.E.D}$$

The equation $x^2+y^2-3x+4y-19=0$ lies on the circle.

$$x^2 + y^2 - 3x + 4y - 19 = 0$$
$$x^2 + y^2 + 2gx + 2fy + c = 0$$

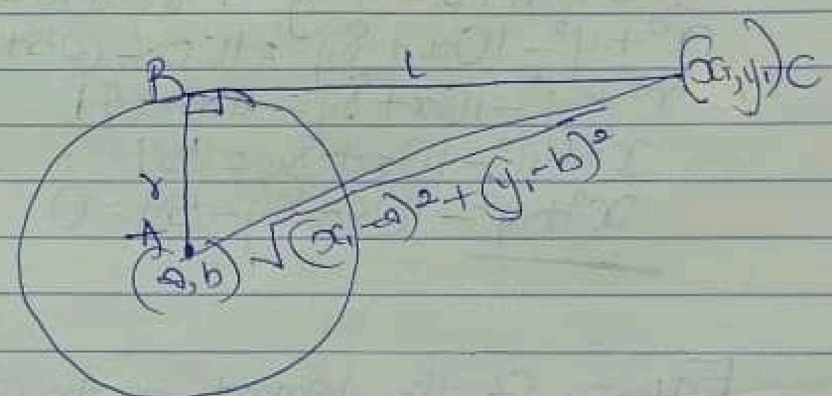
$2g = -3$ $\qquad$ $2f = 4$

$g = \frac{-3}{2}$ $\qquad$ $f = \frac{4}{2} = 2$

$\left(\underset{x_1}{2}, \underset{y_1}{3}\right)$ $\qquad$ $c = -19.$

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$
$$x(2) + y(3) + \left(\frac{-3}{2}\right)(x+2) + 2(y+3) - 19 = 0$$
$$2x + 3y - \frac{3x}{2} - 3 + 2y + 6 - 19 = 0$$
$$4x + 6y - 3x - 6 + 4y + 12 - 38 = 0$$
$$x + 10y - 44 + 12 = 0$$
$$x + 10y - 32 = 0$$

Length of a tangent to a Circle from an external point.



$$|AC|^2 = |AB|^2 + |BC|^2$$
$$|BC| = \sqrt{|AC|^2 - |AB|^2}$$

Find the length Of the tangent to the Circle $x^2 + y^2 - 2x - 4y - 4 = 0$ from the point $(8, 10)$

Solution

$$x^2 + y^2 - 2x - 4y - 4 = 0$$
$$x^2 - 2x + y^2 - 4y = 4$$
$$(x-1)^2 + (y-2)^2 = 4 + 1 + 4$$
$$(x-1)^2 + (y-2)^2 = 9$$
$$(x-1)^2 + (y-2)^2 = 3^2$$

Centre $(1, 2)$; $r = 3$

$$|AC| = \sqrt{(10-2)^2 + (8-1)^2}$$
$$= \sqrt{8^2 + 7^2}$$
$$= \sqrt{64 + 49}$$
$$= \sqrt{113} \text{ units.}$$
$$|BC| = \sqrt{|AC|^2 - |AB|^2}$$
$$= \sqrt{113 - 9}$$
$$= \sqrt{104}$$
$$= \sqrt{4 \times 26}$$
$$= 2\sqrt{26} \text{ units}$$

Loci

The set of all point that satisfy specified condition is called the locus (loci in plural) of the point under the condition.

Example :> Find the locus of point $P(x, y)$ equidistant from point $P_1(1, 0)$ and point $P_2(3, 0)$

Solution

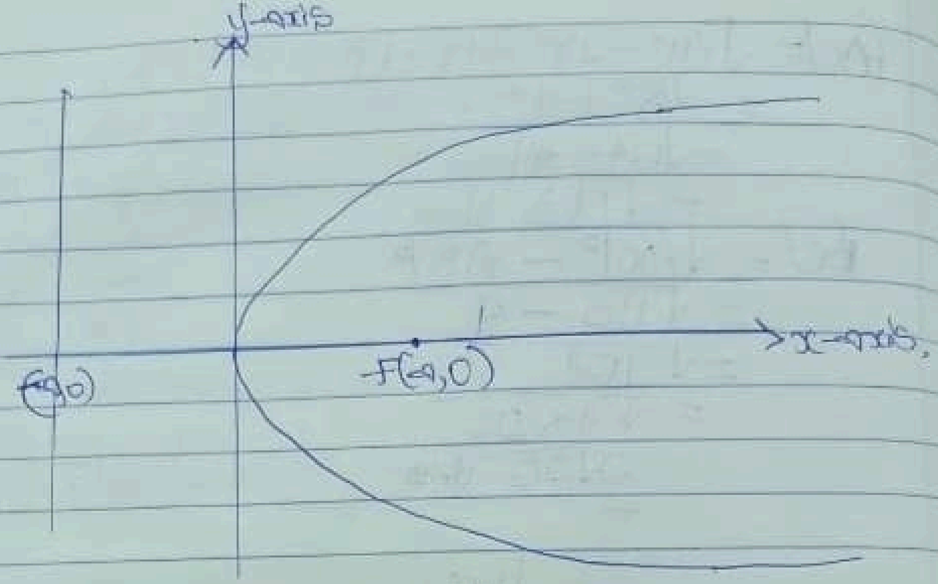
$P_1(1,0)$ $P(x, y)$ $P_2(3,0)$

$$\sqrt{(x-1)^2 + (y-0)^2} = \sqrt{(3-x)^2 + (0-y)^2}$$

$$(x-1)^2 + \cancel{y^2} = (3-x)^2 + \cancel{(-y)^2}.$$
$$(x-1)^2 = (3-x)^2.$$
$$\cancel{x^2} - 2x + 1 = 9 - 6x + \cancel{x^2}.$$
$$-2x + 6x = 9 - 1$$
$$4x = 8$$
$$x = \frac{8}{4} = 2$$

Parabola.

A parabola is defined as the locus of point B equidistant from a given point and a given line i.e $PF = PD$ where F is the given point called focus and PD is the distance to the given line called the directrix. A line through the focus perpendicular to the directrix is called the axis of symmetry and the point on the axis half way between a directrix and the focus is called the vertex.



I Vertex : (0,0)

II Focus: (a,0)

III ~~Axis of~~ Directrix: $x = -a$

IV $y^2 = 4ax \rightarrow$ General eqn Of Parabola.

Replacing x by $x-h$ has the effect of shifting the graph by the absolute value of $|h|$, to the right if h is positive, and/or to the left if h is negative.

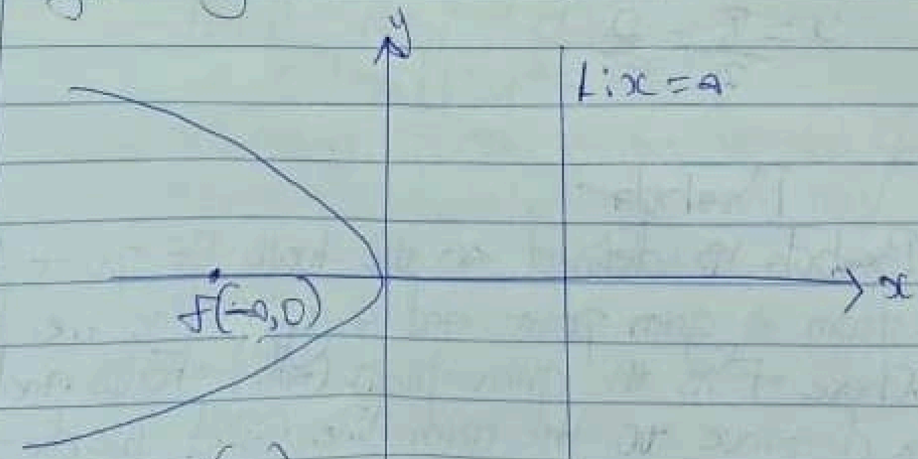
Similarly replacing y by $y-k$ has the effect Of Shifting the graph Of the equation by $|k|$, up if k is positive, down if k is negative.

I Vertex (h, k)

II Focus: $F(h+a, k)$

III Directrix: $x = h-a$

IV Equation: $(y-k)^2 = \pm 4a(x-h)$



i) Vertex : (0,0)

ii) Focus: $-a, 0$

iii) Directrix: $x = a$