NIGERIA MARITIME UNIVERSITY, OKERENKOKO, DELTA STATE
FACULTY OF ENGINEERING
COURSE TITLE: ENGINEERING MATHEMATICS 1V
COURSE CODE: ENG 308.2
SECOND SEMESTER EXAMINATION 2020/2021 SESSION

Instruction: Answer any 5 Questions

Date: 2nd March, 2022. Time: 3 hours

QUESTION 1

Determine the power series solution of the Legendre equation:

$$(1-x^2)y'' - 2xy' + k(k+1)y = 0$$

When C =1

8 marks

Hence, find the series solution at

b. K=0 — 3 marks

c. K=2, up to and including the 5th term — 3 marks

QUESTION 2

a Solve the equation:

$$dz/dx + A(x)z = B(x)\,;\ z(0) = 1$$

Where,

$$A(x) = \begin{cases} 2; & 1 \le x \le 3 \\ 1; & x \ge 3 \end{cases}$$

up

down

and B(x)= cosx — 7 marks

b. A cubical tank of side 3m is filled with water to a height of hm. The water is drained through a circular hole of 0.1m in diameter.

i. Obtain the differential equation relating the height, h of water at time, t

ii. Solve the differential equation for the initial conditions t =0, h =2

iii. How long does it take to empty the tank from 2m full?

iv. State the Engineering statement that justifies (iii) in minutes.

7 marks

2b find $\frac{dh}{dt} = -$

QUESTION 3

a. Solve the equation:

4 marks

$$(x^2 + xy)dy/dx = xy - y^2$$

b. An object of mass, m is allowed to fall from rest in a resisting medium. If the resistance, R is directly proportional to the magnitude of the velocity and the acceleration due to gravity, g is constant.

i. Prove that the equation of motion of this object is given by

$$dv/dt + kv/m = g$$

ii. Obtain the equation for the velocity, v at any time t.

iii. Obtain the equation for the distance, s fallen by the object at any time t.

10 marks

QUESTION 4

a. Solve for z

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$$

4 marks

b. The partial differential equation for a transient one-dimensional heat conduction in a rod is given as $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. The rod of length l with insulated sides is initially at a temperature u. Its end is suddenly cooled to $0° C$ and are kept at that temperature. Prove that the temperature function u (x,t) is given by

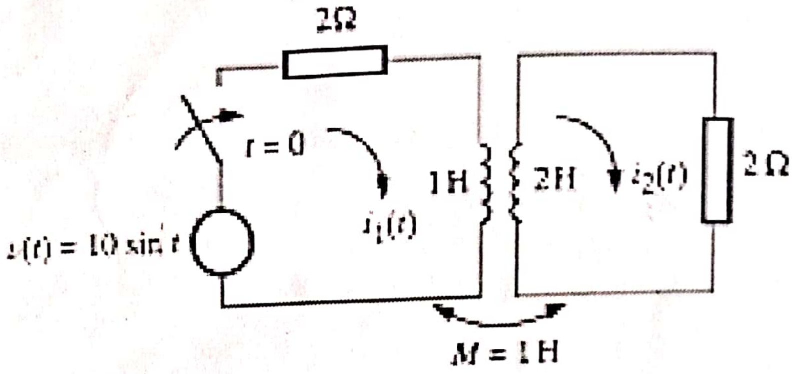
$$u\,(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} . e^{-\frac{c^2\,\pi^2\,n^2\,t}{l^2}}$$, where b_n is a constant determined from the equation. Let each side of the variable be $-p^2$.

Hence show that at initial conditions, $U_0 = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$

10 marks

QUESTION 5

At time t = 0, with no currents flowing a voltage $v(t) = 10 \sin t$ is applied to the primary circuit of a transformer that has a mutual inductance of 1H, as shown in figure below. Denoting the current flowing at time t in the secondary circuit by $i_2(t)$,



Show that

$$i_2(t) = -e^{t} + \frac{12}{37}e^{-6t} + \frac{25}{37}\cos t + \frac{35}{37}\sin t$$

(Hint: Kirchhoff's Voltage Law is useful here. Kindly pay attention to the mutual inductance set-up using Laplace Transform)

10 marks

b. Find the Fourier Transform of the function $F(x) = e^{-a|x|}, -\infty < x < \infty$

4 marks

QUESTION 6

a. The equation of motion of a body performing damped forced vibrations in engine sitting is

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Compute the equation, given that $x=0.1$ and $\frac{dx}{dt} = 0$ when $t=0$.

Write the steady state solution of the engine sitting in the form $K\sin(t + \alpha)$.

8 marks

b. For a horizontal cantilever of length L, with load per unit length, the equation of bending is $EI\frac{d^2y}{dx^2} = W(L-x)^2$

Where E, I and L are constants. If $y = 0$ and $\frac{dy}{dx} = 0$ at $x=0$. Find the value of y in terms of x

Hence find the value of y when $x=L$

6 marks