1

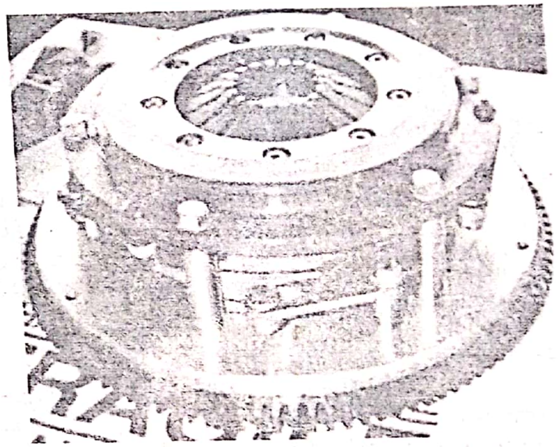
OBJECTIVE TYPE QUESTIONS

1. A spring used to absorb shocks and vibrations is
   (a) closely-coiled helical spring (b) open-coiled helical spring
   (c) conical spring (d) leaf spring
2. The spring mostly used in gramophones is
   (a) helical spring (b) conical spring
   (c) laminated spring (d) flat spiral spring
3. Which of the following spring is used in a mechanical wrist watch?
   (a) Helical compression spring (b) Spiral spring
   (c) Torsion spring (d) Bellevile spring
4. When a helical compression spring is subjected to an axial compressive load, the stress induced in the wire is
   (a) tensile stress (b) compressive stress
   (c) shear stress (d) bending stress
5. In a close coiled helical spring, the spring index is given by D/d where D and d are the mean coil diameter and wire diameter respectively. For considering the effect of curvature, the Wahl's stress factor K is given by
   (a) $\frac{4C-1}{4C+4}+\frac{0.615}{C}$ (b) $\frac{4C-1}{4C-4}+\frac{0.615}{C}$
   (c) $\frac{4C+1}{4C-4}-\frac{0.615}{C}$ (d) $\frac{4C+1}{4C+4}-\frac{0.615}{C}$
6. When helical compression spring is cut into halves, the stiffness of the resulting spring will be
   (a) same (b) double
   (c) one-half (d) one-fourth
7. Two close coiled helical springs with stiffness k_1 and k_2 respectively are conected in series. The stiffness of an equivalent spring is given by
   (a) $\frac{k_1 \cdot k_2}{k_1 + k_2}$ (b) $\frac{k_1 - k_2}{k_1 + k_2}$
   (c) $\frac{k_1 + k_2}{k_1 \cdot k_2}$ (d) $\frac{k_1 - k_2}{k_1 \cdot k_2}$
8. When two concentric coil springs made of the same material, having same length and compressed equally by an axial load, the load shared by the two springs will be .... to the square of the diameters of the wires of the two springs.
   (a) directly proportional (b) inversely proportional
   (c) equal to
9. A leaf spring in automobiles is used
   (a) to apply forces (b) to measure forces
   (c) to absorb shocks (d) to store strain energy
10. In leaf springs, the longest leaf is known as
   (a) lower leaf (b) master leaf
   (c) upper leaf (d) none of these

ANSWERS

1. (e)	2. (d)	3. (c)	4. (c)	5. (b)
6. (b)	7. (a)	8. (a)	9. (c)	10. (b)

24 Clutches



24.1 Introduction

A clutch is a machine member used to connect a driving shaft to a driven shaft so that the driven shaft may be started or stopped at will, without stopping the driving shaft. The use of a clutch is mostly found in automobiles. A little consideration will show that in order to change gears or to stop the vehicle, it is required that the driven shaft should stop, but the engine should continue to run. It is, therefore, necessary that the driven shaft should be disengaged from the driving shaft. The engagement and disengagement of the shafts is obtained by means of a clutch which is operated by a lever.

24.2 Types of Clutches

Following are the two main types of clutches commonly used in engineering practice :

1. Positive clutches, and **2.** Friction clutches.

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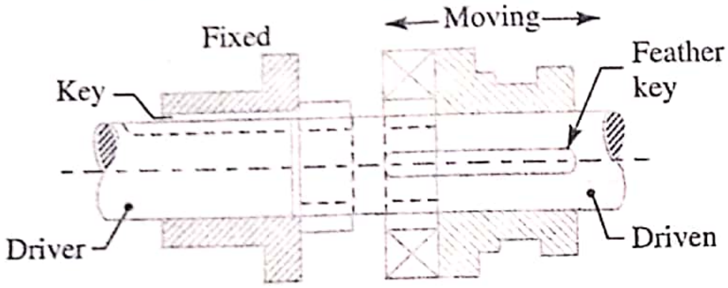
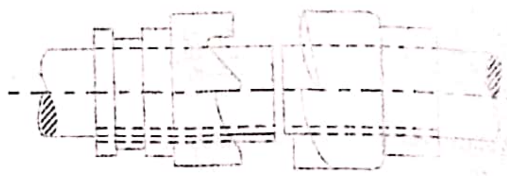
We shall now discuss these clutches in the following pages.

24.3 Positive Clutches

The positive clutches are used when a positive drive is required. The simplest type of a positive clutch is a ***jaw*** or ***claw clutch***. The jaw clutch permits one shaft to drive another through a direct contact of interlocking jaws. It consists of two halves, one of which is permanently fastened to the



(*a*) Square jaw clutch.



(*b*) Spiral jaw clutch.

Fig. 24.1. Jaw clutches.

driving shaft by a sunk key. The other half of the clutch is movable and it is free to slide axially on the driven shaft, but it is prevented from turning relatively to its shaft by means of feather key. The jaws of the clutch may be of square type as shown in Fig. 24.1 (*a*) or of spiral type as shown in Fig. 24.1 (*b*).

A square jaw type is used where engagement and disengagement in motion and under load is not necessary. This type of clutch will transmit power in either direction of rotation. The spiral jaws may be left-hand or right-hand, because power transmitted by them is in one direction only. This type of clutch is occasionally used where the clutch must be engaged and disengaged while in motion. The use of jaw clutches are frequently applied to sprocket wheels, gears and pulleys. In such a case, the non-sliding part is made integral with the hub.

24.4 Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces. In automobiles, friction clutch is used to connect the engine to the drive shaft. In operating such a clutch, care should be taken so that the friction surfaces engage easily and gradually bring the driven shaft up to proper speed. The proper alignment of the bearing must be maintained and it should be located as close to the clutch as possible. It may be noted that :

1. The contact surfaces should develop a frictional force that may pick up and hold the load with reasonably low pressure between the contact surfaces.
2. The heat of friction should be rapidly *dissipated and tendency to grab should be at a minimum.
3. The surfaces should be backed by a material stiff enough to ensure a reasonably uniform distribution of pressure.

24.5 Material for Friction Surfaces

The material used for lining of friction surfaces of a clutch should have the following characteristics :

* During operation of a clutch, most of the work done against frictional forces opposing the motion is liberated as heat at the interface. It has been found that at the actual point of contact, the temperature as high as 1000°C is reached for a very short duration (*i.e.* for 0.0001 second). Due to this, the temperature of the contact surfaces will increase and may destroy the clutch.

24.8 Single Disc or Plate Clutch

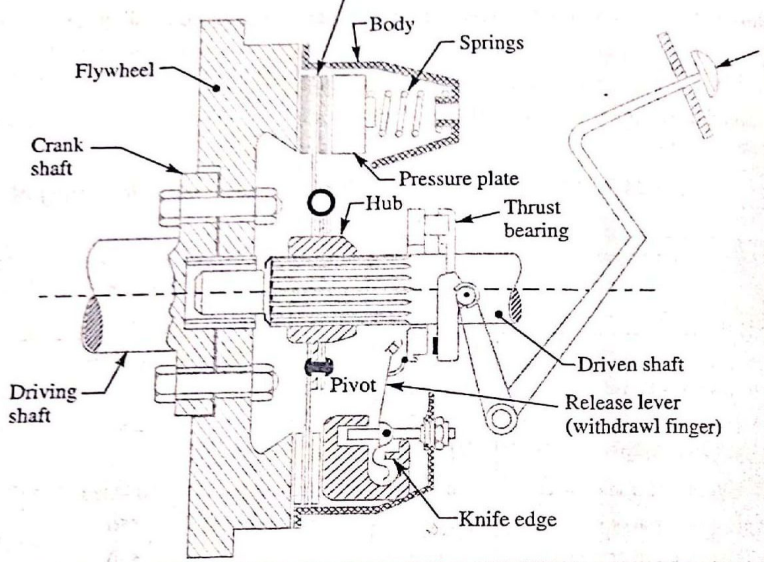
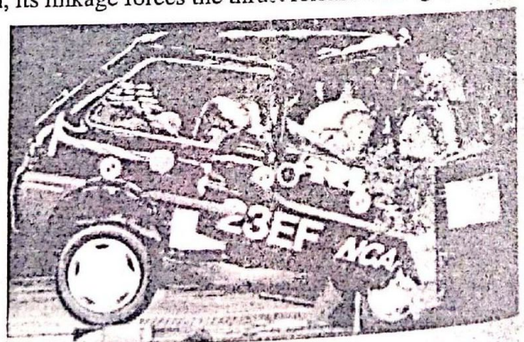


Fig. 24.2. Single disc or plate clutch.

A single disc or plate clutch, as shown in Fig 24.2, consists of a clutch plate whose both sides are faced with a frictional material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel. Both the pressure plate and the flywheel rotate with the engine crankshaft or the driving shaft. The pressure plate pushes the clutch plate towards the flywheel by a set of strong springs which are arranged radially inside the body. The three levers (also known as release levers or fingers) are carried on pivots suspended from the case of the body. These are arranged in such a manner so that the pressure plate moves away from the flywheel by the inward movement of a thrust bearing. The bearing is mounted upon a forked shaft and moves forward when the clutch pedal is pressed.

When the clutch pedal is pressed down, its linkage forces the thrust release bearing to move in towards the flywheel and pressing the longer ends of the levers inward. The levers are forced to turn on their suspended pivot and the pressure plate moves away from the flywheel by the knife edges, thereby compressing the clutch springs. This action removes the pressure from the clutch plate and thus moves back from the flywheel and the driven shaft becomes stationary. On the other hand, when the foot is taken off from the clutch pedal, the thrust bearing moves back by the levers. This allows the springs to extend and thus the pressure plate pushes the clutch plate back towards the flywheel.



When a car hits an object and decelerates quickly the objects are thrown forward as they continue to move forwards due to inertia

The axial pressure exerted by the spring provides a frictional force in the circumferential direction when the relative motion between the driving and driven members tends to take place. If the torque due to this frictional force exceeds the torque to be transmitted, then no slipping takes place and the power is transmitted from the driving shaft to the driven shaft.

24.9 Design of a Disc or Plate Clutch

Consider two friction surfaces maintained in contact by an axial thrust (W) as shown in Fig. 24.3 (a).

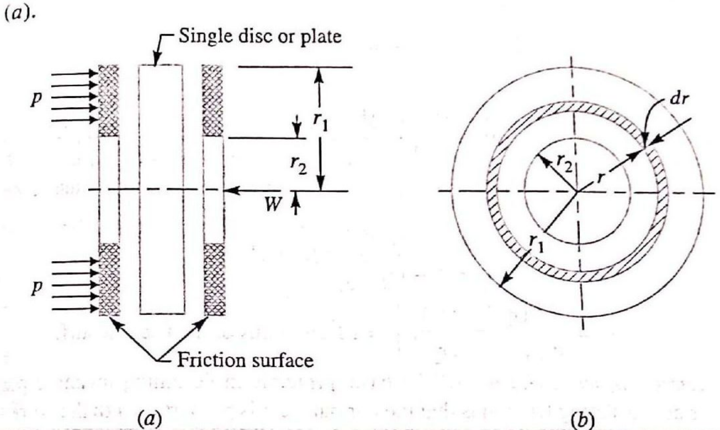


Fig. 24.3. Forces on a disc clutch.

Let T = Torque transmitted by the clutch,

p = Intensity of axial pressure with which the contact surfaces are held together,

r_1 and r_2 = External and internal radii of friction faces,

r = Mean radius of the friction face, and

μ = Coefficient of friction.

Consider an elementary ring of radius r and thickness dr as shown in Fig. 24.3 (b).

We know that area of the contact surface or friction surface

$$= 2\pi\, r.dr$$

∴ Normal or axial force on the ring,

$$\delta W = \text{Pressure} \times \text{Area} = p \times 2\pi\, r.dr$$

and the frictional force on the ring acting tangentially at radius r,

$$F_r = \mu \times \delta W = \mu.p \times 2\pi\, r.dr$$

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p \times 2\pi\, r.dr \times r = 2\,\pi\,\mu\, p.\, r^2.dr$$

We shall now consider the following two cases :

1. When there is a uniform pressure, and
2. When there is a uniform axial wear.

1. ***Considering uniform pressure.*** When the pressure is uniformly distributed over the entire area of the friction face as shown in Fig. 24.3 (a), then the intensity of pressure,

$$p = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]}$$

(4)

where W = Axial thrust with which the friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is

$$T_r = 2\pi\,\mu.p.r^2.dr$$

Integrating this equation within the limits from r_2 to r_1 for the total friction torque.

$\therefore$ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\,\mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\,\mu.p\left[\frac{(r_1)^3-(r_2)^3}{3}\right] = 2\pi\,\mu\times\frac{W}{\pi\,[(r_1)^2-(r_2)^2]}\left[\frac{(r_1)^3-(r_2)^3}{3}\right]$$

... (Substituting the value of p)

$$= \frac{2}{3}\,\mu.W\left[\frac{(r_1)^3-(r_2)^3}{(r_1)^2-(r_2)^2}\right] = \mu.W.R$$

where $R = \dfrac{2}{3}\left[\dfrac{(r_1)^3-(r_2)^3}{(r_1)^2-(r_2)^2}\right]$ = Mean radius of the friction surface.

2. *Considering uniform axial wear.* The basic principle in designing machine parts that are subjected to wear due to sliding friction is that the normal wear is proportional to the work of friction. The work of friction is proportional to the product of normal pressure (p) and the sliding velocity (V). Therefore,

Normal wear $\propto$ Work of friction $\propto p.V$

or $\quad p.V = K$ (a constant) or $p = K/V$ ...(i)

It may be noted that when the friction surface is new, there is a uniform pressure distribution over the entire contact surface. This pressure will wear most rapidly where the sliding velocity is maximum and this will reduce the pressure between the friction surfaces. This wearing-in process continues until the product $p.V$ is constant over the entire surface. After this, the wear will be uniform as shown in Fig. 24.4.

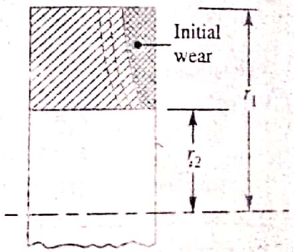


Fig. 24.4. Uniform axial wear.

Let p be the normal intensity of pressure at a distance r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$$p.r = C \text{ (a constant) or } p = C/r \qquad ...(ii)$$

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r}\times 2\pi r.dr = 2\pi C.dr$$

$\therefore$ Total force acing on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi\,C\,dr = 2\pi\,C\,[r]_{r_2}^{r_1} = 2\pi\,C\,(r_1 - r_2)$$

or

$$C = \frac{W}{2\pi\,(r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\,\mu.p.r^2.dr = 2\pi\,\mu\times\frac{C}{r}\times r^2.dr = 2\pi\mu.C.r.dr \qquad ...(\because p = C/r)$$

$\therefore$ Total frictional torque acting on the friction surface (or on the clutch),

$$T = \int_{r_2}^{r_1} 2\pi\,\mu\,C.r.dr = 2\pi\,\mu\,C\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu\,.C\left[\frac{(r_1)^2-(r_2)^2}{2}\right] = \pi\,\mu.C\,[(r_1)^2-(r_2)^2]$$

$$= \pi\mu\times\frac{W}{2\pi\,(r_1-r_2)}[(r_1)^2-(r_2)^2] = \frac{1}{2}\times\mu.W\,(r_1+r_2) = \mu.W.R$$

where $R = \dfrac{r_1+r_2}{2}$ = Mean radius of the friction surface.

Notes : 1. In general, total frictional torque acting on the friction surfaces (or on the clutch) is given by

$$T = n.\mu.W.R$$

where n = Number of pairs of friction (or contact) surfaces, and

R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3-(r_2)^3}{(r_1)^2-(r_2)^2}\right] \qquad \text{... (For uniform pressure)}$$

$$= \frac{r_1+r_2}{2} \qquad \text{... (For uniform wear)}$$

2. For a single disc or plate clutch, normally both sides of the disc are effective. Therefore a single disc clutch has two pairs of surfaces in contact (*i.e.* $n = 2$).

3. Since the intensity of pressure is maximum at the inner radius (r_2) of the friction or contact surface, therefore equation (*ii*) may be written as

$$p_{max}\times r_2 = C \quad \text{or} \quad p_{max} = C/r_2$$

4. Since the intensity of pressure is minimum at the outer radius (r_1) of the friction or contact surface, therefore equation (*ii*) may be written as

$$p_{min}\times r_1 = C \quad \text{or} \quad p_{min} = C/r_1$$

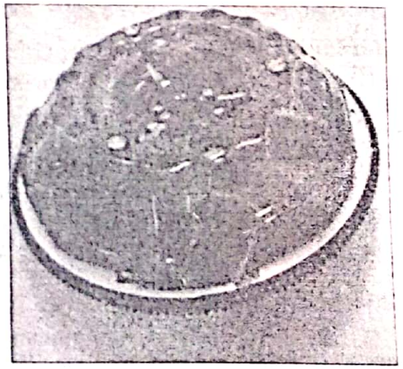
5. The average pressure (p_{av}) on the friction or contact surface is given by

$$p_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}} = \frac{W}{\pi\,[(r_1)^2-(r_2)^2]}$$

6. In case of a new clutch, the intensity of pressure is approximately uniform, but in an old clutch, the uniform wear theory is more approximate.

7. The uniform pressure theory gives a higher friction torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

24.10 Multiple Disc Clutch


A twin disk clutch

A multiple disc clutch, as shown in Fig. 24.5, may be used when a large torque is to be transmitted. The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion (except for the last disc). The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft. The multiple disc clutches are extensively used in motor cars, machine tools etc.

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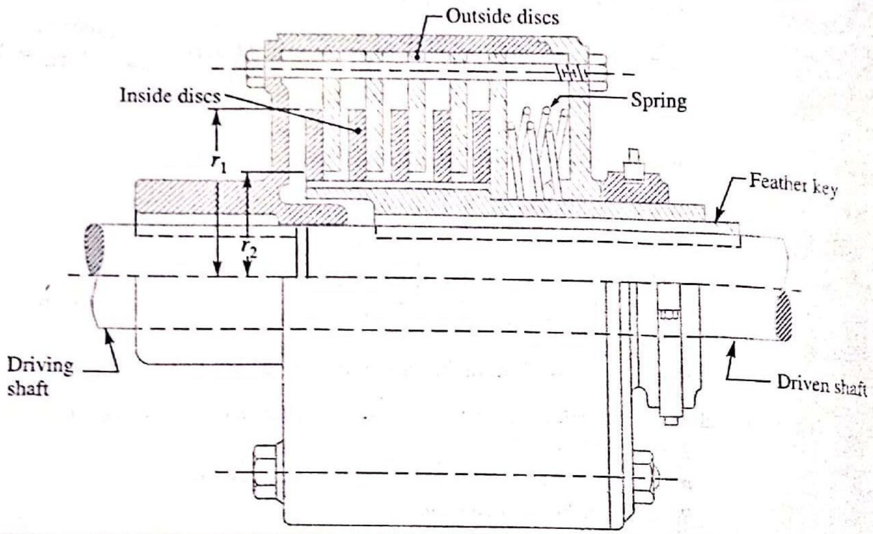

Fig. 24.5. Multiple disc clutch.

Let n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

∴ Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

and total frictional torque acting on the friction surfaces or on the clutch,

$$T = n.\mu.W.R$$

where R = Mean radius of friction surfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \text{... (For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \text{... (For uniform wear)}$$

Example 24.1. *Determine the maximum, minimum and average pressure in a plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.*

Solution. Given : $W = 4$ kN $= 4000$ N ; $r_2 = 50$ mm ; $r_1 = 100$ mm

Maximum pressure

Let p_{max} = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 50\, p_{max}$$

We also know that total force on the contact surface (W),

$$4000 = 2\pi C\,(r_1 - r_2) = 2\pi \times 50\, p_{max}\,(100 - 50) = 15\ 710\, p_{max}$$

$$\therefore \quad p_{max} = 4000 / 15\ 710 = 0.2546 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Minimum pressure

Let p_{min} = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), therefore,

$$p_{min} \times r_1 = C \quad \text{or} \quad C = 100\, p_{min}$$

We know that the total force on the contact surface (W),

$$4000 = 2\pi C\,(r_1 - r_2) = 2\pi \times 100\, p_{min}\,(100 - 50) = 31\ 420\, p_{min}$$

$$\therefore \quad p_{min} = 4000 / 31\ 420 = 0.1273 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surface}} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

$$= \frac{4000}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Example 24.2. *A plate clutch having a single driving plate with contact surfaces on each side is required to transmit 110 kW at 1250 r.p.m. The outer diameter of the contact surfaces is to be 300 mm. The coefficient of friction is 0.4.*

(a) Assuming a uniform pressure of 0.17 N/mm²; determine the inner diameter of the friction surfaces.

(b) Assuming the same dimensions and the same total axial thrust, determine the maximum torque that can be transmitted and the maximum intensity of pressure when uniform wear conditions have been reached.

Solution. Given : $P = 110$ kW $= 110 \times 10^3$ W ; $N = 1250$ r.p.m. ; $d_1 = 300$ mm or $r_1 = 150$ mm ; $\mu = 0.4$; $p = 0.17$ N/mm²

(a) Inner diameter of the friction surfaces

Let d_2 = Inner diameter of the contact or friction surfaces, and

r_2 = Inner radius of the contact or friction surfaces.

We know that the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{110 \times 10^3 \times 60}{2\,\pi \times 1250} = 840 \text{ N-m}$$

$$= 840 \times 10^3 \text{ N-mm}$$

Axial thrust with which the contact surfaces are held together,

$$W = \text{Pressure} \times \text{Area} = p \times \pi\,[(r_1)^2 - (r_2)^2]$$

$$= 0.17 \times \pi\,[(150)^2 - (r_2)^2] = 0.534\,[(150)^2 - (r_2)^2] \quad \text{... (i)}$$

and mean radius of the contact surface for uniform pressure conditions,

$$R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3}\left[\frac{(150)^3 - (r_2)^3}{(150)^2 - (r_2)^2}\right]$$

∴ Torque transmitted by the clutch (T),

$$840 \times 10^3 = n.\mu.W.R$$

$$= 2 \times 0.4 \times 0.534\,[(150)^2 - (r_2)^2] \times \frac{2}{3}\left[\frac{(150)^3 - (r_2)^3}{(150)^2 - (r_2)^2}\right] \quad \text{...}(\because n = 2)$$

$$= 0.285\,[(150)^3 - (r_2)^3]$$

or $$(150)^3 - (r_2)^3 = 840 \times 10^3 / 0.285 = 2.95 \times 10^6$$

$$\therefore \quad (r_2)^3 = (150)^3 - 2.95 \times 10^6 = 0.425 \times 10^6 \text{ or } r_2 = 75 \text{ mm}$$

and $$d_2 = 2r_2 = 2 \times 75 = 150 \text{ mm} \quad \textbf{Ans.}$$

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…m torque …e transmitted

…le Know that the …he axial thrust,

$$W = 0.534\,[(150)^2 - (r_2)^2] \quad \text{... [From equation (i)]}$$
$$= 0.534\,[(150)^2 - (75)^2] = 9011 \text{ N}$$

and mean radius of the contact surfaces for uniform wear conditions,

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 75}{2} = 112.5 \text{ mm}$$

$\therefore$ Maximum torque transmitted,

$$T = n.\mu.W.R = 2 \times 0.4 \times 9011 \times 112.5 = 811 \times 10^3 \text{ N-mm}$$
$$= 811 \text{ N-m Ans.}$$

Maximum intensity of pressure

For uniform wear conditions, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = p_{max} \times 75 \text{ N/mm}$$

We know that the axial thrust (W),

$$9011 = 2\pi\, C\,(r_1 - r_2) = 2\pi \times p_{max} \times 75\,(150 - 75) = 35\,347\, p_{max}$$

$$\therefore \quad p_{max} = 9011 / 35\,347 = 0.255 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Example 24.3. *A single plate clutch, effective on both sides, is required to transmit 25 kW at 3000 r.p.m. Determine the outer and inner diameters of frictional surface if the coefficient of friction is 0.255, ratio of diameters is 1.25 and the maximum pressure is not to exceed 0.1 N/mm². Also, determine the axial thrust to be provided by springs. Assume the theory of uniform wear.*

Solution. Given : $n = 2$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 3000$ r.p.m. ; $\mu = 0.255$; $d_1 / d_2 = 1.25$ or $r_1 / r_2 = 1.25$; $p_{max} = 0.1$ N/mm²

Outer and inner diameters of frictional surface

Let d_1 and d_2 = Outer and inner diameters (in mm) of frictional surface, and

r_1 and r_2 = Corresponding radii (in mm) of frictional surface.

We know that the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\pi N} = \frac{25 \times 10^3 \times 60}{2\pi \times 3000} = 79.6 \text{ N-m} = 79\,600 \text{ N-mm}$$

For uniform wear conditions, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore.

$$p_{max} \times r_2 = C$$

or
$$C = 0.1\, r_2 \text{ N/mm}$$

and normal or axial load acting on the friction surface,

$$W = 2\pi\, C\,(r_1 - r_2) = 2\pi \times 0.1\, r_2\,(1.25\, r_2 - r_2)$$
$$= 0.157\,(r_2)^2 \quad \text{... } (\because r_1 / r_2 = 1.25)$$

We know that mean radius of the frictional surface (for uniform wear),

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\, r_2 + r_2}{2} = 1.125\, r_2$$

and the torque transmitted (T),

$$79\,600 = n.\mu.W.R = 2 \times 0.255 \times 0.157\,(r_2)^2\, 1.125\, r_2 = 0.09\,(r_2)^3$$

$$\therefore \quad (r_2)^3 = 79.6 \times 10^3 / 0.09 = 884 \times 10^3 \quad \text{or} \quad r_2 = 96 \text{ mm}$$

and
$$r_1 = 1.25\, r_2 = 1.25 \times 96 = 120 \text{ mm}$$