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COURSE: PHY 102.2

CONTINUATION OF ELECTROSTATIC

GAUSS' LAW

States that the electric flux through any closed surface is equal to the net charge Q inside the surface divided by the permittivity of free space. For highly symmetric distributions of charge, Gauss's law can be used to calculate electric fields.

The electric flux Φ in an area is defined as the electric field E multiplied by the area A of the surface projected in a plane and perpendicular to the field.

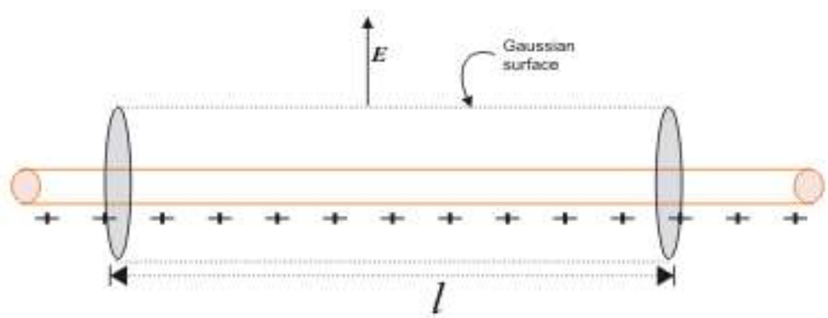
From Gauss' Law

$\Phi = \frac{q}{\varepsilon_0}$ Where

Φ = electric flux, q = charge, ε_0 = permittivity of free space

ε_0 = 8.854 x 10^{-12}F/m

APPLICATION OF GAUSS' LAW



Consider an infinitely long wire with charge density λ and length L. To calculate electric field, we assume a cylindrical Gaussian surface. As the electric field E is radial in direction, the flux through the end of the cylindrical surface will be zero.

This is because the electric field and area vector are perpendicular to each other. As the electric field is perpendicular to every point of the curved surface, we can say that its magnitude will be constant.

The surface area of the curved cylindrical surface is 2πrl. The electric flux through the curve is;

$\Phi = E \times 2\pi rl$

According to Gauss's Law

$\Phi = \frac{q}{\varepsilon_0}$, but q = λl

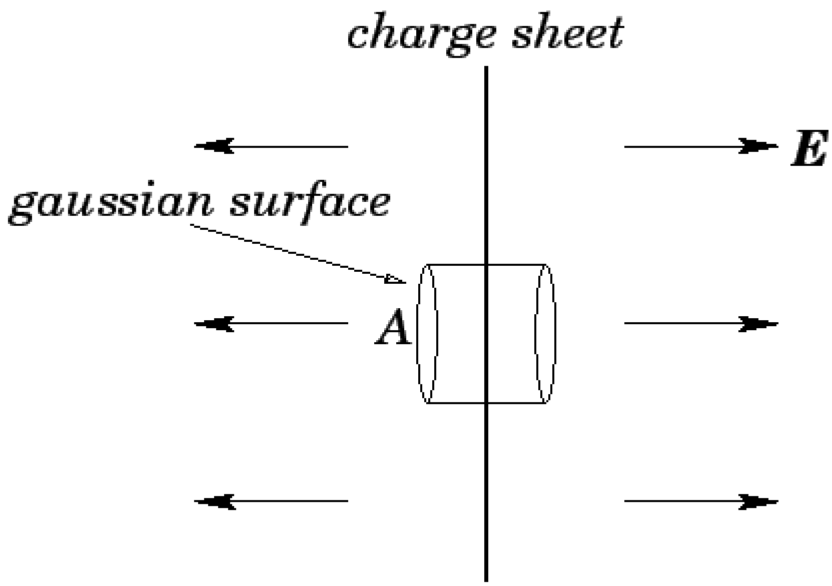
$E \times 2\pi rl = \frac{\lambda l}{\varepsilon_0}$

$E = \frac{\lambda}{2\pi\varepsilon_0 r}$

The direction of electrical field is radially outward if charge density is positive, and radially inward if the charge density is negative.

ELECTRIC FIELD DUE TO INFINITE ELECTRICAL SHEET

Consider an infinite plane sheet, with surface charge density σ and cross-sectional area A. The position of the infinite plane sheet is as below:



The direction of the electric field due to an infinite charge sheet is perpendicular to the plane of the sheet. Let us consider a cylindrical Gaussian surface, whose axis is normal to the plane of the sheet.

From Gauss' Law, we recall that

$\Phi = \frac{q}{\varepsilon_0}$, $q = \sigma A$

So, the net electric flux, which is the electric flux on the both sides of the sheet will be;

$\Phi = EA - (-EA)$

$\Phi = 2EA$

Then

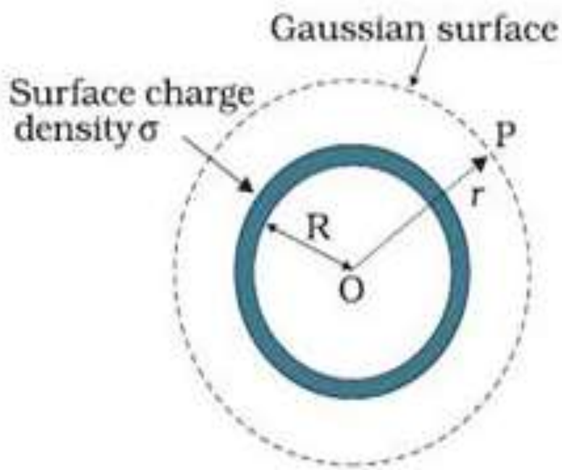
$2EA = \frac{\sigma A}{\varepsilon_0}$

$E = \frac{\sigma}{2\varepsilon_0}$

The areas A cancelled out which showed that electrical field due to an infinite plane sheet is independent of the Cross-sectional area A.

ELECTRIC FIELD DUE TO THIN SPHERICAL SHELL

Let us consider a thin spherical shell of surface charge density σ and radius R.



By observation, we can see that the shell has spherical symmetry. Therefore, we can evaluate the electric field due to the spherical shell in two different positions:

a.) Electric field outside the spherical shell
b.) Electric field inside the spherical shell

Let us look at these two cases in greater detail.

ELECTRIC FIELD OUTSIDE THE SPHERICAL SHELL

To find electric field outside the spherical shell, we take a point P outside the shell at a distance r from the center of the spherical shell. By symmetry, we take Gaussian spherical surface with radius r and center O. The Gaussian surface will pass through P, and experience a constant electric field **E** all around as all points are equally distanced "r'' from the center of the sphere.

According to Gauss' Law; $\Phi = \frac{q}{\varepsilon_0}$

The enclosed charge inside the Gaussian surface q will be $\sigma \times 4\pi R^2$. The total electric flux through the Gaussian surface will be

$$\Phi = E \times 4\pi r^2, \text{ Then}$$

$E \times 4\pi r^2 = \sigma \text{ x } \frac{4\pi R^2}{\varepsilon_0}$

$E = \frac{\sigma R^2}{\varepsilon_0 r^2}$

If the surface charge density σ is given as $q/4\pi R^2$, we can rewrite electric field as

$E = \frac{kq}{r^2}$, $K = \frac{1}{4\pi\varepsilon_0}$

where **r** is the radius vector, depicting the direction of electric field. What we must note here is that if the surface charge density σ is negative, the direction of the electric field will be radially inward.

ELECTRIC FIELD INSIDE THE SPHERICAL SHELL

To evaluate electric field inside the spherical shell, let's take a point P inside the spherical shell. By symmetry, we again take a spherical Gaussian surface passing through P, centered at O and with radius r. Now according to Gauss's Law

$\Phi = \frac{q}{\varepsilon_0}$,

The net electric flux will be $E \times 4 \pi r^2$.

- Since there is no charge inside the spherical shell, electric field E = 0 inside the shell.
- Why is there no electric charge inside the spherical shell?

Answer: The enclosed charge q will be zero, because surface charge density is dispersed outside the surface not inside it, therefore there is no charge inside the spherical shell, i.e E = 0

ELECTRICAL POTENTIAL DIFFERENCE

Electrical potential difference is defined as the work done per unit of charge (Joules per Coulomb) while moving the charge between two points in an electric field. It can also be thought as the change in PE per unit charge between two points in an electric field.

$\Delta V = \frac{Work}{q} = - \frac{Eq\Delta x}{q}$

$= - E\Delta x$

Electric field E could be written as

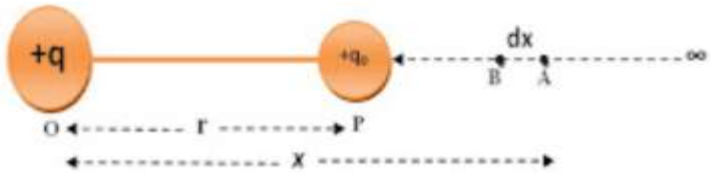
$E = - \frac{\Delta V}{\Delta x}$

The negative sign indicates that the direction of decreasing potential is the same as the field direction.

- The potential difference between the positive and the negative plates depends on the field strength and the plate separation. Potential difference is measured in volts (1V = 1 J/C)

ELECTRIC POTENTIAL DUE TO POINT CHARGE

Work done in moving a unit positive charge against the electric field is store as energy which is called electric potential.



Let us consider a source point charge +q is placed in air and vacuum at point O. Let take a point P at distance r from the source point charged particle $+q_0$ is brought from infinity to point P. If the test charged particle move a very small distance dx from point A to B against the electrostatic force. So electrostatic force at point A which is placed at a distance x from point O

The electric field intensity at P due to point charge +q is;

E = $\frac{kq}{x^2}$, $k = \frac{1}{4\pi\varepsilon_0}$

E = $\frac{q}{4\pi\varepsilon_0 x^2}$

Work done in bringing the point charge from A to B is;

dw = $\overrightarrow{F}.\overrightarrow{dx}$

dw = $\overrightarrow{F}.\overrightarrow{dx}$ cos 180^0

= - $\overrightarrow{F}.\overrightarrow{dx}$, but $\vec{F}$ = Eq_0

dw = E$q_0$dx

Total work done in bringing unit positive test charge from infinity to distance r is;

W = $\int_{\infty}^{r} dw$ = -$\int_{\infty}^{r} Eq_0\, dx$, = -$\int_{\infty}^{r} \frac{qq_0}{4\pi\varepsilon_0 x^2}\, dx$

= - $\frac{qq_0}{4\pi\varepsilon_0} \int_{\infty}^{r} \frac{1}{x^2}\, dx$

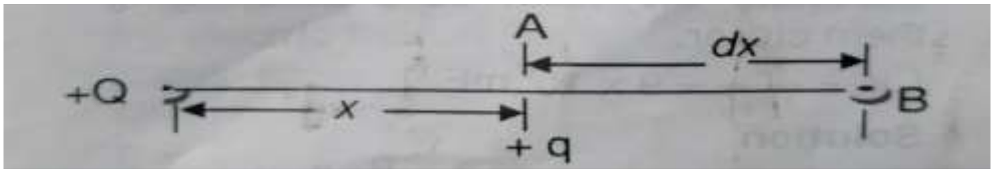
W = - $\frac{qq_0}{4\pi\varepsilon_0}$ $[-\frac{1}{x}]_{\infty}^{r}$

W = $\frac{qq_0}{4\pi\varepsilon_0}\frac{1}{r}$, Recall V = $\frac{W}{q_0}$

V = $\frac{q}{4\pi\varepsilon_0 r}$

POTENTIAL DIFFERENCE DUE TO POINT CHARGE

The potential difference between two points A and B in the field of a small charge, Q.



When a unit positive charge +q is at distance x from the charge Q in free space, the force is given by

$$F = \frac{qQ}{4\pi\varepsilon_o r^2}\text{, Where q = 1}$$

$$\therefore F = \frac{Q}{4\pi\varepsilon_o r^2}$$

The work done in taking the charge from B to A, against the force F over a small distance dx is

dw = dx, over the whole distance AB, therefore, the work done by the force on the unit charge is

$$\int_A^B dw = \int_{x=a}^{x=b} F dx = \int_a^b \frac{Q}{4\pi\varepsilon_0 x^2} dx$$

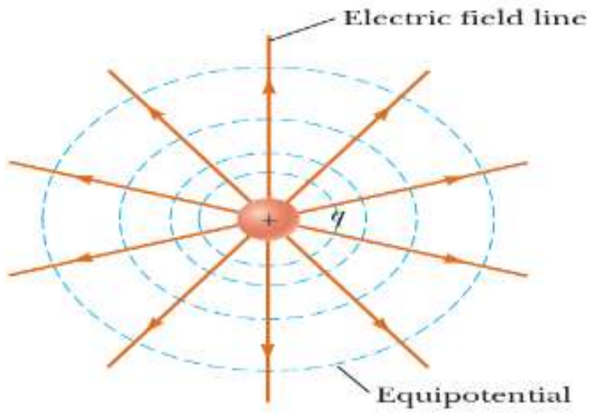
$$= \frac{Q}{4\pi\varepsilon_o}\int_a^b x^{-2} dx$$

$$V = \frac{Q}{4\pi\varepsilon_o}\left[\frac{-1}{x}\right]_a^b, \; = \frac{Q}{4\pi\varepsilon_o}\left[\left(\frac{-1}{b}\right) - \left(\frac{-1}{a}\right)\right]$$

$$V_{AB} = \frac{Q(b-a)}{4\pi\varepsilon_o ab}$$

Thus, then, is the value of the work which an external agent must do to carry a unit positive charge from B to A the work per coulomb is the potential difference V between A and B.

EQUIPOTENTIAL SURFACE



A surface on which all points are at the same potential is called an equipotential surface. The potential difference between any two points on an equipotential surface is zero. Hence, no work is required to move a charge from one point to another at constant speed on an equipotential surface. Equipotential surfaces have a simple relationship to the electric field: The electric field at every point of an equipotential surface is perpendicular to the surface. **NOTE** If the points in an electric field are all at the same electric potential, then they are known as equipotential point. If these points are been connected by lines or curved, it is known as equipotential line. If the points are distributed throughout a space or a volume, it is known as an Equipotential Volume.