STEAM NOZZLE.

The properties of steam can be obtained from tables or from a h–s chart, but in order to find the critical pressure ratio and hence the critical velocity and the maximum mass flow rate, approximate formulae may be used, it is a good approximation to assume that steam follows an Isentropic law $Pv^K =$ constant, where K is an Isentropic Index for steam.

For steam Initially dry saturated, $K = 1.135$ and for steam Initially Superheated, $K = 1.3$,

$$\frac{P_c}{P_1} = \left(\frac{2}{K+1}\right)^{K/K-1}$$

Therefore when the steam entering nozzle is dry saturated

$$\frac{P_c}{P_1} = \left(\frac{2}{2.135}\right)^{1.135/0.135} = 0.577$$

when the steam entering a nozzle is superheated

$$\frac{P_c}{P_1} = \left(\frac{2}{2.3}\right)^{1.3/0.3} = 0.546$$

$$C_c = \sqrt{2(h_1 - h_c)}$$

Example 4: Estimate the critical pressure and the throate area per unit mass flow rate of a convergent-divergent nozzle expanding steam from 10bar, dry saturated down to atmospheric pressure of 1bar. Assume that the inlet velocity is negligible and that the expansion is isentopic.

Solution:

Since it is dry saturated:

$$\frac{P_c}{P_1} = \left(\frac{2}{K+1}\right)^{K/K-1} = 0.577$$

$$P_c = 10 \times 0.577 = \underline{5.77 \text{ bar}}$$

$$C_c = \sqrt{2(h_1 - h_c)}$$

At 1 bar from the steam table $h_c = 2675$

and at 10bar from the steam table $h_1 = 2778$

$$C_c = \sqrt{2(2778 - 2675)} = \underline{454 \text{ m/s}}$$

$$\frac{A}{m} = \frac{V}{C} = \cancel{}$$

$$V_c = x V_g$$

From the Steam Table at 5.77bar, $V_g = 0.328\,m^3/Kg$ (13)

dryness fraction $x = 0.962$

$$V_c = xV_g = 0.962 \times 0.328 = 0.316\,m^3/Kg$$

$$\frac{A}{m} = \frac{V}{C} = \frac{0.316 \times 10^6}{454} = \underline{696\,mm^2}$$

$\therefore$ Throat area per Kilogram per Second $= \underline{696\,mm^2}$

STAGNATION CONDITIONS.

We have assumed all through that the inlet velocity to the nozzle is negligible. when this is not the case, the Concept of Stagnation can be used.

Let a gas moving with velocity, C, at a temperature, T, be brought to rest adiabatically, finally reaching a Temp T_0 when at rest. Then applying the flow equation for a perfect gas

$$C_pT + \frac{C^2}{2} = C_pT_0 \quad \text{or} \quad T_0 = T + \frac{C^2}{2C_p}$$

T_0 = Stagnation Temperature of the moving gas.

$\frac{C_2}{2C_p}$ is called temperature equivalent of Velocity

The Stagnation pressure, P_0 of a gas Stream is defined as the pressure the gas would attain if brought to rest Isentropically.

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} \quad \text{or} \quad \frac{P_0}{P} = \left(1 - \frac{C^2}{2C_pT}\right)^{\frac{\gamma}{\gamma-1}} \quad \text{or} \quad \frac{P_0}{P} = \left[1 + \frac{(\gamma-1)C^2}{2\gamma RT}\right]^{\frac{\gamma}{\gamma-1}}$$

$\boxed{\text{where } C_p = \frac{\gamma R}{\gamma - 1}}$

Velocity of Sound $(a) = \sqrt{\gamma RT}$

$$\frac{P_0}{P} = \left[1 + \frac{(\gamma-1)C^2}{2a^2}\right]^{\frac{\gamma}{\gamma-1}} = \left[1 + \frac{(\gamma-1)(Ma)^2}{2}\right]^{\frac{\gamma}{\gamma-1}}$$

where $Ma = \frac{C}{a}$ = Mach Number.

If the right hand side of the equation is expanded by binomial theorem

$$\frac{P_0}{P} = 1 + \frac{\gamma(\gamma-1)(Ma)^2}{(\gamma-1)\times 2} + \left(\frac{\gamma}{\gamma-1}\right)\left[\left(\frac{\gamma}{\gamma-1}\right)-1\right]\frac{1}{2}\frac{(\gamma-1)^2(Ma)^4}{4} + \ldots$$

when the velocity of gas is Low, and Ma is therefore Small

$$\frac{P_0}{P} = 1 + \frac{\gamma(Ma)^2}{2} = 1 + \frac{\gamma C^2}{2\gamma RT} = 1 + \frac{C^2}{2RT}$$

$$P_0 = P + \frac{C^2}{2}\frac{P}{RT}$$

Now the density $\rho = \frac{1}{V} = \frac{P}{RT}$

$\therefore$ Stagnation pressure $P_0 = P + \frac{\rho C^2}{2}$, where $\frac{\rho C^2}{2}$ = velocity Head

Applying stagnation conditions to flow through a nozzle we have at the Inlet

$$\frac{C_1^2}{2} + C_pT_1 = C_pT_{01}$$

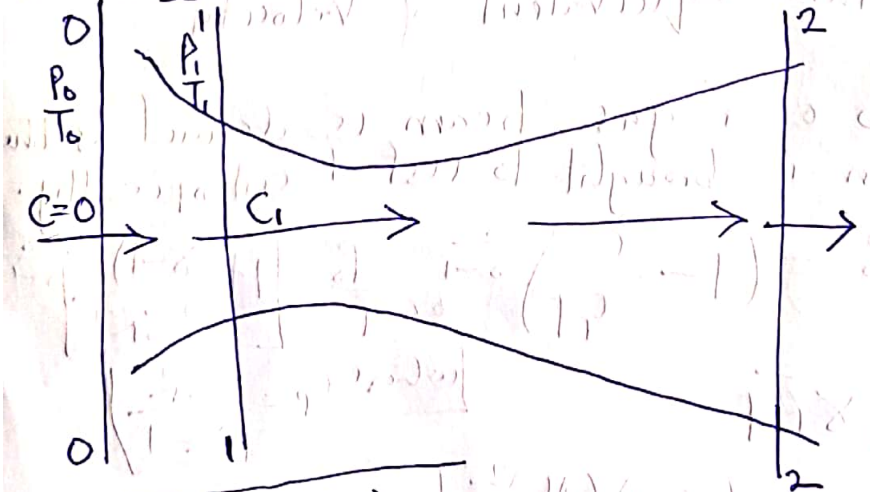
At any other Section of the nozzle where the velocity is C & Temp T

$$\frac{C^2}{2} + C_pT = C_pT_0$$

Therefore

$$\frac{C_1^2}{2} + C_pT_1 = \frac{C^2}{2} + C_pT = C_pT_{01} = C_pT_0$$

Therefore the ~~St~~ Stagnation temperature remain constant through out the nozzle.



$$C = \sqrt{2C_p(T_{01}-T)}$$

$$\frac{P_c}{P_{01}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \quad \text{and} \quad \frac{T_c}{T_{01}} = \frac{2}{\gamma+1}$$

THERMODYNAMICS III

ADVANCED THERMODYNAMICS.

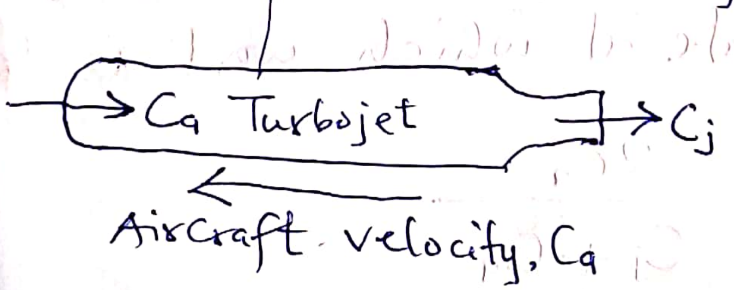
JET PROPULSION.

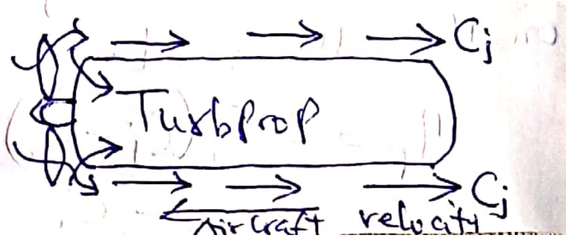
Aircraft propulsion may be achieved by using a heat engine to drive an air screw or propeller, or by allowing a high-energy fluid to expand and leave the aircraft ~~engine~~ in a rearward direction as a high-velocity jet. In the propeller type of aircraft engine, the propeller takes a large mass flow and gives it a moderate velocity backwards relative to the aircraft.

In the jet engine the aircraft induces a comparatively small airflow and gives it a high velocity backwards relative to the aircraft. In both cases the rate of change of momentum of the air provides a reactive forward thrust which propels the aircraft.

The propeller-type engine can be driven by a petrol engine or by a gas turbine unit.

if the velocity of the jet (from propeller or jet engine) backwards relative to aircraft is C_j and the velocity of the aircraft is C_a. then the atmospheric air, initially at rest is given a velocity of $(C_j - C_a)$.





- The Thrust available for propulsion is solely due to the rate of change of momentum of the stream,
  i.e Thrust per Unit mass flow rate $= C_j - C_a$
- The propulsive power is given by.
  Thrust power per Unit mass flow rate $= C_a(C_j - C_a)$

The above equation / power is the rate at which work must be done in order to keep the aircraft moving at the constant velocity C_a against the frictional resistance or drag.

The net work output from the engine is given by the Increase in Kinetic energy, $\frac{(C_j^2 - C_a^2)}{2}$.

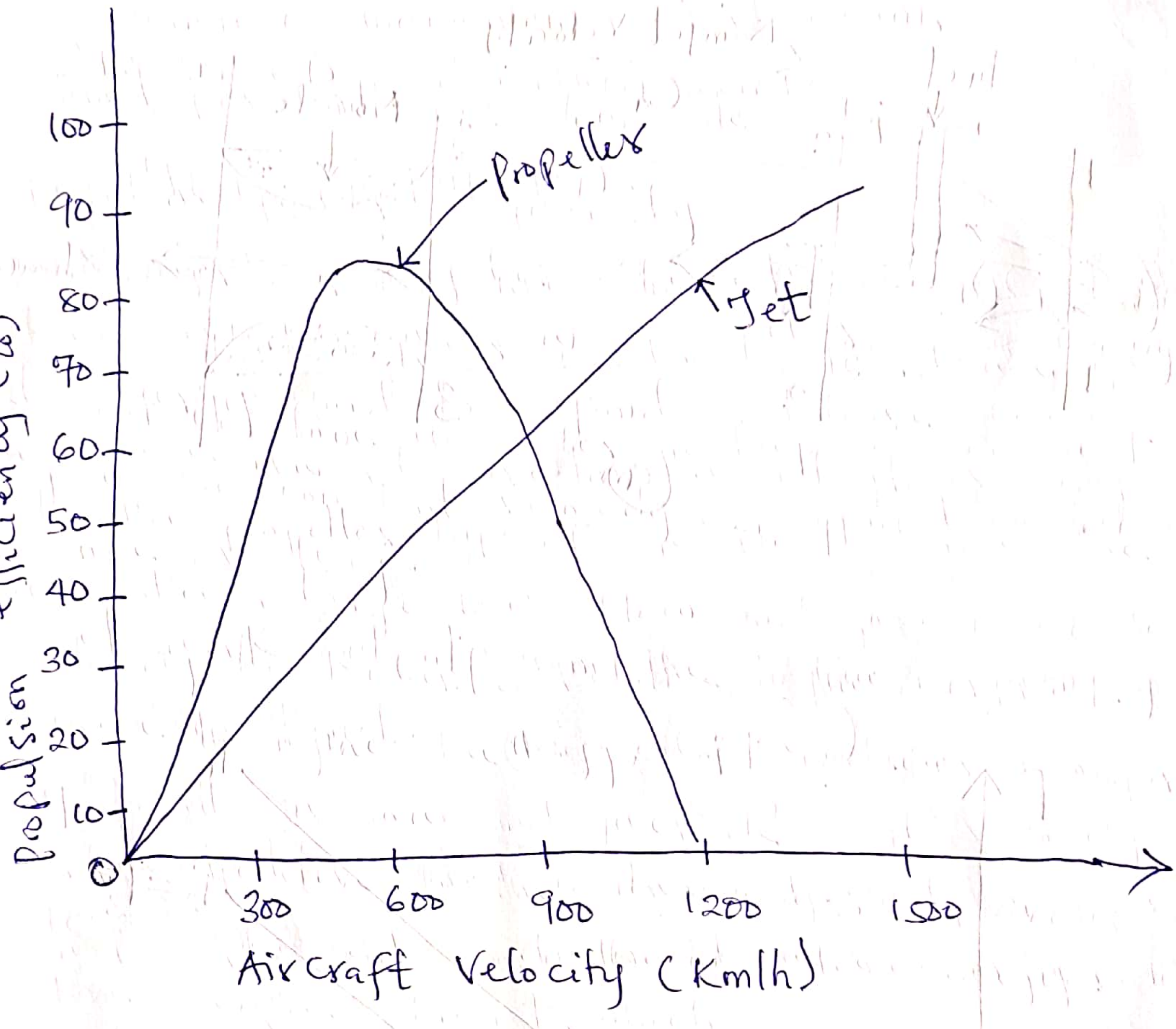
This work output is used in two ways; it provides the thrust work and and the Kinetic energy.

i.e $C_a(C_j - C_a) + \frac{(C_j - C_a)^2}{2} = C_aC_j - C_a^2 + \frac{C_j^2}{2} + \frac{C_a^2}{2} - \frac{2C_jC_a}{2}$

Work output from engine $= \frac{C_j^2 - C_a^2}{2}$

<u>Propulsive Efficiency (η_p)</u>: is defined as the trust work divided by the rate at which work is done on the aircraft.

$$\eta_p = \frac{2C_a(C_j - C_a)}{C_j^2 - C_a^2} = \frac{2C_a}{C_j + C_a}$$



From the figure above it can be seen that for aircraft speeds up to about 850 Km/h the propeller is the more efficient means of propulsion, but for speed above this, the jet engine is superior.

* The simplest form of jet engine is the ramjet. In the ramjet the air is compressed by the conversion of the kinetic energy of the atmospheric air relative to the aircraft. This is known as the ram effect. fuel is then burned in the compressed air stream at approximately constant pressure and the hot gases are allowed to expand through a nozzle, reaching a high velocity

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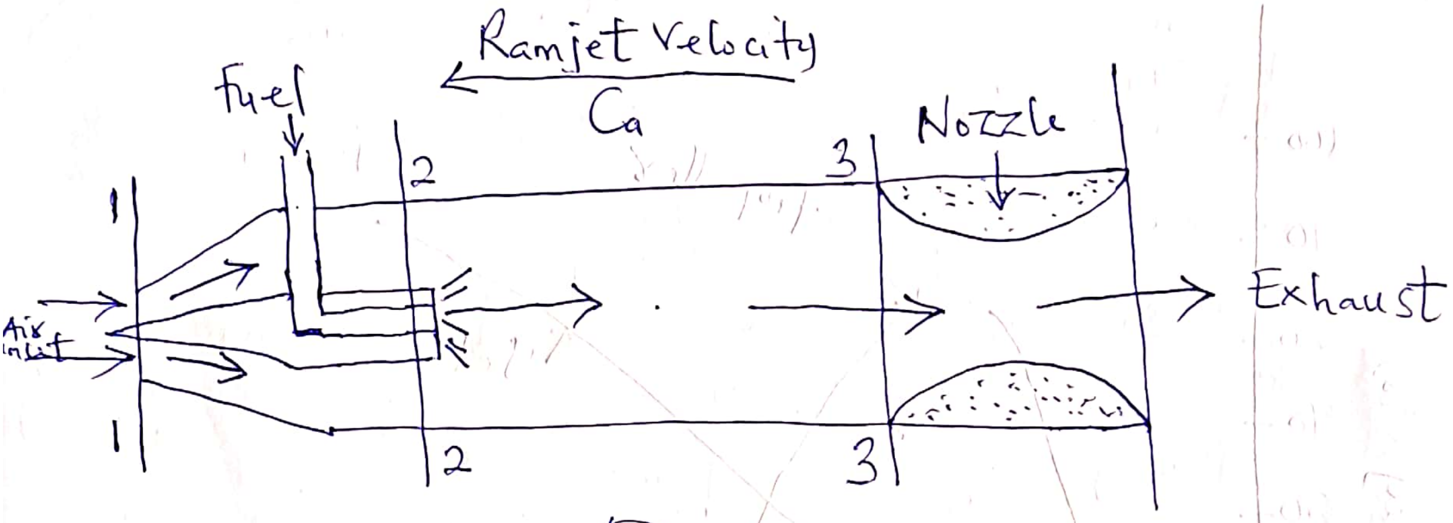
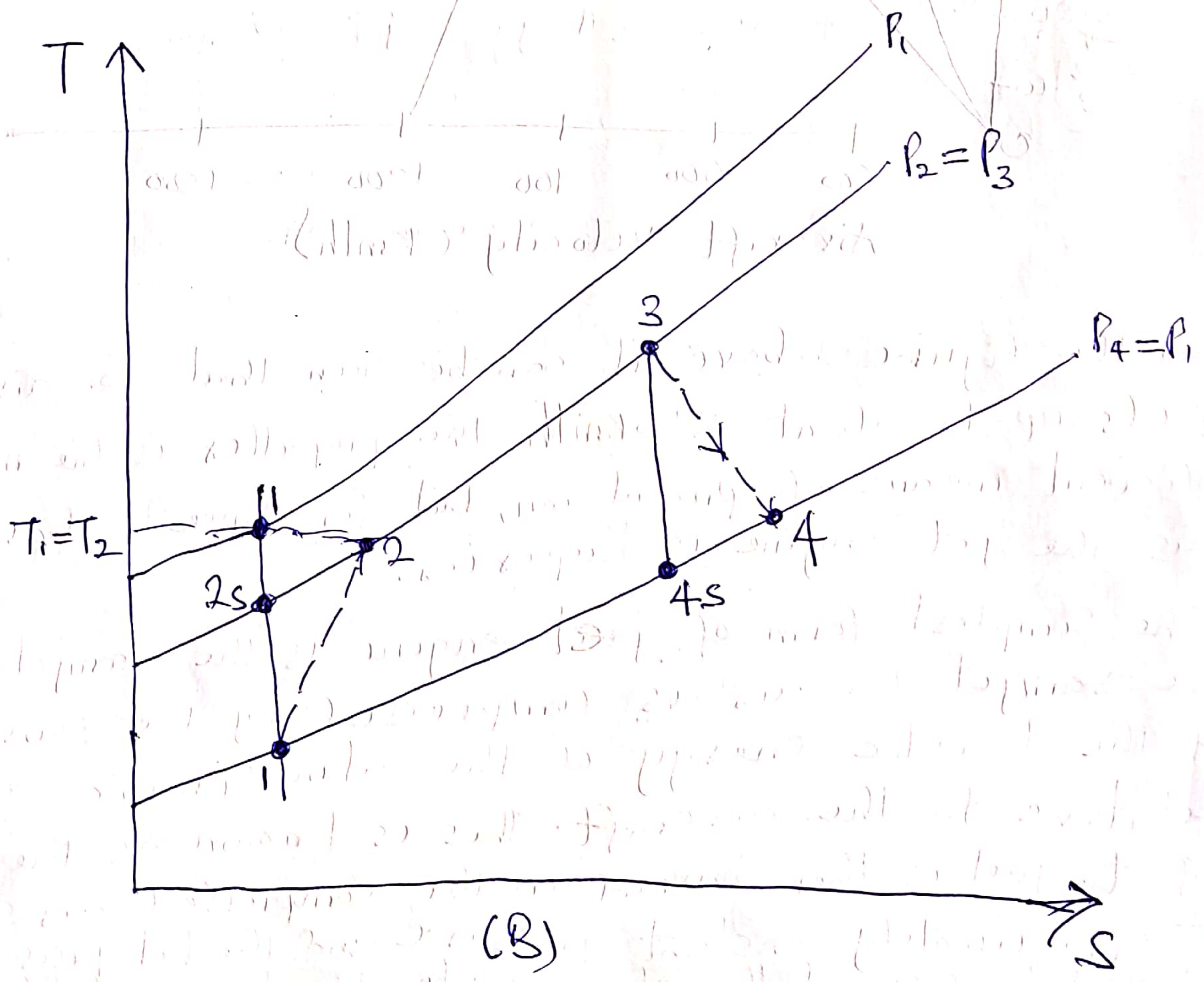
backwards relative to the aircraft.



(a)



(B)

If the ramjet velocity is C_a, then the air enters the diffuser with a Kinetic energy of $C_a^2/2$ per unit mass of air.

Velocity after diffusion $h_1 + \frac{C_a^2}{2} = h_1 + \frac{C_1^2}{2}$

There fore,

$$C_pT_1 + \frac{C_a^2}{2} = C_p\left(T_1 + \frac{C_1^2}{2C_p}\right) = C_pT_0$$

i.e $\frac{C_a^2}{2} = C_p(T_0 - T_1)$

or $T_0 - T_1 = \frac{C_a^2}{2C_p}$

$$\frac{T_{01}}{T_1} = \left(\frac{P_{01}}{P_1}\right)^{(\gamma-1)/\gamma}$$

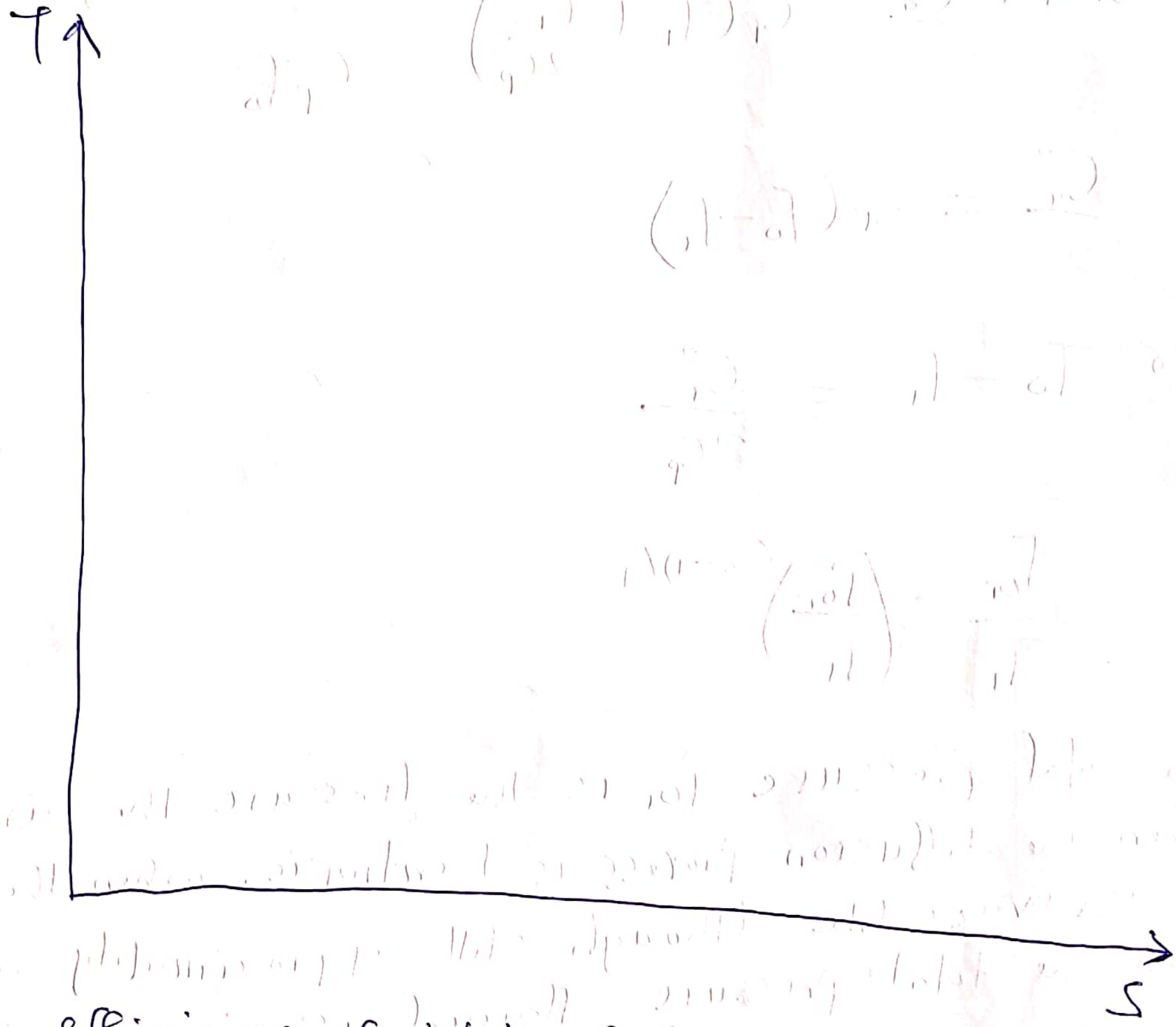
The Total pressure P_{01} is the Pressure the air attains when the diffusion process is Isentropic. when the process is irreversible although still approximately adiabatic then the total pressure attained is P_{02}, which is less than P_{01} as seen in the figure above.

$T_{02} = T_{01}$ Since Kinetic energy $C_a^2/2$ is Same whether or not the process is reversible.

Intake efficiency $= \frac{T - T_1}{T - T_1}$

Turbo Jet.

In the Turbojet the Kinetic energy of the in coming air can be used to obtain a ram compression in the intake duct, thus raising the overall efficiency of the Unit.



Isentropic efficiency of intake duct =

Isentropic efficiency of Compressor =

Isentropic efficiency of Turbine =

~~Isentropic efficiency of~~ —

Jet pipe efficiency =

For adiabatic flow, the total Temperature remain Constant and therefore $T_0 = T_{01}$ and $T_{04} = T_{05}$ for the Intake duct and jet pipe respectively.

PRESSURE THRUST.

It can be assumed in the analysis that the gases expand down to atmospheric pressure in the jet nozzle. In practice particularly in the case of a convergent nozzle, the back pressure will normally be lower than the pressure of the gases at the nozzle outlet; this phenomenon is called underexepansion.

Due to the difference in pressure between the nozzle exist and the atmosphere in which the aircraft is flying there will be an additional thrust called the pressure thrust. Also in the case of the supersonic aircraft, the pressure at the air intake is higher than the atmospheric pressure because of compression through the shock wave formed, this causes a reduction in the net thrust calculated purely from momentum considerations.

Let consider a turbojet, where ~~A = A~~

A_1 = Area of Intake air

P_1 = Pressure " " "

A_2 = Area of exist air

P_2 = Pressure of exist air

P_a = Atmospheric pressure

For a control volume round the working fluid in the aircraft engine, we have using Newton's Second Law

$F - P_1A_1 - P_2A_2 =$ rate of change of momentum of working fluid in the direction of motion of the fluid.

where F is the net force due to hydrostatic pressure and friction excerted by the inside of the aircraft on the working fluid in the direction of it's motion.

$$F - P_1A_1 - P_2A_2 = \dot{m}(C_j - C_a)$$

Therefore

$$F = \dot{m}(C_j - C_a) - P_1A_1 + P_2A_2$$

There is an equal and opposite force, R, exerted by the working fluid on the inside of the aircraft engine

$$R = \dot{m}(C_j - C_a) - P_1A_1 + P_2A_2$$

There are three forces acting on the aircraft. There is the force R, there is a total drag Δ, due to the air resistance and there is a pressure force due to the atmospheric pressure acting on the projected area in the direction of flight.

Assume the aircraft silhouette area in the direction of flight is A, then the net pressure force in the direction of flight is given by.

$$P_a(A - A_2) - P_a(A - A_1) = P_a(A_1 - A_2)$$

Since the aircraft is flying at constant velocity the net force acting is zero.

$$R - \Delta + P_a(A_1 - A_2) = 0$$

Therefore the total thrust required to overcome the total drag force is given by

$$\text{Total thrust} = D = R + P_a(A_1 - A_2)$$
$$= \dot{m}(C_j - C_a) - P_1A_1 + P_2A_2 + P_a(A_1 - A_2)$$

Therefore

$$\text{Total thrust} = \dot{m}(C_j - C_a) + A_2(P_2 - P_a) - A_1(P_1 - P_a)$$

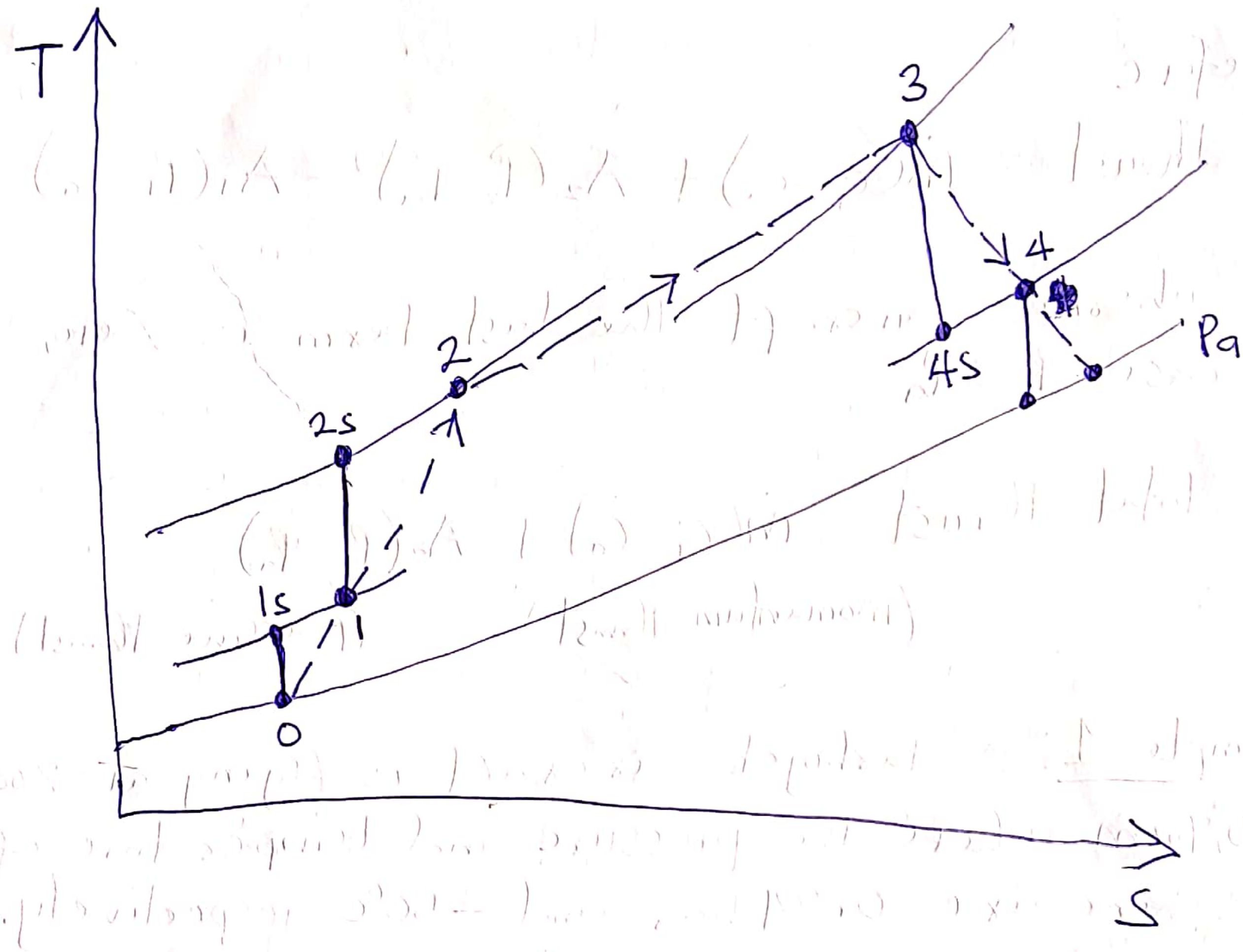
for subsonic aircraft the last term is zero, since in that case $P_1 = P_a$

i.e
$$\text{Total thrust} = \underbrace{\dot{m}(C_j - C_a)}_{\text{(Momentum thrust)}} + \underbrace{A_2(P_2 - P_a)}_{\text{(Pressure thrust)}}$$

<u>Example 1</u>: A turbojet aircraft is flying at 800 Km/h at 10,700m where the pressure and temperature of the atmosphere are 0.24 bar and −50°C respectively. The compressor pressure ratio is 10/1 and the maximum cycle temperature is 820°C. Calculate the thrust developed and the specific fuel consumption, using the following information entry duct efficiency 0.9; isentropic efficiency of compressor 0.9; stagnation pressure loss in the combustion chamber 0.14 bar; calorific value of fuel 43,300 KJ/Kg; combustion efficiency 98%; isentropic efficiency of turbine 0.92; mechanical efficiency of drive 98%; jet pipe efficiency 0.92; nozzle outlet area 0.08 m²; C_p and γ for the compression process 1.005 KJ/KgK and 1.4; C_p and γ for combustion and expansion 1.15 KJ/KgK and 1.333; Assume [illegible]

Solution.

The cycle is shown on a T–s diagram in the figure below. The exhaust condition of the gases leaving the nozzle is not known until it is ascertained whether or not the nozzle is choked.



Kinetic energy of air at inlet

$$= \frac{1}{2}\left(\frac{800 \times 1000}{3600}\right)^2 = \frac{1}{2}(222.2)^2 \text{ Nm/Kg}$$

$$K.E = 24.7 \text{ KJ/Kg}.$$

$$T_{01} - T_0 = \frac{24.7}{C_p} = \frac{24.7}{1.005} = 24.6 K$$

$$T_{01} = (-50 + 273) + 24.6 = 247.6 K$$

Intake efficiency $= \dfrac{T_{01s} - T_0}{T_{01} - T_0} = 0.9$

Therefore $T_{01s} - T_0 = 0.9 \times 24.6 = 22.1K$

$$T_{01s} = (-50 + 273) + 22.1 = 245.1K$$

$$\frac{P_{01}}{P_a} = \left(\frac{T_{01s}}{T_0}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{245.1}{223}\right)^{\frac{1.4}{0.4}} = 1.1^{3.5} = 1.393$$

$$P_{01} = 1.393 \times 0.24 = 0.334 \text{bar}$$

For the compressor we have

$$\frac{T_{02s}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{(\gamma-1)}{\gamma}} = 10^{0.4/1.4} = 1.931$$

Therefore $T_{02s} = 1.932 \times 247.6 = 478K$

Isentropic efficiency $= \dfrac{T_{02s} - T_{01}}{T_{02} - T_{01}} = 0.9$

$$T_{02} - T_{01} = \frac{478 - 247.6}{0.9} = 256K$$

$$T_{02} = 247.6 + 256 = 503.6K$$

Also $P_{02} = 10 \times P_{01} = 10 \times 0.334 = 3.34 \text{bar}$.

Hence $P_{03} = P_{02} -$ (Loss of total pressure in Combustion)

$P_{03} = 3.34 - 0.14 = 3.2$ bar.

Now the turbine develops just enough work to drive the compressor and overcome mechanical losses.

$$C_{pg}(T_{03} - T_{04}) = \frac{C_{pg}(T_{02} - T_{01})}{0.98}$$

Note that the work output from the turbine and the work input to the compressor are given by the product of C_p and the difference in total temperature, when the flow in each is adiabatic. example for a turbine using the flow equation we have.

$$C_{pg}T_3 + \frac{C_3^2}{2} + W = C_{pg}T_4 + \frac{C_4^2}{2}$$

$$-W = C_{pg}(T_{03} - T_{04})$$

Therefore

$$T_{03} - T_{04} = \frac{1.005(503.6 - 247.6)}{1.15 \times 0.98} = 228.3K$$

$$T_{04} = (820 + 273) - 228.3 = \underline{864.7\,K}$$

$$\text{Isentropic efficiency} = \frac{T_{03} - T_{04}}{T_{03} - T_{04s}} = 0.92$$

Therefore $T_{03} - T_{04s} = \dfrac{228.3}{0.92} = 248.2K$

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$T_{04s} = (820 + 273) - 248.2 = 844.9\,K$

Then $\dfrac{P_{03}}{P_{04}} = \left(\dfrac{T_{03}}{T_{04s}}\right)^{\gamma/(\gamma-1)} = \left(\dfrac{1093}{844.9}\right)^{1.333/0.333} = 2.803$

Therefore $P_{04} = \dfrac{3.24}{2.803} = 1.156\,bar$

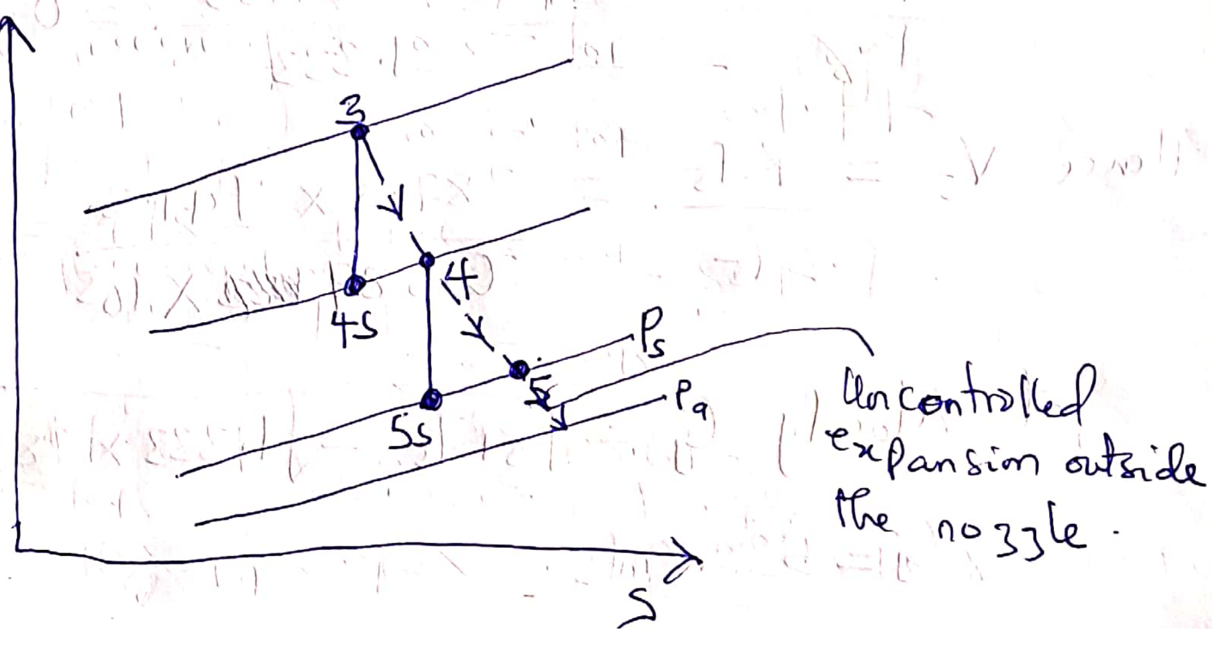
To determined if it is choked, we will solve for the critical pressure

$$\frac{P_c}{P_{04}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{2.333}\right)^{1.333/0.333} = 0.54$$

$$P_c = 1.156 \times 0.54 = 0.624\,bar.$$

Since the atmospheric pressure is 0.24 bar, it follows that the nozzle is choking and hence the actual velocity of gas at exit is sonic.

The expansion in the nozzle is shown in the T-s diagram below.



The Temperature at exit T_5, is solved below.

$$\frac{T_5}{T_{04}} = \frac{2}{\gamma+1} = \frac{2}{1.333+1}$$

$$T_5 = \frac{864.7}{1.1665} = 741.3K$$

Jet pipe efficiency $= \frac{T_{04}-T_5}{T_{04}-T_{5s}} = 0.92$

Therefore $T_{5s} = 864.7 - \frac{(864.7 - 741.3)}{0.92} = 730.6K$

$$\frac{P_{04}}{P_5} = \left(\frac{T_{04}}{T_{5s}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{864}{730.6}\right)^{1.333/0.333} = 1.963$$

Therefore $P_5 = \frac{1.156}{1.963} = 0.589$ bar.

$$R = \frac{C_p(\gamma-1)}{\gamma} = \frac{1.15 \times 0.333}{1.333} = 0.2873 KJ/KgK$$

Hence $V_5 = \frac{RT_5}{P_5} = \frac{287.3 \times 741.3}{0.589 \times 10^5} = 3.616 \, m^3/Kg$

Jet Velocity $C_j = \sqrt{\gamma R T_5} = \sqrt{1.333 \times 287.3 \times 741.3}$

$C_j = \underline{532.8 m/s}$

CS CamScanner

Then Mass flow $= \frac{A C_j}{V_5} = \frac{0.08 \times 532.8}{3.616} = 11.788\ kg/s$

Momentum thrust $= \dot{m}(C_j - C_a) = 11.788(532.8 - 222.2)$
$= \underline{\underline{3661N}}$

Pressure Thrust $= (P_5 - P_a)A = (0.589 - 0.24) \times 0.08 \times 10^5$
$= 2792N$

Total Thrust $= 3661 + 2792 = 6453N$

Heat supplied $= \dot{m} C_{pg}(T_{o3} - T_{o2})$
$= 11.788 \times 1.15(1093 - 503.6) = 7990\ KJ/s$

if curves of theoretical total temperature rise against fuel-air ratio were available for the fuel used, then the fuel consumption could be found in this way. In this case, it is sufficient to write

Heat Supplied $= \dot{m}_f \times$ Calorific Value $= \frac{7990}{0.98}\ KJ/s$

where m_f is the mass of fuel supplied in kg/s

$m_f = \frac{7990}{43300 \times 0.98} = 0.188\ kg/s$

Specific fuel consumption $= \frac{0.188 \times 10^3}{6453} = 0.0291\ kg/kNs$

THE TURBO-PROP

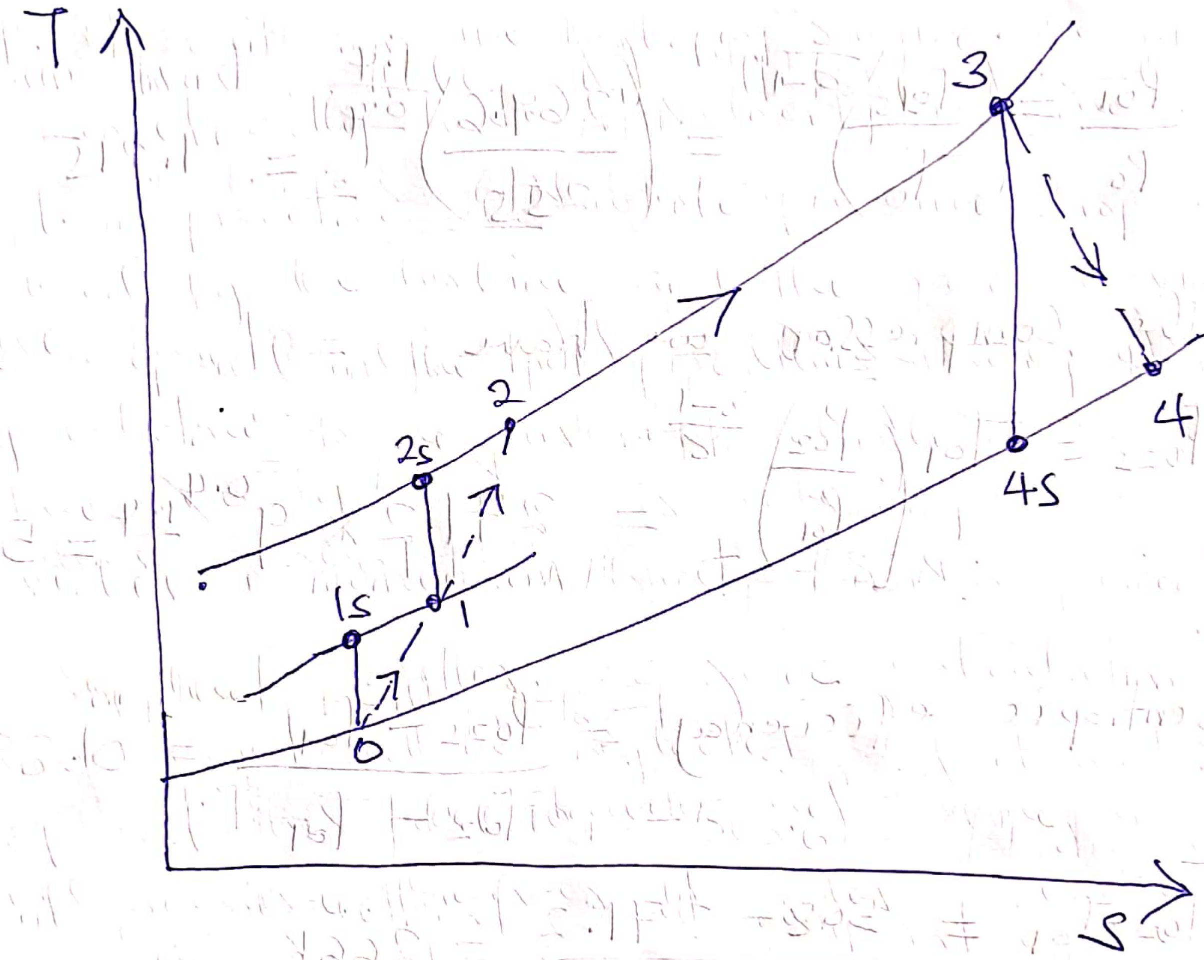
In a turboprop aircraft the ram effect of the incoming air relative to the aircraft can be used as in the turbojet. At exit from the turboprop engine. The gases should theoretically have a velocity relative to the aircraft. In practice the whole pressure drop available is not used by the turbine, and the gases leaving the turbine expand in the jet pipe, thus leaving with a velocity relative to the aircraft which is higher than the relative velocity of the air entering the engine. This provides a momentum thrust which complements the thrust from the propeller, for basic calculations this additional thrust will be neglected, and it will be assumed that the gases leaves the turbine at the ambient pressure. However in order to make use of the stagnation isentropic efficiency of the turbine it is necessary to know the stagnation temperature at turbine exhaust.

<u>Example 5</u>: A turboprop aircraft is flying at 650km/h at an altitude where the ambient temperature is −18°C. The compressor pressure ratio is 9/1 and the maximum cycle temperature is 850°C. The intake duct efficiency is 0.9, and the stagnation isentropic efficiencies of the compressor and turbine are 0.89 and 0.93 respectively. Calculate the specific power output and the cycle efficiency taking a mechanical efficiency of 98% and neglecting the pressure loss in the combustion chamber. Assume that the exhaust gases leave the aircraft at 650km/h relative to the

aircraft, and take C_p and γ as used in the previous examples.

(22)

Solution



Kinetic energy of the air at inlet

$$= \frac{1}{2}\left(\frac{650\times10^3}{3600}\right)^2 = \frac{180.5^2}{2} \text{ Nm/kg}$$

$$T_{01}-T_0 = \frac{180.5^2}{2C_p} = \frac{180.5^2}{2\times10^3\times1.005} = 16.2\text{K}$$

$$T_{01} = (-18+273)+16.2 = 271.2\text{K}$$

Intake efficiency $\dfrac{T_{01s}-T_0}{T_{01}-T_0} = 0.9$

$T_{01s} - T_0 = 0.9 \times 16.2 = 14.6K$

$T_{01s} = 269.6K$

Then $\frac{P_{01}}{P_0} = \left(\frac{T_{01s}}{T_0}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{269.6}{255}\right)^{\frac{1.4}{0.4}} = 1.215$

for the compressor we have

$$T_{02s} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 271.2 \times 9^{0.4/1.4} = 508K$$

$$\text{Isentropic efficiency} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} = 0.89$$

$$T_{02} - T_{01} = \frac{508 - 271.2}{0.89} = 266K$$

$$T_{02} = 271.2 + 266 = 537.2K$$

$$\text{Compressor Work input} = C_p(T_{02} - T_{01}) = 1.005 \times 266$$
$$= 267.5 kJ/kg.$$

Now $\frac{P_{03}}{P_{4}} = \frac{P_{02}}{P_{01}} \times \frac{P_{01}}{P_0} = 9 \times 1.215 = 10.935$

Therefore $T_4 = T_{03}\left(\frac{P_4}{P_{03}}\right)^{\frac{\gamma-1}{\gamma}} = \frac{1123}{10.935^{0.333/1.333}} = 617.8$

$$T_{04} = T_4 + \frac{C_4^2}{2C_{pg}} = 617.8 + \frac{180.5^2}{2\times 10^3 \times 1.15} = 617.8 + 14.2$$

$$T_{04} = 632K.$$

$$\text{Isentropic efficiency} = \frac{T_{03} - T_{04}}{T_{03} - T_{04s}} = 0.93$$

Therefore $T_{03} - T_{04} = 0.93(1123 - 632) = 456.6K$

Then Turbine Work output $= C_p(T_{03} - T_{04}) = 1.15 \times 456.6$
$= 525.1 kJ/kg$.

The Net work output $= (525.1 - 267.5)$
$= 257.6$

Specific power output $= 257.6 \times 0.98 = 252.5 kJ/kg$

Heat supplied $= C_{pg}(T_{03} - T_{02}) = 1.15(1123 - 537.2)$
$= 675 kJ/kg$

$$\text{Thermal efficiency} = \frac{252.5}{675} = 0.374 \text{ or } 37.4\%$$