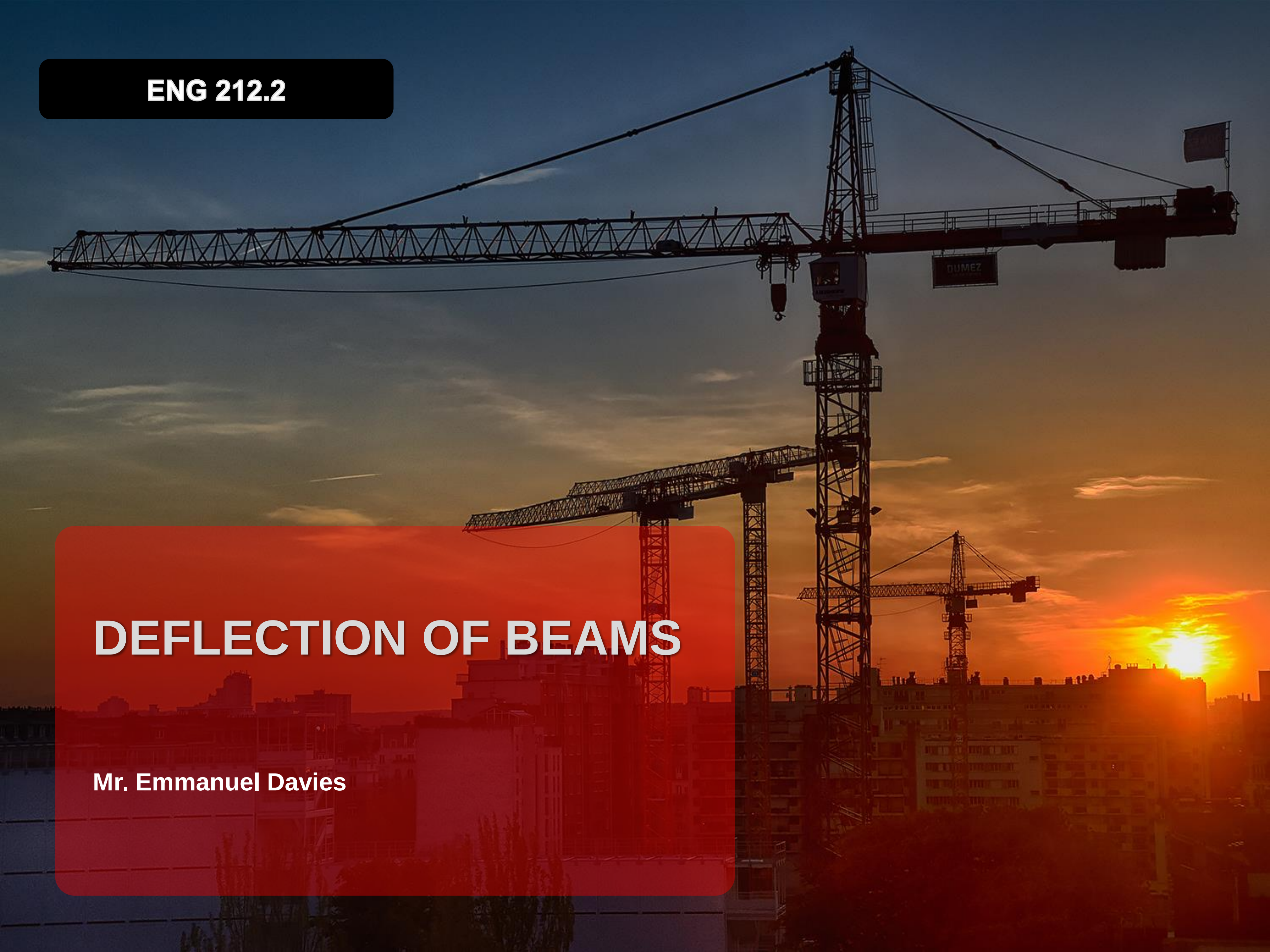


ENG 212.2

DEFLECTION OF BEAMS

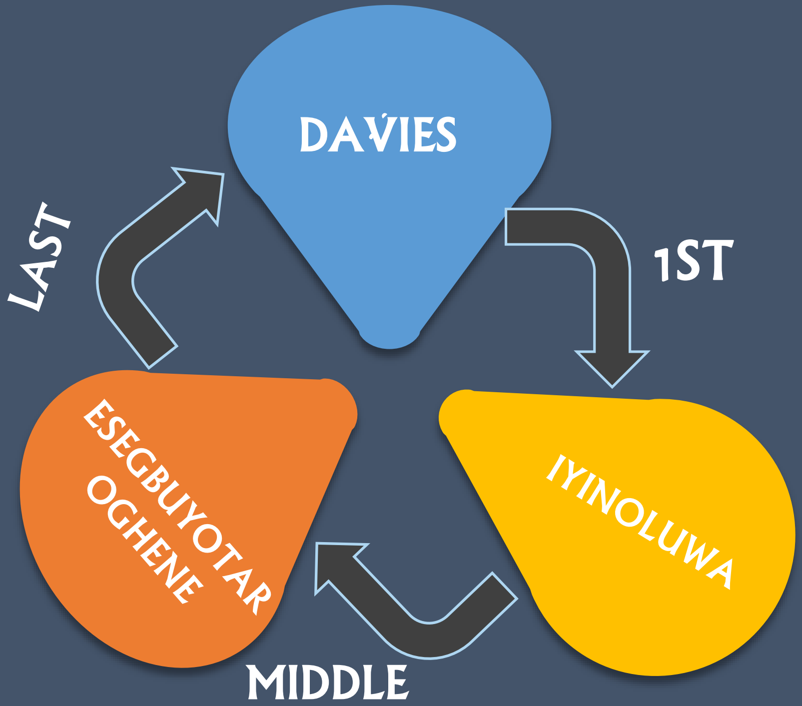
Mr. Emmanuel Davies

Outline

- ✓Introduction
- ✓What is a Beam
- ✓Types of Beam Support
- ✓Types of Beam Loading
- ✓Beam Deflection
- ✓Beam Deflection Methods
- ✓Double Integration Method
- ✓Calculations
- ✓Conclusion

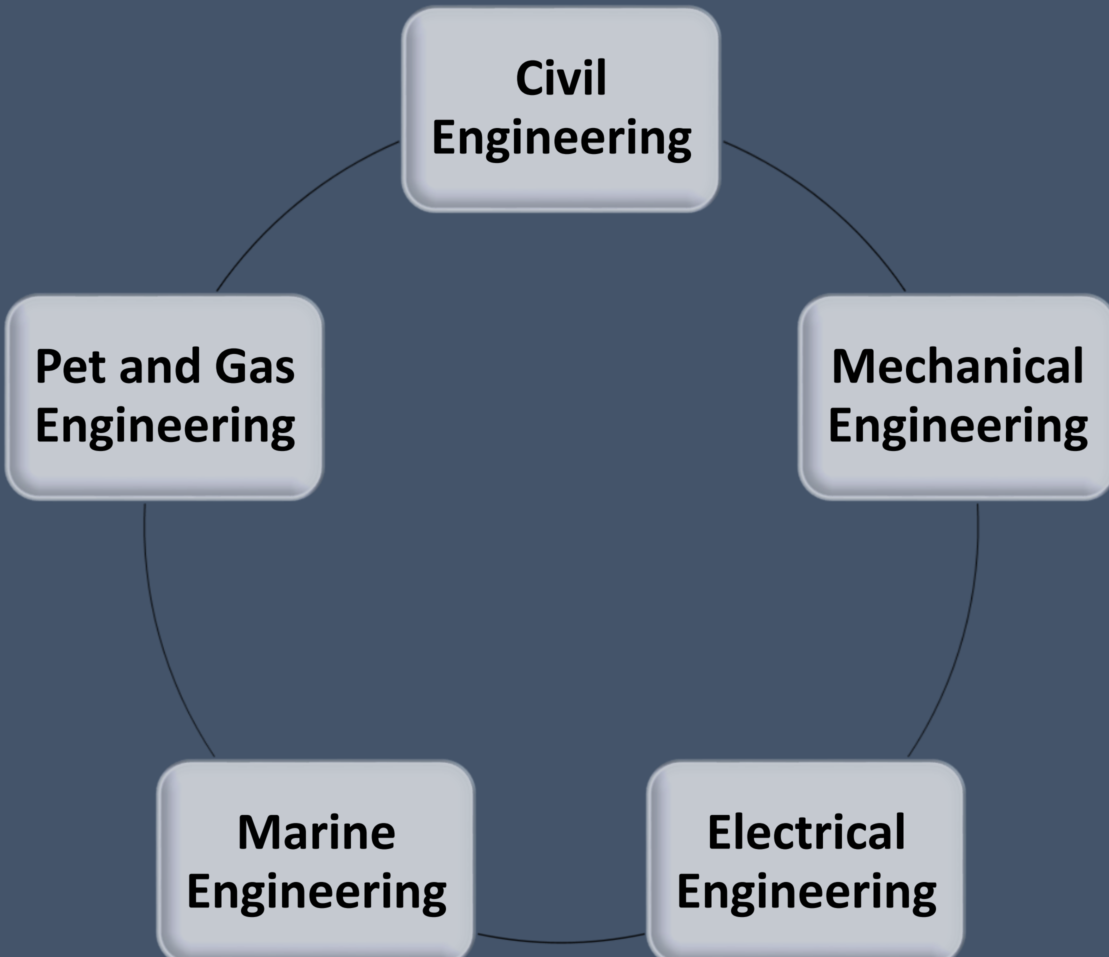
Introduction

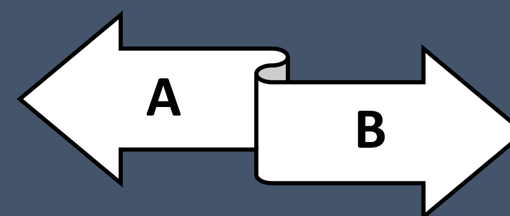
Civil Engineering Department



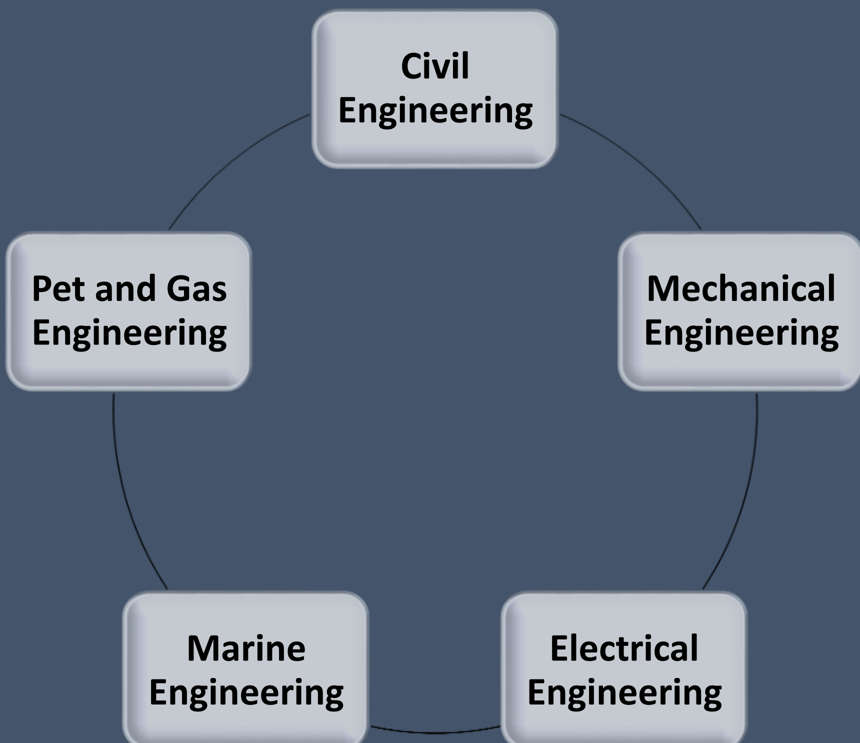
EMMANUEL

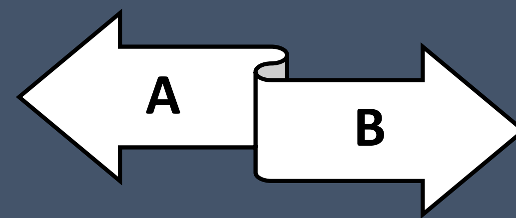
Introduction





Introduction





Group A and Group B Leaders

1. Submit tutorial report and group update to me on WhatsApp (+2347036868768)

Group A and Group B Members

1. Submit practice questions answers to group leaders before any deadline.
2. Attend tutorials.
3. Always indicate your group on your test script.

Beams

A beam is a horizontal structural member that resists applied loads.

Columns or beams supports beams.

Beams

BEAMS

BEAMS

PURPOSE OF BEAMS

1. **Resist Load**
2. **Counter bending moment and shear force**
3. **Connect the structure together**
4. **Provide a uniform distribution of loads**
5. **In concrete buildings, it transfers load from the slab to the column**

CLASSIFICATION OF BEAM S

Based on Material

1. Steel beams
2. Timber beams
3. Concrete beams

1

2

3

CLASSIFICATION OF BEAM S

Based on Support Condition

1. Simply Supported Beams
2. Continuous Beams
3. Fixed Beams
4. Cantilever Beams
5. Overhanging Beams

Beam Based on Support Condition

Real Life Application

Simply Supported Beam - Discontinuity between beams on the support

1

Simply Supported Beam - Discontinuity between beams on the support

1

Beam Based on Support Condition

Real Life Application

Fixed Beam

3

2

Continuous beam – The beam ha
s more than two supports

Beam Based on Support Condition

Real Life Application

Steel beam connected to a steel column by bolt connection

3

Cantilever Beam

4

Beam Based on Support Condition

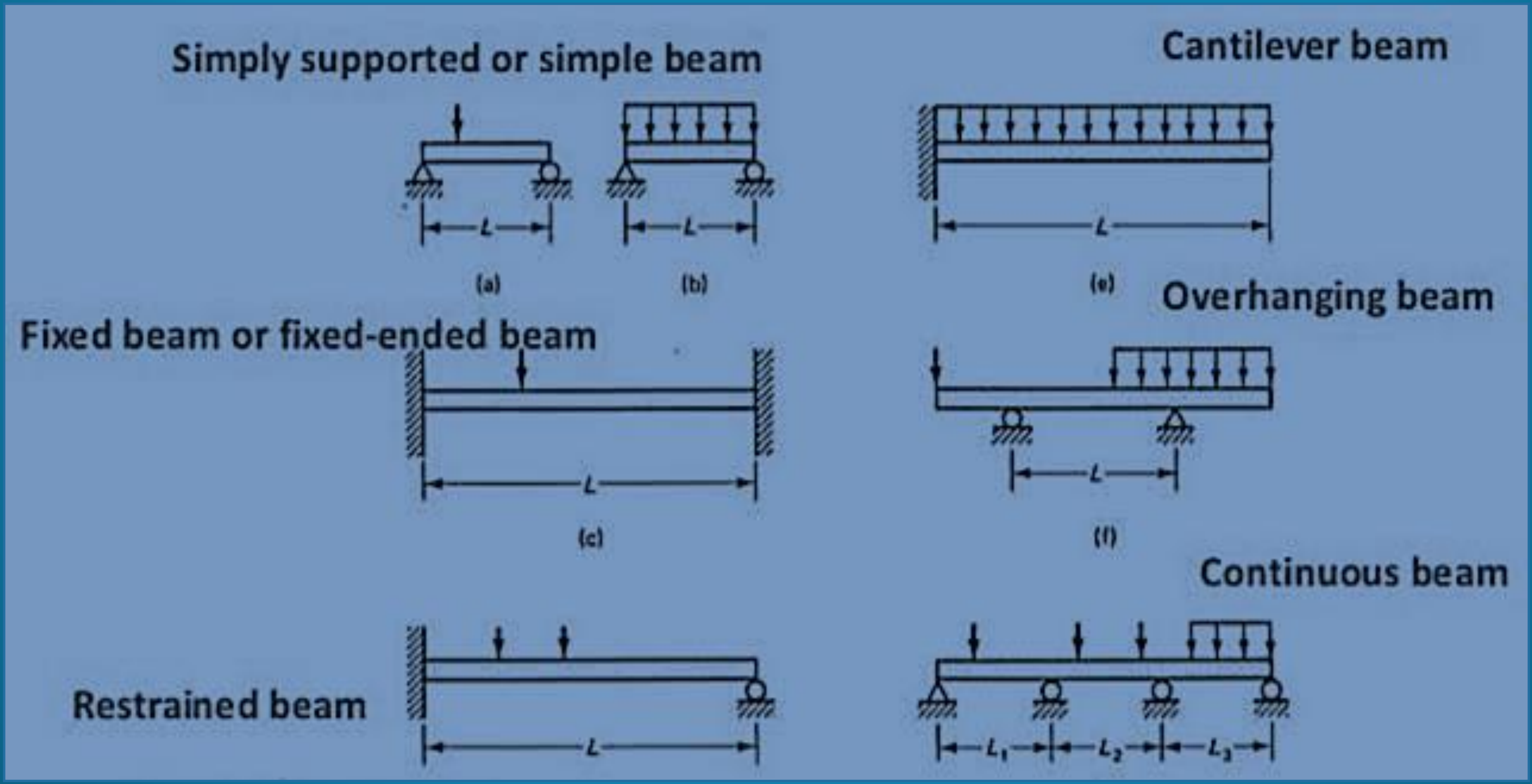
Real Life Application

Overhang Cantilever Beam 5

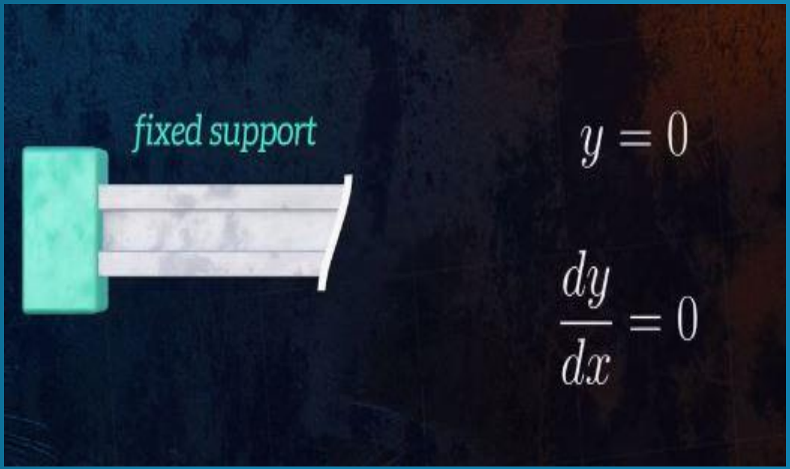
Cantilever Beam 4

Beam Based on Support Condition

Beam Analytical Model



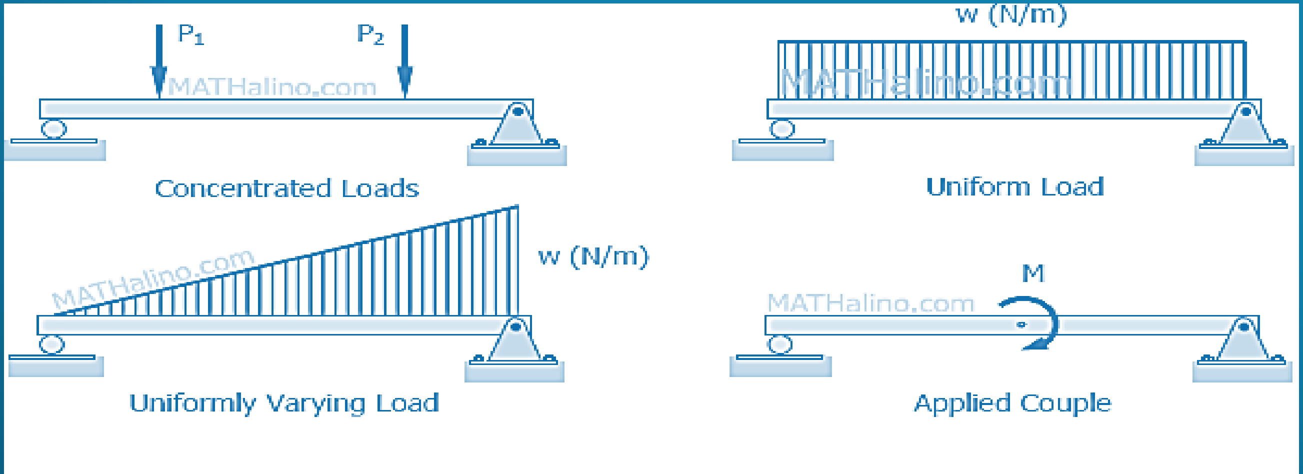
TYPES OF BEAM SUPPORT



Beam Loading

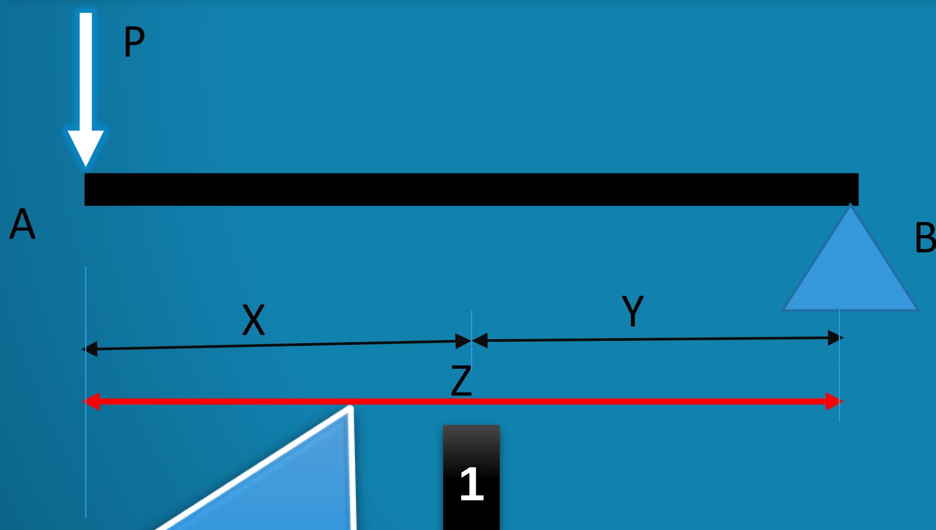
Types of Beam Load

1. Concentrated or point load
2. Uniformly distributed load (U.D.L)
3. Uniformly varying load (U.V.L)
4. Couple load



Bending Moment

The moment about a point in a structure is the product of the force(load) acting on the structure and the perpendicular distance between the force(load) and the point on the structure



X, Y and Z are the perpendicular distance of load P, but Z is the perpendicular distance of load P to the point B.

From our Definition

1. Taking moment about Point B will be;

$$M_b = P \times Z$$

Beam Deflection

Beam deflection is the vertical displacement of a beam subjected to an applied load.

There are five (5) common methods of determining beam deflection;

1. **Double integration method**
2. **Macaulay's Method**
3. **Castigliano's theorem**
4. **Moment Area method**
5. **Superposition method**

In this course, only the double integration method of beam deflection would be discussed.

DOUBLE INTEGRATION
METHOD

MACAULAY'S
METHOD

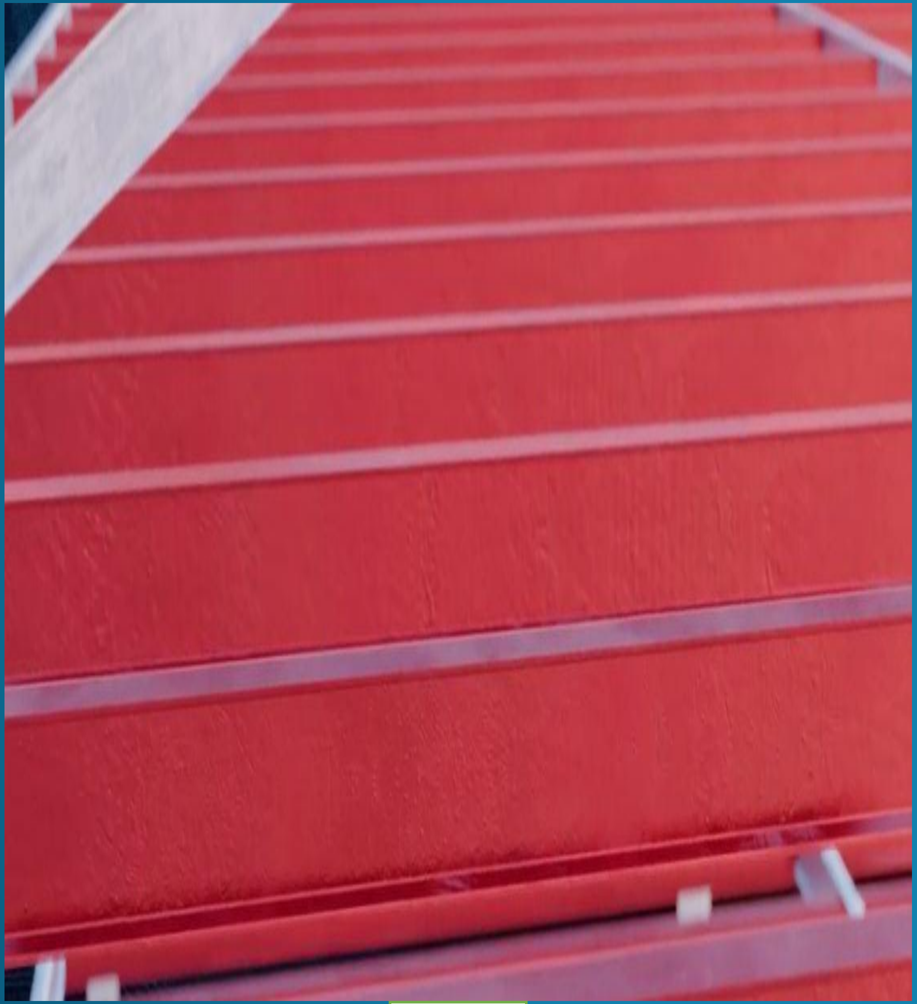
MOMENT-AREA
METHOD

SUPERPOSITION
METHOD

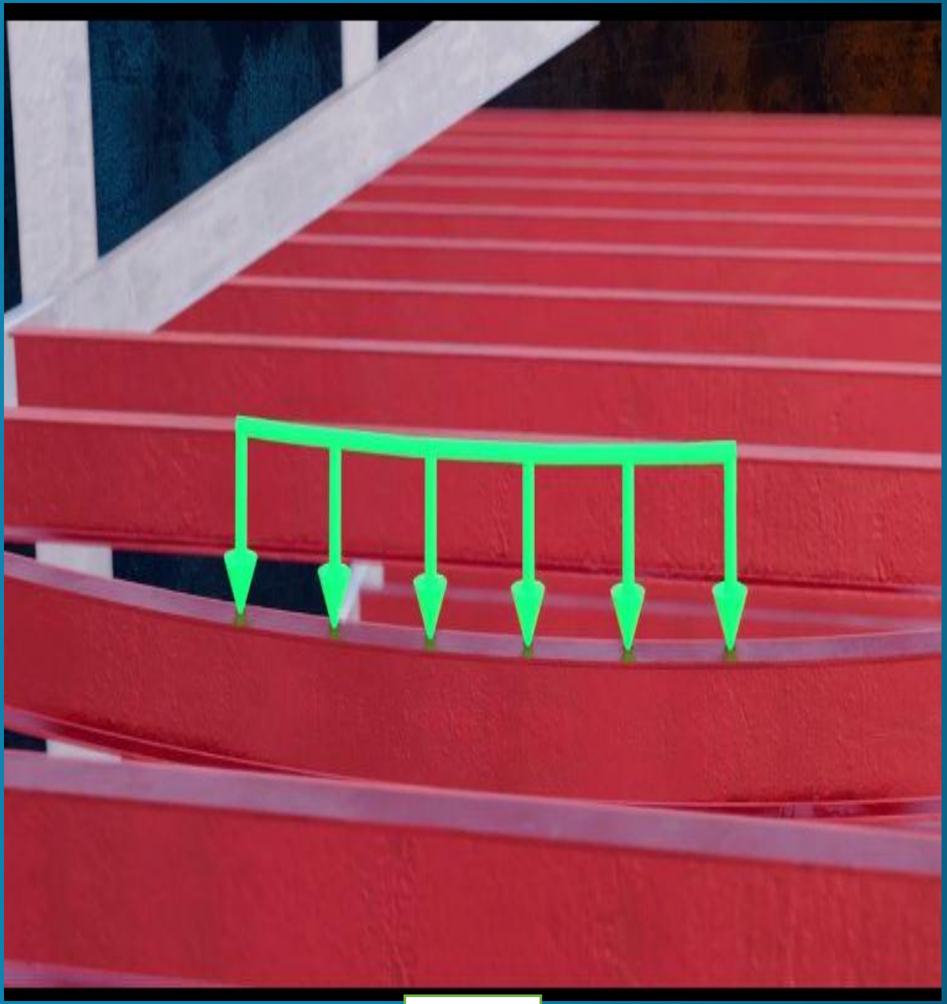
CASTIGLIANO'S
THEOREM

common methods for analysing the deflection of beams

Undeformed Beam (A) and Deformed (deflected) Beam (B)

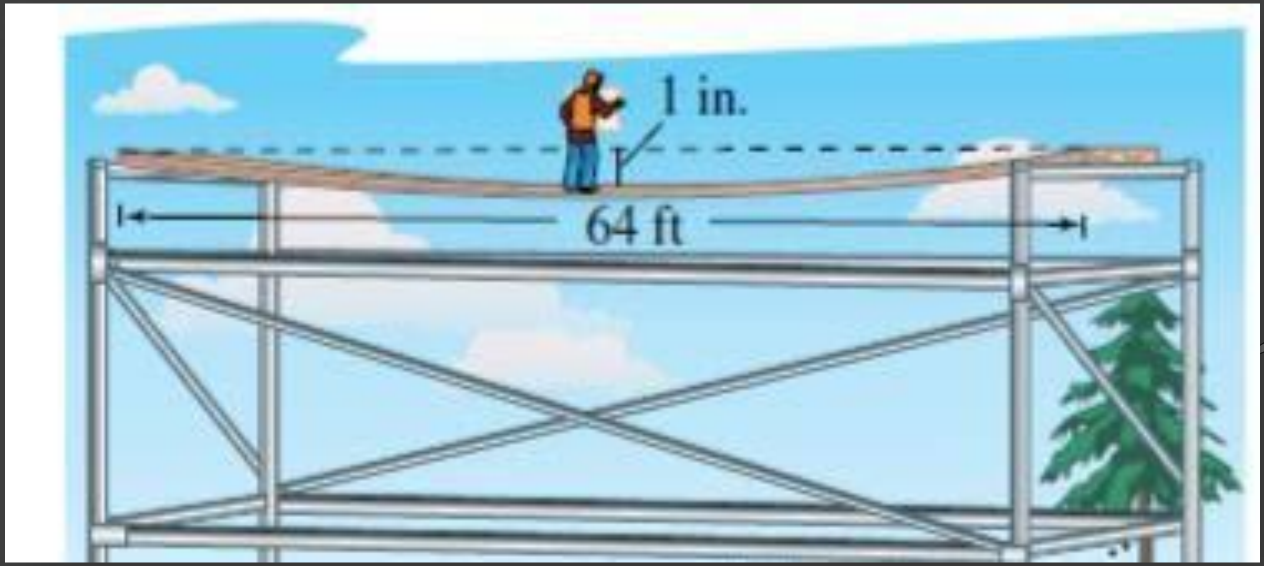


A



B

BEAM DEFLECTION



Deflection under applied load

The figure above illustrates deflection properly. The man standing on the structure is the applied load, 1inch is the deflection of the structure member by the man's weight and 64ft is the length of the structure member.

The moment exerted by the man about any point would be the man's weight multiplied by the distance between the man and that point on the structure.

DOUBLE INTEGRATION METHOD OF BEAM DEFLECTION

The most useful skills in applying the double integration method are moment, bending moment and integration.

The double integration method is not complicated; all you need to do is apply the three basics.

Integration

If Y= $\int 1.dx$	You will have	Y= $x + c$
If A= $\int x.dx$		A= $\frac{1}{2}x^2 + c$
If B= $\int x^2.dx$		B= $\frac{1}{3}x^3 + c$
If C= $\int x^3.dx$		C= $\frac{1}{4}x^4 + c$
If D= $\int 2x^2.dx$		D= $\frac{2}{3}x^3 + c$
If E= $\int 7x.dx$		E= $\frac{7}{2}x^2 + c$

DOUBLE INTEGRATION METHOD OF BEAM DEFLECTION

Integration

If,	You will have
• $\frac{dy}{dx} = 1$	Y= $x + c$
• $\frac{dB}{dx} = x^2$	B= $\frac{1}{3}x^3 + c$
• $\frac{dC}{dx} = x^3$	C= $\frac{1}{4}x^4 + c$
• $\frac{dD}{dx} = 2x$	D= $\frac{2}{2}x^2 + c$
• $\frac{dE}{dx} = 7x$	E= $\frac{7}{2}x^2 + c$

If,	You will have	Lastly
• $\frac{d^2y}{dx^2} = 1$	$\frac{dy}{dx} = x + C_1$	$y = \frac{1}{2}x^2 + C_1x + C_2$
• $\frac{d^2B}{dx^2} = x^2$	$\frac{dB}{dx} = \frac{1}{3}x^3 + C_1$	$B = \frac{1}{12}x^4 + C_1x + C_2$
• $\frac{d^2C}{dx^2} = x^3$	$\frac{dC}{dx} = \frac{1}{4}x^4 + C_1$	$C = \frac{1}{20}x^5 + C_1x + C_2$
• $\frac{d^2D}{dx^2} = 2x$	$\frac{dD}{dx} = \frac{2}{2}x^2 + C_1$	$D = \frac{2}{6}x^3 + C_1x + C_2$
• $\frac{d^2E}{dx^2} = 7x$	$\frac{dE}{dx} = \frac{7}{2}x^2 + C_1$	$E = \frac{7}{6}x^3 + C_1x + C_2$

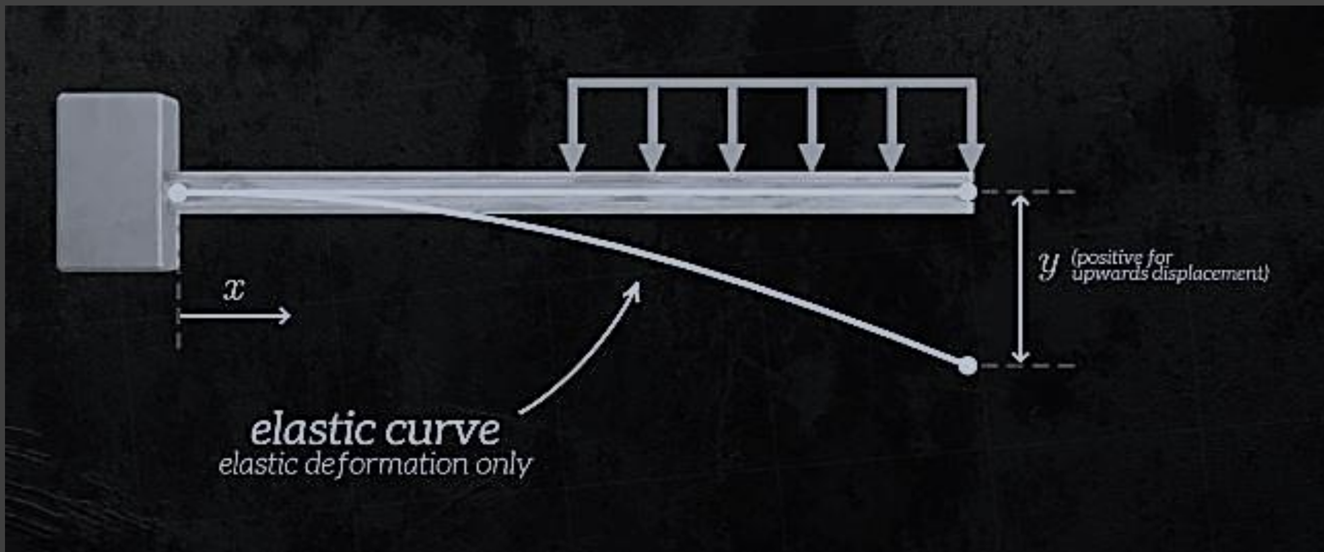
DOUBLE INTEGRATION METHOD OF BEAM DEFLECTION

Now that you know the basics of moment (force × distance), bending moment, and integration, applying the double integration method wouldn't be a problem.

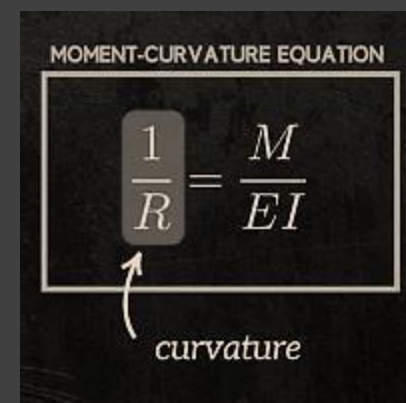
In deriving the formula for both the slope and deflection of a beam in the double integration method, two equations are very important;

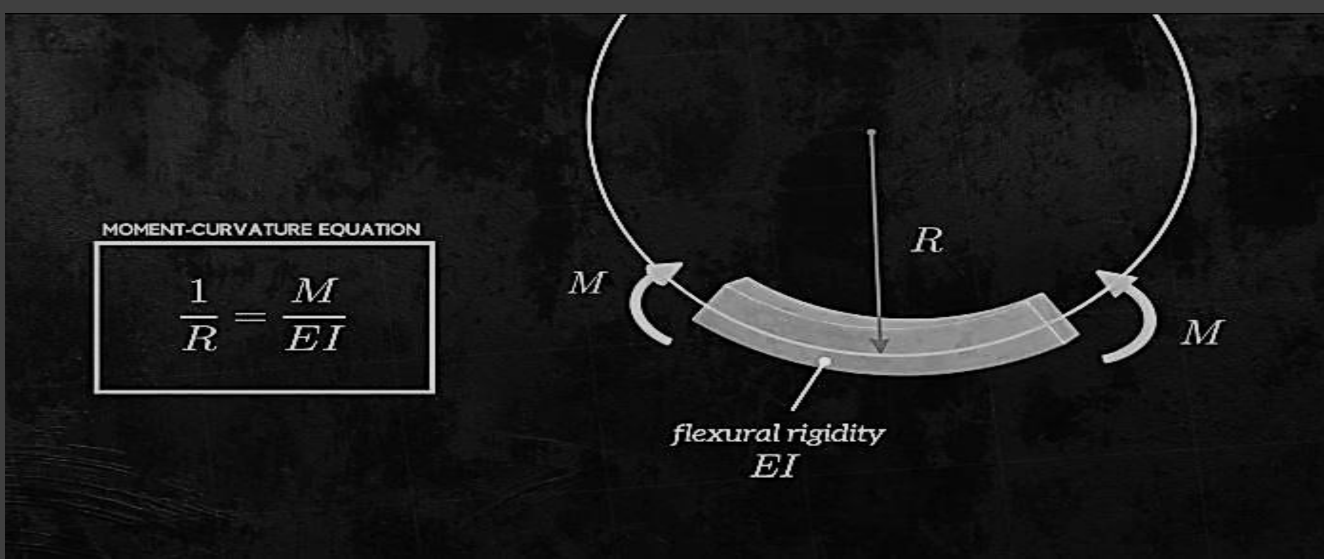
1. **Moment Curvature Equation, and**
2. **Deflection Differential Equation**

BEAM DEFLECTION



Beam deformation or deflection under load





E = Modulus of Elasticity

I = Moment of Inertia

DOUBLE INTEGRATION METHOD OF BEAM DEFLECTION

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

DEFLECTION DIFFERENTIAL EQUATION

$$\boxed{\frac{d^2y}{dx^2} = \frac{M}{EI}}$$

$$\frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}} = \frac{M}{EI}$$

slope is small

DOUBLE INTEGRATION METHOD

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

integrate once

$$\frac{dy}{dx} = \int \frac{M}{EI}\, dx$$

integrate again

$$y = \int\int \frac{M}{EI}\, dx\, dx$$

DOUBLE INTEGRATION METHOD OF BEAM DEFLECTION

As the name implies, double integration has to do with integrating the Deflection Differential Equation twice to derive the deflection equation, y (formula).

The first integration process carried out on the Deflection Differential Equation will give us the slope of the deformed beam, θ or $\frac{dy}{dx}$.

DOUBLE INTEGRATION METHOD

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

integrate once

$$\frac{dy}{dx} = \int \frac{M}{EI}\,dx$$

integrate again

$$y = \int\int \frac{M}{EI}\,dx\,dx$$

BEAM DEFLECTION FORMULA DERIVATION

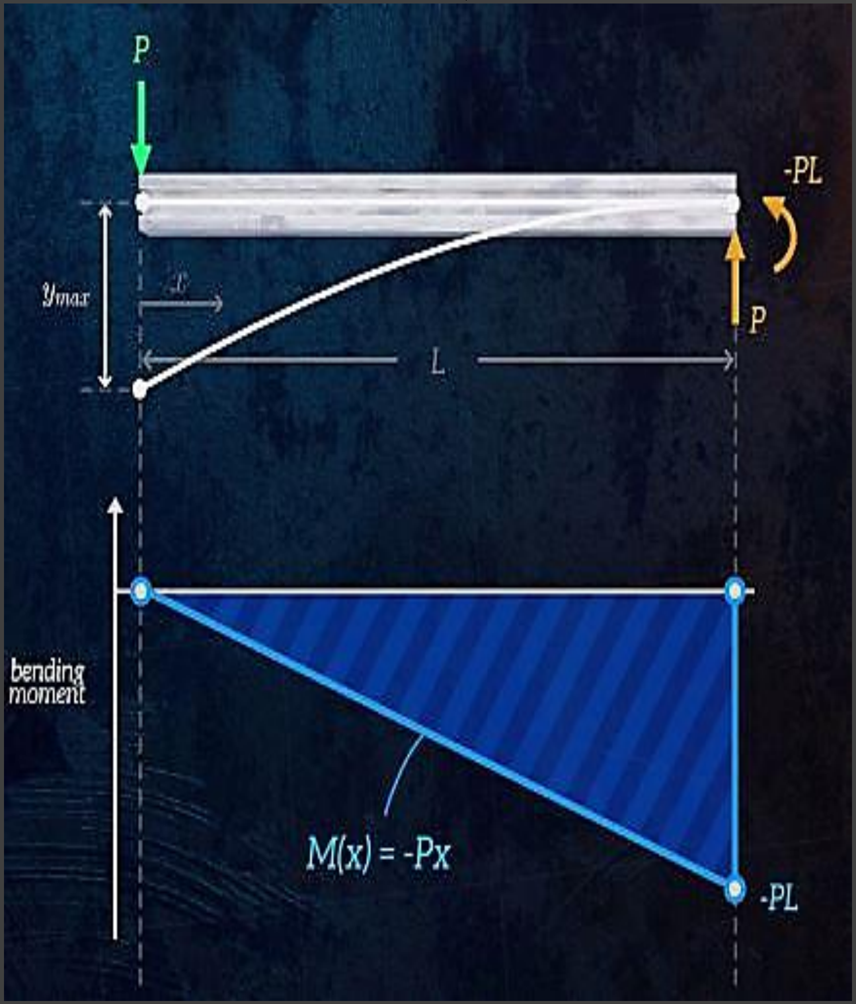
Fig. 1 is a cantilever beam subjected to a concentrated load, P acting at the end of the beam. The cantilever beam length is L, and its maximum deflection is y_{max} .

Recall, moment = Force × Perpendicular Distance.

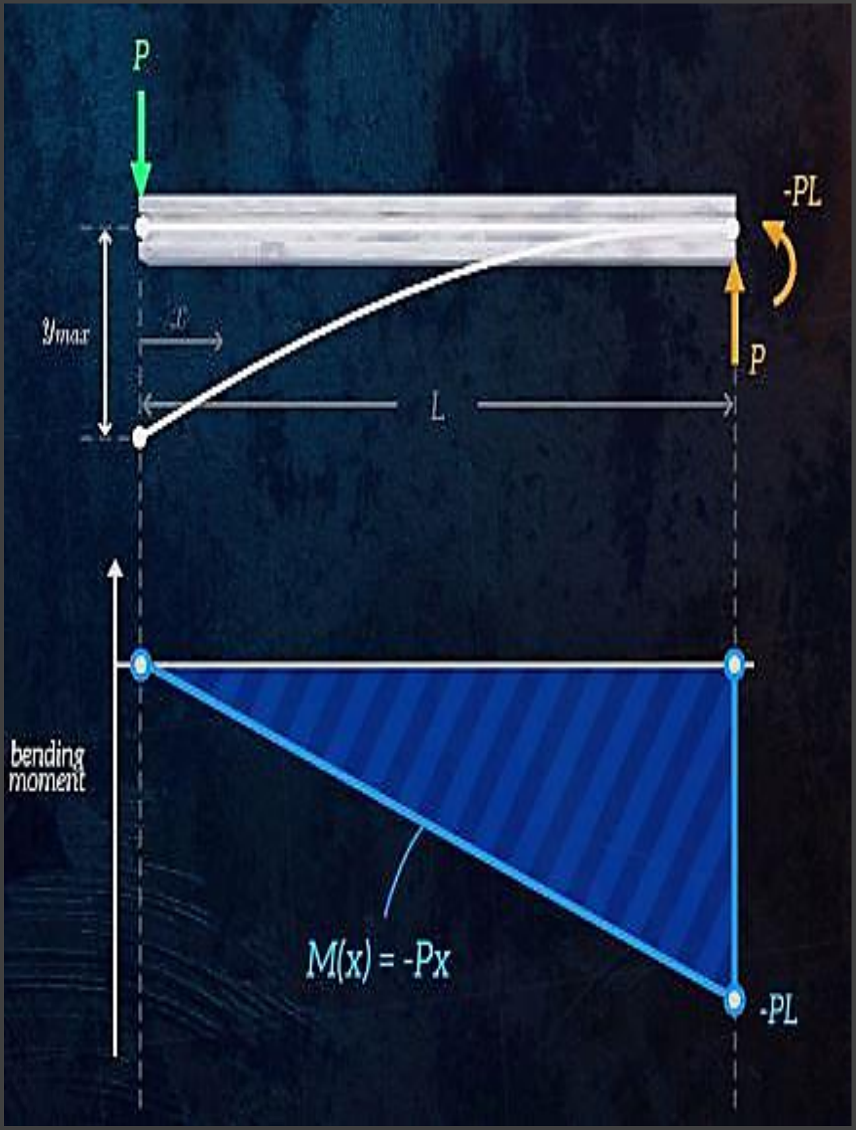
Moment about any distance X on the beam will be = - P × X.

The reason why it is – P is because the force P is acting downwards.

1



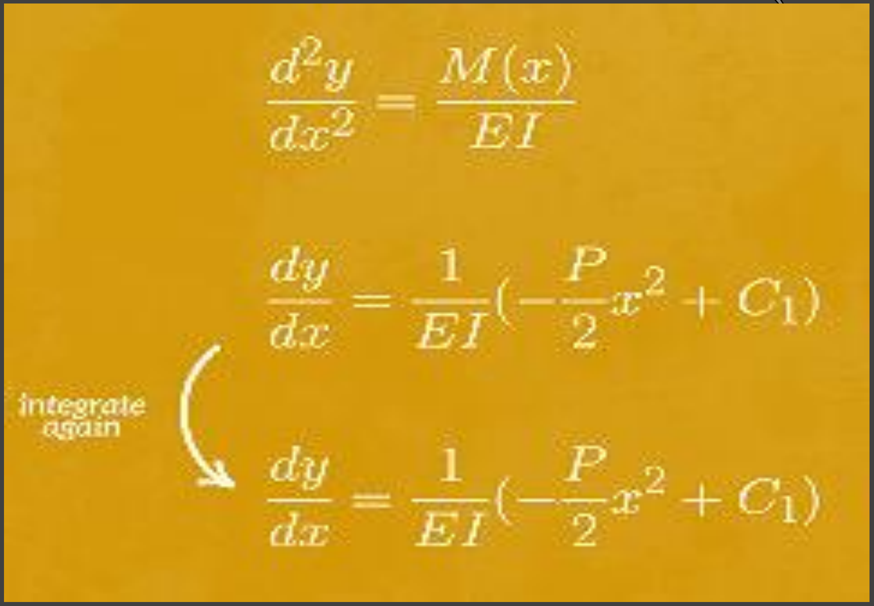
BEAM DEFLECTION



Recall, M(x) in this case is = - P × X

Integrating –Px will give you, (- P × $\frac{x^2}{2}$) + C_1… C_1 is the 1st constant of the integration

2



BEAM DEFLECTION

3

Integrating (- P × $\frac{x^2}{2}$) $+ C_1$, further…

We have

$$(- P \times \frac{x^3}{3 \times 2}) + C_1 x + C_2$$

C_2 is the 2nd constant of the integration (i.e. the constant of the 2nd integration of M(x))

Recall, I have been explaining the solution of the 1st and 2nd integral of M(x).

$$\frac{dy}{dx} = \frac{1}{EI}(-\frac{P}{2}x^2 + C_1)$$

$$y = \frac{1}{EI}(-\frac{P}{6}x^3 + C_1 x + C_2)$$

- At $x = L \rightarrow \frac{dy}{dx} = 0$

$$C_1 = \frac{P}{2}L^2$$

- At $x = L \rightarrow y = 0$

$$y = \frac{1}{EI}(-\frac{P}{6}x^3 + C_1 x + C_2)$$

BEAM DEFLECTION

To determine the value of C_1 and C_2, boundary conditions have to be applied.

The two boundary conditions for this cantilever beam are;

1. At the fixed end of the beam, the deflection or vertical displacement is zero, I.e. y = 0.

2. At the fixed end of the beam, the slope is zero, i.e. $\frac{dy}{dx}$ = 0.

Applying these two conditions the value of C_1 and C_2 can be found easily.

4

$$y = \frac{1}{EI}(-\frac{P}{6}x^3 + C_1 x + C_2)$$

- At $x = L \rightarrow \frac{dy}{dx} = 0$

$$C_1 = \frac{P}{2}L^2$$

- At $x = L \rightarrow y = 0$

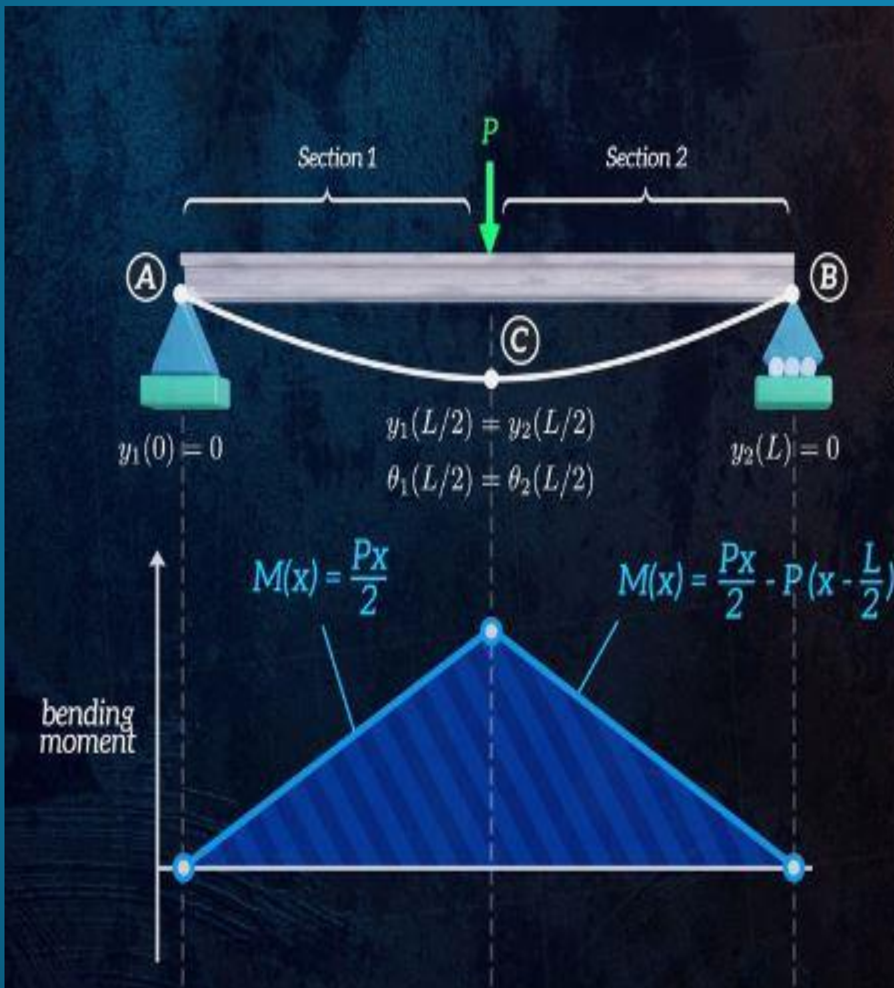
$$y = \frac{1}{EI}(-\frac{P}{6}x^3 + C_1 x + C_2)$$

5

$$y = \frac{P}{6EI}(-x^3 + 3L^2 x - 2L^3)$$

$$y_{max} = y(0) = \frac{-PL^3}{3EI}$$

Beam Deflection Formula Derivation for Simply Supported Beam



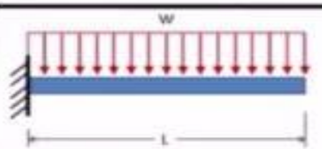
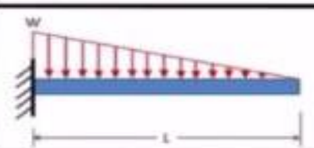
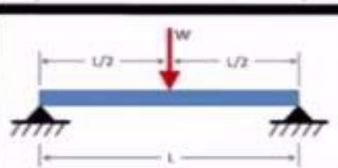
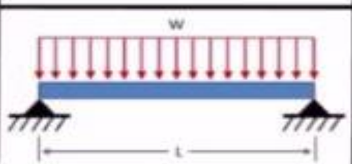
$$y_1 = \frac{1}{EI}\left(\frac{P}{12}x^3 + C_1 x + C_2\right) \qquad (0 \le x \le \tfrac{L}{2})$$

$$y_2 = \frac{1}{EI}\left(\frac{-P}{12}x^3 + \frac{PL}{4}x^2 + C_3 x + C_4\right) \qquad (\tfrac{L}{2} \le x \le L)$$

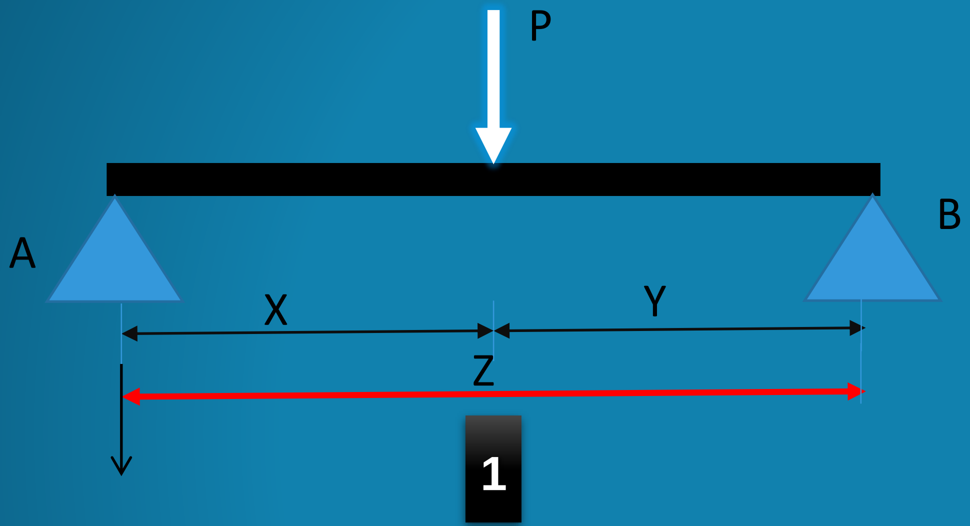
- $y_1(0) = 0 \rightarrow C_2 = 0$
- $\theta_1(L/2) = \theta_2(L/2) \rightarrow C_3 = C_1 - \frac{PL^2}{8}$
- $y_1(L/2) = y_2(L/2) \rightarrow C_4 = \frac{PL^3}{48}$
- $y_2(L) = 0 \rightarrow C_1 = \frac{-PL^2}{16}$

$$y_{max} = y(L/2) = -\frac{PL^3}{48EI}$$

Beam Deflection Formula Table

SR. NO.	TYPE OF BEAM	MAX. BM	SLOPE	DEFLECTON
1		M	$\theta = \frac{ML}{EI} = \frac{ML}{EI}$	$\delta = \theta \times \frac{L}{2} = \frac{ML^2}{2EI}$
2		WL	$\theta = \frac{ML}{2EI} = \frac{WL^2}{2EI}$	$\delta = \theta \times \frac{2L}{3} = \frac{WL^3}{3EI}$
3		$\frac{WL^2}{2}$	$\theta = \frac{ML}{3EI} = \frac{WL^3}{6EI}$	$\delta = \theta \times \frac{3L}{4} = \frac{WL^4}{8EI}$
4		$\frac{WL^2}{6}$	$\theta = \frac{ML}{4EI} = \frac{WL^3}{24EI}$	$\delta = \theta \times \frac{4L}{5} = \frac{WL^4}{30EI}$
5		$\frac{WL}{4}$	$\theta = \frac{ML}{4EI} = \frac{WL^2}{16EI}$	$\delta = \theta \times \frac{L}{3} = \frac{WL^3}{48EI}$
6		$\frac{WL^2}{8}$	$\theta = \frac{ML}{3EI} = \frac{WL^3}{24EI}$	$\delta = \theta \times \frac{5L}{16} = \frac{5WL^4}{384EI}$

CALCULATIONS USING THE FORMULA ON THE TABLE



Q1. If X = 2m, Y = 2m, Z = 4m, P = 30kN. Determine the maximum deflection of the Beam AB.

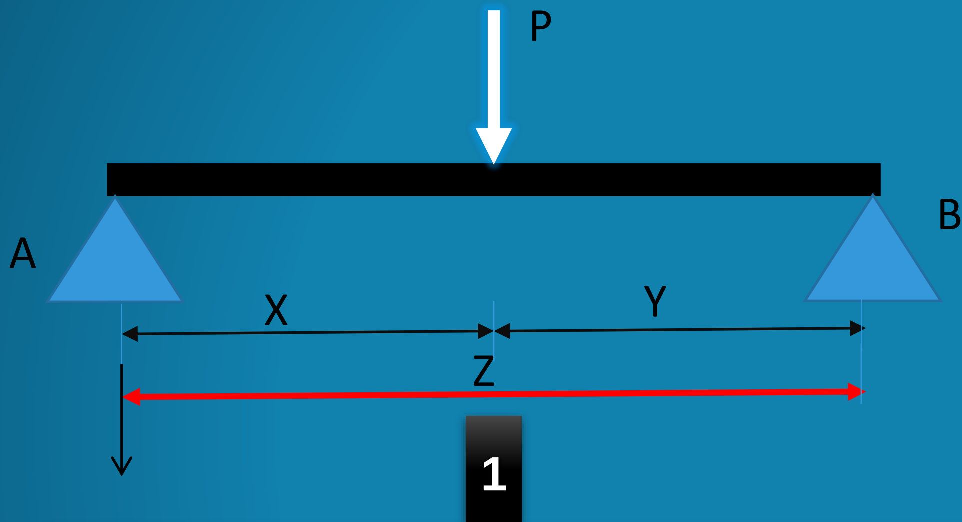
Deflection = $\frac{PL^3}{48EI} = \frac{30\times10^3\times4^3}{48EI} = \frac{40,000}{EI}$

Q2. If X = 5m, Y = 5m, Z = 10m, P = 80kN. Determine the maximum deflection and slope of the beam AB.

Deflection = $\frac{PL^3}{48EI} = \frac{80\times10^3\times10^3}{48EI} = \frac{1,666,666.7}{EI}$

Slope = $\frac{PL^2}{16EI} = \frac{80\times10^3\times10^2}{16EI} = \frac{500,000}{EI}$

CALCULATIONS USING THE FORMULA ON THE TABLE



Q3. If X = 2m, Y = 2m, Z = 4m, P = 30kN, E = 200GPa, I = 1.25× $10^9 mm^4$. Determine the maximum deflection of the Beam AB.

Gpa = $10^9 \frac{N}{m}$

$mm^4 = (10^{-3})^4 m^4 = 10^{-12} m^4$

Deflection = $\frac{PL^3}{48EI}$ = $\frac{30\times10^3\times4^3}{48EI}$ = $\frac{4,000}{48EI}$ =

$\frac{40,000}{200\times10^9\times1.25\times10^9\times10^{-12}} = 160 \times 10^{-6} m$ = 0.16mm

Q4. If X = 5m, Y = 5m, Z = 10m, P = 80kN, E = 200GPa, I = 1.25× $10^9 mm^4$. Determine the maximum deflection and slope of the beam AB.

Deflection = $\frac{PL^3}{48EI}$ = $\frac{80\times10^3\times10^3}{48EI}$ = $\frac{166,666.7}{EI}$ =

$\frac{1,666,666.7}{200\times10^9\times1.25\times10^9\times10^{-12}}$ = $6,666.67\times10^{-6} m = 6.667 mm$

Slope = $\frac{PL^2}{16EI} = \frac{80\times10^3\times10^2}{16EI}$ =

$\frac{500,000}{200\times10^9\times1.25\times10^9\times10^{-12}}$ = $2,000 \times 10^{-6}$ rad = 0.002 rad

Beam Deflection

Calculating beam deflection is not difficult at all.

Thoroughly applying the formula on the table without making mistake is the way to go.

All you need to do to derive the cantilever beam formula is to practice deriving it, a minimum of three (3) times and you are good to go.

All it takes to deriving the cantilever beam formula is understanding the basics; the basics of integration and moment.

Practice Questions for Group A and B

1. What is a beam?
2. Types of beam according to material, support condition and support.
3. Mention three reasons why beams are necessary in structures.
4. What is beam deflection
5. Mention three (3) methods of analyzing beam deflection.
6. Derive the cantilever beam deflection formula five(5) times.
7. Using the formula on the deflection table, determine the deflection and slope of the different types of beam with different assumed values of load (W) and beam length (L). Your E and I should be 200GPa and 1.25$\times 10^9 mm^4, respectively$.

Practice Questions for Group A and B

<u>Submission Time</u>

12pm, Tuesday, 2nd March, 2021.

THANK YOU