Lecture Note One: Introduction to Ship Dynamics

The subject of ship dynamics is concerned with all conditions where the inertia forces interplay a role on ship motions. In the traditional form of the subject the ship is assumed to behave as a rigid body that is static or slowly moving between positions of equilibrium. Thus, ship dynamics account for all operational conditions that differ from the ideal still water condition. Waves, forward speed effects and the influence of elastic distortions on ship dynamic response may also play a role.

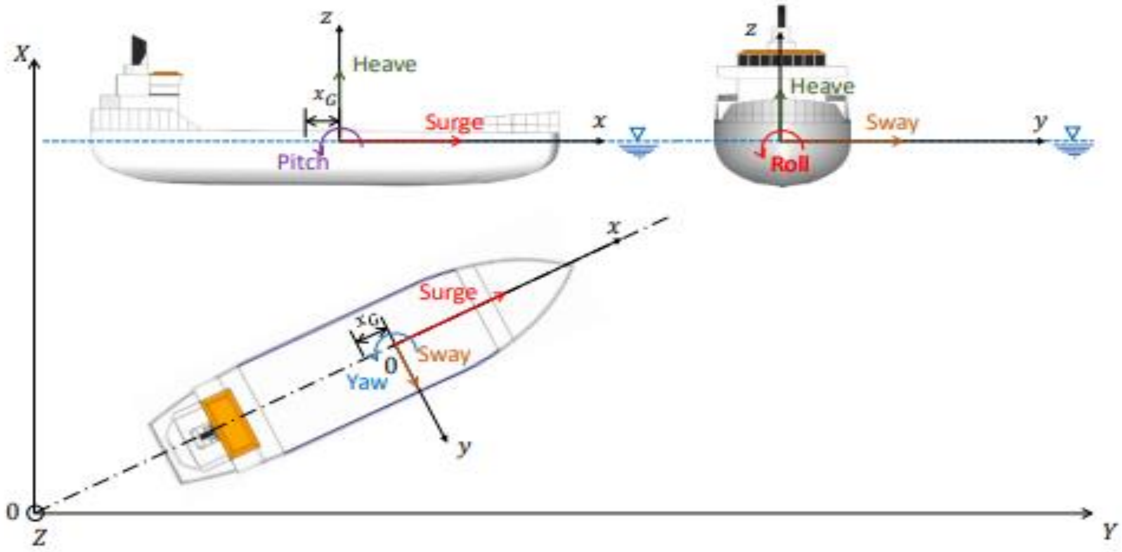
Ship hydrodynamic models comprise of sub-models encompassing the principles of ocean wave mechanics, seakeeping, maneuvering, structural vibration, and dynamic stability. Modelling each of those within the context of marine hydrodynamics is prone to simplifying assumptions that should be thoroughly evaluated before used in design or for operational decision support. Linear hydrodynamic models are today considered mature and can be useful at concept or preliminary design stages. On the other hand, understanding ship dynamics in the time domain remains challenging. This is because numerical methods are not mature or well validated although can help with improved quantification of the influence of environmental and operational conditions on large amplitude ship motions on loads and dynamic stability.

Some Definitions

Seakeeping

refers to a ship’s ability to remain at sea in all conditions and carry her intended mission. Topics such as dynamic stability in waves, strength, maneuvering, added resistance inevitably link with seakeeping dynamics and ship performance in waves. This is because excessive ship motions may have adverse effects on ship design and operations. They may lead to hull rupture, discomfort of the passengers and crew, result in less efficient working conditions and bad worker or customer experience. Added water resistance due to ship motions in waves and propellers exposed to heavy conditions may also result in reduced ship efficiency. This is because severe motions and heavy loading on propulsors may lead to voluntary ship speed loss. To control this problem computerized weather routing systems are now fitted on several ships allowing the master greater control of speed and seaworthiness in demanding or extreme conditions. In traditional seakeeping analysis the ship is modelled as a rigid body moving in six degrees of freedom namely three translations (heave, surge, and sway), and three rotations (roll, pitch, and yaw). For a ship to maintain sound dynamic stability in waves the oscillatory degrees of freedom of roll, pitch and heave should be controlled. In heavy seas a ship's bow may dig into waves and water may be driven over the ship's forecastle deck. The phenomenon is known as deck wetness and may be linked with strongly coupled heave and pitch motions. In such conditions slamming loads may be evident. The main design factors affecting these operational scenarios are the relative motion of the bow, the sea surface and the ship freeboard.

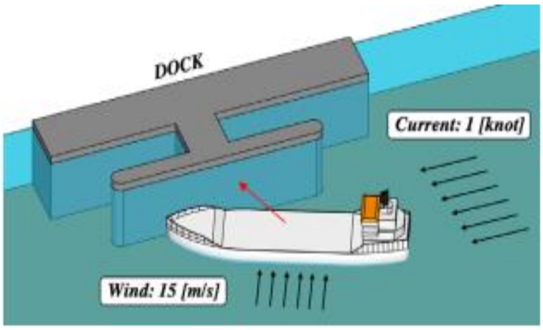
When sailing in congested waterways such as canals or during navigation in harbors, ship control is essential to ensure accurate ship tracking relative to the berth points and safety in relation to other ships in harbor. A ship is said to be directionally stable if a deviation from a set course increases only while an external force or moment is acting to cause the deviation. Conversely, it is said to be unstable if a course deviation begins or continues even in the absence of an external cause. A directionally unstable ship is easy to maneuver, while a stable ship requires less energy to maintain a set course. A compromise between extremes is therefore desirable. Another example are dynamic positioning systems used in offshore vessels or drilling platforms. Such systems help maintain floatability and positioning relative to the seabed. Thus, propulsors producing ahead and astern thrust as well as turning moments and thrust have been developed. The latter (i.e., turning moments and lateral thrust) are provided using rudders directly positioned behind the main propulsors and in some cases additional lateral thrusters are used where higher maneuvering capability is required (e.g., ship bow/stern regions).



Ship maneuvering and control

(positional and directional stability) relate with controlling ship course and speed and may involve the investigation of motions due to disturbing forces from the environment (e.g Sway and Yaw) and or control mechanism such as rudder. The maneuvering characteristics of a vessel are usually defined in still water conditions. Wind effects on maneuvering characteristics are of concern for ships with large superstructures such as cruise ships and ferries. When a ship operates close to banks or at close proximity to another ship, she may experience additional forces and turning moments with significant variations. Thus, the use of time domain hydrodynamic models is typical in manoeuvering but only an option in seakeeping.

Maneuvering is often studied in shallow waters but seakeeping in open seas. Finally, seakeeping is studied by an inertial coordinate system while manoeuvring by a ship fixed system. The simulation of viscous fluid flows implies mathematical difficulties and computational costs. The primary purpose of computing motions and loads of ships in a seaway is to assure the safety of persons on board, the integrity of the ship and the cargo. They also aim to improve performance and efficiency. Excessive motions may cause shift of cargo, damage from loosened deck containers or equipment and dangerously large heel angles and capsizing. Furthermore, ship motions affect the comfort of persons on board, leading to sea sickness or, in extreme cases, to render it impossible for the crew to accomplish a ship's mission. Knowledge of wave-induced loads is necessary to assess the integrity of the ship's structure. Most important for this are vertical and horizontal bending moments, torsional moments, and sometimes shear forces in transverse sections of the hull girder. Wave-induced local pressure acting on the hull determines the necessary strength of plates, stiffeners, and web frames. Furthermore, steady wave- and wind-induced forces and moments should not prevent the ship from arbitrary course changes nor from adjusting the speed ahead.



a) shallow water slow maneuver

b) deep water fast maneuver

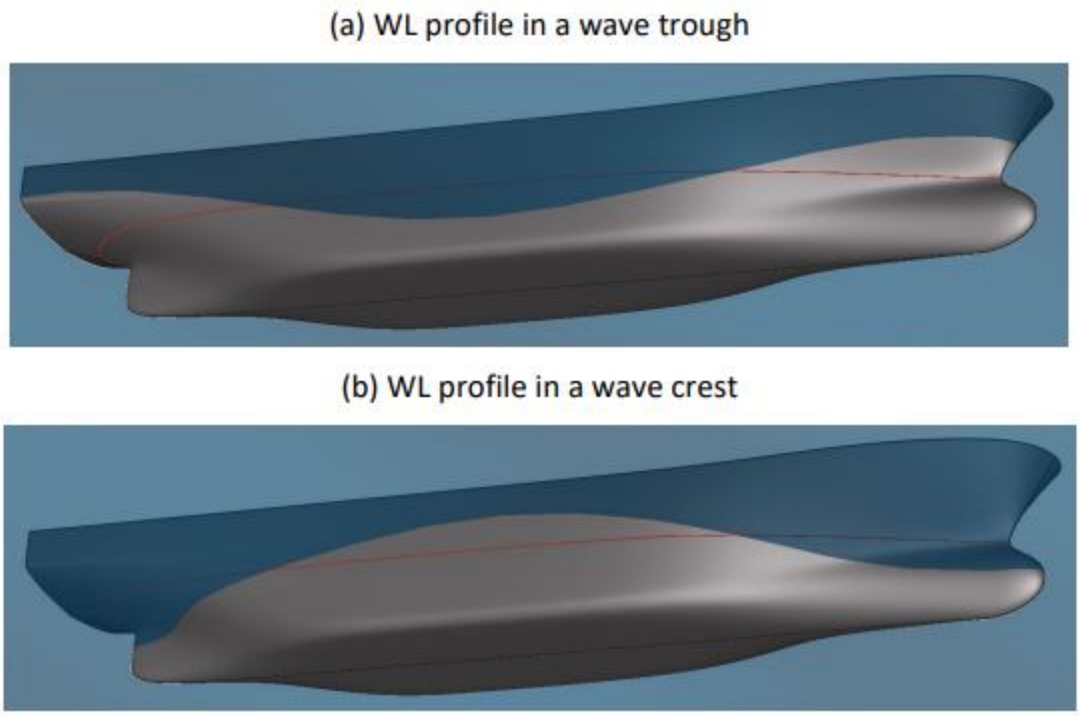
Hydroelasticity of ships: is concerned with the interaction of the ship modelled as an elastic body with her surrounding fluid (Hirdaris et al., 2010). Theoretically, flexible ship dynamics recognize the significant differences in the hydrodynamic, inertia, and elastic forces that may lead to the amplification of wave loads and excessive strains and stresses possibly leading to hull rupture or high fatigue loads. The importance of flexible ship dynamics increased over the last few years as sea transportation and ship sizes increased. Modern ocean carriers are more flexible, and their structural natural frequencies can fall into the range of the encounter frequencies of the sea spectrum. It is now recognized that hydroelastic effects associated with ship slamming or the antisymmetric (i.e. coupled horizontal bending and torsion) dynamics of ships with large openings may influence wave load predictions.

Ship dynamic stability in waves: attempts to investigate roll motions which are subject to heeling moments in the irregular seaway. Investigations therefore include nonlinearities (e.g., roll damping) and provide variation of roll angle in time with the ultimate purpose to investigate whether the ship will capsize. There are also some investigations dealing with the coupled sway-roll-yaw motions; thus, bringing together the subjects of directional and dynamic stability.

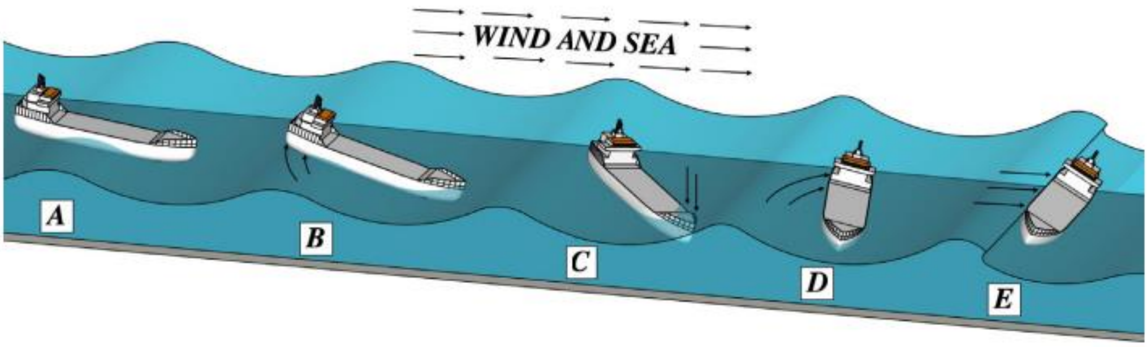
Ship survivability against capsizing: in heavy seas has become one of the areas of primary concern among ship researchers, designers and regulators in recent years. When a ship is subjected to the effect of large waves it may capsize according to several different scenarios, which further depends on the magnitude and direction of the wave excitation and the ship's own capability to resist such excitations.

Resonant or breaking waves: approaching a ship from the ship side (beam seas) have a potential to excite large rolling which could result in capsizing, especially if the intensive oscillation of the ship causes a shift of cargo or a considerable quantity of green water is seen on the deck. More dangerous still can be a group of steep and relatively long waves approaching a ship from the stern (following-seas). Waves of this kind are known to incur significant reductions in roll restoring capability (i.e., the tendency to return to the upright position) for many types of vessels and they may also instigate dangerous coupled motions. In following-seas a ship may capsize in at least two ways known as pure loss of stability and parametric instability. The former is a sudden, non-oscillatory type capsize taking place around a wave crest due to slow passage from a region of the wave where roll restoring has become negative. Parametric instability is the gradual build-up of excessively large rolling created by a mechanism of internal forces, the result of a fluctuating restoring movement that depends on where the ship lies in relation to the wave (ABS, 2004). This phenomenon is related to the periodic change of stability as the ship moves in longitudinal waves at a speed when the ship's wave encounter frequency is approximately twice the rolling natural frequency and the damping of the ship to dissipate the parametric roll energy is insufficient to avoid the onset of a resonant condition.

If a ship is in a wave trough, the average waterplane width is significantly greater than in calm water. The flared parts of the bow and stern are more deeply immersed than in calm water and the wall-sided midship is less deep. This makes the mean, instantaneous waterplane wider than in calm water with the result that the metacentric height increases over the calm water value. When the wave crest is located amidships, the waterplane at the immersed portions of the bow and stern are narrower than in calm water. Consequently, the average waterplane is narrower and the metacentric height decreases in comparison to calm water. As a result, the roll restoring moment of the ship changes as a function of the wave's longitudinal position along the ship. Broaching to relates to an unintentional change in the horizontal-plane kinematics of a ship. Broadly, it may be described as the "loss of heading" by an actively steered ship. It is accompanied by an uncontrollable build-up of a large deviation from the desired course. Broaching to is more commonly occurring in waves which come from behind and propagate in a direction forming a small angle, say 10-30 deg., with the longitudinal axis of the ship.



Ship stability in longitudinal waves (calm water line – WL is denoted in red)

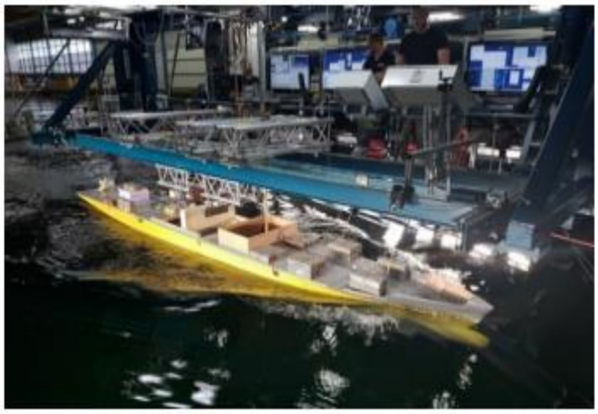


Stages of a Broaching to scenario: (a) the ship may run on crest; (b) ship stern gets too high and thus the rudder losses effect; (c) the bow pitches into trough and buries; (d) stern swings round bringing ship abeam to elements; (e) next wave will possibly break over the ship and cause severe damage

The influence of ship dynamics on ship design development

Ship safety in design is assured by the IMO and Classification Societies. The Class Societies develop rules regarding ship loads. Traditional Classification rules are based on accident records and experiences with ships in operations, as well as theoretical and experimental studies leading to close form and empirical criteria. Ship safety criteria that relate with maritime operations (e.g. maneuvering and stability requirements) are introduced by the International Maritime Organization (IMO) and developed in association with Flag Administrations, Classification Societies, academia and industry including non-governmental organizations (IMO, 2017). In the last 20 years, computational methods have been used to improve and extend the rules related to wave loads and seakeeping and to investigate wave responses for newbuild ships that differ substantially from those for which the rules were prepared. For reliable load predictions, it may be advantageous to apply advanced and possibly costly computations to reduce a ship's scantlings or the probability of structural failures. Numerical simulations may help to estimate the probability of excessive motions and accelerations with respect to ship motions. This may help to extend the safe limits of metacentric height (Lloyd'sRegister, 2018). Ship dynamics can be assessed by using full-scale measurements, model tests, and numerical methods. Despite advances in theoretical ship hydrodynamics the design of novel hull forms at preliminary design stage makes use of model scale experiments. Development of wave basin models are cut from a plan re-drawn from the hull lines and may be costly unless 3D printing methods are employed. From naval architecture perspective it is imperative to realize that ship models used in model tests should be as large as possible to minimize viscosity scale effects. Yet, increased model size should not influence ship dynamics in restricted waters and the size of a stock propeller is to be taken into consideration when the scale for a ship model is selected. The material of which the model is made is not important provided the model is sufficiently rigid. Wood, wax, high density closed cell foam and fiber reinforced plastic are commonly used. Model test results can be converted to full-scale data except for the influence of viscosity, which is small in most cases. More important is the limited size of the model basin, the degree of sophistication of the equipment of the test facility, and cost and time to perform such experiments. In irregular seaways, long test runs are required to obtain representative results. Thus, for seakeeping models today, experiments are used mainly to validate numerical methods. An exception is the sloshing of fluids in tanks, where small-scale effects like wave breaking and the collapse of bubbles may be important for practical questions but are difficult or impossible to simulate accurately.

The range of model tests carried out depend on the type of the analysis or the sub-model by which the ship behavior is investigated. As an example, model tests that aim to predict powering performance of a ship comprises the resistance test, the self-propulsion test and the propeller open-water test. Seakeeping model tests usually employ self-propelled models in narrow towing tanks or broad, rectangular seakeeping basins. The models are sometimes completely free, being kept on course by a rudder operated in remote control or by an autopilot. In other cases, some degrees of freedom are suppressed e.g., by wires. If internal forces and moments are to be determined, the model is divided into sections. The individual watertight sections are coupled to each other by gauges consisting of two rigid frames connected by stiff flat springs with strain gauges. Model motions are then determined either directly or by measuring the accelerations and integrating them twice in time. Waves and relative motions of ships and waves are measured using two parallel wires penetrating the water surface. The change in the voltage between the wires is then correlated to the depth of submergence in water. The accuracy of ultrasonic devices is slightly worse. The model position in the tank can be determined from the angles between the ship and two or more cameras at the tank side. Either lights or reflectors on the ship give the necessary clear signal. Full-scale measurements (FSMs) are possible only if a ship, or her sister to this vessel, are build. FSMs may be expensive, the wave conditions cannot be controlled and assessing the wave conditions during the measurements with the required accuracy may be difficult. During FSMs ship motions are measured by accelerometers and gyros., Global and local loads are measures by strain gauges and loss of speed, propeller rpm and torque are all monitored. Recording the seaway can be done either by recording measurements over many years of operation, or by deducing the maximum values during the lifetime of the ship and then by extrapolating the recorded distribution of long-term measurements.



(a) Hydroelastic ship model **b) Ship resistance model**

Model testing remains key part for the validation of ship dynamics of novel hull forms

The random variation of the actual sea states encountered by a ship may introduce considerable inaccuracies for the predicted extreme values even if several years of measurements are available. Although model tests and FSMs can provide useful information of a ship's performance, designs and operational conditions may differ. Hence it is not certain whether the elaborated ship hull form together with the designed propeller and appendages will ensure efficient performance of any ship in all conditions. Such possibilities are offered by numerical methods such as computational fluid dynamics and finite element analysis. **Linear hydrodynamic models** are used to determine motions and structural hull girder loads for ships advancing at constant forward speed in small amplitude regular waves under various combinations of wave frequency and heading. For any seaway described by a wave spectrum, the results are combined to obtain root mean square values of loads extrapolated linearly over wave amplitude. Results for different seaway conditions are then combined to a long-term probability distribution of loads. For suitably selected design conditions, nonlinear corrections to the linear loads can be applied. If more accuracy is required, solvers for Navier-Strokes or Euler equation may be applied, which consider the water and air interface. Full understanding and an accurate prediction of hydrodynamic wave body interactions is challenging. The associated nonlinear effects become critical when large-amplitude body motion or high surface waves are involved. Recent research advances in the area of **ship damage stability** suggest that implementation of marine hydrodynamics and structural dynamics in ship crashworthiness assessment will influence ship design development and assessment. The use of big data analytics may also prove useful in terms of defining direct assessment methods accounting for the influence of hydro-meteorological conditions on the probability of ship collision and grounding. It is envisaged that these developments will impact upon future IMO ship damage stability standards.

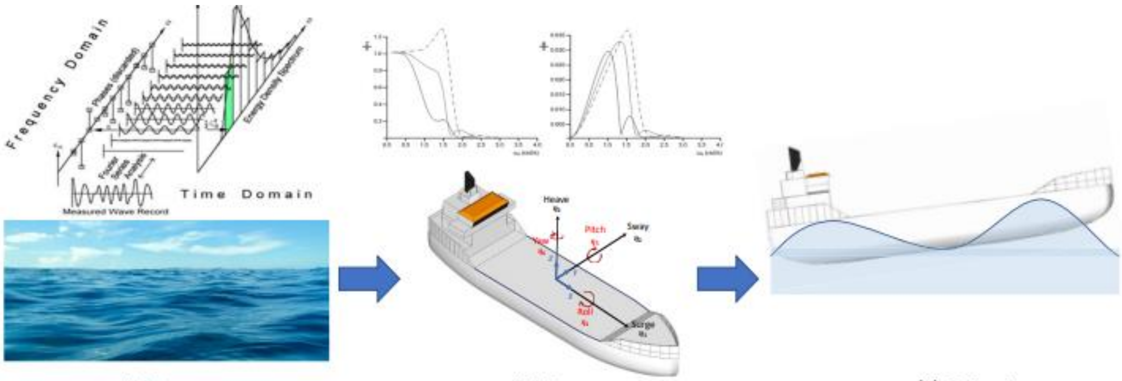
Questions

1. What are the key terms for ship dynamics and which ones affect the design of the ship?
2. What is the difference between seaworthiness and seakeeping?
3. Which phenomena are frequently simplified when creating ship dynamic simulations ?
4. Why would shipyards and shipowners have different maneuvering requirements for vessels?
5. What are the main categories of wave loads and which methods are used to model them?
6. Which organizations are responsible for ship safety?
7. What is the difference between rules and regulations? How they relate with ship dynamics?
8. What types of engineering tools are used to generate and collect ship dynamic data?
9. Name the advantages and limitations of full-scale measurements.
10. Which non-linear effects are important to the operation of a ship and why?

Lecture Note Two: Introduction to ship motions

Introduction

Ship seakeeping is a term that reflects the ability of a vessel to withstand rough conditions at sea. It therefore involves the study of the motions of a ship when subjected to waves, and the resulting effects on humans, systems and mission capability (Lloyd, 1989). With fast computers and sophisticated software readily available to designers, it is now possible for a vessel's seakeeping characteristics to be addressed much earlier in the design spiral. As shown in Figure below there are three main components that influence ship seakeeping responses namely, (a) the waves as input to the system, (b) ship system characteristics and (c) ship motions.



(a) Input
Waves and Winds

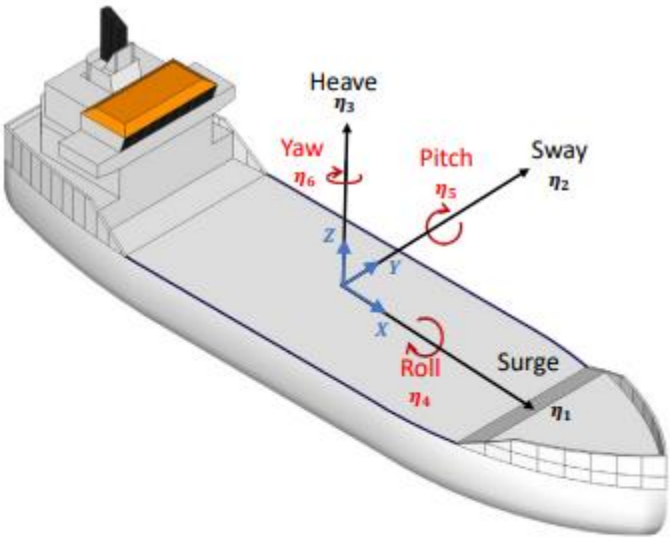
(b) System
RAOs

(c) Output
Motions and structural loads

A vessel's general particulars (e.g., length, beam, draft), hull form and metacentric height influence seakeeping responses and in turn ship safety and performance (Zhang et al., 2021). For example, small length ships with classic hull forms possibly including bow flare while progressing at medium to high forward speeds suffer from large motions. On the other hand, long and bulkier ships experience lower motion amplitudes. Shallow drafts may lead to higher risks of keel emergence and bow slamming loads in rough seas. In turn, motions may also vary due to loading conditions and operational factors. Small hull form adjustments (e.g., reduction of the radius of curvature in way of the bilges) can marginally influence ship motions. Notwithstanding this, a large forward waterplane can reduce overall motions and the probability of keel emergence. Changes to overall ship proportions (e.g., beam to length or beam to draft ratios) may have important consequences. For example, reducing the draft of a ship (for a given length and beam) may reduce ship motion amplitudes. The ship beam relates with metacentric height. Whereas a large metacentric height improves initial stability, it may also lead to high hull natural frequencies which are usually associated with poor motion sickness indices. On the other hand, if the metacentric height is too small motions are smoother but the risk of capsize increases dramatically.

Basic Ship Motion Definitions

A ship is a six degree of freedom (6-DoF) rigid body system. Motions (1 - 3) are linear displacements (translations) known as surge, sway and heave. Motions (4 - 6) are rotations known as roll, pitch and yaw. All motions are measured relative to the ship as shown in Figure below.



Ship seakeeping degrees of freedom

Surge describes the horizontal oscillations of the ship toward the bow and the stern.

Sway is a side - side motion. A vessel moving to starboard travels in positive sway direction.

Heave is the vertical motion. By convention, positive heave points downwards (toward the water bottom). So, a vessel that is sinking into the water (i.e. increasing her draft) is moving in the positive heave direction.

Roll is a rotational motion about the surge axis. If the starboard and port sides move vertically but in opposite directions (i.e. the starboard side is moving up while the port side is moving down). By convention positive roll angles correspond to the starboard moving downwards while the port side moves in the opposite direction.

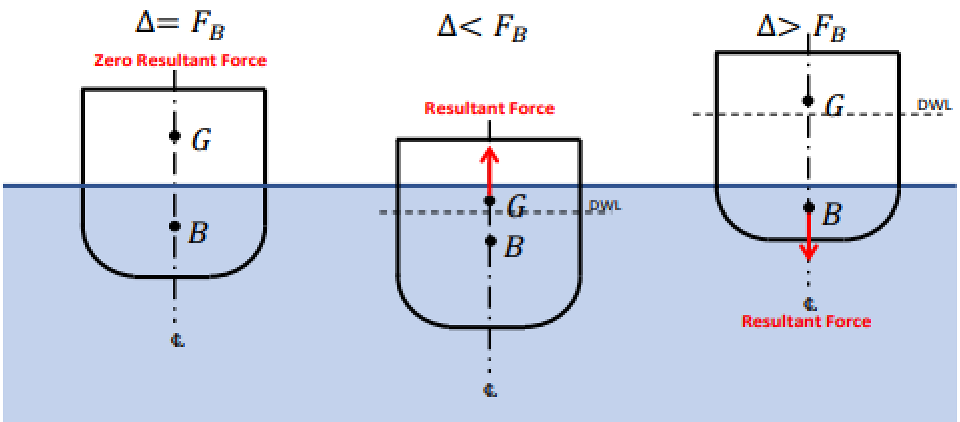
Pitch is the rotational motion about the sway axis. When pitching, the bow and stem are moving vertically in opposite directions (i.e. when the bow is moving up and the stem is moving down). Pitch is positive when the bow is pointing upwards in relation to a level ship.

Yaw is the rotational motion about the heave axis. It describes the turning motion of the ship.

When the bow moves in the starboard direction, we assume that the yaw angle is positive.

Amongst the above mentioned 6 - DoF the most significant ones are those that have a restoring force associated with them. For example, a wave push to the vessel's side (known as the sway motion effect) may be inconvenient in terms of navigation. However, the effect is limited in time as there are no restoring forces. On the other hand, if a ship is pushed over so that her starboard deck drops while waves pass over, returning to her original upright position is critical in terms of avoiding capsize. Figure below illustrates an example of restoring forces emerging from heave movements. In linear seakeeping we can assume that heave is a rigid body response proportional to the distance displaced.

This is because of the disparity between displacement and buoyancy forces that may be considered linear for different waterlines. Of course, ships that have a large water plane area for their displacement will experience much greater heave restoring forces than ships with small water plane areas. So "beamy" ships such as tugs and fishing vessels will suffer short period heave oscillations and high heave accelerations. Conversely, ships with small water plane areas will have much longer heave periods and experience lower heave accelerations. In general, as acceleration reduces, comfort is reassured. This concept is taken to extremes in the case of offshore floating platforms that have very small water plane area compared to their displacement.



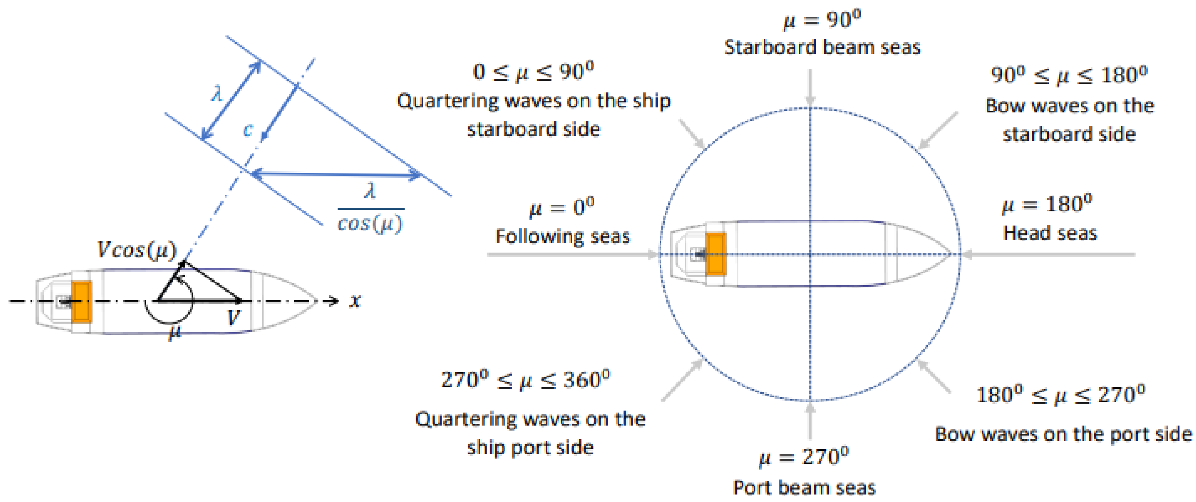
Simplistic idealization of the heave restoring force ($F_B = Buoyancy$; $\Delta = Displacement$)

When ship dynamics are accounted for, the encounter frequency (ω_e) with the waves is used instead of the absolute wave frequency (ω). This is because a ship that is moving relative to the waves, will meet successive peaks and troughs in short or long-time intervals. Her dynamic behavior depends on whether she is advancing into the waves or travelling in their direction. If we assume that the waves and the ship are on a straight course, the frequency with which the ship will encounter a wave crest depends on the wavelength (λ)(defined as the distance between the wave crests, the speed (or celerity) of the waves (c), the speed of the ship (U) and the relative angle between the ship heading with the wave heading (μ), see Figure below. This is the reason why the encounter period is defined as the distance traveled (λ) divided by the speed the ship encounters the waves ($c - U\cos(\mu)$). Therefore, the encounter frequency is defined as

$$\omega_e = \frac{2\pi}{T_E} = \frac{2\pi}{\lambda}(c - Ucos(\mu)) = k\big(c - Ucos(\mu)\big) = \omega - kUcos\mu \tag{5.1}$$

In deep waters the wave number $k = \frac{\omega^2}{g}$ leading to

$$\omega_e = \omega - \frac{\omega^2}{g}Ucos(\mu) \tag{5.2}$$



Ship seakeeping encounters idealizations

To describe the position and orientation of a ship, different coordinate systems may be used (see Figure below). The earth fixed inertial coordinate system {n} is used to define the position of the vessel on the earth, the direction of wind, waves and current. It is determined by a tangent plane attached at a point of interest (O_n). The positive unit vector (n_1) points towards the true North, (n_2) points towards the East, and (n_3) points towards the interior of the earth. When using such system, the inertial assumption is considered reasonable because the velocity of marine vehicles is relatively small and thrust forces due to the rotation of the Earth may be considered negligible relatively to the hydrodynamic forces. The body-fixed coordinate system (O_b) is fixed to the hull and is used to express velocity and acceleration measurements taken onboard or for the idealization of performance motion indices. The positive unit vector (b_1) points towards the bow, (b_2) points towards starboard and (b_3) points downwards. For ships, the axes of this frame are often chosen to coincide with the principal axes of inertia. The seakeeping coordinate system (S) (located at the center of gravity of the vessel moves at the average speed of the vessel and follows her path. It is used to define the wave elevation at the vessel's average location and to compute the hydrodynamic forces using software. This system is fixed to the vessel's equilibrium state, which is defined by the average speed and heading. The positive unit vector (S_1) points forward and is aligned with the average forward speed. The positive unit vector (S_2) points towards starboard, and (S_3) points downwards.

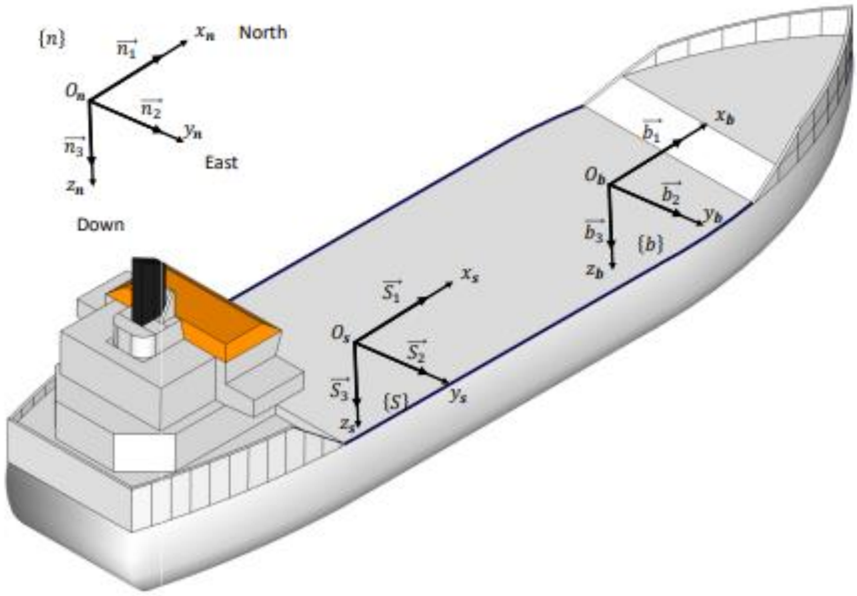
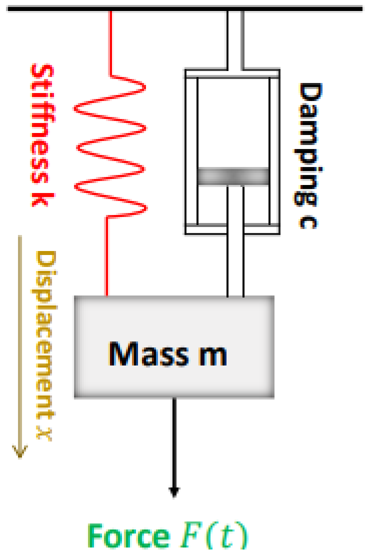


Illustration of the three reference frames

The dynamics of the rigid ship

The fundamental principles discussed in this section are discussed in various basic textbooks dealing with structural dynamics and stochasticity (Brouwers, 2006; Newland, 2012). However, for practical reasons the discussion is presented in an analogous format to principles of naval architecture dynamics and seakeeping in rough weather, see more in (Lloyd, 1989). The seakeeping behavior of a ship is similar to the classic oscillatory response of a damped spring-mass system. If we consider the general form of the typical single degree of freedom (1-DOF) system of such kind with force excitation varying in time while the mass is displaced in either direction, the spring will be compressed or placed in tension as shown in Figure below). This will generate a “restoring force” that attempts to return the ship to her original location. Provided that the spring remains within its linear operating region, the size of the force will be proportional to the amount of displacement. However, because of inertia effects, the mass will overshoot from its original point of reference; hence the spring oscillations shall generate another linear restoring force in the opposite direction that enacts to restore the mass to its central position. This dynamic behavior will be repeated until the effects of the damper dissipate the energy stored by the system oscillations.



Typical spring-mass system with damper

For such system idealization Newton's 2nd law of motion applies as follows;

$$\sum \vec{F} = m\ddot{x} \tag{5.3}$$

where $\sum \vec{F}$ is the total force, m is the mass of the body and $\ddot{x}$ is the acceleration. Decomposition of Eq (5-2) leads to

$$\begin{gathered} -kx - c\dot{x} + F = m\ddot{x} \Rightarrow \\ \Rightarrow m\ddot{x} + c\dot{x} + kx = F(t) \end{gathered} \tag{5.4}$$

where c is the damping coefficient and k is the stiffness. From a physical viewpoint what is presented in Equation (5-3) is similar to the case of a ship floating on waves as an 1-DOF system. The stiffness term is mainly attributed to buoyancy force. To realize the significance of this term just imagine the ideal case for which a ship undergoes pure heave motion. If you push the ship downwards in the water, based on "Archimedes Principle", there will be an extra buoyant force acting upwards in excess of the ship's displacement. If you then release the downward force on the ship, she will move up. Likewise, lifting a ship out of the water will result in lower buoyancy force than the ship's displacement. So, when released the ship will move down.

Free undamped vibration of 1- DOF system

If we assume the ship is a conservative system (i.e., no energy losses occur), Equation (5.4) becomes

$$m\ddot{x} + kx = 0 \tag{5.5}$$

Rigid body dynamic response can be extracted by assuming a sinusoidal solution $x = e^{\lambda t}$ leading to

$$\lambda^2 m + k = 0,\ \lambda = \pm\sqrt{\frac{k}{m}} = \pm j\omega_n \tag{5.6}$$

where $\omega_n = \sqrt{k/m}$ represents that natural frequency of the oscillation. The response may then be defined as

$$x = A_1 e^{j\omega_n t} + B_1 e^{-j\omega_n t} =$$
$$= A\,sin(\,\omega_n t) + B\,cos(\,\omega_n t) = \tag{5.7}$$
$$= X\ sin(\,\omega_n t + \phi)$$

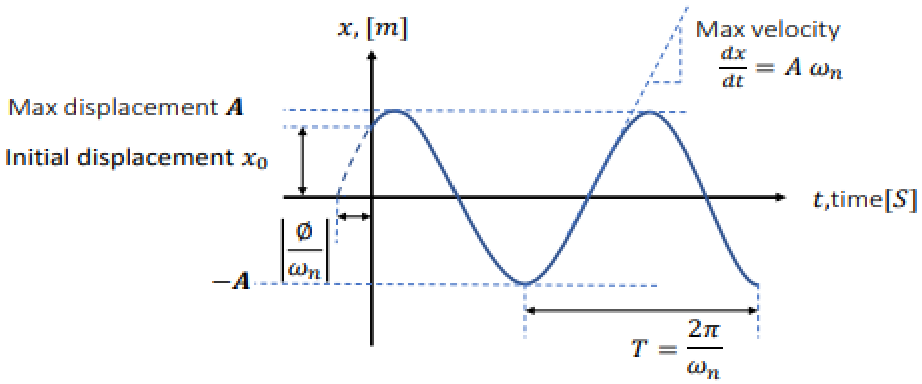
In Eq (5-6) the amplitude $X = \sqrt{A^2 + B^2}$ and the phase $\phi = tan^{-1}\,(B/A)$. If at the start of the oscillation (i.e., at t = 0) the ship displacement is x_0 the velocity becomes $\dot{x}(t = 0) = v_0$, leading to

$$X = \frac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n} \tag{5.8}$$

$$\phi = tan^{-1}(\frac{\omega_n x_0}{v_0}) \tag{5.9}$$

Thus, the final solution of the system displacement, Figure 5. 7, velocity and acceleration become

$$\begin{aligned} x(t) &= X\sin(\omega_n t + \phi) = \frac{1}{\omega_n}\sqrt{\omega_n^2 x_0^2 + v_0^2}\,\sin(\omega_n t + \phi) \\ \dot{x}(t) &= X\omega_n \cos(\omega_n t + \phi) = \sqrt{\omega_n^2 x_0^2 + v_0^2}\,\cos(\omega_n t + \phi) \\ \ddot{x}(t) &= -X\omega_n^2 \sin(\omega_n t + \phi) = -\omega_n\sqrt{\omega_n^2 x_0^2 + v_0^2}\,\sin(\omega_n t + \phi) \end{aligned} \tag{5.10}$$



Free undamped vibration response

Free damped vibration of single DOF system

In reality, the ship will not behave as a conservative system; i.e. the amplitude of oscillation will reduce with time due to damping effects. Even a low level of damping will allow for several oscillations before she comes to rest. Thus, if we may still assume free oscillations and accordingly Equation (5.7) takes the form

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{5.11}$$

Assuming sinusoidal solution $x = e^{\lambda t}$ the equation becomes

$$m\lambda^2 + c\lambda + k = 0, \quad \lambda_{1,2} = \frac{-c + \sqrt{c^2 - 4mk}}{2m} \tag{5.12}$$

There are three different types of motions associated to the above namely

- if $\lambda_{1,2} \in \Re$ then the determinant $c^2 - 4mk > 0$ and the system is considered **overdamped** ; i.e. the response is very slow and looks like an exponential decay signal.
- if $\lambda_{1,2} \in \Im\mathfrak{m}$ the determinant $c^2 - 4mk < 0$ and the system is **underdamped**; i.e. the response is very fast and looks like a rapidly decaying oscillation where the amplitudes of oscillation look smaller and smaller until equilibrium is reached.
- if $\lambda_1 = \lambda_2 \in \Re$ and $c^2 - 4mk = 0$ then the damping factor becomes critical, $c_{cr} = \sqrt{4mk} = 2m\omega_n$. In this case the system is **critically damped**; i.e. the system is allowed to overshoot and then come back to equilibrium state (i.e. at rest) relatively fast and without any oscillations.

A dimensionless system parameter that describes how rapidly the oscillations decay is the damping ratio (ζ) defined as

$$\zeta = \frac{c}{c_{cr}} = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{km}} \rightarrow c = 2m\omega_n\zeta \tag{5.13}$$

where *c* is the damping coefficient, c_{cr} is the critical damping coefficient. If we use this dimensionless notation the roots of Equation (5.12) can be expressed as

$$\lambda_{1,2} = \omega_n[-\zeta \pm \sqrt{\zeta^2 - 1}] \tag{5.14}$$

Accordingly, the three types of motions can be defined as (1) $\zeta > 1$ for the overdamped case ; (2) $0 < \zeta < 1$ for the underdamped case and (3) $\zeta = 1$ for the critically damped case. The linear sinusoidal response of the system in terms of the roots expressed in Equation (5.14) is defined as

$$x(t) = a_1 e^{\lambda_1 t} + a_2 e^{\lambda_2 t} \tag{5.15}$$

Therefore, λ_1 and λ_2 are part of the solution, for an underdamped case this leads to:

$$\lambda_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -\zeta\omega_n \pm \omega_d j \tag{5.16}$$

where

$$\omega_d = \sqrt{1 - \zeta^2} \tag{5.17}$$

If we follow similar process to the one demonstrated in Section 3.2, we can obtain the two unknowns A and ϕ in equation (5.18). Hence, the response becomes

$$x(t) = Ae^{-\zeta\omega_n t} sin(\omega_d t + \phi)$$

for

$$x(t) = \sqrt{\frac{(v_0 + x_0\zeta\omega_n)^2 + (x_0\omega_d)^2}{\omega_d^2}} e^{-\zeta\omega_n t} \tag{5.18}$$

$$\text{and } sin\left(\omega_d t + tan^{-1}\left(\frac{x_0\omega_d}{v_0 + x_0\zeta\omega_n}\right)\right)$$

The response for overdamped / critically damped cases are given by Eqs. (5.19) and (5.20) respectively

$$x(t) = a_3 e^{(-\zeta\omega_n + \omega_d)t} + a_4 e^{(-\zeta\omega_n - \omega_d)t} \tag{5.19}$$

$$x(t) = [x_0 + (v_0 + \omega_n x_0)t]e^{-\omega_n t} \tag{5.20}$$

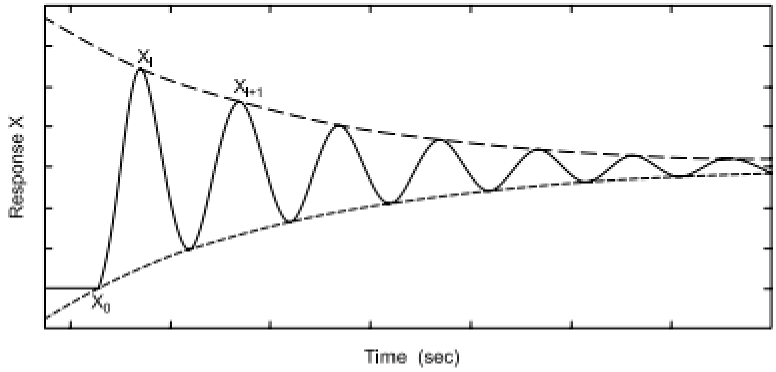
Noted that the response of the overdamped solution is not oscillatory, which is considered a preferable case however it is difficult to achieve. Underdamped response is also non-oscillatory, but it provides the fastest solution that return to zero after time. A practical way to assess damping that is broadly applicable in ship dynamics is the damping decay Figure below. This can be mathematically expressed using the log decrement ∇ that is the natural logarithm of the ratio of two successive amplitudes. The natural logarithm of the ratio of the first two successive amplitudes X_1 and X_2 defined based on the underdamped solution as follows

$$\delta = ln\frac{X_1}{X_2} = ln\frac{Ae^{-\zeta\omega_n t_1}}{Ae^{-\zeta\omega_n (t_1+T_d)}} = ln\, e^{\zeta\omega_n T_d} = \zeta\omega_n T_d \tag{5.21}$$

$$T_d = \frac{2\pi}{\omega_d} \text{ and therefore } \delta = \frac{2\pi\zeta\omega_n}{\omega_d} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \tag{5.22}$$

Since the damping ratio is very small in that case, the log decrement can be approximated by

$$\delta = 2\pi\zeta \tag{5.23}$$



Roll angle decay response (left) of a tanker ship model

Forced vibration of 1- DOF system

In practice ships never operate in conditions that there is no heaving, rolling or pitching. Therefore, to suitably idealize ship oscillations in time, it is necessary to account for the energy from waves. This energy is required to overcome the energy dissipated because of damping. In practical terms, fluid forces from the wave environment representing “the injected energy” should be accounted for when evaluating the ship motions that depend on the mass of the ship. To maintain ship oscillations, a force having the same frequency as the “simple harmonic motion” of the system is required. To illustrate the above principle let us consider adding a harmonic excitation to the vibration system where $F(t)$ (varies in sinusoidal manner instead of being arbitrary function in time. In this case Equation (5-7) becomes.

where F_o is the forcing amplitude and ω_e the encounter frequency representing the frequency at which the waves past the ship. The solution to this equation will be a system that experiences transient dynamic to the point that the ship’s natural buoyant or damping response to the initial displacement and then an equilibrium solution will have the same frequency as the excitation force. Implementation of the same process followed leads to the expression below.

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = f_0 \, cos(\omega t) \text{ for } f_0 = F_0/m \tag{5.25}$$

The general solution of the 2nd order differential Equation (5.25) is given when

$$\ddot{x}_g(t) + 2\zeta\omega_n\dot{x}_g(t) + \omega_n^2 x_g(t) = 0 \tag{5.26}$$

leading to,

$$\begin{gathered} x_g(t) = Ae^{-\zeta\omega_n t} \, sin(\omega_d t + \phi), \\ \omega_d = \omega_n\sqrt{1-\zeta^2} \end{gathered} \tag{5.27}$$

where the terms A and ϕ represent the amplitude and phase of the response. The particular solution of Equation (5.25) is given by solving the differential equation

$$\ddot{x}_p(t) + 2\zeta\omega_n\dot{x}_p(t) + \omega_n^2 x_p(t) = f_0 \, cos(\omega t) \tag{5.28}$$

There are two possible trial solutions namely,

$$\begin{gathered} x_p(t) = A_s \, cos(\omega t) + B_s \, sin(\omega t) \text{ or} \\ x_p(t) = X \, cos(\omega t - \theta) \end{gathered} \tag{5.29}$$

Where,

$$X^2 = A_s^2 + B_s^2, \qquad \theta = tan^{-1}(B_s/A_s) \tag{5.30}$$

If we substitute the trial solution in the equation of motion we get,

$$\begin{gathered} (-A_s\omega^2 + 2B_s\zeta\omega_n\omega + A_s\omega_n^2 - f_0) \cdot cos(\omega t) + (-B_s\omega^2 - 2A_s\zeta\omega_n\omega \\ + Bs\omega_n^2) \cdot sin(\omega t) = 0 \end{gathered} \tag{5.31}$$

For this equation to be zero at any time t, the two coefficients multiplied by $sin(\omega t)$ and $cos(\omega t)$ must be zero. Thus,

$$-A_s\omega^2 + 2B_s\zeta\omega_n\omega + A_s\omega_n^2 - f_0 = 0 \tag{5.32}$$

$$-B_s\omega^2 - 2A_s\zeta\omega_n\omega + Bs\omega_n^2 = 0 \tag{5.33}$$

Solving these two equations we can find the two unknowns

$$\begin{gathered} A_s = \frac{(\omega_n^2 - \omega^2)f_0}{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2} \\ B_s = \frac{2\zeta\omega_n\omega f_0}{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2} \end{gathered} \tag{5.34}$$

The particular solution after solving the unknowns becomes,

$$x_p(t) = A_s\, cos(\omega t) + B_s\, sin(\omega t)$$

$$x_p(t) = \frac{(\omega_n^2 - \omega^2) f_0}{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2} cos(\omega t) + \frac{2\zeta\omega_n\omega f_0}{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2} sin(\omega t) \tag{5.35}$$

or

$$x_p(t) = X\, cos(\omega t - \theta)$$

where $X = \frac{f_0}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}}$ and $\theta = tan^{-1}[\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}]$ (5.36)

$$x_p(t) = \frac{f_0}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}}\, cos\, \left(\omega t - arctan\left[\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}\right]\right)$$

Eventually, the full solution is the summation of the general solution and the particular solution

$$x(t) = x_g(t) + x_p(t) \tag{5.37}$$

$$x(t) = Ae^{-\zeta\omega_n t} sin(\omega_d t + \phi) + X\, cos(\omega t - \theta)$$

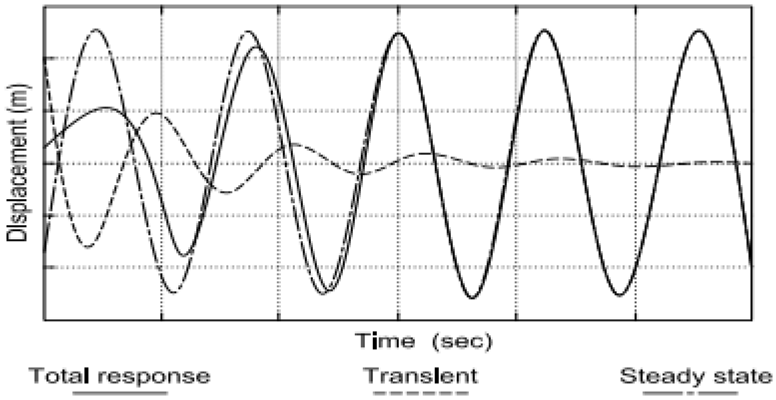
If we solve A and ϕ using the initial conditions $x(0) = x_0$, and $\dot{x}(0) = v_0$

$$\phi = arctan[\frac{\omega_d(x_0 - X\, cos\, \theta)}{v_0 + (x_0 - X\, cos\, \theta)\zeta\omega_n - \omega X\, sin\, \theta}] \tag{5.38}$$

$$A = \frac{x_0 - X\, cos\, \theta}{sin\, \phi} \tag{5.39}$$

The first term in the full solution is the transient solution, which tends to zero as the time goes to infinity, while the second term is the steady oscillatory solution (see Figure 5. 10). The second term is of more importance as it is the steady solution. In many cases, we neglect the transient solution. The full solution then reduces to the particular solution presented in Equation (5.36).

$x_p(t) = X\,cos(\,\omega t - \theta).$



The concept of dynamic magnification factor

As explained the equation of motion subject to a sinusoidal force is

$$m\ddot{x} + c\dot{x} + kx = F(t) = F_0\,cos(\,\omega_e t) \tag{5.40}$$

where F_0 is the forcing amplitude and ω_e the encounter frequency representing the frequency at which the waves past the ship. The solution to this equation expresses the dynamics of a system that experiences transient excitations to the point that the natural buoyant / damping response to the initial displacement and then an equilibrium solution will have the same frequency as the excitation force. A trial solution of the order $x = x_0 cos(\omega_e t - \phi)$, leads to $\dot{x} = -\omega_e x_0 sin(\omega_e t - \phi)$ and $\ddot{x} = -\omega_e^2 x_0 cos(\omega_e t - \phi)$. Thus, the solution to Equation (5.36) becomes

$$X_0 = \frac{F_0}{\sqrt{(k - m\omega_e^2)^2 + c^2\omega_n^2}} \tag{5.41}$$

$$\phi = tan^{-1}\left(\frac{c\omega_n}{k - m\omega_e^2}\right) \tag{5.42}$$

In practice, the natural frequency defined as $\omega_n = \sqrt{\frac{k}{m}}$ expresses the frequency at which the system of stiffness (k) and mass (m) oscillates on its own when disturbed from equilibrium. On the other hand, the frequency ratio tuning factor $(\Lambda = \frac{\omega_e}{\omega_n})$ can be defined as the aspect ratio of the excitation to the natural frequency of the system and the damping ratio defined as $\zeta = \frac{c}{2\sqrt{km}}$ expresses the lost energy encompassed in same system with damping factor c .Therefore, Equations(5.41) and (5.42) take the form

$$X_0 = \frac{F_0}{k\sqrt{(1-\Lambda^2)^2 + (2\zeta\Lambda)^2}} cos(\omega_e t - \phi) \quad (5.43)$$

$$\phi = tan^{-1}\left\{\frac{2\zeta\Lambda}{1-\Lambda^2}\right\} \quad (5.44)$$

The amplitude of the response can be represented in dimensionless form by the so-called magnification factor (Q)

$$Q = \frac{Amplitude\ of\ oscilation}{Equivalent\ static\ displacement} = \frac{X_0}{F_0/_K} = \frac{1}{\sqrt{(1-\Lambda^2)^2 + (2\zeta\Lambda)^2}} \quad (5.45)$$

Equation (5.45) may be used to measure the amplitude and phase angle of the ship response in dimensionless format. This means that for a given forcing amplitude, Fo, the response amplitude changes depending on the damping factor and the tuning factor. The damping factor (ξ)(relates to how much damping there is in the system. The larger the damping factor the smaller the magnification factor (Q). Increasing damping reduces the magnitude of the response. So, the tuning factor relates to how close the excitation frequency is to the natural frequency. When ω_e = ω_n = 1, in the absence of damping the response may go to infinity. The presence of damping reduces the response amplitude, but the max response will occur at $\Lambda = 1$. This peak is called resonance. Systems that are overdamped do not show any response amplitudes greater the static response. For over damped systems there is no magnification and no resonance.

Added Mass

Water is a dense and viscous fluid. Suitable idealization of ship motions is inextricably linked with wave induced hydrodynamics and associated floating body accelerations. Hydrodynamic actions are facilitated in the equations of motion as an addition to the mass of the object. This is known as the added mass effect. The added mass represents the weighted integration of the entire fluid mass effected by the accelerating object. Accordingly, Newton's equation of motion can be simplified to read

$$(a+m)\ddot{x} + b\dot{x} + cx = F_o \sin \omega t \quad (5.46)$$

where a stands for added mass, b is the hydrodynamic damping, c is the stiffness, F is the excitation due to external environment (assumed herby sinusoidal) and the x - variables represent the response (acceleration, $\ddot{x}$ velocity $\dot{x}$ and displacement x .(Both added mass and hydrodynamic damping coefficients are a function of the frequency of oscillation. However, the added mass depends primarily on the shape of the object, the type of motion (linear or rotational), and the direction of the motion. In this way, it differs from mass which is a quantity independent of motion. Hydrodynamic damping is related to the viscosity of the fluid (and hence the frictional drag). However, when a free surface is involved, the damping is dominated by the generation of waves. The larger the waves generated, the larger the hydrodynamic damping. Each degree of freedom that has a restoring force has an associated natural frequency. So, for a ship, there is a natural frequency in heave, roll, and pitch. These natural frequencies depend on the mass and stiffness properties of the system. For a ship with port-starboard symmetry (e.g. typical ocean going or naval vessel) the coupled motions of heave – pitch and sway – roll – yaw can be examined separately during seakeeping analysis. Of these five motions only heave pitch and roll have a restoring force or moment. The forces provided due to the effects of added mass and damping are referred to as hydrodynamic forces. They arise from pressure distribution around the oscillating hull.

Question

1. Plot the displacement, velocity and acceleration of a free undamped system assuming A=1, $\omega_n = 12\, {}^{rad}/_{s}$, $x_o = 1m$ and $v_o = 1\, {}^{m}/_{s}$. What is the relationship between displacement, velocity and acceleration?

2. Plot the transient, steady state and the full solution against time for a damped system under harmonic excitation. Assume $A = X = 1; \omega = 2\omega_n = \pi; \varphi = \frac{\pi}{6}$ and ξ=0.1

3. Plot the amplitude ratio $X\, {}^{\omega_n{}^2}/_{f_o}$ and phase θ against frequency ratio for a damped system under harmonic excitation when $\xi = 0.1, 0.25, 0.5\ and\ 0.7$. How does the damping factor affect the magnification factor?