FILE NO.	VOL NO.

NIGERIA MARITIME UNIVERSITY, OKERENKOKO, DELTA STATE, NIGERIA

FILE TITLE

FACULTY OF ENGINEERING
DEPARTMENT OF ELECTRICAL ENGINEERING

TO	PAGE	DATE	SIGN	TO	PAGE	DATE	SIGN

EEE327.2

ELECTRICAL MACHINES II

2. INDUCTION MACHINES

12-09-22

2.1. Introduction

In this section we would be analyzing the behaviour of the induction machines. Our analysis would start from the single phase machine equivalent circuit which shows the similarity of an induction machine to a transformer. The equivalent circuits can be used to study the electro-mechanical characteristics of an induction machine and also the properties of the machine as a load.

An induction machine is one in which alternating current is applied to the stator directly and passed on to the rotor by electromagnetic induction or transformer action from the stator. When the stator is excited from a balanced polyphase source it will produce a magnetic field in the air gap rotating at synchronous speed, determined by the number of stator poles and the applied stator current frequency.

The polyphase induction motor is by far the most widely used AC motor (about 90% of mechanical power in industry is provided by 3-Ø induction motors). The reasons for their popularity are low cost, simple and rugged construction, absence of commutator, relative good power factor, sufficiently high efficiency and good speed regulation (few percent speed drop from no-load to full-load).

Induction machines are sometimes called asynchronous machines as they run at a speed which is slightly lower than the synchronous speed of the rotating field developed by the stator currents. Like other machines the induction machine is reversible. That is it can operate as both a motor and a generator. The mode of operation is determined by the speed of the rotor in relation to the rotating field

There is no simple and cheap method of controlling the speed of the induction motor as is the case with a DC shunt motor. Also the torque developed in an induction motor is not as strong as that of a DC shunt motor, hence the induction motor still finds stiff competition from the induction motor in industrial application.

2.2. Construction of Induction Machines

Compared to a DC machine or synchronous machine the induction machine has a simple construction. The essential features of a polyphase induction machine are;

1. A laminated stator core carrying a polyphase winding.
2. A laminated rotor core carrying either a cage or polyphase winding.
3. A frame to form the stator housing and carry the end covers, bearings and terminal box
4. A stiff shaft

Non-salient pole construction is used for all polyphase induction motors.

The Frame: This is the outer body of the machine. Its functions include to support the stator core and winding, to protect the inner parts of the machine and serve as a ventilation housing. Depending on the capacity of the machine the frame could be die-cast or fabricated. For small machines of up to 50kW the frame is die-cast with the stator built in. However for medium and large machines (250kW - 10,000 kW) the frames are almost always fabricated.

The Stator Core/Winding: The stator core of an induction machine is similar in construction to that of a 3-Ø synchronous machine. The function of the stator core is to carry the alternating flux produced by the current flowing in the stator winding. Movement of alternating flux through the stator core produces eddy current and hysteresis losses which are reduced by making use of laminations in the construction of the stator core. Slots are punched out on the inner periphery of the stator laminations to accommodate the stator windings, sometimes called primary windings

In a polyphase induction machine the stator winding is usually 3-Ø winding supplied from a 3-Ø supply mains Depending on the method of starting the 3-Ø can be star or delta connected. The squirrel-cage motors are usually started by star-delta starters and their stators are designed for delta connection and six terminals (two per phase) are brought out to be connected to the started. The wound rotor motors on the other hand are started by inserting resistance in the rotor circuit. Therefore the stator winding could be connected in either star or delta. Stator windings are made for a definite number of poles, depending on speed requirement. For a given supply frequency the greater the number of poles the lesser the speed and vice versa.

The ~~Rotor~~ / Rotor Windings: The rotor of an induction machine is a cylindrical laminated core, like that of a DC machine. The rotor has slots around the core which carry the rotor windings. The same sheet steel laminations are employed for the rotor core as for the stator core, though thicker laminations can be employed without excessive iron loss, because of the lower frequencies of the rotor flux. According to type of winding used two types of rotors are used in 3-Ø induction machines. These are the squirrel cage rotor and wound rotor.

i. Squirrel Cage rotor: A large amount of induction machines are fitted with squirrel cage rotors because very simple and almost indestructible construction. In the squirrel cage rotor construction the rotor conductors are made of copper, brass or aluminium bars placed parallel or almost parallel to the machine shaft (one bar per slot), close to the rotor surface. At both ends of the rotor, the rotor bars (conductors) are all short circuited by end rings made from similar material as the rotor bar. The name squirrel cage is gotten from the fact that the rotor bars and their end rings form a closed circuit which ressembles a squirrel cage. The slots on the rotor are not always parallel to the machine shaft but usually skewed in order to obtain a uniform torque, reduce the magnetic locking of the stator and rotor, and reduce the magnetic humming noise while the motor is running.

The squirrel cage rotor bars are perfectly symmetrical and have the advantage of being adaptable to any number of pole pairs. However, since the rotor bars are permanently short circuited there is no possibility of adding any external resistance to the rotor circuit.

ii. Wound rotor: As the name suggests this rotor has windings of insulated conductors similar to that of the stator and since the connection of the rotor windings to the external terminals is made through slip rings and brushes these type of machines are also called slip-ring induction machines. As in a transformer, a large number of rotor turns increases the rotor voltage and decreases the rotor current that flows through the slip rings. The magnitude of secondary or rotor voltage determines the insulation that must be provided. Also the voltage and current determine the value of external resistance to be connected across the slip rings. The stand still open circuit slip ring voltage is usually 100 to 400V for small machines and up to 1kV for large machines

The rotor in wound rotor induction machines must be arranged to produce the same number of poles as is in the stator. Also the rotor winding is always 3-ø regardless of whether the stator is wound for 2-ø or 3-ø. The rotor winding could be star or delta connected, although star connection is prefered.

Shafts and Bearings: The rotor shaft is supported by bearings housed in the end shield. Since the air-gap of an induction machine is kept very small the rotor shaft must be kept short and stiff to prevent the rotor from having any deflections during rotation. A small deflection of the shaft could result in large irregularities which could lead to unbalanced magnetic pull and the possibility of rotor and stator fouling with each other.

Ball and roller bearings are generally used in induction machines as they make accurate centering much simpler than journal bearings. Also the reduce the over-all length of the machine. For large and heavy rotors journal bearings of self aligning spherical seated type are used.

2.3. Types of Three Phase Induction Motors

3-Ø induction motors are of two types

1. Squirrel cage induction motors.
2. Wound rotor or slip ring induction motor.

While both motors are similar in operation principle and stator construction they differ in rotor construction. While the squirrel cage induction motor make use of a squirrel cage rotor the wound rotor induction motor make use of slip ring rotor. The slip ring induction motor is less common compaired to their squirrel cage counterpart due to high cost of purchase and also high maintenance cost. The slip ring or wound rotor induction motors are therefore used only when speed control and high starting torque are required.

2.4. Principle of Operation of an Induction Motor

When the stator or primary winding of a 3-Ø induction motor is connected to a 3-Ø AC supply, a rotating magnetic field, which rotates at synchronous speed, is set up. The direction of rotation of this field would depend on the phase sequence of the primary currents which depends on the order of connection of the primary terminals to the supply. The direction of rotation can therefore be reversed by interchanging the connection of any two terminals of the induction motor. The number of magnetic poles of the revolving field will be the same as the number of poles for which each phase of the primary (stator) is wound. The speed at which this primary field would revolve is called the synchronous speed of the motor and is given as;

$$\text{Synchronous speed}, N_s = \frac{120f}{P}$$

Where f is the supply frequency and P is the number of poles on the stator

As the revolving magnetic field produced by the stator current cuts across the rotor (secondary) conductors an emf is induced in the rotor conductors just as in the case of a transformer secondary winding. Since the secondary winding of an induction motor is short circuited the emf induced in it would cause a current to flow in the secondary (rotor) windings.

<u>Torque Production</u>: Figure 2.1 presents a section of an induction motors stator and rotor, with the magnetic field assumed to be rotating in the clockwise direction and with the rotor still stationary. The relative motion of the rotor with respect to the stator is anti-clockwise. By applying Flemings right hand rule the direction of induced emf and current in the rotor conductor is found to be outward. Hence the direction of rotor flux is found to be anti-clockwise as shown.

Now by applying Flemings left hand rule or by the combined effect of stator and rotor fields the rotor conductor shown would experience a force tending to move it to the right

For simplicity only one rotor conductor is shown in figure 2.1. However it is safe to assume that adjacent conductors would experience a similar force to the right since they carry current in same direction. One half cycle later, the stator field direction would have reversed, but since the rotor current also reverses the force on the stator is still in the same direction. All rotor conductors will thus have a force exerted upon them all tending to turn the rotor clockwise. If the torque developed is large enough to overcome the resisting torque of the load the rotor will accelerate in the clockwise direction, same as the rotating stator field.

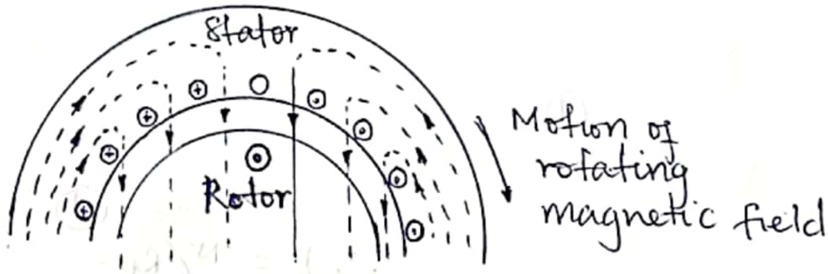


Fig. 2.1. Section of part of an induction machine.

* When the rotor is stationary and just about to start rotation, the frequency of induced emf in the rotor is equal to that of the supply to the stator, because relative motion is at synchronous speed. However as the rotor picks up speed the relative motion between the rotor and the rotating magnetic field reduces and the frequency of the induced emf in the rotor decreases. In the case where the relative motion is zero ie the rotor runs at synchronous speed there would be no induced emf, no current

in rotor conductors, no rotor field and hence no torque.. We therefore see that the induction motor cannot run at synchronous speed.*

2.5. Slip

As stated in the previous section the speed of a 3-Ø induction motor must always be less than the synchronous speed. Also as the load on the motor is increased the speed of the motor will decrease. The decrease in speed from no-load to full load is about 4-5 % for small and medium sized motors and varies between 2 - 2.25 % for large induction motors.

The difference the speed of the stator field, known as synchronous speed (N_s) and the actual speed of the rotor (N) is know as the slip (s). The slip is usually given as a fraction or percentage of synchronous speed. Therefore, the slip is given as;

$$\text{Fractional slip, } s = \frac{N_s - N}{N_s}$$

$$\text{Percentage slip, } s = \frac{N_s - N}{N_s} \times 100$$

At normal load the slip of an induction machine is between 2 and 5 per cent. However at no-load the slip is as small as 0.5 per cent.* As the motor is loaded it slows down due to the opposing effect of load torque. As the motor slows down, slip increases along with current and torque until the driving torque of the machine balances the retarding torque of the load. This determines the speed at which the machine would run on load. The induction motor is thus substantially a constant speed motor and can function just like the DC shunt motor.*

Example : Find the running speed of a motor having 4 poles and a slip of 3% if it operates on a 50Hz supply.

Solution

$$\text{Slip } s = \frac{N_s - N}{N_s}$$

$$\therefore \text{Running speed, } N = N_s - sN_s$$

$$= N_s(1 - s)$$

$$\text{But synchronous speed, } N_s = \frac{120f}{P}$$

$$= \frac{120 \times 50}{4} = 1{,}500 \text{ rpm}$$

$$\therefore \text{Running speed, } N = 1500(1 - 0.3)$$

$$= 1{,}455 \text{ rpm}$$

Example: A 6-pole induction machine is supplied by a 10-pole alternator which is driven at 600 rpm. If the ~~motor~~ runs at 970 rpm, determine the percentage slip.

Solution

$$\text{Percentage slip, } s = \frac{N_s - N}{N_s} \times 100$$

$$N = 970 \text{ rpm}$$

$$N_s = \frac{120f}{P} = \frac{120f}{6}$$

From the alternator we have that

$$\text{Spee, } N = \frac{120f}{P}$$

$$\therefore \text{frequency, } f = \frac{NP}{120}$$

$$= \frac{600 \times 10}{120}$$

$$= 50 \text{ Hz}$$

$$\therefore \text{Synchronous speed, } N_s = \frac{120 \times 50}{6}$$

$$= 1000 \text{ rpm}$$

$$\therefore \text{Percentage slip, } s = \frac{1000 - 970}{1000} \times 100$$

$$= 3\%$$

Measurement of Slip

The various methods of measuring the slip are describe in this section and they include;

1. Measurement of rotor speed: In this method the actual speed of the rotor is measure and then subtracted from the synchronous speed to obtain the slip

$$\text{Therefore slip, } s = N_s - N$$

2. Measurement of rotor frequency: The slip can be evaluated from the following relationship.

$$\text{slip, } s = \frac{\text{Rotor frequency}}{\text{Supply frequency}}$$

3. Stroboscope method: In this method a disc known as a stroboscope, with geometrical patterns on it, is rigidly attached to the shaft of the motor whose slip is to be determined. The complete apparent revolutions of the disc are counted for a given time and slip is determined by the following relation;

$$\text{Slip, } s = \frac{\text{Apparent revolution in the given time}}{\text{time in seconds}} \times \frac{\text{pairs of poles}}{\text{supply frequency}}$$

2.6. Frequency of Rotor Current or Emf

As established earlier the rotor runs in the direction of the rotating magnetic field. When the rotor is stationary the rotor conductors will be cut by the rotating flux at synchronous speed given as

$$N_s = \frac{120f}{P} \quad \therefore f = \frac{PN_s}{120}$$

Therefore the frequency of the induced rotor current or emf will be the same as the supply frequency. However, when the rotor starts rotating, the rate at which rotor conductors will be cut by the rotating magnetic field depends on the relative speed between the rotor and the rotating magnetic field, called the slip speed. The frequency of induced rotor current or emf due to the relative motion between rotor conductors and stator revolving field is given by;

$$\text{Frequency, } f' = \frac{\text{slip speed in rpm}}{120/P} = \frac{N_s - N}{120/P}$$

$$\text{But slip, } s = \frac{N_s - N}{N_s} \quad \therefore N_s - N = sN_s = \frac{s\,120f}{P}$$

Therefore, frequency $f' = \frac{s\,120f}{P} \times \frac{P}{120}$

$$f' = sf$$

Thus, we see that the frequency of the induced emf or current in the rotor conductors is given as the product of slip, s and the supply frequency, f. Hence it is also called slip frequency.

<u>Example</u>

A 2-pole, 3 phase induction motor is connected to a 400V, 50Hz supply. Calculate the actual rotor speed and the frequency of induced emf in the rotor, when the slip is 4%.

<u>Solution</u>

Supply frequency, $f = 50\text{Hz}$; rotor speed, $N = ?$
slip, $s = 4\%$; frequency of induced emf, $f' = ?$

$$\text{Slip, } s = \frac{N_s - N}{N_s} \quad \therefore \quad N = N_s(1-s)$$

but synchronous speed, $N_s = \frac{120f}{P} = \frac{120 \times 50}{2} = 3{,}000 \text{ rpm}$

$\therefore$ Rotor speed, $N = 3000(1-0.04) = 2{,}880$ rpm.

Frequency of induced emf, $f' = 0.04 \times 50$
$= 2\text{Hz}$

Example:

A 6-pole, three phase induction motor is fed from a 50Hz supply. If the frequency of the rotor emf at full load is 2 Hz, find full load slip and speed.

Solution

Supply frequency, $f = 50$ Hz

frequency of rotor emf, $f' = 2$ Hz

Full load slip, $S_F = \dfrac{N_S - N_F}{N_S}$

$\therefore$ Full load speed, $N_F = N_s(1 - S_F)$

But full load slip, $S_F = \dfrac{\text{freq. of induced emf}, f'}{\text{supply frequency}, f} = \dfrac{2}{50} = 0.04$

$\therefore$ Full load speed, $N_F = N_s(1 - 0.04)$

But synchronous speed, $N_s = \dfrac{120f}{P} = \dfrac{120 \times 50}{6} = 1000$ rpm

$\therefore$ Full load speed, $N_F = 1000(1 - 0.04)$

$= 960$ rpm.

2.7. Speed of the Rotor Field or MMF

As already established the rotating magnetic field set up by the stator currents rotates at synchronous speed, N_s rpm relative to the stator surface. Also the rotor currents having a frequency of $f' = sf$ would set up the rotor magnetic field which would rotate at a speed of sN_s rpm relative to the rotor surface, in same direction as the rotor. However the rotor itself runs at a speed of N rpm relative to the stator.

Therefore we find that the speed of rotor magnetic field with respect to stator surface is equal to the sum of actual rotor speed N and the rotor field speed sN_s with respect to the rotor surface. Hence the speed of rotor field or mmf relative to stator surface is given as;

$$N + sN_s = N_s(1-s) + sN_s = N_s \text{ rpm}$$

Thus we see that the rotor magnetic field rotates at the same speed and in same direction as the stator field. Since the stator and rotor fields rotate at same synchronous speed they are stationary relative to each other. These two synchronously rotating field superimpose on each other to produce the rotating field which corresponds to the magnetizing current of the stator winding.

<u>Example</u>

A 3-Ø induction motor runs at almost 1000 rpm on no load and 950 rpm on full load when powered from a 50Hz 3-Ø line. Find;

i. How many poles are on the motor,
ii. The percentage slip on full load,
iii. The frequency of rotor induced voltage,
iv. The speed of the rotor field relative to rotor surface,
v. The speed of the rotor relative to the stator,
vi. The speed of the rotor field relative to the stator field,
vii. The rotor frequency at a slip of 10%.

<u>Solution</u>

Supply frequency, $f = 50\,Hz$

Speed of motor on no load, $N_o = 1000$ rpm

Speed of motor on full load, $N_F = 950$ rpm

i. Synchronous speed, $N_s = \frac{120f}{P}$

$\therefore$ No. of poles, $P = \frac{120f}{N_s}$

Since an induction motor on no load runs at approximately synchronous speed we have that

Synchronous speed, $N_s \simeq$ no load speed $= 1000$ rpm

$\therefore$ No. of poles, $P = \frac{120 \times 50}{1000} = 6$ poles

ii. Percentage slip on full load, $S_F = \frac{N_s - N_F}{N_s} \times 100$

$$= \frac{(1000 - 950)}{1000} \times 100$$

$$= 5\%$$

iii. The frequency of rotor induced voltage, $f' = sf$

$$= 0.05 \times 50$$

$$= 2.5 \text{ Hz}$$

iv. The speed of the rotor field relative to rotor surface is given as;

$$N_{rf} = \frac{120f'}{P} = \frac{120 \times 2.5}{6} = 50 \text{ rpm}$$

v. Speed of rotor relative to stator = speed of rotor on full load

$$= 950 \text{ rpm}$$

vi. Since the rotor and stator field rotate at synchronous speed the speed of rotor field relative to stator field would be zero.

vii. Rotor frequency at a slip of 10% $= sf$

$$= 0.1 \times 50$$
$$= 5\,Hz$$

Example

A 3-Ø, 50Hz induction motor has a full load speed of 960 rpm. Calculate the speed of the rotor field (i) with respect to the rotor surface (ii) with respect to the stator surface (iii) with respect to the stator field

Solution

i) Speed of rotor field with respect to the rotor surface is given as;

$$N_{rf} = sN_s = N_s - N_r = N_s - 960$$

but $N_s = \frac{120f}{P} = \frac{120 \times 50}{P}$

The number of even poles to give a synchronous speed closest to but higher than the actual motor speed of 960 rpm is 6 poles.

$$\therefore N_s = \frac{120 \times 50}{6} = 1000\,rpm$$

$\therefore$ speed of rotor field, $N_{rf} = 1000 - 960 = 40\,rpm$

ii) Speed of rotor field with respect to stator structure = speed of stator field with respect to stator structure $= N_s$ rpm

iii) Speed of rotor field with respect to stator field is zero since they both rotate at synchronous speed, N_s.

2.8. Rotor EMF, Rotor Current and Power Factor

At the instant, just before the rotor begins to rotate, when it is still at standstill the motor is equivalent to a 3-phase transformer with secondary winding short circuited. Therefore induced emf in the rotor per phase is given by;

$$E_2 = E_1 \times \frac{N_2}{N_1}$$

Where,

E_1 = supply voltage per phase of stator (primary) winding
N_1 = number of turns per phase on the stator
N_2 = number of turns per phase on the rotor

When the rotor begins to rotate the relative speed of the rotor with respect to the stator field drops thus the induced emf in the rotor drops in direct proportion to the relative speed or slip s. Hence for a slip s the induced emf in the rotor is s times the induced emf in the rotor at stand still.

Therefore,

$$E_2' = sE_2$$

Where

s = slip
E_2' = induced emf in the rotor for the given slip s
E_2 = induced emf in rotor at standstill.

Rotor current and power factor

Consider the rotor equivent circuit shown in figure 2.2. below. Let the resistance and inductance per phase of the rotor R_2 and L_2 respectively and induced emf per phase in the rotor at standstill be E_2.

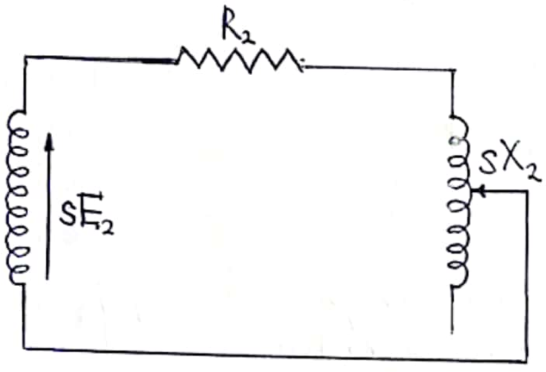


Fig. 2.2. Rotor equivalent circuit

At standstill

Induced emf/phase in rotor = E_2

Rotor winding resistance/phase = R_2

Rotor winding reactance/phase = X_2

$$X_2 = 2\pi f L_2$$

Where f is supply frequency

Rotor winding impedance/phase = Z_2

$$Z_2 = \sqrt{R_2^2 + X_2^2}$$

$\therefore$ Rotor current, $I_2 = \dfrac{E_2}{Z_2} = \dfrac{E_2}{\sqrt{R_2^2+X_2^2}}$

Power factor of rotor current, $\cos\theta_2 = \dfrac{R_2}{Z_2}$

$$= \frac{R_2}{\sqrt{R_2^2+X_2^2}}$$

At a given slip s

Induced emf/phase in rotor winding = sE_2

Rotor winding resistance/phase = R_2

Rotor winding reactance/phase = $2\pi f' L_2$

$$= 2\pi s f L_2 = s(2\pi f L_2) = sX_2$$

Rotor winding impedance/phase = $Z_2 = \sqrt{R_2^2+(sX_2)^2}$

Rotor current/phase, $I_2 = \dfrac{sE_2}{\sqrt{R_2^2+(sX_2)^2}}$

Power factor of rotor current, $\cos\theta_2 = \dfrac{R_2}{\sqrt{R_2^2+(sX_2)^2}}$

Example: In a 6-pole, 3-phase, 50 Hz motor with star connected rotor, the rotor resistance per phase is 0.3 Ω, the reactance per phase is 1.5 Ω and the emf between the slip rings on open circuit is 175 V. Calculate;
(i) Slip at a speed of 950 rpm
(ii) Rotor emf per phase
(iii) Rotor frequency and reactance at a speed of 950 rpm.

Solution
i. Slip, $s = \dfrac{N_s - N}{N_s}$

$$N = 950 \text{ rpm}; \quad N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$\therefore \text{Slip, } s = \frac{1000 - 950}{1000} = 0.05$$

ii. Rotor emf/phase, $E_2 = \dfrac{\text{line emf}}{\sqrt{3}} = \dfrac{175}{\sqrt{3}} = 101 \text{ V}$

iii. Rotor frequency at 950 rpm, $f' = sf = 0.05 \times 50$
$= 2.5 \text{ Hz}$

Rotor reactance at a speed of 950 rpm $= sX_2$
$= 0.05 \times 1.5$
$= 0.75 \, \Omega$

Example
A 1,100 V, 50 Hz delta-connected induction motor has a star-connected slip-ring rotor with a phase transformation ratio of 3.8. The rotor resistance and standstill leakage reactance 0.012 Ω and 0.25 Ω per phase respectively. Neglecting stator impedance and magnetization current (i) the rotor current at start with slip ring shorted (ii) the rotor pf at start with slip rings shorted (iii) the rotor current at 4% slip with slip rings shorted (iv) the rotor power factor at 4%

slip with slip rings shorted (v) the external rotor resistance/phase required to obtain a starting current of 100 A in the stator supply lines.

Solution

i. Rotor current, $I_2 = \frac{E_2}{Z_2}$

where, E_2 = emf induced per phase in the rotor

Z_2 = rotor impedance per phase $= R_2 + jX_2$

$$E_2 = \frac{\text{Voltage applied to stator/phase}}{\text{phase transformation ratio}} = \frac{E_1}{3.8} = \frac{1,100}{3.8}$$

$\therefore E_2 = 289.5\ V$

$$Z_2 = R_2 + jX_2 = 0.012 + j0.25$$

$$|Z_2| = \sqrt{R_2^2 + X_2^2} = \sqrt{(0.012)^2 + (0.25)^2} = 0.25\ \Omega$$

$$\therefore \text{Rotor current, } I_2 = \frac{289.5}{0.25} = 1,156.6\ A$$

ii. Rotor power factor, $\cos\theta_2 = \frac{R_2}{Z_2} = \frac{0.012}{0.25} = 0.048$

iii. At a slip of 4%

$$\text{Rotor current, } I_2 = \frac{sE_2}{Z_2} = \frac{0.04 \times 289.5}{Z_2}$$

$$Z_2 = \sqrt{R_2^2 + (sX_2)^2} = \sqrt{(0.012)^2 + (0.04 \times 0.25)^2}$$

$$= 0.0156\ \Omega$$

$$\therefore I_2 = \frac{0.04 \times 289.5}{0.0156} = 742.3\ A$$

iv. Rotor power factor at 4% slip, $\cos\theta_2 = \frac{R_2}{Z_2} = \frac{0.012}{0.0156}$

$= 0.768$

v. Rotor resistance corresponding to a stator line current of 100 A is evaluated thus;

Rotor resistance, $R_2 = \sqrt{Z_2^2 - X_2^2}$

$$X_2^2 = (0.25)^2$$

$$Z_2 = \frac{E_2}{I_2} = \frac{289.5}{I_2}$$

I_2 = rotor current per phase corresponding to stator line current of 100 A

$$I_2 = \frac{100/\sqrt{3}}{1/3.8} = 219.4A$$

$$\therefore \quad Z_2 = \frac{289.5}{219.4} = 1.32\,\Omega$$

$\therefore$ Required rotor resistance/phase $= R_2 = \sqrt{Z_2^2 - X_2^2}$

$$= \sqrt{(1.32)^2 - (0.25)^2}$$

$$= 1.2956\,\Omega$$

$\therefore$ External resistance required/phase $= 1.2956 - 0.012$

$$= 1.2836\,\Omega$$

2.9. Rotor Torque

As established earlier the rotor torque is developed as a result of the interaction between rotor and stator fields. However, the magnitude of the rotor torque is proportional to;

i. The rotor current, I_2

ii. Stator flux per pole, ϕ and

iii. Power factor of the rotor circuit, $\cos\theta_2$

Therefore,

$$\text{Rotor torque, } T \propto \phi I_2 \cos\theta_2$$

Since the rotor emf per phase is proportional to stator flux, $\varnothing$ we can say that

$$\text{Rotor torque, } T \propto E_2 I_2 \cos\theta_2$$
$$\text{or } T = K E_2 I_2 \cos\theta_2$$

Torque under running conditions

As established earlier under running conditions

$$\text{Rotor current, } I_2 = \frac{sE_2}{\sqrt{R_2^2 + s^2X_2^2}}$$

$$\text{Power factor, } \cos\theta_2 = \frac{R_2}{\sqrt{R_2^2 + s^2X_2^2}}$$

Substituting for I_2 and $\cos\theta_2$ in our torque equation we have;

$$T = KE_2 \times \frac{sE_2}{\sqrt{R_2^2 + s^2X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + s^2X_2^2}}$$

$$\therefore T = \frac{ksR_2E_2^2}{R_2^2 + s^2X_2^2}$$

Since the rotor induced emf E_2 is proportional to the emf E_1 supplied to the stator we can conclude that the running torque is proportional to the square of the supply voltage.

Starting torque

At start, when the rotor is still stationary, the rotor speed is zero. Therefore slip, $s = 1$. Substituting $s = 1$ into the torque equation we have that starting torque is given as;

Starting torque, $T_{st} = \dfrac{KR_2E_2^2}{R_2^2+X_2^2} = \dfrac{KRE_2^2}{Z_2^2}$

Condition for maximum running torque

As established earlier the expression for the torque of an induction motor under running conditions is given as;

$$T = \frac{KR_2sE_2^2}{R_2^2+s^2X_2^2}$$

Assuming the supply voltage is fixed then E_2 will be constant and torque T would be maximum when

$$\frac{sR_2}{R_2^2+s^2X_2^2} = \frac{R_2}{\frac{R_2^2}{s}+sX_2^2} = \frac{R_2}{\left(\frac{R_2}{\sqrt{s}}-X_2\sqrt{s}\right)^2+2R_2X_2} \quad \text{is maximum}$$

Therefore,

$$\frac{R_2}{\sqrt{s}} - X_2\sqrt{s} = 0$$

and $\quad s = \dfrac{R_2}{X_2}$

substituting $s = \dfrac{R_2}{X_2}$ into the torque equation we hav

$$T_{max} = \frac{KR_2^2E_2^2/X_2}{R_2^2+R_2^2} = \frac{KE_2^2}{2X_2}$$

From the equation of T_{max} above we deduce as follows

i. Maximum torque is independent of rotor circuit resistance.

ii. The slip at which maximum torque occurs is controlled by the rotor resistance. Therefore by varying the rotor resistance (possible with wound rotors) maximum torque can be made to occur at any slip or motor speed.

iii. Maximum torque varies inversely as the reactance of the rotor.

iv. Maximum torque varies directly as the square of supply voltage.

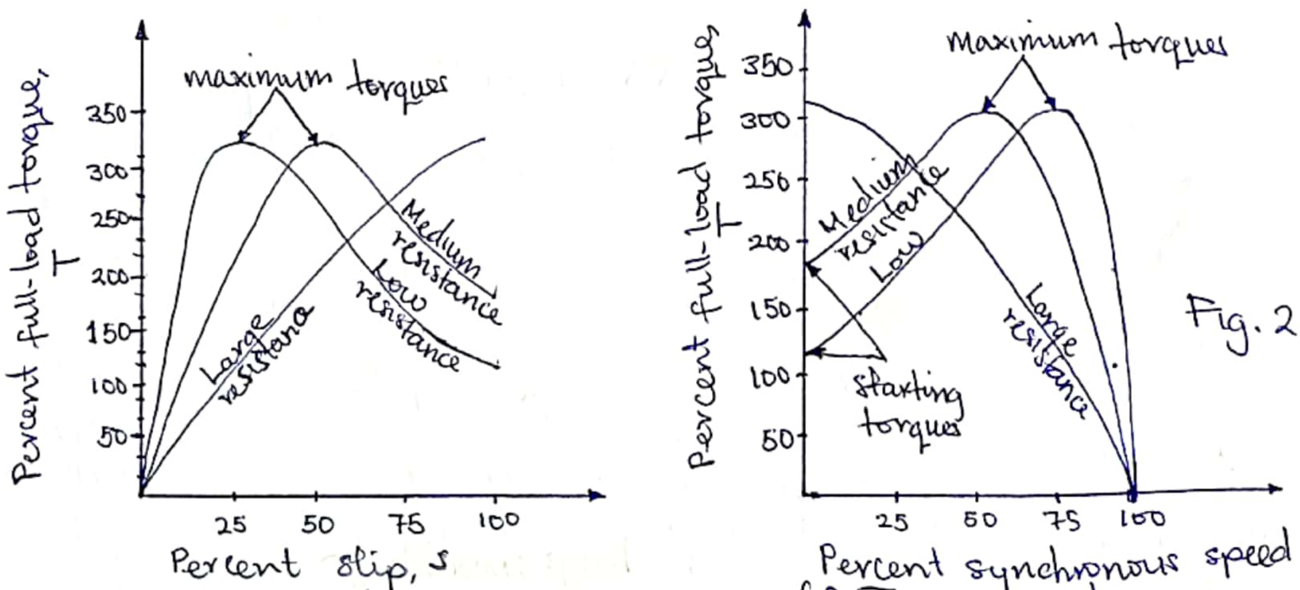
Condition for maximum starting torque

Starting torque would be maximum when

$$s = \frac{R_2}{X_2} = 1 \quad \therefore \quad R_2 = X_2$$

Since rotor resistance, R_2 is usually not more than 1 or 2 percent of leakage reactance X_2 extra resistance is required in the rotor start circuit at start and removed gradually as the rotor picks up speed. This is only possible with wound rotor motors. Hence they are used in applications where heavy loads need to be accelerated at start, such as cranes, elevators etc.

2.9. Torque-Slip and Torque-Speed Characteristics



(a) Torque-slip curve

(b) Torque-speed curve

Fig. 2.3

The torque-slip and torque-speed characteristics are produced from the torque equation developed earlier;

$$T = \frac{K_s R_2 E_2^2}{R_2^2 + s^2 X_2^2}$$

From the torque equation and as seen in figures 2.3 (a) and (b) we can deduce as follows;

(i) When speed of the rotor is synchronous ie when slip is zero, the torque is zero. Therefore the torque-slip curve starts from the origin.

(ii) When the speed is very near synchronous speed that is when the slip is still very small the term sX_2 will be negligibly small compaired with the rotor resistance R_2. If we assume that the rotor resistance is constant then torque, T, would be directly proportional to slip s and this portion of the torque slip and torque-speed curves are approximately straight lines.

iii. As the speed drops and slip increases with increasing load, torque increases reaching a maximum value at $s = \frac{R_2}{X_2}$. This maximum torque is known as breakdown or pull-out torque and the corresponding slip is called breakdown slip, s_b.

iv. With further increase in slip or drop in speed above and beyond the maximum torque values the torque begins to decrease. The result is that the motor slows down and stops. Thus the motor operates for the value of slip between zero and the value corresponding to breakdown torque.