PHY 102 (GENERAL PHYSICS)
NUCLEAR PHYSICS

Constituents of Nucleus

Nucleus: The nucleus contains proton and neutron. The proton are heavy particles which are about 1836 times more massive than electrons. Protons are positively charged. The magnitude of the charged on a proton is equal to the magnitude of the charge on an electron. Neutrons have masses nearly to that of the protons. However, neutrons do not possess any electric charge.

Nucleon: This is the collective name for protons and neutrons. The total number of nucleons in an atom is called nucleon number denoted by **A**. Nucleon number was formerly called **atomic *mass number***. while the total number of protons alone is called ***proton (atomic) number*** denoted by **Z**. Proton number was formerly called atomic number. Proton number gives a measure of the total positive electric charge in an atom. The proton number is different for atoms of different elements. The number of neutrons in the nucleus is given by (**A – Z**) because the particles in the nucleus which are not protons must be neutrons. The number of electrons in an atom is equal to the number of protons and the electronic charge is numerically equal but opposite to the charge on a proton.

Nuclide: This is a special nucleus with a specified number of protons and neutrons. Any nuclide may be represented by its chemical symbol together with its proton and nucleon number **Z** and **A** respectively. Thus, the representation is $^{A}_{Z}\mathbf{X}$ for example, $^{14}_{7}N$ is for nitrogen.

Example: How many protons, neutrons and electrons are present in $^{235}_{92}\mathrm{U}$, $^{14}_{6}C$, $^{14}_{7}N$?

Isotopes: the isotopes of an element are two or more atoms or nuclide which have the same proton number **Z** but different nucleon number **A** (or have different number of neutrons). Isotopes also have same chemical properties but different masses. Examples; $^{35}_{17}\mathbf{Cl}$ and $^{37}_{17}\mathbf{Cl}$ two isotopes of chlorine as well as $^{1}_{1}\mathbf{H}$, $^{2}_{1}\mathbf{H}$ and $^{3}_{1}\mathbf{H}$ three isotopes of hydrogen (hydrogen, deuterium and tritium)

Isobars: these are two or more nuclides of different elements with same nucleon number. example; $^{14}_{6}\mathbf{C}$ and $^{14}_{7}\mathbf{N}$

1.2 Nuclear Mass

Masses of atom are expressed in terms of atomic mass unit, u.

Mass of the proton, $m_p = 1.007277\ u\ or\ 1.67252\ \times 10^{-24}\ kg\ or\ 938.3\ MeV/c^2$ with a charge $1.602 \times 10^{-19}C$

Mass of the neutron, $m_n = 1.008665\ u\ or\ 1.67489\ \times 10^{-24}\ kg\ or\ 939.6\ MeV/c^2$ with no charge.

$i.e. 1u = 1.6605 \times 10^{-27}kg$ and $1eV = 1.6022 \times 10^{-19}J\ or$

Nuclear Radius

To a first approximation the atomic nucleus can be considered to be a sphere of radius R giving by the expression $r = r_0 A^{\frac{1}{3}}\ m.$ (where r_0 is constant with a value $1.25 \times 10^{-15}\ m\ or\ 1.25\ fm,$ A = N + Z so as Z increases it adds to a little contribution to r).

The volume, V of a nucleus is proportional to r^3 and so V almost equal to A. The A/V = constant for all nuclei which suggests uniform density matter and implies that the nuclei is similar to little liquid drop (liquid drop model).

Example: Find the radius of an iron-56 nucleus.

Solution

The radius of a nucleus is given by $r = r_0 A^{1/3}$. Substituting the values for R_0 and A yields

$R = (1.2fm)(56)^{1/3} = (1.2fm)(3.83) = 4.6\ fm$

Example: Find the approximate density of $^{56}_{26}Fe$ in kg/m^3, approximating the mass of $^{56}_{26}Fe$ to be 56 u.

Solution

Density is defined to be $\rho = \frac{m}{V}$, which for a sphere of radius r is

$\Rightarrow \rho = \frac{m}{V} = \frac{m}{(4/3)\pi r^3}$

Substituting known values gives

$\Rightarrow \rho = \frac{56\ u}{(1.33)(3.14)(4.6\ fm)^3} = 0.138\ u/fm^3$

Converting to units of kg/m^3, we find

$\Rightarrow \rho = (0.138\ u/fm^3)(1.66 \times 10^{-27}kg/m^3) = 2.3 \times 10^{17}/m^3$

Mass Defect

The mass of all nuclei are slightly less than the sum of the masses of the neutrons and protons contained in them. This mass difference is called "**Mass Defect**" given as;

$\boldsymbol{\Delta m = Zm_p + (A-Z)m_n - m(A,Z)}$

Where M_A is the mass of the nucleus.

Example: Find the mass defect of a copper-63 nucleus if the actual mass of a copper-63 nucleus is $62.91367\ u$.

Solution

Fist, find the composition of the copper-63 nucleus and determine the combined mass of its components.

Copper has 29 protons and also has (63-29) neutrons.

Mass of the of 29 proton, $29m_p = 29 \times 1.007277\ u = 29.211033\ u$

Mass of the 34 neutron, $34m_n = 34 \times 1.008665\ u = 34.29461\ u$

$thus, Zm_p + (A-Z)m_n = 29.211033\ u + 34.29461\ u = 63.50590\ u$

∴ mass defect, $\Delta m = 63.50590\ u - 62.91367\ u\ = 0.59223\ amu$

Binding Energy

When Δm is expressed in energy units, it is equal to the energy which is necessary to break the nucleus into its constituent protons and neutrons, this is known as the **binding energy** of the system, since it represents the energy with which the nucleons is held together. On the other hand, when a nucleus is produced from A nucleons, ΔM is equal to the energy released in the process.

In order to determine the energy, there is need to multiply the mass by the square of the speed of light in air $(2.9979 \times 10^8 m/s)$. i. e. (**Binding Energy** $E = mc^2$)

$\Longrightarrow (1.6605 \times 10^{-27}kg)(2.9979 \times 10^8 m/s)^2 = 14.924 \times 10^{-11} kg.m^2/s^2$

$\Longrightarrow 14.924 \times 10^{-11} J$, but $1\ kg.m^2/s^2 = 1\ joule.$

The most convenient unit of energy to use is the electron volt. To express the energy in electron volts, then there is need to conversion from joules to electron volts.

$\Longrightarrow E = (14.923 \times 10^{-11})(1\ eV/1.6022 \times 10^{-19} J) = 9.3149 \times 10^8\ eV\ = 931.49\ MeV$

Hence, 1 u of nuclear mass is equivalent to $\mathbf{931.49\ MeV}$

Example: If the mass of tritium is 3.016049 u, the mass of a proton is 1.007825 u, the mass of a neutron is 1.008665 u. Find the nuclear mass defect and binding energy.

Solution

$mass\ of\ 1\ proton = 1.007825\ u$

$mass\ of\ 2\ neutrons = 2.017330\ u$

$mass\ of\ tritium = 3.016049\ u$

Thus, the mass defect is

$\Delta M = (1.007825 + 2.017330) - (3.016049)$

$\Longrightarrow \Delta M = 3.025155 - 3.016049 = 0.009106\ u$

Since Binding energy of $1\ u = 931.49\ MeV$, The binding energy, E is

$E = (-0.009106\ u)(931.49\ MeV/u) = 8.482\ MeV$

Virtual Energy

The binding energy of the least bound nucleon in a nucleus is called '**Virtual energy'**. This is the minimum energy which must be added to the nucleus in order to remove a nucleon, and it is entirely analogous to the first ionization energy of an atom. Nuclear excited states above the virtual energy are called '**Virtual State'** or Virtual Level. It is possible for nuclei in virtual states to decay by nucleon emission, whereas this is not possible for nuclei in bound states.

Radioactivity

Radioactivity can be defined as the spontaneous disintegration of unstable atomic nuclei, with the emission of $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ radiations with the released of energy. Radioactivity is governed by one fundamental law that a nucleus decays by a constant factor independent of time. This constant is called 'Decay constant, λ'.

Examples of Natural radioactivity

${}^{222}_{86}Rn \longrightarrow {}^{222}_{88}Ra\ + 2{}^{\ 0}_{-1}e + energy$

${}^{234}_{90}Th \longrightarrow {}^{234}_{91}Pa + {}^{\ 0}_{-1}e + energy$

${}^{238}_{92}U \longrightarrow {}^{230}_{90}Th + 2({}^{4}_{2}He)\ + 2{}^{\ 0}_{-1}e\ + energy$

Examples of Artificial Radioactivity
$^{6}_{3}Li + ^{1}_{0}n \longrightarrow ^{3}_{1}H + ^{4}_{2}He + energy$
$^{9}_{4}Be + ^{4}_{2}He \longrightarrow ^{12}_{6}C + ^{1}_{0}n + energy$
$^{24}_{12}Mg + ^{1}_{0}n \longrightarrow ^{24}_{11}Na \rightarrow ^{1}_{1}H + energy$

Alpha Decay
Let's first consider alpha decay, $^{A}_{Z}X \longrightarrow ^{A-4}_{Z-2}X' + ^{4}_{2}He$.
Alpha particles are helium nuclei, $\mathbf{^{4}_{2}H}$. they contain two protons and two neutrons. This means that they have charge +2e, and nucleon number 4. (doubly ionized helium ions which are moving with great speed). Unstable nuclei simply cannot overcome the proton repulsion and an α particle ultimately succeeds in escaping the nucleus indefinitely.
Example: How many $\alpha - particle$ are emitted in the radioactive decay of $^{234}_{88}Ra$ to $^{214}_{82}Ra$

Solution
$^{234}_{88}Ra \longrightarrow ^{214}_{82}Ra + ^{x}_{y}He$
$234 = 214 + x; x = 20$
$\therefore$ number of $\alpha - particle$ emitted is $\frac{20}{4} = 5$

Beta particles
There are three related nuclear decay processes which are all mediated by the **weak nuclear interaction**.

1. $^{A}_{Z}X \rightarrow {}_{Z+1}{}^{A}X' + {}_{-1}^{0}e + \bar{v}$ (called β^{-} decay)
   e. g , $n \rightarrow p + \bar{e} + \bar{v}$ the simplest beta decay

2. $^{A}_{Z}X \rightarrow {}_{Z-1}{}^{A}X' + {}_{+1}^{0}e + \nu_e$, (called β^{+} decay or **positron emission**).
   e. g. $p \longrightarrow n + e^{+} + \nu_e$

3. $^{A}_{Z}X + {}_{-1}^{0}e \rightarrow {}_{Z-1}{}^{A}X' + \nu_e$, (called **electron capture**).
   e. g. $p + e^{-} \longrightarrow n + \nu_e$

Note that all of the three transition $-\boldsymbol{\beta}^{-}$, $\boldsymbol{\beta}^{+}$ and $\boldsymbol{e}^{-}$ capture –leaves the mass number unchanged. The parents and daughters' nuclei are **isobars**. Beta decay involves the weak nuclear force. This is one of the four fundamental forces in the world, and its small strength means that the decay happens much more slowly than most other reactions.

Example: Calculate the number a and b from the equation $^{23}_{11}Na + Proton = ^{a}_{b}Z + \alpha - particle$

Solution
$^{23}_{11}Na + Proton = ^{a}_{b}Z + \alpha - particle$
$\Rightarrow ^{23}_{11}Na + ^{1}_{1}H = ^{a}_{b}Z + ^{4}_{2}He$
$from\ equation\ above$
$\Longrightarrow 23 + 1 = a + 4$
$\Rightarrow 24 - 4 = a,$
$\Longrightarrow a = 20$
$also,$
$\Longrightarrow 11 + 1 = b + 2$
$\Longrightarrow b = 12 - 2,$
$\Longrightarrow b = 10$
$\therefore (20, 10)$

Gamma ray
Gamma decays are electromagnetic transitions, and found when excited nuclear states relax to their ground states. Just as for an atom, a nucleus can only exist in certain definite energy states. When a nucleus goes from one state to the other, it can emit a photon (γ –ray).
$^{A}_{Z}X \longrightarrow ^{A}_{Z}X' + ^{0}_{0}\gamma$
e.g. $\boldsymbol{e}^{-} \longrightarrow \boldsymbol{e}^{-} + \gamma$
$e^{+} + e^{-} \longrightarrow \gamma$

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Properties of $\alpha - particles, \beta - particles$ and $\gamma - rays$

	$Property$	$\alpha - particles$	$\beta - particles$	$\gamma - rays$
1.	Nature	Helium nucleus 2n +2p	Electron	High-energy Electromagnetic radiation
2.	Deflection	Slightly deflected by magnetic field	Strongly deflected by magnetic field	Not deflected by magnetic field
3.	Charge	+2	-1	Zero
4.	Absorption	Paper	~2 mm Al	~5 cm lead
5.	Ionization	High	Medium	Low
6.	Penetrating Power	Low	Medium	High
7.	Energy	Order of 5.5 MeV	Order of 0.3 MeV	High energy
8.	Mass and momentum	Great deal of mass and momentum	Fairly deal of mass and momentum	Massless
9.	Velocity	Order of 10^7 ms^{-1}	Up to 10^8 ms^{-1} but variable	3 X 10^8 ms^{-1}
8.	Spread of Velocity	One or few definite velocities	Widespread	All the same
9.	Examples of suitable source	${}^{241}_{95}Americium$	${}^{90}_{39}Strontium$	${}^{60}_{27}Cobait$

Decay Law

The rate of disintegration of radioactive sample at any instant is directly proportional to the number of undisintegrated nuclei present in the sample at that instant i.e. $\boldsymbol{\frac{dN}{dt} \propto \lambda N}$,

The number of particles remaining at time t is governed by the decay law:

$\boldsymbol{\frac{dN}{dt} = -\lambda N}$,

Where the constant λ is the decay per nucleus and the negative sign indicate reduction in same of the atomic nuclei. The equation is easily integrated to give

$\boldsymbol{N(t) = N_0 e^{-\lambda t}}$

Where N_0 is the original or initial number of the nuclei at time $t = 0(t_0)$, N(t) is the number of the atomic nuclei at time $\boldsymbol{t}$. The SI unit for the measurement of activity is $'becqurel'$ and is defined as,

$1\ becquerel = 1\ decay\ per\ second\ and\ 1\ Curie = 3.7 \times 10^{10} Bq$

Example: A radioactive source has a half-life of 40days, compare the initial rate of disintegration of the atoms with the rate after 3days.

<u>Solution</u>

$T_{1/2}$ = 4days t = 3 days

But $T_{1/2} = \frac{0.693}{\lambda}$, $\lambda = \frac{0.693}{4} \Longrightarrow \lambda = 0.1733$

Using $N = N_0 e^{-\lambda t}$, $\Longrightarrow N = N_0 e^{-0.1733 \times 3} = N_0 e^{-0.52}$

Dividing both sides by $N_0 e^{-0.52}$

$\Longrightarrow \frac{N}{N_0 e^{-0.52}} = \frac{N_0 e^{-0.52}}{N_0 e^{-0.52}} \Rightarrow \frac{N}{N_0} = e^{-0.52}$

$\Longrightarrow \frac{N_0}{N} = e^{0.52} = 1.6811$

Half Life

The time during which the activity of the radioactive sample falls by a factor 2 is known as the half-life, denoted as $T_{1/2}$. i. e.

$T_{1/2} = \frac{\text{In}2}{\lambda} = \frac{\text{In}2}{0.693}$. where n($T_{1/2}$) = $\frac{n_0}{2}$

Example: A radius sources has a decay constant 1.36 x 10^{-11}s^{-1}. Calculate the half-life of the radius.

<u>Solution</u>

λ = 1.36 x 10^{-11}s^{-1}, $T_{1/2}$ = ?, Using $T_{1/2} = \frac{\text{In}2}{\lambda} = \frac{0.693}{1.36 \times 10^{-11}} = 5.10 \times 10^{10} s = \frac{5.10 \times 10^{10}}{60 \times 60 \times 24 \times 365} = 1615.8 years$

Example: 16gm of a radioactive element was isolated and stored. It was discovered that after 40 days 1gm of the element remained undecayed. Calculate the half-life of the element.

Solution

$N_0 = 16g$, $t = 40 days$, $N = 1g$ $T_{1/2} = ?$

$Using \quad N = N_0 e^{-\lambda t}$

$\Longrightarrow 1 = 16 e^{-\lambda \times 40}$

$\Longrightarrow \frac{1}{16} = \frac{16 \times e^{-\lambda \times 40}}{16}$

$\Longrightarrow 2^{-4} = e^{-\lambda \times 40}$

Take the In of both sides

$\Longrightarrow \ln 2^{-4} = \ln e^{-\lambda \times 40}$

$\Longrightarrow -4 \ln 2 = -\lambda \times 40$

$\Longrightarrow \lambda = \frac{4 \ln 2}{40}$

$Also, since \quad T_{1/2} = \frac{\ln 2}{\lambda}$

$\Longrightarrow T_{1/2} = \frac{\ln 2}{\frac{4 \ln 2}{40}} = 10\ days$

Probability

Now, let $P(t)$ be the probability that a nucleus decay in the time dt between t and $t + dt$. In other word, $P(t)dt$ is the probability that a nucleus survives up to a time, t and then decays in the interval t and $t + dt$. This evidently equals to the probability that the nucleus has not decayed up to the time t times the probability that it does in fact decay in the additional time dt. It follows that

$\text{P(t)} = \lambda e^{-\lambda t} dt$

Integrating above gives unity, thus,

$$\int_0^\infty \text{P(t)dt} = \lambda \int_0^\infty e^{-\lambda t} dt = 1$$

This is the total probability that a radioactive nucleus eventually decays and is equal to unity

Mean Life

The mean life of a nucleus can be determined by finding the average value of time, t over the probability distribution P(t). denoting mean life τ, τ by can be written as

$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44 T_{1/2}$

Example: Calculate the activity of a $30 - Bq$ source of $Na - 24$ after 2.5 d. What is its mean life τ

Solution: $T_{1/2} = 15\ h,\quad N_0 = 30\ Bq,\quad t = 2.5d \times 24\ h/d = 60\ h$

$but\ \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{15} = 0.0462 h^{-1}$

$from\ \ N_0 = N e^{-\lambda t}$

$\Longrightarrow N_0 = 30 e^{-(0.0462 \times 60)} = 1.88\ Bq$

$Thus, \tau = \frac{1}{0.0462} = 21.64\ h$

Uses of radioactivity

Dating once-living objects, medical applications, food preservation, tracers, industrial purposes, academic research, etc.