# *Introduction to IRT Using R (2PL)*

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Introduction

Our focus is on 2PL model only. The formula is given as:

$$P(\theta) = \frac{1}{1 + e^{-a(\theta - b)}}$$

1PL (Rasch), 3PL & 4PL have to learn on your own.

A short introduction and comparison between item analysis and item response theory analysis is presented by Arifin and Yusoff (in press).

## Preliminary readings

- "The Basics of Item Response Theory" @ http://echo.edres.org: 8080/irt/baker/final.pdf
- "Applying item response theory (IRT) modeling to question-naire development, evaluation, and refinement" @ https://www.researchgate.net/profile/Maria\_Edelen/publication/6432794\_Applying\_item\_response\_theory\_(IRT)\_modeling\_to\_questionnaire\_development\_evaluation\_and\_refinement/links/09e415092a186a0ccb000000.pdf

# Important cutoff values

#### Discrimination:

- De Ayala (2008, pg. 101): Good .8 to 2.5
- Baker (2001, pg. 35): o: None, .01 to .34: Very low, .35 to .64: Low, .65 to 1.34: Moderate, 1.35 to 1.69: High, > 1.70: Very high, + infinity: Perfect
- Hambleton (1991, pg. 15): Usual range o to 2

#### Difficulty:

- De Ayala (2008, pg. 15: Theoretical range -inf to + inf; typical range -3 to +3; < -2.0: Easy; -2.0 to 2.0: Average, > 2.0: Hard
- Hambleton (1991, pg. 13): -2 (very easy) to +2 (very difficult)
- Baker (2001, pg. 32-33): Theoretical range -4 to +4, typical range -3 to +3, usual range -2.8 to + 2.8

Practical session

Install packages

```
install.packages(c("psych", "ltm", "irtoys", "mirt", "latticeExtra"))
```

Load required libraries

```
library("psych")
library("ltm")
library("irtoys")
library("mirt")
library("latticeExtra")
```

Load data

Read data set "mtf.csv" into "data.mtf" data frame

```
data.mtf = read.csv("mtf.csv", header = TRUE) # Includes headers
head(data.mtf) # View first 6 students in the data set
##
     Q1A Q1B Q1C Q1D Q1E Q2A Q2B Q2C Q2D Q2E
## 1
              0
                          0
## 2
          0
              0
                          0
                             0
                                         1
## 3
      0 1 0
                0
                    1
                          1
                             0
                                1
                                    1
                                         0
                                 0
## 4
      1
         1 0
                 1
                      1
                          0 1
                                    1
                                         1
                  0
                      1
                                 1
                                     1
                                         0
## 5
      1
          1
                             1
## 6
          1
                  1
                      1
                                 1
                                     1
                                         1
names(data.mtf) # List down variables in the data set
   [1] "Q1A" "Q1B" "Q1C" "Q1D" "Q1E" "Q2A"
  [7] "Q2B" "Q2C" "Q2D" "Q2E"
dim(data.mtf) # Data set consists of 10 variables and 160 students
## [1] 160 10
```

Descriptive statistics

Percentages of correct answers (1) by questions

```
Perform analysis by 1tm
# Basic statistics using ltm, focus on percentages of correct answers (1) by questions
descript(data.mtf)
##
## Descriptive statistics for the 'data.mtf' data-set
##
## Sample:
  10 items and 160 sample units; 0 missing values
## Proportions for each level of response:
##
            0
                   1 logit
## Q1A 0.3062 0.6938 0.8177
## Q1B 0.2562 0.7438 1.0656
## Q1C 0.3750 0.6250 0.5108
## Q1D 0.4062 0.5938 0.3795
## Q1E 0.1625 0.8375 1.6397
## Q2A 0.2500 0.7500 1.0986
## Q2B 0.2688 0.7312 1.0010
## Q2C 0.3438 0.6562 0.6466
## Q2D 0.4750 0.5250 0.1001
## Q2E 0.4812 0.5188 0.0750
##
##
## Frequencies of total scores:
       0 1 2 3 4 5 6 7 8 9 10
## Freq 2 1 1 7 6 19 38 31 27 16 12
```

response.frequencies(data.mtf)

1 miss

0

0

0

0

0

0

## Q1A 0.30625 0.69375 ## Q1B 0.25625 0.74375 ## Q1C 0.37500 0.62500

## Q1D 0.40625 0.59375

## Q1E 0.16250 0.83750 ## Q2A 0.25000 0.75000

## Q2B 0.26875 0.73125

## Q2C 0.34375 0.65625 ## Q2D 0.47500 0.52500 ## Q2E 0.48125 0.51875

IRT analysis, 2-PL model

##

```
##
##
## Point Biserial correlation with Total Score:
       Included Excluded
## Q1A
        0.4259
                 0.2084
## Q1B
        0.3381
                 0.1236
## Q1C
        0.5585
                0.3539
## Q1D
        0.4177
                 0.1832
## Q1E
        0.3819
                 0.2068
## Q2A
        0.3798
                 0.1712
        0.3497
## Q2B
                 0.1327
## Q2C
        0.4403
                 0.2175
## Q2D
        0.4895
                 0.2623
## Q2E
        0.4619
                 0.2296
##
##
## Cronbach's alpha:
##
                 value
## All Items
                0.5033
## Excluding Q1A 0.4772
## Excluding Q1B 0.5018
## Excluding Q1C 0.4277
## Excluding Q1D 0.4857
## Excluding Q1E 0.4791
## Excluding Q2A 0.4881
## Excluding Q2B 0.4995
## Excluding Q2C 0.4743
## Excluding Q2D 0.4589
## Excluding Q2E 0.4702
##
##
## Pairwise Associations:
     Item i Item j p.value
          3
                    1.000
## 1
                 5
## 2
          6
                 8 1.000
          2
## 3
               10
                    1.000
## 4
          4
                 5
                    0.978
## 5
          2
                 3 0.963
## 6
          7
                10
                    0.945
                 7
## 7
          6
                    0.918
## 8
          1
                 4 0.836
## 9
          1
                 5 0.803
```

## 10

5

8

0.800

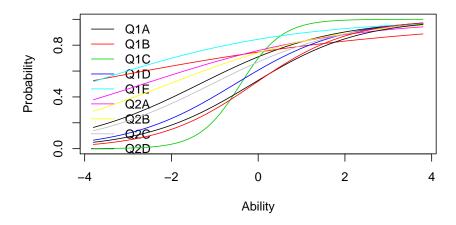
```
# Perform the analysis with ltm(), and save the results in "irt.mtf"
irt.mtf = ltm(data.mtf ~ z1, IRT.param = TRUE)
coef(irt.mtf) # Obtain difficulty and discrimination parameter estimates
##
           Dffclt
                     Dscrmn
## Q1A -1.34813444 0.6637809
## Q1B -4.20384339 0.2572201
## Q1C -0.40398852 2.0871272
## Q1D -0.53216018 0.8114138
## Q1E -3.96868564 0.4283655
## Q2A -2.64619595 0.4320399
## Q2B -2.05447180 0.5154989
## Q2C -1.06267344 0.6670853
## Q2D -0.13935513 0.8074819
## Q2E -0.09428061 0.9122159
summary(irt.mtf) # Obtain LL, SE & z.vals
##
## Call:
## ltm(formula = data.mtf ~ z1, IRT.param = TRUE)
##
## Model Summary:
     log.Lik
                 AIC
                           BIC
##
   -956.738 1953.476 2014.979
##
## Coefficients:
               value std.err z.vals
## Dffclt.Q1A -1.3481 0.5674 -2.3758
## Dffclt.Q1B -4.2038 4.1223 -1.0198
## Dffclt.Q1C -0.4040 0.1527 -2.6457
## Dffclt.Q1D -0.5322 0.2748 -1.9362
## Dffclt.Q1E -3.9687 2.8787 -1.3786
## Dffclt.Q2A -2.6462 1.6249 -1.6285
## Dffclt.Q2B -2.0545 1.0384 -1.9785
## Dffclt.Q2C -1.0627 0.4600 -2.3101
## Dffclt.Q2D -0.1394 0.2281 -0.6108
## Dffclt.Q2E -0.0943 0.2059 -0.4578
## Dscrmn.Q1A 0.6638 0.2876 2.3077
## Dscrmn.Q1B 0.2572 0.2558 1.0055
## Dscrmn.Q1C 2.0871 0.9927 2.1025
```

## Dscrmn.Q1D 0.8114 0.3007 2.6980 ## Dscrmn.Q1E 0.4284 0.3284 1.3045 ## Dscrmn.Q2A 0.4320 0.2762 1.5640

```
## Dscrmn.Q2B 0.5155 0.2721 1.8949
## Dscrmn.Q2C 0.6671 0.2762 2.4148
## Dscrmn.Q2D 0.8075 0.3078 2.6232
## Dscrmn.Q2E 0.9122 0.3166 2.8810
##
## Integration:
## method: Gauss-Hermite
## quadrature points: 21
##
## Optimization:
## Convergence: 0
## max(|grad|): 0.0083
## quasi-Newton: BFGS
```

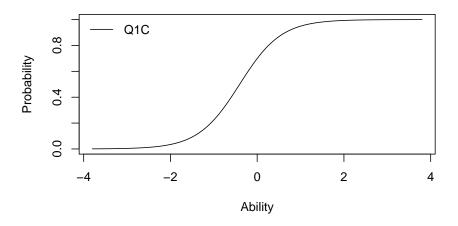
plot(irt.mtf, type = "ICC", legend = TRUE) # Item Characteristic Curves

#### **Item Characteristic Curves**



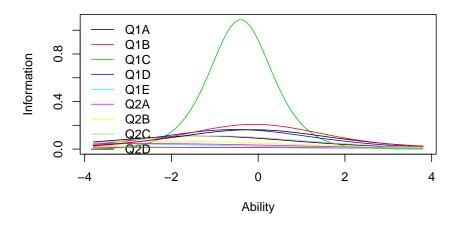
```
plot(irt.mtf, type = "ICC", legend = TRUE, items=3) # Q1c
```

# **Item Characteristic Curves**



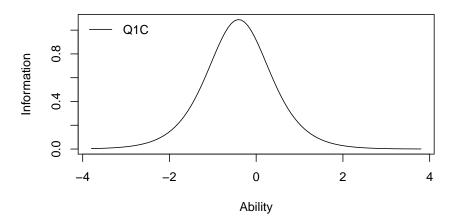
plot(irt.mtf, type = "IIC", legend = TRUE) # Item Information Curves

# **Item Information Curves**



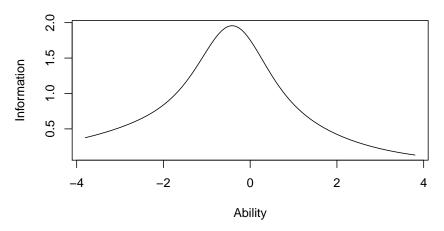
```
# or Item Information Function
plot(irt.mtf, type = "IIC", legend = TRUE, items=3) # Q1c
```

#### **Item Information Curves**



plot(irt.mtf, items = 0, type = "IIC") # Test Information Function

### **Test Information Function**



information(irt.mtf, c(-3,3)) # Test information between -3 to +3 ability range

```
##
## Call:
## ltm(formula = data.mtf ~ z1, IRT.param = TRUE)
##
## Total Information = 7.46
## Information in (-3, 3) = 5.87 (78.7\%)
## Based on all the items
```

```
# "irtoys" package
plot(trf(est(data.mtf, model = "2PL", engine = "ltm"))) #Test Characteristic Curve
```

### **Test response function**

```
10
Expected score
       \infty
       9
        4
       7
       0
                                     -2
                                                           0
                                                                                 2
                -4
                                                        Ability
```

```
# or Test Response Function
# Item fit
item.fit(irt.mtf) # df = 10-2 = 8
##
## Item-Fit Statistics and P-values
##
## Call:
## ltm(formula = data.mtf ~ z1, IRT.param = TRUE)
##
## Alternative: Items do not fit the model
## Ability Categories: 10
##
           X^2 Pr(>X^2)
## Q1A 14.3105
                  0.074
## Q1B 24.0446
                 0.0023
## Q1C 33.1838
                 0.0001
## Q1D 14.9949
                 0.0592
## Q1E 12.8225
                 0.1181
## Q2A 16.4653
                 0.0362
## Q2B 19.8424
                 0.0109
## Q2C 15.9399
                 0.0432
## Q2D 15.8849
                 0.0441
## Q2E 15.2307
                 0.0548
# Fit for margins
margins(irt.mtf)
```

##

## Call:

```
## ltm(formula = data.mtf ~ z1, IRT.param = TRUE)
## Fit on the Two-Way Margins
##
## Response: (0,0)
    Item i Item j Obs
                       Exp (0-E)^2/E
         5
                6 14 7.24
## 1
                                6.32 ***
## 2
         2
                5 13 7.11
                                4.87 ***
## 3
         4
              7 15 20.18
                                1.33
##
## Response: (1,0)
    Item i Item j Obs
                       Exp (0-E)^2/E
## 1
         2
                5 13 18.89
                                1.84
## 2
         5
                6 26 32.78
                                1.40
## 3
         4
                7 28 22.83
                                1.17
##
## Response: (0,1)
    Item i Item j Obs Exp (0-E)^2/E
         5
                6 12 18.77
                                2.44
## 2
         2
               5 28 33.89
                                1.02
## 3
         7
             10 23 19.30
                                0.71
##
## Response: (1,1)
    Item i Item j Obs
                        Exp (0-E)^2/E
                6 108 101.22
## 1
         5
                                 0.45
## 2
         4
                7 67 72.09
                                 0.36
## 3
         2
              5 106 100.10
                                0.35
##
## '*** denotes a chi-squared residual greater than 3.5
table(data.mtf[,5], data.mtf[,6])
##
##
        0
          1
##
    0 14 12
    1 26 108
##
# Personfit
person.fit(irt.mtf)
##
## Person-Fit Statistics and P-values
## Call:
```

```
## ltm(formula = data.mtf ~ z1, IRT.param = TRUE)
##
## Alternative: Inconsistent response pattern under the estimated model
##
##
        Q1A Q1B Q1C Q1D Q1E Q2A Q2B Q2C Q2D Q2E
## 1
          0
               0
                    0
                        0
                             0
                                  0
                                       0
                                           0
                                                0
                                                     0
## 2
          0
                    0
                             0
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## 3
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## 4
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## 6
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                        1
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                                           0
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                        1
## 8
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## 9
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                    1
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## 11
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## 19
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## 20
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## 21
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## 27
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## 29
          0
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## 30
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## 31
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## 32
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               1
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                                                1
                                                     1
## 33
                             0
                                  1
          0
               1
                    1
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                                                     1
## 34
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                    1
                        1
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                                       1
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##
   35
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               1
                    1
                        1
                             1
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## 36
          0
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## 37
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               1
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                             1
                                  1
                                       0
                                           0
                                                     0
## 38
          0
               1
                    1
                        1
                             1
                                  1
                                       0
                                                0
                                                     1
## 39
          0
               1
                    1
                        1
                             1
                                  1
                                       0
                                           1
                                                0
                                                     1
```

##	40	0	1	1	1	1	1	1	0	0	0
##	41	0	1	1	1	1	1	1	1	0	0
##	42	0	1	1	1	1	1	1	1	1	0
##	43	0	1	1	1	1	1	1	1	1	1
##	44	1	0	0	0	0	0	1	1	0	0
##	45	1	0	0	0	1	0	0	1	1	1
##	46	1	0	0	0	1	1	0	1	1	1
##	47	1	0	0	0	1	1	1	0	0	1
##	48	1	0	0	0	1	1	1	0	1	1
##	49	1	0	0	0	1	1	1	1	0	0
##	50	1	0	0	1	0	1	1	1	1	1
##	51	1	0	0	1	1	0	1	0	1	0
##	52	1	0	0	1	1	1	0	1	1	0
##	53	1	0	0	1	1	1	1	1	0	0
##	54	1	0	1	0	0	1	0	1	1	0
##	55	1	0	1	0	0	1	1	0	1	0
##	56	1	0	1	0	1	0	1	0	1	0
##	57	1	0	1	0	1	0	1	1	1	0
##	58	1	0	1	0	1	1	1	0	0	1
##	59	1	0	1	0	1	1	1	0	1	1
##	60	1	0	1	0	1	1	1	1	0	1
##	61	1	0	1	1	0	0	1	1	0	0
##	62	1	0	1	1	0	0	1	1	0	1
##	63	1	0	1	1	0	1	0	0	1	1
##	64	1	0	1	1	1	1	0	0	0	1
##	65	1	0	1	1	1	1	0	1	1	0
##	66	1	0	1	1	1	1	1	0	0	1
##	67	1	0	1	1	1	1	1	0	1	1
##	68	1	0	1	1	1	1	1	1	0	0
##	69	1	0	1	1	1	1	1	1	1	1
##	70	1	1	0	0	1	0	0	0	0	0
##	71	1	1	0	0	1	0	1	1	0	0
##	72	1	1	0	0	1	0	1	1	0	1
##	73	1	1	0	0	1	0	1	1	1	0
##	74	1	1	0	0	1	1	0	0	0	0
##	75	1	1	0	0	1	1	0	1	0	1
##	76	1	1	0	0	1	1	1	0	0	0
##	77	1	1	0	0	1	1	1	0	1	0
##	78	1	1	0	0	1	1	1	1	0	0
##	79	1	1	0	0	1	1	1	1	0	1
##	80	1	1	0	0	1	1	1	1	1	1
##	81	1	1	0	1	0	1	0	1	0	0
##	82	1	1	0	1	0	1	0	1	1	0
##	83	1	1	0	1	0	1	1	1	0	0

```
## 84
          1
               1
                    0
                         1
                              1
                                  0
                                       1
                                            0
                                                 1
                                                      1
## 85
          1
                              1
                                  0
                                            1
                                                 0
               1
                    0
                         1
                                       1
                                                      1
## 86
          1
               1
                    0
                              1
                                  1
                                       0
                                            0
                                                 1
                                                      1
                         1
## 87
           1
               1
                    0
                         1
                              1
                                   1
                                       0
                                            1
                                                 0
                                                      1
## 88
          1
               1
                    0
                         1
                              1
                                  1
                                       0
                                            1
                                                 1
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## 89
                    0
                                            0
                                                 0
                                                      0
          1
               1
                         1
                              1
                                  1
                                       1
## 90
                    0
                                  1
          1
               1
                         1
                              1
                                       1
                                            0
                                                 0
                                                      1
## 91
          1
               1
                    0
                         1
                              1
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## 119
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##
              LO
                       Lz Pr(<Lz)
## 1
        -5.3491 0.1492 0.5593
## 2
        -5.7587 0.0019
                            0.5008
        -7.1412 -0.8338
                            0.2022
## 3
## 4
        -6.2277 0.0566
                            0.5226
        -6.0112 0.1226
## 5
                            0.5488
        -9.3691 -2.7050
                            0.0034
## 6
## 7
        -7.0497 -0.8796 0.1895
```

```
## 8
       -6.0974 -0.0330
                         0.4868
## 9
       -5.8550 -0.1146
                         0.4544
       -6.4326 -0.1797
                         0.4287
## 10
       -6.7261 -0.3680
## 11
                         0.3564
## 12
       -4.4674 1.0838
                         0.8608
       -7.3993 -1.0325
## 13
                         0.1509
## 14
       -4.5660
               1.2017
                         0.8853
## 15
       -4.8410
                1.1836
                         0.8817
## 16
       -5.8466
                0.4695
                         0.6806
## 17
       -4.2362
                1.6547
                          0.951
## 18
       -4.5394
                1.6404
                         0.9495
## 19
       -5.7648
                0.2894
                         0.6139
## 20
                0.7744
                         0.7806
       -5.5319
       -6.2890
                0.0724
## 21
                         0.5289
## 22
       -5.0006
                1.2492
                         0.8942
## 23
       -5.7161
                0.6577
                         0.7446
## 24
       -6.1913
                0.0810
                         0.5323
       -4.9991
                1.3733
## 25
                         0.9152
## 26
       -5.4548
                0.8436
                         0.8006
## 27
       -7.8996 -1.5247
                         0.0637
       -6.2894 -0.3985
## 28
                         0.3451
## 29
       -6.0336 0.1110
                         0.5442
## 30
       -5.9842 0.1842
                         0.5731
## 31
       -4.9444 0.3598
                         0.6405
       -8.2693 -1.8092
                         0.0352
## 32
## 33
       -5.9333 -0.4310
                         0.3332
## 34
       -5.9688 -0.1709
                         0.4322
       -5.9748 -0.1397
## 35
                         0.4445
## 36
       -5.3796 -0.0839
                         0.4666
## 37
       -6.5403 -0.2283
                         0.4097
       -6.6974 -0.5251
## 38
                         0.2998
       -5.9111 -0.1006
## 39
                         0.4599
## 40
       -5.6659 0.4669
                         0.6797
## 41
       -5.0117 0.7396
                         0.7702
       -4.6559 0.6006
## 42
                         0.7259
       -3.7934 0.6764
## 43
                         0.7506
       -6.9109 -0.6356
## 44
                         0.2625
       -7.9170 -1.5621
## 45
                         0.0591
       -7.0419 -0.7051
                         0.2404
## 46
## 47
       -6.0914 0.2702
                         0.6065
## 48
       -6.6073 -0.2652
                         0.3954
## 49
       -5.2154 1.1030
                          0.865
## 50
       -7.5369 -1.2126
                         0.1126
       -7.0246 -0.6586
## 51
                         0.2551
```

```
## 52
      -6.6411 -0.3033
                        0.3808
## 53
      -5.5093 0.8545
                         0.8036
       -8.2484 -1.8260
                         0.0339
## 54
       -7.8653 -1.4952
## 55
                         0.0674
## 56
       -7.3076 -0.9952
                         0.1598
       -6.6629 -0.5472
## 57
                         0.2921
## 58
       -6.2595 -0.1931
                         0.4234
## 59
       -5.9749 -0.2324
                         0.4081
## 60
       -5.4343 0.1994
                          0.579
       -8.0274 -1.6300
                         0.0516
## 61
## 62
       -7.8626 -1.5206
                         0.0642
       -8.3552 -1.8750
                         0.0304
## 63
      -6.8642 -0.7376
## 64
                         0.2304
       -5.9563 -0.2370
## 65
                         0.4063
## 66
       -5.6539 -0.0117
                         0.4953
## 67
       -4.9732 0.0941
                         0.5375
       -5.1215 0.4673
                         0.6798
## 68
       -3.4214
                0.6813
                         0.7522
## 69
## 70
       -5.0246
                0.8719
                         0.8084
## 71
       -5.0779
                1.1979
                         0.8845
       -5.8639
## 72
                0.5036
                         0.6927
## 73
       -5.7578
                0.6147
                         0.7306
## 74
       -4.6496
                1.3514
                         0.9117
## 75
       -5.7167
                0.6555
                         0.7439
## 76
       -4.3491
                         0.9654
                1.8175
## 77
       -5.1259
                1.2456
                         0.8935
## 78
       -4.3989
                1.9559
                         0.9748
       -4.9956
## 79
                1.3034
                         0.9038
## 80
       -5.1697
               0.8453
                          0.801
## 81
       -6.5941 -0.2354
                          0.407
       -7.1560 -0.8032
                         0.2109
## 82
       -6.0012 0.3694
                         0.6441
## 83
## 84
       -6.5220 -0.2712
                         0.3931
## 85
       -5.8787 0.3542
                         0.6384
## 86
       -6.4110 -0.1536
                         0.439
       -5.7623 0.4852
                         0.6862
## 87
## 88
       -5.8264 0.1775
                         0.5704
## 89
       -4.8102 1.5624
                         0.9409
       -5.3456 0.9240
                         0.8222
## 90
## 91
       -4.8045 1.2109
                          0.887
## 92
       -6.9988 -0.9553
                         0.1697
## 93
       -6.2213 -0.0917
                         0.4635
## 94
       -5.5539 0.5654
                         0.7141
## 95
       -5.1456 0.6381
                         0.7383
```

```
## 96 -4.7604 0.4832 0.6855
## 97 -4.5552 1.1784 0.8807
## 98 -4.2418 0.9028 0.8167
## 99 -4.2654 0.9445 0.8275
## 100 -3.4982 0.9309 0.8241
## 101 -8.3867 -2.0246 0.0215
## 102 -6.9665 -0.7589
                       0.224
## 103 -6.6777 -0.7364 0.2307
## 104 -6.0762 -0.2961 0.3836
## 105 -5.7898 -0.0038 0.4985
## 106 -5.1965 0.0318 0.5127
## 107 -4.7628 0.4177
                       0.6619
## 108 -3.7415 0.5882 0.7218
## 109 -5.1815 0.0768 0.5306
## 110 -4.6903 0.4544
                      0.6752
## 111 -4.7336 0.4765 0.6831
## 112 -3.7708 0.6136 0.7303
## 113 -4.8260 0.8868 0.8124
## 114 -4.4388 0.6942 0.7562
## 115 -3.6114 0.7856 0.7839
## 116 -3.9443 1.1638 0.8777
## 117 -3.1747 1.1231
                       0.8693
## 118 -3.2570 1.1337 0.8715
## 119 -1.9076 1.3843 0.9169
# Unidimensional test
#unidimTest(irt.mtf) # This takes a long time to run
  Repeat analysis with mirt
# "mirt" package
# simple way to fit the model
mirt.mtf = mirt(data.mtf, 1, itemtype = "2PL")
coef(mirt.mtf, IRTpars = T, simplify = T)
## $items
##
          а
                 bgu
```

## Q1A 0.663 -1.349 0 1
## Q1B 0.257 -4.207 0 1
## Q1C 2.093 -0.404 0 1
## Q1D 0.811 -0.532 0 1
## Q1E 0.428 -3.973 0 1
## Q2A 0.432 -2.648 0 1

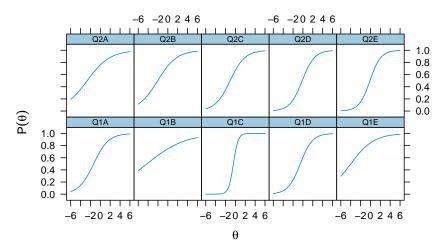
```
## Q2B 0.516 -2.053 0 1
## Q2C 0.667 -1.063 0 1
## Q2D 0.807 -0.140 0 1
## Q2E 0.912 -0.094 0 1
##
## $means
## F1
##
##
## $cov
      F1
## F1 1
```

```
# test info
areainfo(mirt.mtf, c(-3,3))
```

```
## LowerBound UpperBound Info TotalInfo
##
                      3 5.877779 7.467052
           -3
## Proportion nitems
     0.787162
```

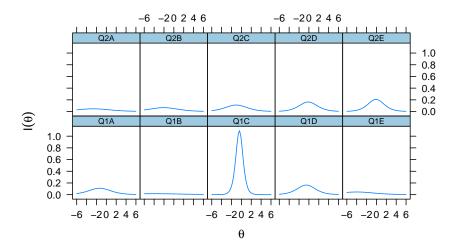
```
# plots
plot(mirt.mtf, type = "trace")
```

### **Item trace lines**



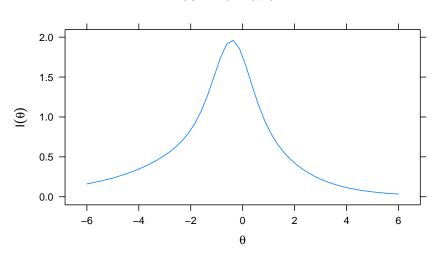
```
plot(mirt.mtf, type = "infotrace")
```

# Item information trace lines



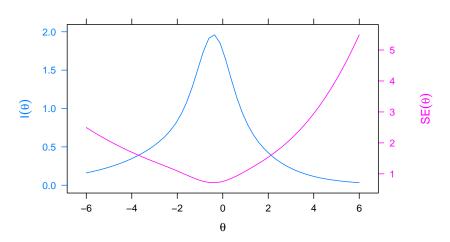
plot(mirt.mtf, type = "info")

# **Test Information**



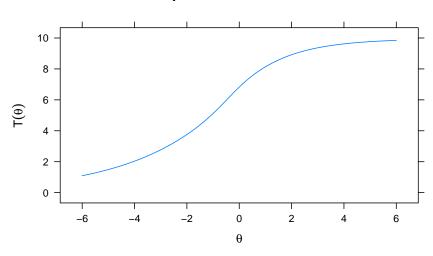
plot(mirt.mtf, type = "infoSE")

#### **Test Information and Standard Errors**



plot(mirt.mtf)

# **Expected Total Score**



```
# model fit
M2(mirt.mtf)
              # M2 nsig.
```

## Warning in (1 - accel) \* longpars: Recycling array of length 1 in array-vector arithmetic is depreca Use c() or as.vector() instead.

## Warning in accel \* preMstep.longpars: Recycling array of length 1 in array-vector arithmetic is depr Use c() or as.vector() instead. ##

## M2 df RMSEA p ## stats 45.27155 35 0.1145428 0.04296208 RMSEA\_5 RMSEA\_95 ## 0 0.07530942 0.07488569 ## stats

```
CFI
##
               TLI
## stats 0.7982402 0.8430757
```

#### itemfit(mirt.mtf)

```
item S_X2 df.S_X2 p.S_X2
##
## 1
      Q1A 1.353
                      5 0.929
## 2
      Q1B 9.367
                       5 0.095
## 3
      Q1C 0.132
                      3 0.988
## 4
      Q1D 9.299
                      5 0.098
      Q1E 3.241
                       4 0.518
## 5
## 6
      Q2A 10.220
                      5 0.069
## 7
      Q2B 1.231
                      5 0.942
## 8
      Q2C 2.134
                       5 0.830
## 9
      Q2D 6.402
                       5 0.269
## 10 Q2E 4.570
                       4 0.334
```

#### personfit(mirt.mtf)

```
##
                Zh
      -0.59920953
## 1
## 2
      -1.55278581
## 3
      0.48361416
## 4
      -0.28806330
## 5
      0.86186607
## 6
      -0.12945241
## 7
      0.43249649
## 8
      -0.21983954
## 9
       -0.28447053
       1.16550221
## 10
## 11
        0.41626410
## 12
        0.35450391
        1.07463370
## 13
## 14
        0.41054254
## 15
        1.63477195
## 16
        1.35254110
## 17
        1.20135830
## 18
        0.07666715
## 19
       -0.15323259
## 20
        0.06945484
        1.06490231
## 21
## 22
        0.05086960
## 23
        0.64776424
## 24 -0.76909670
```

- ## 25 -0.23601304
- -2.02591373 ## 26
- -0.34175787 ## 27
- ## 28 0.14815545
- ## 29 -0.15091822
- ## 30 0.20695314
- ## 31 1.25676002
- ## 32 1.80902937
- ## 33 0.53015360
- ## 34 1.10417602
- ## 35 0.81470044
- ## 36 0.04837985
- ## 37 1.35254110
- -0.28002229 ## 38
- 0.39074013 ## 39
- ## 40 1.25676002
- ## 41 -0.78175354
- ## 42 0.13335641
- ## 43 1.06490231
- ## 44 1.35254110
- ## 45 1.22027754
- ## 46 0.15455766
- ## 47 1.96074685
- ## 48 0.57072655
- ## 49 -0.07278620
- ## 50 -0.76577439
- ## 51 -1.82302879
- -1.22236752 ## 52
- 0.86186607 ## 53
- ## 54 -1.52812052
- ## 55 1.35254110
- 1.35445353 ## 56
- ## 57 -0.01659993
- ## 58 1.06490231
- 0.29636756 ## 59
- 0.87044347 ## 60
- ## 61 0.64776424
- ## 62 0.78414741
- ## 63 1.35254110
- ## 64 -1.49875377 ## 65 -0.34604077
- 1.05858201 ## 66
- ## 67 1.10994010
- ## 68 1.96074685

- ## 69 -0.15091822
- ## 70 1.25041255
- ## 71 0.86424000
- ## 72 -0.07278620
- ## 73 0.87044347
- -0.79381635 ## 74
- ## 75 0.57336889
- ## 76 1.35254110
- ## 77 -0.21477371
- ## 78 -0.12483915
- ## 79 -0.76909670
- ## 80 -0.05696428
- ## 81 0.81470044
- ## 82 0.13335641
- ## 83 1.38353967
- ## 84 0.62672420
- ## 85 0.67157818
- ## 86 0.67025597
- ## 87 0.62716282
- ## 88 0.64776424
- ## 89 -1.87225814
- ## 90 -0.21983954
- ## 91 0.84039329
- 1.35254110 ## 92 ## 93 0.20695314
- ## 94 -1.63070670
- ## 95 1.35254110
- 0.73355830 ## 96
- ## 97 1.25676002
- ## 98 0.51453078
- ## 99 0.89232188
- 0.39954995 ## 100
- ## 101 1.35254110
- ## 102 -1.50017716
- ## 103 0.38152393
- ## 104 0.54912052
- ## 105 -0.30169107
- ## 106 -0.57831523
- ## 107 0.47169486
- ## 108 0.41626410
- ## 109 0.82230273
- ## 110 -0.55239952
- ## 111 0.07850495
- ## 112 1.57198816

- ## 113 0.35450391
- ## 114 -0.16844663
- ## 115 1.06490231
- ## 116 -0.18877086
- ## 117 1.18950617
- ## 118 0.86186607
- ## 119 0.86186607
- ## 120 1.05858201
- ## 121 0.52909299
- ## 122 0.80017804
- ## 123 -0.64344399
- ## 124 0.29636756
- ## 125 -0.70571658
- ## 126 1.35254110
- ## 127 0.91572740
- ## 128 -0.26241847
- ## 129 -1.00637158
- ## 130 0.31503532
- ## 131 -0.79758405
- ## 132 0.33709094
- ## 133 0.13929727
- ## 134 -1.52812052
- ## 135 0.57072655
- ## 136 -1.52812052
- ## 137 0.86186607 ## 138 -0.98137057
- ## 139 1.30235700
- ## 140 -0.06599807
- ## 141 -0.90361524
- ## 142 -2.67731095
- ## 143 0.28574583
- ## 144 0.02564499
- ## 145 1.35254110
- ## 146 0.62129773
- ## 147 -0.46778735
- ## 148 0.66826134
- ## 149 -1.01123703
- ## 150 0.07850495
- ## 151 0.82230273
- ## 152 -0.16561168
- ## 153 -0.23118694
- ## 154 1.35254110
- ## 155 1.96074685
- ## 156 -0.44042666

```
## 157 0.08194588
## 158 -1.80566517
## 159 1.65468756
## 160 1.11965959
# reliabilities: marginal & empirical
marginal_rxx(mirt.mtf) # 0.5574205
## [1] 0.5574205
theta_se = fscores(mirt.mtf, full.scores.SE = T)
empirical_rxx(theta_se) # 0.5681729
##
          F1
## 0.5681729
```

Refer to Brown (2014) for explanations abour marginal and empirical reliabilities.

#### References

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