Optik 165 (2018) 240–247

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Numerical calculation of the reflection, absorption and transmission of a nonuniform plasma slab based on FDTD

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ARTICLE INFO

Article history:
Received 20 February 2018
Accepted 27 March 2018

Keywords:
Finite-difference time-domain (FDTD)
Plasmas
Reflection coefficient
Transmission line theory

ABSTRACT

In this paper, reflection, absorption and transmission characteristics of nonuniform plasma layer with varying electron number density are analyzed. The plasma number density profile is parabolic. Finite-difference time-domain (FDTD) and subslabs approximation methods are utilized and the results are validated. In FDTD, we use auxiliary differential equation (ADE) to simulate the plasma dispersive characteristics. In subslabs method, the plasma layer is divided into several thin subslabs with constant electron number density in each subslab. Partial reflection coefficients at each subslab boundary with different electromagnetic parameters are calculated. The total reflected and transmitted coefficients are then deduced using transmission line method. The reflected, transmitted and absorption power ratio with respect to the incident power is acquired. Their functional dependence on the number density, collision frequency and the distribution of number density, collision frequency is studied.

1. Introduction

In recent years there has been a definite trend toward using a plasmas as absorbers or reflectors of the electromagnetic radiation depending on a specified application. It is mainly due to their tunable reflection and absorption characteristics offering some advantages [1]. Such study is very important to find out the suitable parameters of the plasma which affect the reflection, absorption and transmission of the electromagnetic energy.

Studying the electromagnetic waves interaction with a stratified layered media can be carried out using either analytical or numerical methods. Reflection, absorption and transmission of electromagnetic waves by a magnetized nonuniform plasma slab are analyzed [2]. Scattering matrix method (SMM) was used to determine nonuniform magnetized plasma characteristics [3]. They have shown an easy way to analyze propagation of an electromagnetic wave through a plasma slab by dividing it into many subslabs. By using SMM method, it is possible to determine partial absorption in each subslabs. Interaction of an electromagnetic wave with a magnetized nonuniform plasma slab having parabolic electron number density is studied [4]. SMM method was used for the analysis of electromagnetic wave propagation in planar bounded plasma region [5]. Luebbers have studied the reflection and transmission coefficients of plasma slab based on the finite-difference time-domain (FDTD) method in one-dimension [6]. In all these studies, the aim was to determine the reflection or absorption performance of the plasma layer.

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https://doi.org/10.1016/j.ijleo.2018.03.118

In this paper, we study the reflection, absorption and transmission of electromagnetic waves by a plasma slab. In our paper, the incident wave is assumed to be a plane wave incident at a plasma slab normally. The plasma density profile is parabolic. The plasma is cold, weakly ionized, steady state, unmagnetized, nonuniform and collisional. FDTD and subslabs method are utilized to simulate the electromagnetic wave propagation [7,8].

In subslabs method, the plasma is modeled as a series of two-dimensional subslabs with the wave being absorbed in each slab and reflected at each boundary. This model is acceptable as an approximation under the assumption that the plasma properties vary very slowly along the wave propagating path. The partial reflection coefficients at each slab boundary are computed by the method of transmission line. The total reflection coefficients at each slab boundary are then calculated by iteration. The total transmission coefficient is calculated based on the total reflection coefficients at each boundary.

In FDTD method, every wavelength with respect to the maximum frequency is divided to 40 Yee cells. Auxiliary differential equation (ADE) is used to calculate the dispersive characteristics of plasma [9]. The calculation area is truncated with perfectly matched layer (PML) to simulate infinitely large space [10,11]. The whole area is divided into total field and scattering field by the total field boundary. The fields of every time step at the two boundaries of the plasma slab are recorded and then transformed to frequency domain to obtain the reflected and transmitted coefficients.

The reflected and transmitted power ratio can be obtained by square the abstract of the reflection and transmission coefficients. Because of energy conservation, the absorption power can be calculated by subtracting incident power and the reflected and transmitted power. The dependence of the reflected, absorbed and transmitted power on the plasma number density, the collision frequency is investigated.

In this paper, the time dependence $e^{j\omega t}$ is assumed, where ω is angular frequency.

2. Formulation

2.1. Plasma model

A plane wave propagating in a lossy medium, such as plasma, obeys the following Maxwell's equations

$$\nabla \times \boldsymbol{H} = j\omega\varepsilon_{\mathrm{r}}\varepsilon_0\boldsymbol{H} \tag{1a}$$

$$\nabla \times \boldsymbol{E} = -j\omega\mu_{\mathrm{r}}\mu_0\boldsymbol{H} \tag{1b}$$

where $\boldsymbol{E}$ is electric field, $\boldsymbol{H}$ is magnetic field, ε_{r} is relative permittivity, μ_{r} is relative permeability, ε_0 is permittivity in free space, μ_0 is permeability in free space, respectively. For plasma medium, we have

$$\varepsilon_{\mathrm{r}} = 1 + \frac{\omega_{\mathrm{p}}^2}{j\omega(j\omega + \nu_{\mathrm{c}})} \tag{2a}$$

$$\mu_{\mathrm{r}} = 1 \tag{2b}$$

where ν_{c} is collision frequency, ω_{p} is plasma frequency, respectively. ω_{p} can be expressed by the plasma density N_0, which is

$$\sigma_{\mathrm{p}} = \frac{e^2N_0}{m\varepsilon_0} \tag{3}$$

where e and m are the charge and the mass of electron, respectively.

In plasma, the complex wavenumber k is

$$k = k_0\sqrt{\varepsilon_{\mathrm{r}}\mu_{\mathrm{r}}} \tag{4}$$

where $k_0 = c/\omega$ is wavenumber in free space and c is speed of light in free space.

2.2. Subslabs method

The plasma slab can be approximated as several adjacent, homogenous two-dimensional plasma slabs. Every two slabs have a boundary at which reflection may happen because of mismatch of impedance. Total reflection coefficient can be calculated by transmission line method. To be more clear, we deduce a general solution to arbitrary layers with arbitrary dielectric media.

Assume there are N parallel boundaries and thus the space is divided to $N+1$ areas with homogeneous medium in them, which is shown in Fig. 1. The permittivity in each areas are $\varepsilon_0, \varepsilon_2, \ldots, \varepsilon_{N+1}$, respectively. The wavenumber in the areas are $k_0, k_1, \ldots, k_{N+1}$, respectively. The first and the last areas are infinitely thick and occupy half space. The thickness of the other $N-1$ layers are $d_1, d_2, \ldots, d_{N-1}$, respectively.

First, the partial reflection coefficients R_n of each n boundary are calculated by the formula

$$R_n = \frac{\sqrt{\varepsilon_{n-1}} - \sqrt{\varepsilon_n}}{\sqrt{\varepsilon_{n-1}} + \sqrt{\varepsilon_n}}, \quad n = 1, 2, \ldots, N \tag{5}$$

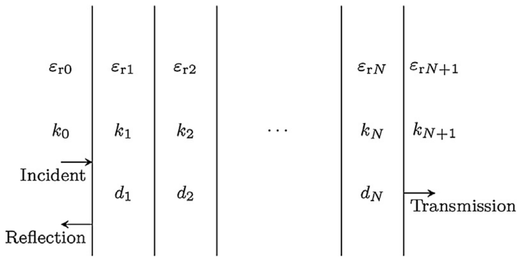

Fig. 1. Subslabs and the parameters.

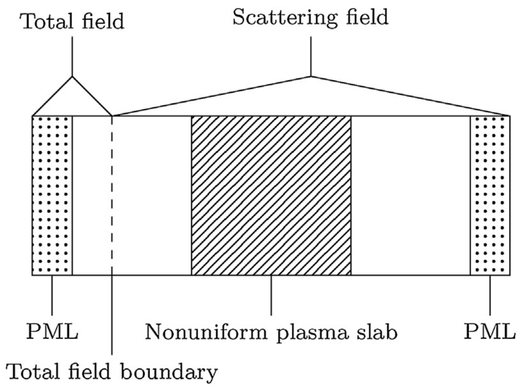

Fig. 2. FDTD calculation area division.

The total reflection coefficients R_n' at each boundary are calculated by transmission line method. For the last boundary, we have $R_N' = R_N$. For $n = N-1, N-2, \ldots, 1$, the total reflection coefficients can be calculated iteratively by

$$R_n' = \frac{R_n + R_{n+1}' e^{-2jk_n d_n}}{1 + R_n R_{n+1}' e^{-2jk_n d_n}} \tag{6}$$

The total transmission coefficients of each boundary can be calculated based on the total reflection coefficients. The transmission coefficient of the first layer is $T_1' = 1 + R_1'$. for $n = 2, 3, \cdots, N$, the transmission coefficients are

$$\begin{aligned} T_n' &= \frac{T_{n-1}' e^{-jk_{n-1}d_{n-1}}(1 + R_n')}{1 + R_n' e^{-2jk_{n-1}d_{n-1}}} \\ &= \frac{T_{n-1}'(1 + R_n')}{e^{jk_{n-1}d_{n-1}} + R_n' e^{-jk_{n-1}d_{n-1}}} \end{aligned} \tag{7}$$

Thus, the total transmission coefficient at the Nth boundary is

$$T_N' = \frac{\prod_{n=1}^{N}(1 + R_n')}{\prod_{n=1}^{N-1}(e^{jk_n d_n} + R_{n+1}' e^{-jk_n d_n})} \tag{8}$$

The ratios of reflected and transmitted power with respect to the incident power are

$$R = \left|R_1'\right| \tag{9a}$$

$$T = \left|T_N'\right| \tag{9b}$$

The absorbed power is the incident power subtracted by the reflected power and the transmitted power. Thus, the ratio of absorbed power with respect to the incident power is

$$A = 1 - R - T \tag{10}$$

2.3. Finite-difference time-domain

FDTD is utilized to calculated the reflected and transmitted power to validate with the subslabs method. The calculated area is divided into total field area and scattering field area by total boundary condition, illustrated in Fig. 2. The calculation area is truncated by 10 layer perfectly matched layer (PML). Nonuniform plasma slab is placed in the total area.

Auxiliary differential equation (ADE) method is used to calculate the plasma's dispersive characteristics. Maxwell's equation can be written as

$$\nabla \times \boldsymbol{H} = j\omega\varepsilon_0\boldsymbol{E} + \boldsymbol{J}_\mathrm{p} \tag{11}$$

where $\boldsymbol{J}_\mathrm{p}$ is polarized current, which is

$$\boldsymbol{J}_\mathrm{p} = \frac{\varepsilon_0\omega_\mathrm{p}^2}{j\omega + \nu_\mathrm{c}}\boldsymbol{E} \tag{12}$$

$\boldsymbol{J}_\mathrm{p}$ is sampled at half integer time step. The spatial position is the same as $\boldsymbol{E}$. By discretizing $\boldsymbol{J}_\mathrm{p}$ and $\boldsymbol{E}$, we get

$$\frac{\boldsymbol{J}_\mathrm{p} - \boldsymbol{J}_\mathrm{p}}{\Delta t} + \nu_\mathrm{c}\frac{\boldsymbol{J}_\mathrm{p} + \boldsymbol{J}_\mathrm{p}}{2} = \varepsilon_0\omega_\mathrm{p}^2\boldsymbol{E} \tag{13}$$

where Δt is temporal increment. Then we get the explicit iterative equation

$$\boldsymbol{J}_\mathrm{p}^{n+1/2} = \frac{2 - \nu_\mathrm{p}\Delta t}{2 + \nu_\mathrm{p}\Delta t}\boldsymbol{J}_\mathrm{p}^{n+1/2} + \frac{2\varepsilon_0\omega_\mathrm{p}^2\Delta t}{2 + \nu_\mathrm{p}\Delta t}\boldsymbol{E}^n \tag{14}$$

where n is time step number.

The incident wave is excited by the differential Gaussian wave, which is

$$f(t) = \left(\frac{t - t_0}{\tau}\right) e^{-4\pi\left(\frac{t-t_0}{\tau}\right)^2} \tag{15}$$

where τ is the width of Gaussian wave, t_0 is time shift.

The incident and reflected wave at the left boundary of the plasma at each time step are recorded, denoted as $\boldsymbol{E}_\mathrm{i}^n$ and $\boldsymbol{E}_\mathrm{r}^n$, respectively. The transmitted wave at the right boundary of the plasma at each time step is recorded, denoted as $\boldsymbol{E}_\mathrm{t}^n$. The time varying of the fields are then transformed to frequency domain by Fourier transformation. The frequency domain of the three fields are

$$\boldsymbol{E}_\mathrm{i}(\omega) = \int \boldsymbol{E}_\mathrm{i}(t)e^{-j\omega t}\mathrm{d}t = \sum_{n=1}^{N_t}\boldsymbol{E}_\mathrm{i}^n e^{-j\omega n\Delta t}\Delta t \tag{16a}$$

$$\boldsymbol{E}_\mathrm{r}(\omega) = \int \boldsymbol{E}_\mathrm{r}(t)e^{-j\omega t}\mathrm{d}t = \sum_{n=1}^{N_t}\boldsymbol{E}_\mathrm{r}^n e^{-j\omega n\Delta t}\Delta t \tag{16b}$$

$$\boldsymbol{E}_\mathrm{t}(\omega) = \int \boldsymbol{E}_\mathrm{t}(t)e^{-j\omega t}\mathrm{d}t = \sum_{n=1}^{N_t}\boldsymbol{E}_\mathrm{t}^n e^{-j\omega n\Delta t}\Delta t \tag{16c}$$

where N_t is total simulation time step number in the FDTD simulation.

In our problem, incident, reflected and transmitted electric fields are all along the same direction. Thus, they can be denoted as E_i, E_r and E_t, respectively. The total reflection and transmission coefficients are

$$R^{\mathrm{FDTD}} = \frac{E_\mathrm{r}(\omega)}{E_\mathrm{i}(\omega)} \tag{17a}$$

$$T^{\mathrm{FDTD}} = \frac{E_\mathrm{t}(\omega)}{E_\mathrm{i}(\omega)} \tag{17b}$$

3. Numerical results and discussion

3.1. Plasma slab's parameters

In this section, the subslab and FDTD simulation results are presented and discussed. The reflected, absorbed and transmitted power ratio calculated for the plasma slab are presented and their dependence on plasma parameters such as number density and collision frequency is investigated.

The thickness of the slab is 12 cm. In subslab method, to calculate the reflection coefficient, the plasma is divided into 24 slabs with boundary separating them and each slab is 0.5 cm thick. The number density is constant in each slab, but the overall density profile follows some curve. In FDTD method, the plasma number density distributes strictly as parabolic function.

There are 25 boundaries by 24 slabs, so 25 partial reflection coefficients are calculated. The total reflected and transmitted power ratio are then computed by the method of transmission line. The absorbed power ratio is then acquired.

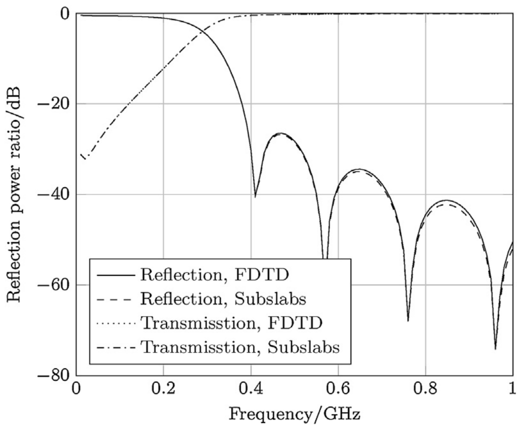

Fig. 3. Reflection and transmission coefficient amplitude value by variation of frequency for different methods.

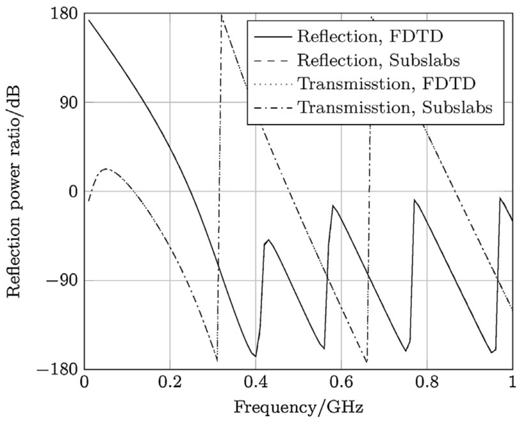

Fig. 4. Reflection and transmission phase value by variation of frequency for different methods.

3.2. Numerical validation

The plasma slab with $\nu_c = 0.1$ GHz and center number density of $1 \times 10^{15}\ \mathrm{m}^{-3}$ is calculated to validate the algorithm. The FDTD and the subslabs results are imposed in Fig. 3. The results are presented as plots of the ratio of the reflected, transmitted and absorbed power by the incident power against wave frequency. We can see that the results of the two methods are agree well with each other. The reflected and transmitted phases are also calculated and compared in Fig. 4. The results of the two methods agree well with each other.

3.3. Influence of the collision frequency

The plasma number density is chosen for a center density of $N_0 = 1 \times 10^{15}\ \mathrm{m}^{-3}$. We get corresponding plasma frequency $\omega_p = 3.18\ \mathrm{Grad\ s}^{-1}$. Fig. 5 shows the reflected power fraction versus frequency for three values of the collision frequency, which are $\nu_c = 0.1$ GHz, $\nu_c = 0.5$ GHz, $\nu_c = 1$ GHz, respectively. From Fig. 5 it can be seen that the reflected power decreases for increasing collision frequency when the incident frequency is less than 0.4 GHz. At the four resonance frequencies of 0.41 GHz, 0.57 GHz, 0.76 GHz and 0.96 GHz, the reflected power increase for increasing collision frequency. At other frequency when incident frequency is greater than 0.4 GHz, the reflected powers are almost the same for different collision frequency.

Fig. 6 shows the transmitted power fraction versus frequency for three values of the collision frequency, which are $\nu_c = 0.1$ GHz, $\nu_c = 0.5$ GHz, $\nu_c = 1$ GHz, respectively. The transmitted power greatly increases with increasing collision frequency when the incident frequency is less than 0.2 GHz. However, the transmitted power increases with increasing collision frequency when the incident frequency is greater than 0.2 GHz.

Fig. 7 shows the absorbed power fraction versus frequency for three values of the collision frequency, which are $\nu_c = 0.1$ GHz, $\nu_c = 0.5$ GHz, $\nu_c = 1$ GHz, respectively. It is seen that the peak absorbed power increase with increasing collision frequency. This is an intuitive result, since as the collision frequency increases, the electromagnetic energy exhausts more effectively.

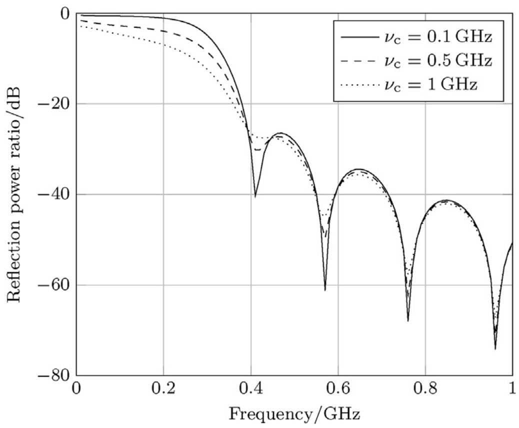

Fig. 5. Reflection value by variation of frequency for different collision frequency.

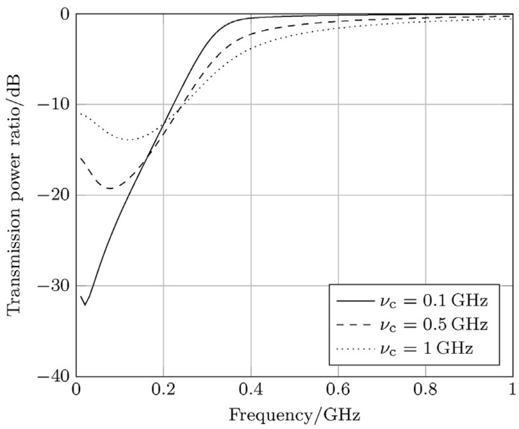

Fig. 6. Transmission value by variation of frequency for different collision frequency.

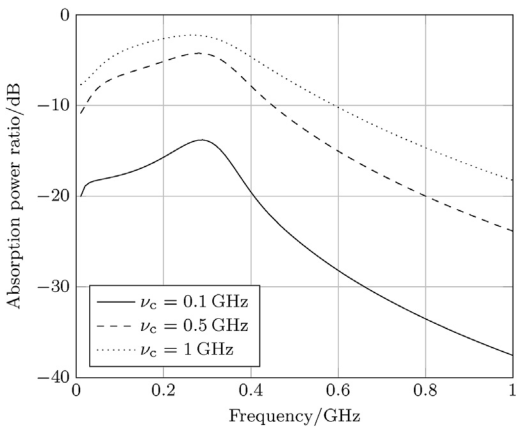

Fig. 7. Absorption value by variation of frequency for different collision frequency.

3.4. Influence of the plasma number density

Fig. 8 shows the reflected power fraction versus frequency for a collision frequency of 0.1 GHz and three values of the center number density, which are $N_0 = 1\times10^{15}$ m^{-3}, $N_0 = 5\times10^{15}$ m^{-3}, $N_0 = 10\times10^{15}$ m^{-3}, respectively. It is clear that the reflected power greatly increases with increasing number density. It can be concluded that a low density and highly collisional plasma can greatly reduce the reflected power. However, the two requirements, namely, low density and high collision frequency, are hard to achieve simultaneously.

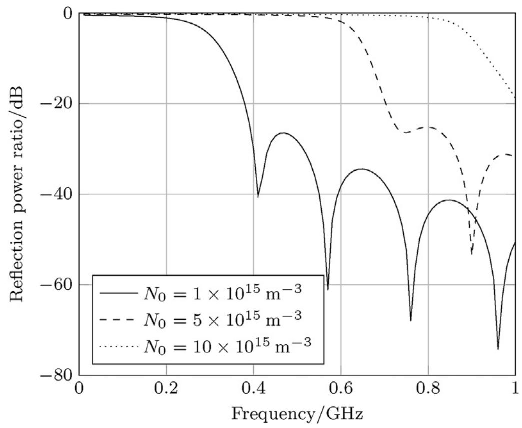

Fig. 8. Reflection value by variation of frequency for different collision frequency.

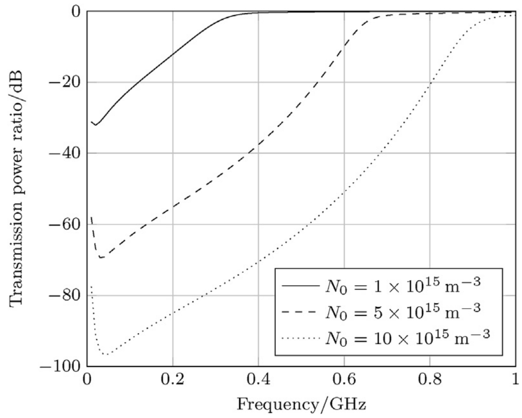

Fig. 9. Transmission value by variation of frequency for different collision frequency.

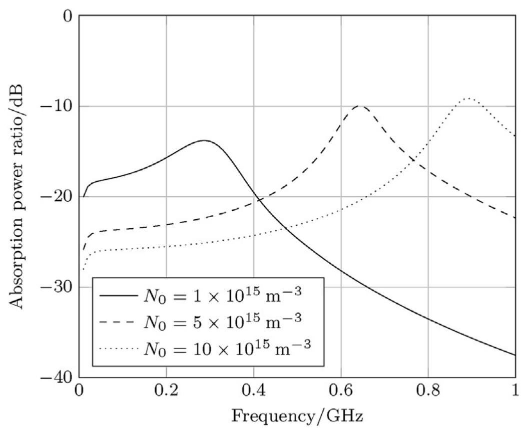

Fig. 10. Absorption value by variation of frequency for different collision frequency.

Fig. 9 shows the transmitted power fraction versus frequency for a collision frequency of 0.1 GHz and three values of the center number density, which are $N_0 = 1 \times 10^{15}$ m^{-3}, $N_0 = 5 \times 10^{15}$ m^{-3}, $N_0 = 10 \times 10^{15}$ m^{-3}, respectively. It can be seen that the transmitted power greatly decreases with increasing number density. The mechanism is that as the number density increases, the plasma is more and more like metal and is more conductive. As the number density trends infinitely large, the plasma trends to perfectly electric conductor and can shield electromagnetic wave.

Fig. 10 shows the absorbed power fraction versus frequency for a collision frequency of 0.1 GHz and three values of the center number density, which are $N_0 = 1 \times 10^{15}$ m^{-3}, $N_0 = 5 \times 10^{15}$ m^{-3}, $N_0 = 10 \times 10^{15}$ m^{-3}, respectively. It can be seen that the amount of absorbed power increases slightly with number density. The peak absorption frequencies are 0.3 GHz, 0.64 GHz and 0.89 GHz, which are eventually increase.

4. Conclusion

This study of reflection, transmission and absorption of electromagnetic waves by a nonuniform unmagnetized plasma has led to several results. We showed the effects of two plasma parameters, namely the number density and the collision frequency. It is found that high number density is more reflective and transmits less power. The influence of number density on absorption is mainly on the peak absorption frequency. High collision frequency leads to high absorbed power. However, the collision frequency's influence on the reflected and transmitted power is time dependence. When incident wave frequency is less than 0.4 GHz, the increasing collision frequency make the reflected power decrease and when the frequency is greater than 0.4 GHz, the collision frequency only make the reflected power change at some resonant frequencies. When incident wave frequency is less than 0.2 GHz, the transmitted power increase as the increasing of collision frequency. However, when the incident wave frequency is greater than 0.2 GHz, the transmitted power decrease as the increasing of collision frequency.

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