

TOWARDS A FRAMEWORK FOR OPERATIONAL RISK IN THE BANKING
SECTOR

by

Mphekeleli Hoohlo

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Approved:

Eric Schaling, Ph.D.
Major Professor

Thanti Mthanti, Ph.D.
Committee Member

Odongo Kodongo, Ph.D.
Committee Member

Thabang Mokoaleli-Mokoteli, Ph.D.
Committee Member

Christopher Malikane, Ph.D.
Committee Member

Paul Alagidede, Ph.D.
Vice President for Research and
Dean of the School of Graduate Studies

UTAH STATE UNIVERSITY
Logan, Utah

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ABSTRACT

Towards a Framework for Operational Risk
in the Banking Sector

by

Mphekeleli Hoohlo

University of the Witwatersrand, 2019

Major Professor: Eric Schaling, Ph.D.
Supervisor: Thanti Mthanti, Ph.D.
Department: Law, Commerce & Management

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Shefrin (2016) notes that people would rather incur greater risks to hold on to things they already have, than the risks they would taken to get into that position in the first place, thereby risking a banks' survival, rather than expose their trading losses by consciously deceiving senior management to hide unethical operational practices. In this paper the application of machine learning techniques on the observed data demonstrates how these issues can be resolved given their flexibility to different types of empirical data.

(116 pages)

PUBLIC ABSTRACT

Towards a Framework for Operational Risk
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The purpose of this research is to provide clarity; based on theory and empirical evidence, on how to tackle the specific problems in the *operational risk* (OpRisk) literature, which have earned a place in modern day recourse in risk and finance, due to how significantly its importance has increased over the last few decades. During this period, until present day, there have been and continues to be series of destructive events that have threatened the stability of financial systems due to OpRisk. In most, if not all of these cases, human error is at the center of the chain of events that lead or may lead to (OpRisk) losses. There are many attitudes that can potentially infect organisational processes, the most persistent of these attitudes stem from human failings that are exploitable Barberis & Thaler (2003), thus forming a basis for the theoretical foundation of OpRisk.

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DEDICATION

Dedicate it.

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Acknowledge those acknowledged individuals and things.

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CHAPTER 1 INTRODUCTION

Purpose of the study

The purpose of this research is to apply a generalised linear model (GLM) suitable for exposure-based operational risk (EBOR) treatments within the operational risk management framework (ORMF), effectively replacing historical loss severity curves obtained from historical loss counts, by forward-looking measures using event frequencies based on actual operational risk (OpRisk) exposures. Preliminary work on EBOR models was undertaken by (Einemann, Fritscher, & Kalkbrener, 2018). Secondly, this study provides a comprehensive computational comparison of various data-intensive techniques amongst each other, and versus *classical* statistical estimation methods for classification and regression performances.

Our understanding of existing ORMF to date is limited to the assumption that financial institutions (FI's) are risk-neutral. Thirdly, in lieu of the afore-mentioned, this study finally seeks to invalidate the risk-neutral assumption, by means of various unsupervised learning techniques, by proposing that FI's are more risk-averse; this can be measured by analysing subtle patterns between data features and trends in the allocated risk capital estimates. In theory, a risk manager who experiences persistent/excessive losses due to particular risk events, would over-compensate cover for these particular risk types, and this would show in reduced losses in these types over time.

Fundamentals of ORMF's

Most banks' estimates for their risk are divided into credit risk (50%), market risk (15%) and OpRisk (35%). Cruz (2002) postulated that OpRisk, which focuses on the human side of risk management is difficult to manage with the reduced ability to measure it. The process of OpRisk, that is, the how manifests in conscious and/or unconscious states of the risk manager/s (Hemrit & Arab, 2012), and encompasses approaches and theories that focus on how one will choose when faced with a decision, based on how comfortable they are with the situation and the variables that are present.

Definition of operational risk

Operational risk (OpRisk) is defined as: *The risk of loss resulting from inadequate or failed internal processes, people and systems, and from external events. This definition includes legal risk, but excludes strategic and reputational risk.* (Risk, 2001).

A major managerial concern for businesses is an inability to identify and account for their susceptibility to OpRisk events following a number of very costly and highly publicized operational losses, in particular, it became popular following a fraudulent trading incident which was responsible for a catastrophic loss that led to the collapse of Barings Bank (the UK's oldest bank) in 1995.

The term OpRisk began to be used after the afore-mentioned and similar types of OpRisk events became more common. A (rogue) trader (Nick Leeson), who risked the banks' survival rather than expose his trading losses, by consciously deceiving senior management to hide his unethical acts, was found to have been

responsible for unethical trading practices when he created illegal trades in his account, then used his position in the front and back offices of the bank to hide his trading losses. Worse still, he incurred a greater risk to the bank by lying in order to give a false impression of his profits. Shefrin (2016) notes that people would rather incur greater risks to hold on to things they already have, than the risks they would taken to get into that position in the first place.

It was later discovered that he was placing illegal bets in the Asian-markets, and kept these contracts out of sight from senior management to cover up his illegal activity. When his fraudulent behaviour was discovered (after an earthquake hit at Kobe in Japan, that collapsed the Osaka Securities Exchange) he succumbed to unrecoverable losses due to trading positions he had accumulated, which resulted in a loss of around £1.3 billion to the bank, thus resulting in it's collapse. In most, if not all of these cases, human error is at the center of the chain of events that lead or may lead to OpRisk losses.

Since then, there have been a series of destructive events that have threatened the stability of the financial system due to OpRisk. Large fines have been imposed on the culprits and regulatory scrutiny has been heightened as a result of a number of operational events, e.g. the January 2016 "Dark Pool" trading penalties suffered by Barclays (\$70mn) and Credit Suisse (\$85mn), imposed by the United States (US) based securities exchange commision (SEC). These OpRisk loss events were due to fraudulent trading activity consisting of rogue traders dealing in illegally placed high frequency trades for private clients where prices were hidden.

In South Africa (SA), there is an upcoming case of price fixing and market allocation in trading foreign exchange (FX) currency pairs, reffered to the SA based competition tribunal for prosecution. Absa bank, Standard bank & Investec may be liable to payment of an admistrative penalty equal to 10% of their annual turnover

in 2016, following accusations by the local based competition commission in February 2017, of rogue traders manipulating the price of the rand through buying and selling US dollars in exchange for the rand at fixed prices. According to the competition commission, it has been alleged that currency traders have been colluding or manipulating the price of the rand through these buy and sell orders to change supply of the currency.

This has compromised the quality and accuracy of risk management's advisory service and pedigree, and aroused huge interest as the value of the rand has implications on South African's. Furthermore, this kind of behaviour can lead to catastrophic operational losses, as with the case for the Barings event, resulting is a mismatch between business' expectations and the value the risk management practice was able to deliver, which is prevalent across FI's and remains unchanged. There are many attitudes that can potentially infect organisational processes, the most persistent of these attitudes stem from human failings that are exploitable (Barberis & Thaler, 2003); i.e. humans' propensity to be deceitful during periods of distress, thus forming a basis for a theoretical foundation of OpRisk management.

Basel Committee's quantitative operational risk management framework

The Bank for International Settlements (BIS) is an organisation consisting of a group of central bank governors and heads of supervision of central banks around the world who represent an authority on good risk management in banking. More specifically, the BIS oversee the duties of the Basel Committee on Banking Supervision (BCBS)/Basel Committee. The role of the BCBS is to set out guidelines on international financial regulation to cover risks in the banking sector. There have been three banking accords from the BCBS under the supervision of the BIS in dealing with financial regulation, viz., Basel I, Basel II & Basel III. These accords

describe an overview of capital requirements for financial institutions (FI's) in order to create a level playing field, by making regulations uniform throughout the world.

The Capital Adequacy Accord (Basel I)

Basel I was established in 1988. Basel I meant that FI's were required to assign capital for credit risk to protect against credit default. In 1996, an amendment to Basel I imposed additional requirements to cover exposure due to market risk as well as credit risks. Basel I effectively minimised rules that favoured local FI's over potential foreign competitors, by opening up global competition so that these banks could buffer against international solvency. In 2001, the Risk (2001) consultative package provided an overview of the proposed framework for regulatory capital (RC) charge for OpRisk. A financial institution (FI) has an OpRisk component, which constitutes a substantial risk component other than credit and market risk. There are two types of OpRisk's viz., potential high severity risk where the probability of an extreme loss is very small but costly, and high frequency/low severity risk where frequency plays a major role in the OpRisk capital charge calculation.

New Capital Adequacy Accord (Basel II)

The framework for Basel II was implemented in June 2006. The rationale for Basel II is to introduce risk sensitivity through more restrictive capital charge measures and flexibility with specific emphasis on OpRisk. The structure of the new accord is built upon a three-pillar framework: Pillar I stipulates minimum capital requirements for the calculation of regulatory capital for credit risk, market risk and OpRisk in order to retain capital to ward against these risks. Pillar II imposes a supervisory review process through which additional requirements can be imposed,

such as the bank's internal capital assessments, or to act on needed adequate capital support or best practice for mitigating their risks. Pillar III relates to market discipline, i.e. transparency requirements which require banks to publicly provide risk disclosures to keep them in line by enabling investors to form an accurate view of their capital adequacy, in order to reward or punish them on the basis of their risk profile.

Basel III

Basel III establishes tougher capital standards through more restrictive capital definitions, higher RWA's, additional capital buffers, and higher requirements for minimum capital ratios (Dorval, 2013). Through Basel III, the BCBS is introducing a number of fundamental reforms grouped under three main headings (Committee & others, 2010): 1] A future of more capital through incremental trading book risk (credit items in trading book treated in the same way as if they were in banking book), 2] More liquidity through the introduction of a global liquidity risk standard (Basel III will push banks toward holding greater levels of liquid instruments, such as government bonds and more liquid corporate instruments), and 3] Lower risk under the new requirements of the capital base, i.e., establish more standardized risk-adjusted capital requirements.

Regarding the sequence Basel I and Basel II: Regulation begins as a qualitative recommendation which requires banks to have an assets-to-capital multiple of at least 20, then focuses on ratios in which both on-balance sheet and off-balance sheet items are used to calculate the bank's total risk-weighted assets (RWA's)¹, then on tail risk. In other words, auditors' discretion is replaced by market perception of capital, meaning there is a market risk capital charge for all items in the

¹Also referred to as risk-weighted amount, it is a measure of the bank's total credit exposure

trading business line, then exciting new static risk management approaches which involve calculating a 99.9 percentile left tail confidence interval to measure OpRisk value-at-risk (VaR) and convert it into a RC charge.

The future regulatory environment requires OpRisk professionals, who are not only intelligent, creative and motivated but also have the courage to uphold the OpRisk advisory service standards. Businesses that want to successfully manage risk, would be well advised to utilize new theoretical and empirical techniques, such that large and small scale experiments play an important role in risk analysis and regulatory research.

Modern OpRisk measurement frameworks (ORMF's)

Basel II describes three methods of calculating capital charge for OpRisk RC viz., the standardised approach (SA), the basic indicator approach (BIA) and the internal measurement approach (IMA). The basic indicator approach (BIA) sets the OpRisk RC equal to a percentage (15%) of the annual gross income of the firm as a whole to determine the annual capital charge. The SA is similar to the BIA except the firm is split into eight business lines and assigned a different percentage of a three year average gross income per business line, the summation of which is the capital charge (Hoohlo, 2015). In the IMA, the bank uses it's own internal models to calculate OpRisk loss.

Advanced Measurement Approach (AMA)

The advanced measurement approach (AMA) is an IMA method which applies estimation techniques of OpRisk capital charge derived from a bank's internal risk measurement system Cruz (2002). Basel II proposed measurement of OpRisk

to define capital requirements against unexpected bank losses whereas the unexpected loss (UL) is the quantile for the level α minus the mean. According to the AMA, which is thought to outperform the simpler SA approach and the BIA, RC requirements are defined according to the UL limit in one year and the loss distribution at a 99.9% confidence level ($\alpha = 0.01\%$) aggregate loss distribution² used as a measure of RC. The BCBS proposes to define RC as $RC = UL$. This involves simulations based on historical data to establish frequency and severity distributions for losses. In this case the RC is a VaR measure.

The Basel III capital adequacy rules permit model-based calculation methods for capital, including the AMA for OpRisk capital. Under Basel III, standardised methods for OpRisk capital have been overhauled, however for a while there was no prospect of an overhaul of the AMA. Given the relative infancy of the field of OpRisk measurement, banks are mostly free to choose among various AMA principle-based frameworks to a significant degree of flexibility (Risk, 2016). A bank that undertakes an AMA should be able to influence their capital requirements through modeling techniques resulting in lowered pressure on OpRisk capital levels, which in turn has a positive impact on the bank.

A FI's ability to determine the framework used for its regulatory OpRisk RC calculation, evolves from how advanced the FI is along the spectrum of available approaches used to determine capital charge (Risk, 2001). BCBS recognizes that a variety of potentially credible approaches to quantify OpRisk are currently being developed by the industry, and that these R&D activities should be incentivised. Increasing levels of sophistication of OpRisk measurement methodologies should generally be rewarded with a reduction in the regulatory OpRisk capital requirement.

²The aggregate loss distribution is obtained by convoluting a loss event frequency distribution and a loss severity distribution by means of the random sums method.

The standardised measurement approach (SMA)

The flexibility of internal models was expected to narrow over time as more accurate OpRisk measurement was obtained and stable measures of RC were reached, ultimately leading to the emergence of best practice. Instead, internal models produced wildly differing results of OpRisk RC capital from bank to bank, contrary to the expectations of the BCBS. In March 2016, the BCBS published for consultation a standardised measurement approach (SMA) for OpRisk RC; that proposes to abandon the freedom of internal modelling (thus ending the AMA) approaches for OpRisk RC, in exchange for being able to use a simple formula to facilitate comparability across the industry.

Under the SMA, RC will be determined using a simple method comprising of two components: A stylised systemic risk model (business indicator component), and an idiosyncratic risk model (loss component), which are combined via an internal loss multiplier (ILM), whose function is to link capital to a FI's operational loss experience to determine SMA capital.

The SMA formula is thought to be consistent with regulators' intent for simplification and increased comparability across most banks. However, there is a feeling from some in the banking industry that the SMA is disadvantaged as it is not the same as measuring OpRisk. Mignola, Ugoccioni, & Cope (2016) and Peters, Shevchenko, Hassani, & Chapelle (2016) identified that the SMA does not respond appropriately to changes in the risk profile of a bank i.e., it is unstable viz., two banks of the same risk profile and size can exhibit OpRisk RC differences exceeding 100%, and risk insensitive; that SMA capital results generally appear to be more variable across banks than AMA results, where banks had the option of fitting the loss data to statistical distributions.

Argument

Over the last twenty years, hard-won incremental steps to develop a measure for the size of OpRisk exposure along with the emergence of promising technologies presents a unique opportunity for bankers and treasurers - traditionally risk-averse players - to develop a novel type of way of looking at decision making under risk/uncertainty. New technologies have been introduced which make use of up to date technical solutions (such as homo heuristics developed by Gigerenzer & Brighton (2009), who maintain their methods solve practical finance problems by simple rules of thumb, or Kahneman (2003)'s intuitive judgements and deliberate decision making), argued to more likely represent the true embedded OpRisk in financial organisations as these methods are designed to fit normal behavioral patterns in their formulation, which is consistent with how decisions are made under risk/uncertainty.

What are the important steps toward completing the post crisis reforms during the current year? Should the risk management fraternity follow the chartered³ path followed in the Risk (2016) consultative document, scrapping away twenty years of internal measurement approaches (such as the AMA), or should the focus of financial regulators shift toward improving on what they see fit within current existing AMA frameworks. The question is should OpRisk managements' focus be on stimulating active discussions on practical approaches to quantify, model and manage OpRisk for better risk management and improved controls, or abandon the adoption of innovative measurement approaches, such as the AMA, in exchange for being able to use a simple formula across the whole industry?

³Meaning as of the publication [risk2016supporting] the methods brought forth in the consultative document have not been approved for the public, the ideas within an experimental (leased) phase for the exclusive use of BCBS and certain FI's

Context of the study

Regulatory reforms are designed and fines imposed to protect against operational errors and other conduct costs connected with wrongdoing and employee misconduct. Despite the introduction and use of these seemingly robust strategies, regulations, processes and practices relating to managing risk in FI's, bank losses continue to occur at a rather distressing frequency. A cyclical pattern of OpRisk loss events still persists; as evidenced in the recent price fixing and collusion cases, defeating the explicit objectives of risk management frameworks. This demonstrates a scourge of reflexivity prevailing in financial markets emphasising that, there are theories that seem to work for a time only to outlive their use and become insufficient for the complexities that arise in reality.

Why OpRisk?

A forceful narrative in management theory is that an organisation running effective maintenance procedures combined with optimal team and individual performers i.e., the right balance of skills in the labour force and adequate technological advancements, means systems and services can be used to more efficiently produce material gains, enhance organisational effectiveness, meet business objectives and increase investment activity. Conversely, the risk of the loss of business certainty associated with lowered organisational competitiveness and inadequate systems technology that underpins operations and services is a key source leading to a potential breakdown in investment services activity (Hoohlo, 2015). In fact OpRisk control could set banks apart in competition. This serves as an incentive to support regulation, particularly Basel III recovery and resolution processes.

Consider the case of a regulator in a financial system, who assumes that he/she is consciously and accurately analysing an observed subject, trusting the validity and relying on the visual information that their sense of sight reveals. In the absence of visual confirmation they are hindered from extracting and/or analysing information about the system and their efforts to regulate could potentially fail. The organisational methods and functioning of current information systems in this industry sector obscure the full extent of OpRisk challenges from the eyes of the risk practitioner.

When an attack such as an operational error occurs at a speed that the OpRisk agent (an individual legal entity or a group) is unable to react quickly enough, due to limitations of their processing speed, and they are not able to process all the information in the given time span, they could lose control/fail to comply with regulatory standards. The latter case is more often than not the most accurate reflection of current risk management practices. The agent represents one end of the spectrum of a risk management strategy, which mitigates risk and enforces regulation, dependent on the information received. The other end of the spectrum is one which does not react at all to changes in the system environment.

Current conventional financial systems where information processing is slow and have a tendency to rely on manual, uncertain, unpredictable and unrealistic controls, obscure risk management reporting and produce undesirable market conditions. The OpRisk management function should be able to assist the firms' ability to mitigate risks by acquiring and/or refining risk management solutions which deliver reliable and consistent benefits of improved control and management of the risks inherent in banking operations (Dorval, 2013). This proposal attempts to fill the gap in the current system where there is a risk management information lag or an obstruction from the eyes of the risk practitioner.

Analysis and interpretation issues with behavioral finance theory

Behavioral management theory is very much concerned with social factors such as motivation, support and employee relations. A critical component of behavioral finance is building models which better reflect actual behavior. Studies have revealed that these social factors are not easy to incorporate into finance models or to understand in the traditional framework.

The traditional finance paradigm seeks to understand financial markets using models in which agents are “rational”. According to Barberis & Thaler (2003), this means that agents update their beliefs on the onset of new information, and that given their beliefs, they make choices that are normatively acceptable, and that most people do this most of the time. Neoclassical theory has grown to become the primary take on modern-day economics formed to solve problems for decision making under uncertainty/risk. Expected Utility Theory (EUT) has dominated the analysis and has been generally accepted as the normative model of rational choice, and widely applied as a descriptive model of economic choice (Kahneman & Tversky, 2013).

Expected utility theory

Expected utility theory⁴ (EUT): We see a fundamental relation for expected utility (Expectation) of a contract X , that yields outcome x_i with probability p_i , where $X = (x_1, p_1; \dots; x_n, p_n)$ and $p_1 + p_2 + \dots + p_n = 1$ given by:

$$U(x_1, p_1; \dots; x_n, p_n) = p_1 u(x_1) + \dots + p_n u(x_n) \quad (1.1)$$

⁴Expected utility theory provides a model of rationality based on choice.

corroborated by Morgenstern & Von Neumann (1953); Friedman & Savage (1948); Kahneman & Tversky (2013) & others.

A common thread running through the rational viz., the neoclassical take of modern-day economics vs the non-neoclassical schools of thought are findings of behavioral economics which tend to refute the notion that individuals behave rationally. Many argue that individuals are fundamentally irrational because they do not behave rationally giving rise to a literature and debates as to which heuristics and sociological and institutional priors are rational (Altman, 2008).

In the real world there is a point of transition between the traditional (neoclassical) approach to decision making, based on data and data analysis (logic and rational), by adding new parameters and arguments that are outside rational conventional thinking but are also valid. For example, that neoclassical theory makes use of the assumption that all parties will behave rationally overlooks the fact that human nature is vulnerable to other forces, which causes people to make irrational choices.

An essential ingredient of any model trying to understand trading behavior is an assumption about investor preferences (Barberis & Thaler, 2003), or how investors evaluate risky gambles. Investors systematically deviate from rationality when making financial decisions, yet as acknowledged by Kuhnen & Knutson (2005), the mechanisms responsible for these deviations have not been fully identified. Some errors in judgement suggest distinct mental operations promote different types of financial choices that may lead to investing mistakes. Deviations from the optimal investment strategy of a rational risk neutral agent are viewed as risk-seeking mistakes and risk-aversion mistakes (Kuhnen & Knutson, 2005).

Theoretical investigations for the quantification of modern ORMF

Kuhnen & Knutson (2005) explain that these risk-seeking choices (such as gambling at a casino) and risk-averse choices (such as buying insurance) may be driven by distinct neural⁵ phenomena, which when activated can lead to a shift in risk preferences. Kuhnen & Knutson (2005) found that certain areas of the brain precede risk-seeking mistakes or risky choices and other areas precede risk-aversion mistakes or riskless choices. A risk-aversion mistake is one where a gamble on a prospect of a gain is taken by a risk-averse agent in the face of the chance of a prospective loss. The fear of losing prohibits one's urge to gamble, but people engage in gambling activity anyway. Barberis & Thaler (2003) show that people regularly deviate from the traditional finance paradigm evidenced by the extensive experimental results compiled by cognitive psychologists on how people make decisions given their beliefs.

Kahneman & Tversky (2013) maintains, preferences between prospects which violate rational behaviour demonstrate that outcomes which are obtained with certainty are overweighted relative to uncertain outcomes. This will contribute to a risk-averse preference for a sure gain over a larger gain that is merely probable or a risk-seeking preference for a loss that is merely probable over a smaller loss that is certain. As a psychological principle, overweighting of certainty favours risk-aversion in the domain of gains and risk-seeking in the domain of losses.

The present discussion replicates the common behavioral pattern of risk aversion, where people weigh losses more than equivalent gains. Furthermore, neuroeconomic research shows that this pattern of behavior is directly tied to the brain's greater sensitivity to potential losses than gains (Tom, Fox, Trepel, & Poldrack,

⁵As recent evidence from human brain imaging has shown [Kuhnen2005neural] linking neural states to risk-related behaviours [Paulus2003increased].

2007). This provides a target for investigating a more comprehensive theory of individual decision-making rather than the rational actor model and thus yield new insights relevant to economic theory⁶ (Kuhnen & Knutson, 2005).

If people are reasonably accurate in predicting their choices, the presence of systematic violations of risk neutral behavior provides presumptive evidence against this i.e., people systematically violate EUT when choosing among risky gambles. This seeks to improve and adapt to reality and advance different interpretations of economic behaviour; viz., to propose a more adequately descriptive model, that can represent the basis for an alternative to the way the traditional model is built for decisions taken under uncertainty. This has led some influential commentators to call for an entirely new economic paradigm to displace conventional neoclassical theory with a psychologically more realistic preference specification (List, 2004).

A new class of ORMF models approach

A substantial body of evidence shows that decision makers systematically violate EUT when choosing between risky prospects. Indeed, people would rather satisfy their needs than maximise their utility, contravening the normative model of rational choice (i.e., EUT) which has dominated the analysis of decision making under risk. In recent work (Barberis & Thaler, 2003) in behavioral finance, it has been argued that some of the lessons learnt from violations of EUT are central to understanding a number of financial phenomena. In response to this, there has been several theories put forward advocating for the basis of a slightly different interpretation which describes how individuals actually make decisions under uncertainty/risk. Of all the non-EUT's, we focus on Prospect Theory (PT) as this

⁶Representing ability of FI's financial market models to characterise the repeated decision-making process that applies to loss aversion

framework has had most success matching most empirical facts⁷.

Kahneman & Tversky (2013) list the key elements of PT, which are 1] a value function, and 2] a non-linear transformation of the probability scale, that factors in risk aversion of the participants. According to Kahneman & Tversky (2013), the probability scale overweights small probabilities and underweights high probabilities. This feature is known as loss/risk aversion: This means that people have a greater sensitivity to losses (around 2.5 times more times) than gains, and are especially sensitive to small losses unless accompanied by small gains⁸. Loss aversion is a strong differentiator when it comes to explaining exceptions to the general risk patterns that characterize prospect theory.

Prospect theory

By relaxation of the expectation principle in equation 1.6.1, the over-all value \bigvee of the regular prospect $(x, p; y, q)$: In such a prospect, one receives x with probability p , y with probability q , and nothing with probability $1 - p - q$, is expressed in terms of two scales, $\pi(\cdot)$, and $\nu(\cdot)$, where $\pi(\cdot)$ is a decision weight and $\nu(\cdot)$ a number reflecting the subjective value of the outcome. Then \bigvee is assigned the value:

$$\bigvee = \pi(p)\nu(x) + \pi(q)\nu(y) \quad \text{iff} \quad p + q \leq 1 \quad (1.2)$$

The scale, π , associates with each probability p a decision weight which reflects the impact of p on the over-all value of the prospect. The second scale, ν , assigns to each outcome x a number $\nu(x)$, which measures the value of deviations from a reference point i.e., gains or losses. π is not a probability measure and $\pi(p) +$

⁷OpRisk loss events in FI's are largely due to human failings that are exploitable e.g., fraudulent trading activity, and PT is based on the same behavioural element of how people make financial decisions about prospects

⁸Diminishing marginal utility for gains but opposite for losses.

$\pi(1 - p) < 1$. Through PT we add new parameters and arguments to improve the mathematical modelling method for decisions taken under risk/uncertainty, such that the value of each outcome is multiplied by a decision weight, not by an additive probability.

PT looks for common attitudes in people (in FI's) with regard to their behaviour toward taking financial risks or gambles that cannot be captured by EUT. In light of this view, people are not fully invested in either of the perceived outcomes x and y , Which tells us that $p + q \leq 1$. In lieu of this, an FI using (internal) historical OpRisk loss data to model future events; say a historical case of fraud at the FI occurs and is incorporated in the model, the probability of making the same error in future is provided for in the model versus risk events that haven't happened. The modelled future should over-provide for the loss events that have already occurred, which fits normal patterns around individuals psychological make up and is consistent with risk-averse behavior. The idea at the basis of PT is that a better modeling method can be obtained which leads to a closer approximation of the over-all-value of OpRisk losses.

Modeling

In this study, an important new algorithm for ORMFs and is laid out coupled with data intensive estimation techniques; viz. Generalised Additive Models for location Scale & Shape (GAMLSS), Generalized Linear Models (GLDs), Artificial Neural Networks (ANNs), Random Forest (RF) & Decision Trees (DTs), which have capabilities to tease out the deep hierarchies in the features of covariates irrespective of the challenges associated with the non-linear or multi-dimensional nature of the underlying problem, at the same time supporting the call from industry for a new class of EBOR models that capture forward-looking aspects. Machine Learn-

ing (ML) is used as a substitute tool for the traditional model based Autoregressive Moving Average (ARMA) used for analysing and representing stochastic processes. As opposed to the statistical tool, ML does not impose a functional relationship between variables, the functional relationship is determined by extracting the pattern of the training set and by learning from the data observed.

Using computationally intensive (using ML techniques on historical data) OpRisk measurement techniques and mixing with a theory is not a new approach for modeling, particularly in calculating OpRisk RC; as evidenced through Agostini, Talamo, & Vecchione (2010) in a study whereby the LDA model for forecasting OpRisk RC, via VaR, was implemented in conjunction with the use of advanced credibility theory (CT). The idea at the basis of their use of CT, is to advance the very recent literature that a better estimation of the OpRisk RC measurement can be obtained by integrating historical data and scenario analysis i.e., combining the historical simulations with scenario assessments through formulas that are weighted averages of the historical data entries and scenario assessments, advocating for the combined use of both experiences.

However, applying ML is an original way of looking at the approximation issue as opposed to advanced CT. The essential feature of PT are assumptions which are more compatible with basic principles of perception and judgement for decisions taken under uncertainty, whereas ML will reveal additional chance probabilities determined through the natural clusters of unknown data feature findings from which new discoveries are made.

According to Kahneman & Tversky (2013), the decision maker, who is a risk agent within the FI, constructs a representation of the losses and outcomes that are relevant to the decision, then assesses the value of each prospect and chooses according to the losses (changes in wealth), not the overall financial state of the FI. We wish to bring the prescribed model to equilibrium, by applying a method

that tries to establish what accurately ascribes to decision rules that people wish to obey, in made predictions about what operational loss events might result in the future, then use empirical data to test this idea in a way that is falsifiable.

Problem statement

Main problem

The existing models of OpRisk VaR measurement frameworks assume FI's are risk neutral, and do not learn from past losses/mistakes: We address weaknesses in current OpRisk VaR measurement frameworks by assuming that FI's are more risk averse. Furthermore, introducing exposure-based operational risk modeling, we gain an understanding of how capturing past losses and exposures of forward looking aspects affect risk attitudes using machine learning techniques. As a consequence, projected future losses are estimated through a learning algorithm adapting capital estimates to changes in the risk profile, i.e. in the introduction of new products or changes in the business mix of the portfolio (e.g. mergers, trade terminations, allocations or disinvestments), providing sufficient incentives for OpRisk management to mitigate risk.

Objectives of the study

The research objectives are three-fold:

Exposure-based OpRisk (EBOR) models

To quantify OpRisk losses by introducing generalised additive models for location, scale and shape (GAMLSS) in the framework for OpRisk management, that captures exposures to forward-looking aspects of the OpRisk loss prediction problem. EBOR treatments effectively replace historical loss severity curves obtained from historical loss counts, by looking into deep hierarchies in the features of covariates in investment banking (IB), and by forward-looking measures using event frequencies based on actual operational risk (OpRisk) exposures in the business environment and internal control risk factors (BEICF) thereof.

Modeling OpRisk depending on covariates

To investigate the performance of several supervised learning classes of data-intensive methodologies for the improved assessment of OpRisk against current *traditional* statistical estimation techniques. Three different machine learning techniques viz., DTs, RFs, and ANNs, are employed to approximate weights of input features (the risk factors) of the model. A comprehensive list of user defined input variables with associated root causes contribute to the *frequency* of OpRisk events of the underlying value-adding processes. Moreover, the *severity* of OpRisk is also borne out through loss impacts in the dataset . As a consequence of theses new methodologies, capital estimates should be able to adapt to changes in the risk profile of the bank, i.e. upon the addition of new products or varying the business mix of the bank providing sufficient incentives for ORMF to mitigate risk (Einemann et al., 2018).

Interpretation Issues using cluster analysis

To identify potential flaws in the mathematical framework for the loss distribution approach (LDA) model of ORM, which is based the derivation of OpRisk losses based on a risk-neutral measure \mathbb{Q} , by employing Cluster Analysis (CA). The study addresses weaknesses in the current *traditional* LDA model framework, by assuming managerial risk-taking attitudes are more risk averse. More precisely, CA learns the deep hierarchies of input features⁹ that constitute OpRisk event *frequencies & severities* of losses during banking operations. In theory, a risk manager who experiences persistent/excessive losses due to particular risk events, would over-compensate cover for these particular risk types. This would show in reduced losses in those loss event types over time, subsequently determining whether risk adverse techniques over-compensate for persistent losses.

Significance of the study

This study fills a gap in that advancing OpRisk VaR measurement methods beyond simplistic and traditional techniques, new data-intensive techniques offer an important tool for ORMFs and at the same time supporting the call from industry for a new class of EBOR models that capture forward-looking aspects of ORM (Embrechts, Mizgier, & Chen, 2018). The current *traditional* approach consists of a loss data collection exercise (LDCE) which suffers from inadequate technologies at times relying on spreadsheets and manual controls to pull numbers together, and therefore do not support the use of data intensive techniques for the management of financial risks. In this study, a new dataset with unique feature characteristics

⁹A typical approach taken in the literature is to use an unsupervised learning algorithm to train a model of the unlabeled data and then use the results to extract interesting features from the data [coates2012learning]

is developed using an automated LDCE, as defined by Committee & others (2011) for internal data. The dataset in question is at the level of individual loss events, it is fundamental as part of the study to know when they happened, and be able to identify the root causes of losses arising from which OpRisk loss events.

This study will provide guidance on combining various supervised learning techniques with extreme value theory (EVT) fitting, which is very much based on the Dynamic EVT-POT model developed by Chavez-Demoulin, Embrechts, & Hofert (2016). This can only happen due to an abundance of larger and better quality datasets and which also benefits the loss distribution approach (LDA) and other areas of OpRisk modeling. In Chavez-Demoulin et al. (2016), they consider dynamic models based on covariates and in particular concentrate on the influence of internal root causes that prove to be useful from the proposed methodology. Moreover, EBOR models are important due to wide applicability beyond capital calculation and the potential to evolve into an important tool for auditing process and early detection of potential losses, culminating in structural and operational changes in the FI, hence releasing human capital to focus on dilemmas that require human judgement.

Organisation of the study

This study is made up of seven chapters. The introductory chapter is to the purpose, overview, research problem & objectives, and the significance of the study. The introductory chapter is succeeded by a general literary review chapter (two) followed by three stand alone chapters each focusing on the three research objectives regarding the issues in OpRisk capital requirement estimation.

Chapter one begins with an account of significance and a commentary on the nature and scope of the practical problem. It then provides a background of current

issues when dealing with OpRisk measurement, the research problem and research questions thereof. Chapter two gives an overview of the literature concerning the LDA, an AMA technique used in the generation of OpVaR. It concludes by proposing the a research methodology in which a combination of ML techniques and statistical theory underlying ORMF's would benefit measurement of capital requirements for OpVaR.

Chapter three looks at the methodological and empirical determinants of OpRisk measurement. It explores the different dataset...

CHAPTER 2

LITERATURE REVIEW

Introduction

A look into literary sources for OpRisk indicates (Acharyya, 2012) that there is insufficient academic literature that looks to characterize its theoretical roots, as it is a relatively new discipline, choosing instead to focus on proposing a solution to the quantification of OpRisk. This chapter seeks to provide an overview of some of the antecedents of OpRisk measurement and management in the banking industry. As such, this chapter provides a discussion on why OpRisk is not trivial to quantify and attempts to understand its properties in the context of risk aversion with the thinking of practitioners and academics in this field.

According to Cruz (2002), FI's wish to measure the impact of operational events upon profit and loss (P&L), these events depict the idea of explaining the *volatility of earnings* due to OpRisk data points which are directly observed and recorded. By seeking to incorporate data intensive statistical approaches to help understand the data, the framework analyses response variables that are decidedly non-normal (including categorical outcomes and discrete counts) which can shed further light on the understanding of firm-level OpRisk RC. Lastly, a synopsis of gaps in the literature is presented.

The theoretical foundation of OpRisk

Hemrit & Arab (2012) argue that common and systematic operational errors in hypothetical situations poses presumptive evidence that OpRisk events, assuming that the subjects have no reason to disguise their preferences, are created subconsciously. This study purports, supported by experimental evidence, behavioural finance theories should take some of this behaviour into account in trying to explain, in the context of a model, how investors maximise a specific utility/value function.

Furthermore its argued by integrating OpRisk management into behavioral finance theory,¹, that it may be possible to improve our understanding of firm level RC by refining the resulting OpRisk models to account for these behavioral traits - implying that people's economic preferences described in the model, have an economic incentive to improve the OpRisk RC measure.

Wiseman & Catanach Jr (1997) suggest that managerial risk-taking attitudes are influenced by the decision (performance) context in which they are taken. In essence, managerial risk-taking attitude is considered as a proxy for measuring OpRisk (Acharyya, 2012). In so doing, Wiseman & Catanach Jr (1997) investigate more comprehensive economic theories, viz. prospect theory and the behavioural theory of the firm, that prove relevant to complex organizations who present a more fitting measure for OpRisk.

In a theoretical paper, Wiseman & Catanach Jr (1997) discussed several organizational and behavioural theories, such as PT, which influence managerial risk-taking attitudes. Their findings demonstrate that behavioural views, such as PT and the behavioural theory of the firm explain risk seeking and risk averse behaviour

¹In behavioral finance, we investigate whether certain financial phenomena are the result of less than fully rational thinking [barberis2003survey]

in the context of OpRisk even after agency based influences are controlled for. Furthermore, they challenge arguments that behavioral influences are masking underlying root causes due to agency effects. Instead they argue for mixing behavioral models with agency based views to obtain more complete explanations of risk preferences and risk taking behavior (Wiseman & Catanach Jr, 1997).

Despite the reality that OpRisk does not lend itself to scientific analysis in the way that market risk and credit risk do, someone must do the analysis, value the RC measurement and hope the market reflects this. Besides, financial markets are not objectively scientific, a large percentage of successful people have been lucky in their forecasts, it is not an area which lends itself to scientific analysis.

Overview of operational risk management

It is important to note how OpRisk manifests itself: King (2001) has established the causes and sources of operational loss events as observed phenomena associated with operational errors and are wide ranging. By definition, the occurrence of a loss event is due to P&L volatility from a payment, settlement or a negative court ruling within the capital horizon over a time period (of usually one year) (Einemann et al., 2018). As such, P&L volatility is not only related to the way firms finance their business, but also in the way they *operate*.

In operating practice, one assumes that on observing or on following instructions we are consciously analysing and accurately executing our tasks based on the information. However, the occurrence of operational loss events indicates that there are sub-conscious faults in information processing, which we are not consciously aware of. These operational loss events are almost always initiated at the dealing phase of a trading process, which more often than not implicates front office (FO) personnel to bear the responsibility for the losses e.g., during the trading process

in cases where OpRisk events occur as a result of a mismatch between the trade booked (booking in trade feed) and the details agreed by the trader. The middle office (MO) and back offices (BO) conduct the OpRisk management, who undertake a broad view of P&L attribution carried out from deal origination to settlement within the perspective of strategic management, and detects the interrelationships between OpRisk factors with others to conceptualise the potential overall consequences (Acharyya, 2012) e.g., in the afore-mentioned example, human error (a subconscious phenomenon) is usually quoted as the source of error, and the trade is fixed by “amending” or manually changing the trade details.

Furthermore, Acharyya (2012) recognised that organizations may hold OpRisk due to external causes, such as failure of third parties or vendors (either intentionally or unintentionally), in maintaining promises or contracts. The criticism in the literature is that no amount of capital is realistically reliable for the determination of RC as a buffer to OpRisk, particularly the effectiveness of the approach of capital adequacy from external events, as there is effectively no control over them.

The loss collection data exercise (LCDE)

The main challenge in OpRisk modeling is in poor loss data quantities, and low data quality. There are usually very few data points and are often characterised by high frequency low severity (HFLS) and low frequency high severity (LFHS) losses. It is common knowledge that HFLS losses at the lower end of the spectrum tend to be ignored and are therefore less likely to be reported, whereas low frequency high severity losses (LFHS) are well guarded, and therefore not very likely to be made public.

In this study, a new dataset with unique feature characteristics is developed

using the official loss data collection exercise (LDCE), as defined by Committee & others (2011) for internal data. The dataset in question is at the level of individual loss events, it is fundamental as part of the study to know when they happened, and be able to identify the root causes of losses arising from which OpRisk loss events.

The LCDE is carried out drawing statistics directly from the trade generation and settlement system, which consists of a tractable set of documented trade detail extracted at the most granular level, i.e. on a trade-by-trade basis [as per number of events (frequencies) and associated losses (severities)], and then aggregated daily. The dataset is split into proportions and trained, validated and tested. The aforementioned LDCE, is an improved reflection of the risk factors by singling out the value-adding processes associated with individual losses, on a trade-by-trade level.

Current operational risk measurement modeling framework

Historical severity curves obtained from historical loss counts have been widely considered to be the most reliable models when used in OpRisk loss estimation. However they have not been successful when used as measures capturing forward-looking aspects of the OpRisk loss prediction problem.

In this paper, we develop data intensive analysis techniques which yield a more realistic estimation for underlying risk factors, through linking risk factors to covariates based on internal control vulnerabilities (ICV's). ICV's are selected as measures of trading risk exposure, business environment and internal control factors (BEICF's) i.e., trade characteristics and causal factors. For each loss event, information such as unique trade identifier, trader identification, loss event capture personnel, trade status and instrument type, loss event description, loss amount, market variables, trading desk and business line, beginning and ending date and

time of the event, and settlement time are given.

AMA's allow banks to use their internally generated risk estimates Under Basel II; a first attempt internal measurement approach (IMA) capital charge calculation for OpRisk (i.e.) is similar to the Basel II model for credit risk, where a loss event is a default in the credit risk jargon. There are generally seven event type categories (Risk, 2001) and eight business lines. Potential losses are decomposed into several ($7 \times 8 = 56$) sub-risks using event types and business line combinations: e.g., execution, delivery & process management is one such category defined the risk that operational losses/problems would take place in the banks transactions, given as:

$$\mathcal{C}_{OpRisk}^{IMA} = \sum_{i=1}^8 \sum_{k=1}^7 \gamma_{ik} \epsilon_{ik} \quad (2.1)$$

where ϵ_{ik} : expected loss for business line i , risk type k

γ_{ik} : scaling factor

The business line/ event type (BL/ET) matrix

The 3-dimensional diagram, Figure ?? depicts the formation of the *BL/ET* matrix: Duration (time $T + \tau$) is represented along the depth ordinate.

Loss Distribution Approach (LDA)

The Loss Distribution Approach (LDA) is an AMA method whose main objective is to provide realistic estimates to calculate VaR for OpRisk RC in the banking sector and it's business units based on loss distributions that accurately reflect the frequency and severity loss distributions of the underlying data. Having calculated

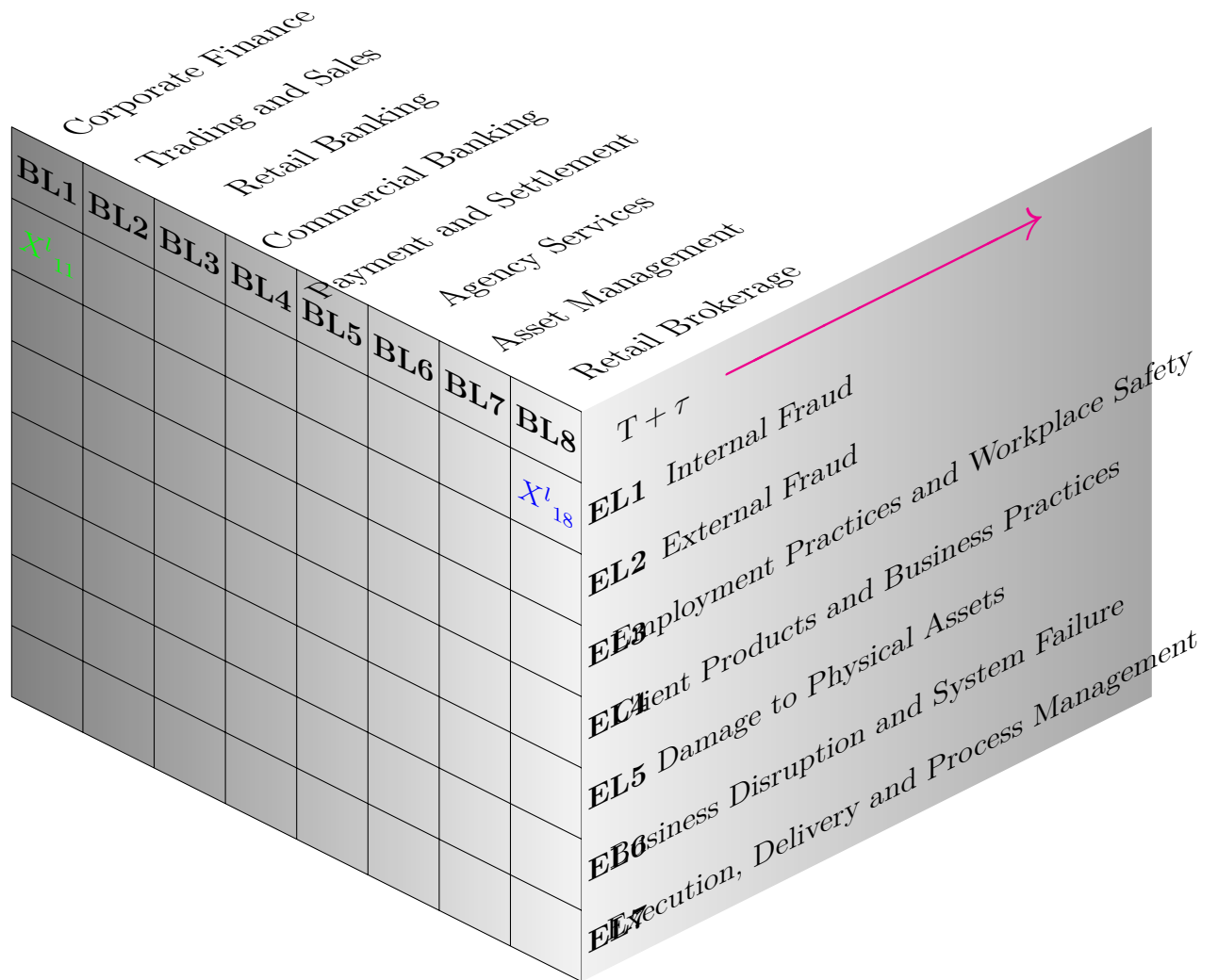


Figure 5.1: The 3-Dimensional grid of the BL/ET matrix for 7 event types and 8 business lines

separately the frequency and severity distributions, we need to combine them into one aggregate loss distribution that allows us to produce a value for the OpRisk VaR.

We begin by defining some concepts:

- In line with Basel II, and according to @frachot2001loss, we consider a matrix consisting of business lines BL and (operational) event types ET . The bank estimates, for each business line/event type (BL/ET) cell, the probability functions of the single event impact and the event frequency for the next three months. More precisely, in each cell of the BL/ET matrix separate distributions for loss frequency and severity are modeled and aggregated to a loss distribution at the group level. The aggregated operational losses can be seen as a sum S of a random number N of individual operational losses (X_1, \dots, X_N) . This sum can be represented by:

$$S = X_1, \dots, X_N, \quad N = 1, 2, \dots \quad (2.2)$$

- Three month daily statistics are taken of the time series of internal processing errors (frequency data) and their associated severities and used in each cell of the BL/ET matrix. Frequency refers to the number of events that occur within the specified time period (daily buckets) T and $T + \tau$ and severity refers to the P&L impact resulting from the frequency of events. The time (1 day bucket) period is chosen in order to ensure that the number of data points is sufficient for statistical analysis.

Computing the frequency distribution

- Let N_{ij} be variable in random selection, representing **the number of times of process risk event failures** between times T & $T + \tau$. Suppose subscript

i refers to the BL which ranges from $1, \dots, k$ and subscript j to ET ($j = 1$ for process risk). We have taken a random sample implying that the observations N_{ij} , where $i, j = (1, 1), \dots, (k, 1)$ are independent and identically distributed (i.i.d).

- The random variable N_{i1} ² has distribution function³ The random variable has distribution function (d.f.) $\mathbf{P}_{i1}(n/\theta_0)$, where θ_0 is an unknown parameter of the estimated distribution. The unknown parameter θ_0 may be a scalar or a vector quantity θ_0 , for example, The Poisson distribution depends on one parameter called λ whereas the univariate normal distribution depends on two parameters, μ and σ^2 , the mean and variance. These parameters are to be estimated in some way. We use the Maximum Likelihood Estimate (m.l.e) which is that value of θ that makes the observed data “most probable” or “most likely”.
- The d.f. $\mathbf{P}_{i1}(n/\theta_0)$, is the probability that N_{i1} takes a value less than or equal to n , where n is a small sample from the entire population of observed frequencies, i.e.

$$\mathbf{P}_{ij}(n) = Pr(N_{ij} \leq n) \quad i, j = (1, 1), \dots, (k, 1) \quad (2.3)$$

- The probability density function (p.d.f) : A density function is a non-negative function $p(n)$ whose integral, extended over the entire x axis, is equal to 1 for a given continuous random variable X . i.e. it is the area under the probability density curve, of the discrete random variable N_{i1} takes discrete values of n with finite probabilities. In the discrete case the term for p.d.f. is the probability function (p.f.) also called the probability mass function, i.e. N_{i1} is given by the probability that the variable takes the value n , i.e.

² N_{ij} where subscript $j = 1$ since we are only dealing with 1 event type i.e. process risk

³The term distribution function is monotonic increasing function of n which tends to 0 as $n \rightarrow -\infty$, and to 1 as $n \rightarrow \infty$

$$p_{ij}(n) = \Pr(N_{ij} = n), \quad i, j = (1, 1), \dots, (k, 1) \quad (2.4)$$

- The r.h.s of equation (2.3) is the summation of the r.h.s of equation (2.4), we derive a relation for the **loss frequency distribution** in terms of the (p.f):

$$\mathbf{P}_{ij}(n) = \sum_{k=1}^{n_k} p_{ij}(n) \quad i, j = (1, 1), \dots, (k, 1) \quad (2.5)$$

Computing the severity distribution

- Suppose X_{ij} is a random variable representing **the amount of one loss event** in a cell of the BL/ET matrix. Define next period's loss in each cell (i, j) , where i is the number of business line cells, $L^{T+1}_{i,j}$: Operational loss for loss type $j = 1$ (process risk). One models the amount of the total operational loss of type j at a given time T & $T + 1$, over the future (say 3 months), as:

$$L^{T+1} = \sum_{i=1}^k L^{T+1}_{i1} = \sum_{i=1}^2 \sum_{l=1}^{N_{i1}^{T+1}} X^l_{i1} \quad l = 1, 2, \dots, N_{i1} \quad (2.6)$$

- Let N_1, N_2, \dots, N_m (where m is the number of combinations in the BL/ET matrix) be random variables that represent the loss frequencies. It is usually assumed that the random variables X_{i1} are independently distributed and independent of the number of events N_m . A fixed number of a particular loss type would be denoted by X^1_{i1} , i.e the random variable X^l_{i1} , represents random samples of the severity distribution [aue2006lda].

The **loss severity distribution** is denoted by \mathbf{F}_{i1} . Since loss severity variate X is continuous (i.e. can take on any real value), we define a level of precision h such that the probability of X being within $\pm h$ of a given number x tends to zero. The loss severity, X_{i1} has a (d.f.) $\mathbf{F}_{i1}(x/\theta_1)$, where θ_1 is an unknown

parameter and x is a small sample from the entire population of loss severity.

- We define probability density in the continuous case as follows:

$$\begin{aligned}
 f_X(x) &= \lim_{h \rightarrow 0} \frac{Pr[x < X \leq x + h]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{F_X(x + h) - F_X(x)}{h} \\
 &= \frac{dF_X(x)}{dx}
 \end{aligned} \tag{2.7}$$

operate with $\int dx$ on both sides of 2.7

$$\mathbf{F}_{X_{ij}}(x) = \int_{k=1}^{\infty} f_{X_{ij}}(x) dx \quad i, j = (1, 1), \dots, (k, 1) \tag{2.8}$$

where $f_{X_{ij}}(x)$ is the probability density function (p.d.f.). Once again, the subscript X identifies the random variable for severity (P&L impact) of one loss event while the argument x is an arbitrary sample of the severity events.

Formal Results

Having calculated both the frequency and severity process we need now to combine them in one aggregate loss distribution that allows us to predict an amount for the operational losses to a degree of confidence. There is no simple way of aggregating the frequency and severity distribution. Numerical approximation techniques (computer algorithms) successfully bridge the divide between theory and implementation for the problems of mathematical analysis.

The aggregated losses at time t are given by $\vartheta(t) = \sum_{n=1}^{N(t)} X_n$ (where X represents individual operational losses). Frequency and severity distributions are estimated, e.g., the poisson distribution is a representation of a discrete variable commonly used to model operational event frequency (counts), and a selection from

continuous distributions which can be linear (e.g. gamma distribution) or non-linear (e.g. lognormal distribution) for operational loss severity amounts. The compound loss distribution $\mathbf{G}(t)$ can now be derived. Taking the aggregated losses we obtain:

$$\mathbf{G}_{\vartheta(t)}(x) = Pr[\vartheta(t) \leq x] = Pr \left(\sum_{n=1}^{N(t)} X_n \leq x \right) \quad (2.9)$$

For most choices of $N(t)$ and X_n , the derivation of an explicit formula for $\mathbf{G}_{\vartheta(t)}(x)$ is, in most cases impossible. $\mathbf{G}(t)$ can only be obtained numerically using the Monte Carlo method, Panjer's recursive approach, and the inverse of the characteristic function [Frachot, Georges, & Roncalli (2001); Aue & Kalkbrener (2006); Panjer (2006); & others].

- We now introduce the aggregate loss variable at time t given by $\vartheta(t)$. This new variable represents **the loss for business line i and event type j** . The aggregate loss is defined by $\vartheta(t) = \sum_{n=1}^{N(t)} X_n$ (where X represents individual operational losses). Once frequency and severity distributions are estimated, the compound loss distribution $\mathbf{G}(t)$ can be derived. Taking the aggregated losses we obtain:

$$\mathbf{G}_{\vartheta(t)}(x) = Pr[\vartheta(t) \leq x] = Pr \left(\sum_{n=1}^{N(t)} X_n \leq x \right) \quad (2.10)$$

- The derivation of an explicit formula for $\mathbf{G}_{\vartheta(t)}(x)$ is, in most cases impossible. Again we implicitly assume that the processes $\{N(t)\}$ and $\{X_n\}$ are independent and identically distributed (i.i.d). Deriving the analytical expression for $\mathbf{G}_{\vartheta(t)}(x)$, we see a fundamental relation corroborated by @frachot2001loss, @cruz2002modeling, @embrechts2013modelling, & others:

$$\mathbf{G}_{\vartheta(t)}(x) = \left\{ \begin{array}{ll} \sum_{n,k=0,1}^{\infty} p_k(n) \mathbf{F}_X^{k*}(x) & x > 0 \\ p_k(0) & x = 0 \end{array} \right\} \quad (2.11)$$

where \star is the *convolution* operator on d.f.'s, $\mathbf{F}^{k\star}$ is the k -fold convolution of \mathbf{F} with itself. The convolution of two functions $f(x)$ and $g(x)$ is the function

$$\int_0^x f(t)g(x-t)dt \quad (2.12)$$

, i.e. $\mathbf{F}_X^{k\star}(x) = Pr(X_1 + \dots + X_k \leq x)$, the d.f. of the sum of k independent random variables with the same distribution as X .

- The aggregate loss distribution $\mathbf{G}_{\vartheta(t)}(x)$ cannot be represented in analytic form, hence approximations, expansions, recursions of numerical algorithms are proposed to overcome this problem. For purposes of our study, an approximation method will do. One such method consists of taking a set $\langle \vartheta_1, \dots, \vartheta_s \rangle$, otherwise known as the ideal generated by elements $\vartheta_1, \dots, \vartheta_s$ which are s simulated values of the random variable ϑ_{ij} for $s = 1, \dots, S$ [@fraleigh2003first].

This method is popularly known as Monte Carlo simulation coined by physicists in the 1940's, it derives its name and afore-mentioned popularity to its similarities to games of chance. The way it works in layman's terms is; in place of simulating scenario's based on a base case, any possible scenario through the use of a probability distribution (not just a fixed value) is used to simulate a model many times. In the LDA separate distributions of frequency and severity are derived from loss data then combined by Monte Carlo simulation.

Dependence Effects (Copulae)

The standard assumption in the LDA is that frequency and severity distributions in a cell are independent and the severity samples are i.i.d. According to Basel II, dependence effects in OpRisk are not considered. Economic capital allocation however, could benefit if it were determined in a way that recognises the risk-

reducing impact of correlation effects between the risks of the BL/ET combinations. Concluding remarks from a study by Urbina & Guillén (2014) allude that failure to account for correlation may lead to risk management practices that are unfair, as evidenced in an example using data from the banking sector.

One of the main issues we are confronted with in OpRisk measurement is the aggregation of individual risks (in each BL/ET element). A powerful concept to aggregate the risks – the *copula* function – has been introduced in finance by Embrechts, McNeil, & Straumann (2002). Copulas have been used extensively in finance theory lately and are sometimes held accountable for recent global financial failures, e.g. the global credit crunch of 2008 - 2009. They are nevertheless still applicable and in use for OpRisk as operational risk models follow a different stochastic process to other areas of risk, e.g. operational VaR is subject to more jumps than market VaR and is thought to be discrete whereby market VaR is continuous.

Copulas are functions which conveniently incorporate correlation into a function that combines each of the frequency (marginal) distributions to produce a single bivariate cumulative distribution function. Our model is used to determine the aggregate (bivariate) distribution of a number of correlated random variables through the use of a Clayton copula. Dependence matters due to the effect of the addition of risk measures over different risk classes (cells in the BL/ET matrix).

More precisely, the frequency distributions of the individual cells of the BL/ET matrix are correlated through a Clayton copula in order to replicate observed correlations in the observed data. Let m be the number of cells, $\mathbf{G}_1, \mathbf{G}_2, \dots, \mathbf{G}_m$ the distribution functions of the frequency distributions in the individual cells and \mathbf{C} the so-called copula. Abe Sklar proved in 1959 through his theorem (Sklar's Theorem) that for any joint distribution \mathbf{G} the copula \mathbf{C} is unique. \mathbf{C} is a distribution function on $[0, 1]^m$ with uniform marginals. We refer to a recent article by Chavez-

Demoulin, Embrechts, & Nešlehová (2006) for further information: It is sufficient to note that \mathbf{C} is unique if the marginal distributions are continuous.

$$\mathbf{G}(x_1, \dots, x_m) = \mathbf{C}(\mathbf{G}_1(x_1), \dots, \mathbf{G}_m(x_m)) \quad (2.13)$$

Conversely, for any copula \mathbf{C} and any distribution functions $\mathbf{G}_1, \mathbf{G}_2, \dots, \mathbf{G}_m$, the functions $\mathbf{C}(\mathbf{G}_1(x_1), \dots, \mathbf{G}_m(x_m))$ is a joint distribution function with marginals $\mathbf{G}_1(x_1), \dots, \mathbf{G}_m(x_m)$. Moreover, combining given marginals with a chosen copula through Equation 2.13 always yields a multivariate distribution with those marginals. The copula function has then a great influence on the aggregation of risk.

LDA model shortcomings

After most complex banks adopted the LDA for accounting for RC, significant biases and delimitations in loss data remain when trying to attribute capital requirements to OpRisk losses (Frachot et al., 2001). OpRisk is related to the internal processes of the FI, hence the quality and quantity of internal data (optimally combined with external data) are of greater concern as the available data could be rare and/or of poor quality. Such expositions are unsatisfactory if OpRisk, as Cruz (2002) professes, represents the next frontier in reducing the riskiness associated with earnings.

Opdyke (2014) advanced studies intending on eliminating bias apparently due to heavy tailed distributions to further provide insight on new techniques to deal with the issues that arise in LDA modeling, keeping practitioners and academics at breadth with latest research in OpRisk VaR theory. Recent work in LDA modeling has been found wanting (Badescu, Lan, Lin, & Tang, 2015), due to the very complex characteristics of data sets in OpRisk VaR modeling, and even when studies

used quality data and adequate historical data points, as pointed out in a recent paper by Hoohlo (2015), there is a qualitative aspect in OpRisk modeling that is often ignored, but whose validity should not be overlooked.

Opdyke (2014), Agostini et al. (2010), Jongh, De Wet, Raubenheimer, & Venter (2015), Galloppo & Previati (2014), and others explicate how greater accuracy, precision and robustness uphold a valid and reliable estimate for OpRisk capital as defined by Basel II/III. Transforming this basic knowledge into “risk culture” or firm-wide knowledge for the effective management of OpRisk, serves as a starting point for a control function providing attribution and accounting support within a framework, methodology and theory for understanding OpRisk measurement. FI’s are beginning to implement sophisticated risk management systems similar to those for market and credit risk, linking theories which govern how these risk types are controlled to theories that govern financial losses resulting from OpRisk events.

Jongh et al. (2015) and Galloppo & Previati (2014) sought to address the shortcomings of Frachot et al. (2001) by finding possible ways to improve the problems of bias and data delimitation in operational risk management. They follow the recent literature in finding a statistical-based model for integrating internal data and external data as well as scenario assessments in an endeavor to improve on accuracy of the capital estimate.

A new class of models capturing forward-looking aspects

Agostini et al. (2010) also argued that banks should adopt an integrated model by combining a forward-looking component (scenario analysis) to the historical operational VaR, further adding to the literature through their integration model which is based on the idea of estimating the parameters of the historical and subjective distributions and then combining them by using the advanced CT.

The idea at the basis of CT is that a better estimation of the OpRisk measure can be obtained by combining the two sources of information: The historical loss data and expert's judgements, advocating for the combined use of both experiences. Agostini et al. (2010) seek to explain through a weight called the credibility, the amount of credence given to two components (historical and subjective) determined by statistical uncertainty of information sources, as opposed to a weighted average approach chosen on the basis of qualitative judgements.

Thus generating a more predictable and forward looking capital estimate. He deemed the integration method as advantageous as it is self contained and independent of any arbitrary choice in the weight of the historical or subjective components of the model.

EBOR methodology for capturing forward-looking aspects of ORM

In a theoretical paper, Einemann et al. (2018) construct a mathematical framework for an EBOR model to quantify OpRisk for a portfolio of pending litigations. Their work unearths an invaluable contribution to the literature, discussing a strategy on how to integrate EBOR and LDA models by building hybrid frameworks which facilitate the migration of OpRisk types from a *classical* to an exposure-based treatment through a quantitative framework, capturing forward looking aspects of BEICF's (Einemann et al., 2018), a key source of the OpRisk data. As mentioned in their paper (Einemann et al., 2018), they were first lay the groundwork for future development across industry and to establish a common language through a strategy for integrating EBOR and LDA models, the former used for predictable loss types e.g., a portfolio of pending litigations; a predictable loss type in so far as the event triggering the filing of the litigation has already happened and only the final outcome of the court case has to be modeled, and the latter which

cover risks that are well reflected through historical events.

Benefits and Limitations

These approaches in 2.5, were found to have significant advantages over conventional LDA methods, proposing that an optimal mix of the two modeling elements could more accurately predict OpRisk VaR over traditional methods. Particularly Agostini et al. (2010), whose integration model represents a benchmark in OpRisk measurement by including a component in the AMA model that is not obtained by a direct average of historical and subjective VaR.

Instead, the basic idea of the integration methodology in 2.8 is to estimate the parameters of the frequency and severity distributions based on the historical losses and correct them; via a statistical theory, to include information coming from the scenario analysis. The method has the advantage of being completely self contained and independent of any arbitrary choice in the weight of the historical or subjective component of the model, made by the analyst. The components weights are derived in an objective and robust way, based on the statistical uncertainty of information sources, rather than through risk managers choices based on qualitative motivations.

However, they could not explain the prerequisite coherence between the historical and subjective distribution function needed in order for the model to work; particularly when a number of papers (Chau, 2014), propose using mixtures of (heavy tailed) distributions commonly used in the setting of OpRisk capital estimation (Opdyke, 2014).

In 3.2, their model (Einemann et al., 2018) is particularly well-suited to the specific risk type dealt with in their paper i.e., the portfolio of litigation events, due

to better usage of existing information and more plausible model behavior over the litigation life cycle, but is bound to under-perform for many other OpRisk event types, since these EBOR models are typically designed to quantify specific aspects of OpRisk - litigation risk have rather concentrated risk profiles. However, EBOR models are important due to wide applicability beyond capital calculation and its potential to evolve into an important tool for auditing process and early detection of potential losses.

Gap in the Literature

There is cognitive pressure which seeks to remove information which we are largely unaware of, because they are undetectable to human senses that no one could ever see them. We seek to remove this pressure, effectively lowering uncertainty and allowing us to position ourselves to develop a defense against our cognitive biases. It is through patterns in that information that we are largely unaware of that predictions could arise; or that, OpRisk management incorporates rather than dismiss the many alternatives that were not imagined, the possibility of market inefficiencies or finding value in unusual places.

Conclusion

A substantial body of evidence suggests that loss aversion, the tendency to be more sensitive to losses than to gains plays an important role in determining how people evaluate risky gambles. In this paper we evidence that human choice behaviour can substantially deviate from neoclassical norms.

PT takes into account the loss avoidance agents and common attitudes to-

ward risk or chance that cannot be captured by EUT; which is not testing for that inherent bias, so as to expect the probability of making the same operational error in future to be overcompensated for i.e., If an institution suffers from an OpRisk event and survives, it's highly unlikely to suffer the same loss in the future because they will over-provide for particular operational loss due to their natural risk aversion. This is a testable proposition which fits normal behavioral patterns and is consistent with risk averse behaviour.

CHAPTER 3

EXPOSURE-BASED OPERATIONAL RISK ANALYSIS

Introduction

The fundamental premise in the nature behind ORMFs, is to provide an exposure-based treatment of OpRisk losses which caters to modeling capital estimates for forward-looking aspects of ORM. This proves tricky due to the need of specific knowledge about potential loss events as a requirement, from the time the loss event occurs and the underlying loss-generating mechanisms until the actual realised loss is realised. By its very nature, this OpRisk type is characterised by a significant lag between the moment the event is conceived to the point the event is observed and accounted for.

For example, in the case of rogue trading, there is a frequency exposure associated with traders *going rogue*, due to a probability of rogue events happening between a specific group of traders, which is then modeled for each rogue trading event and the impact (severity based on the size of the position) of the loss when it is realised (at time of detection). This timing paradox often results in questionable capital estimates, especially for those near misses, pending and realised losses that need to be captured in the model.

Applicability of EBOR methodology for capturing forward-looking aspects of ORM

By its very nature, OpRisk has a significant lag usually characterised by the time delay between the moment the OpRisk event is conceived up until the moment

the event is observed and accounted for, i.e., there is a gap in time between the moment the risk is conceived and the realised losses. A fundamental premise behind the tricky nature of ORMF, is to provide an exposure-based treatment of OpRisk losses which caters for modeling capital estimates of forward-looking aspects of ORM, due to the lag in the loss data.

Einemann et al. (2018) works unearth an invaluable contribution to the literature, whereby the EBOR model is considered as an additional cell, analogously to the BL/ET matrix combinations in a classical LDA model, therefore building hybrid frameworks when integrated with the LDA model facilitating the migration of OpRisk types from a classical to an exposure-based treatment through a quantitative framework (Einemann et al., 2018). Conceptually, the EBOR component can be extended to include potential future events e.g., future litigations, based on some underlying property, capturing forward looking aspects of business environment and internal control factors (BEICF's) thereof.

The fundamental premise behind the LDA is that each firm's OpRisk losses are a reflection of its underlying Oprisk exposure. OpRisk events relating to a varying or constant degree of exposure need to take this into account when modeling count or frequency of an event occurring. In particular, the assumption behind the use of the Poisson distribution in the model to estimate the frequency of losses, is that both the intensity (or rate) of occurrence and the opportunity (or exposure) for counting can assume either of these two forms, for all available observations (Dobson & Barnett, 2008). In the former case the varying degrees of exposure impact on the rate of events, whereas in the latter case the exposure is constant hence not relevant to the model.

When observed counts all have the same exposure, modeling the mean count μ as a function of explanatory variables x_1, \dots, x_p is the same as modeling the rate R . The actual measure of exposure we need to use depends specifically on project-

ing the count of OpRisk events (frequency of realised losses) as the target variable in the model as opposed to the measure if the target variable were the severity of the losses, whereby modeling for each rogue trading event the severity based on size of loss position as severity exposure, and time to detection or CapturedBy as severity risk factors.

Definition of exposure

Exposure is residual risk, or the risk that remains after risk treatments have been applied. In the ORMF context, it is defined as:

Definition 3.2.1.1 *The **exposure** of risk type i , d_i is the time interval, expressed in units of time, from the initial moment when the event happened, until the occurrence of a risk correction.*

As per definition 3.2.1, the lag represents exposure; we need historical exposure for experience rating because we need to be able to compare the loss experience of different years on a like-for-like basis and to adjust it to current exposure levels (Parodi, 2014).

Definition of rate

Often the poisson count λ needs to be described as a rate; for example the OpRisk hazard rate can be specified as the rate per day. More generally, the rate is specified in terms of units of *exposure*; The **rate**, R is defined as:

Definition 3.2.2.1 *the **rate** is the mean count per unit exposure*
i.e.,

$$R = \frac{\mu}{\tau} \quad \text{where} \quad R = \text{rate}, \quad \tau = \text{exposure}, d_i \quad \text{and} \\ \mu = \text{mean count over an exposure duration of } d = [T, T + \tau]$$

For example, in OpRisk hazard rates, each potential OpRisk transaction event is “exposed” over the period $[T, T + \tau]$; it’s detection life cycle period, and a P&L impact determined, So the rate may be defined in terms of transaction-days *at risk*.

Limitations of the EBOR model

In their model (Einemann et al., 2018), the definition of exposure, Definition 3.2.1, is particularly well-suited to the specific risk type dealt with in their paper i.e., the portfolio of litigation events, due to better usage of existing information and more plausible model behavior over the litigation life cycle. However, it is bound to under-perform for many other OpRisk event types since these EBOR models are typically designed to quantify specific aspects of OpRisk i.e., litigation risk have rather concentrated risk profiles. Furthermore, EBOR models are important due to wide applicability beyond capital calculation and its potential to evolve into an important tool for auditing process and early detection of potential losses.

Generalised Linear Models (GLM’s)

Many of the ideas and concepts of linear modelling carry over to generalized linear modelling, however the “generalized” term is used to refer to all linear models other than simple straight lines found in the “general” case. In the case of the OpRisk dataset, the relationship between outcomes and drivers of risk are frequently

not normal, therefore models of the form

$$E(\mathbf{Y}_i) = (\mu_i) = \mathbf{x}_i^T(\mathbf{B}); \quad \mathbf{Y}_i \sim \mathbf{N}(\mu_i, \sigma^2), \quad (3.1)$$

where random variables \mathbf{Y}_i are independent, are not applicable. The transposed vector \mathbf{x}_i^T represents the i th row of the dataset \mathbf{X} . In such cases, due to recent advances in statistical theory and computational techniques, generalised linear models (GLM); which are analogous to linear models, are used to assess and quantify the relationships between a target variable and explanatory variables. GLM's differ in that (Dobson & Barnett, 2008)

- The distribution of the target variable is chosen from the exponential family
- A transformation of the mean of the response is linearly related to the explanatory variables, however their association need not be of the simple linear form in equation 3.1

Operational riskiness in FIs grows as trading transactions grow in complexity, i.e. the more complex and numerous trading activity builds the higher the rate at which new cases of OpRisk events occur. Therefore, it is likely that the rate of operational hazard may be increasing exponentially over time. The scientifically interesting question is whether the data provides any evidence that the increase in the underlying operational hazard generation is slowing. The aforementioned postulate provides a plausible model to start investigating this question.

Exponential family of distributions

As with the linear model, consider independent rv's \mathbf{Y}_i not i.i.d, whose probability depends on a parameter θ_i . The choice of parameter θ_i determines the response distribution who is assumed to have the same form as the exponential fam-

ily, in turn characterising the statistical unit i . Thus, the exponential family representation depend on a constant scale parameter ϕ , and varying parameters θ_i . the pdf of Y_i is

$$f(y_i; \theta_i; \phi) = e^{\left[\frac{a(y_i)b(\theta_i) - c(\theta_i)}{\phi} - d(y_i, \phi) \right]}, \quad y_i \in X \quad (3.2)$$

where a , b , c , & d are regarded as known functions. Expanding the expression in equation 3.3 yields

$$\begin{aligned} f(y_i; \theta_i; \phi) &= e^{\left[\frac{a(y_i)b(\theta_i) - c(\theta_i)}{\phi} - d(y_i, \phi) \right]} \\ &= \frac{1}{e^{d(y, \phi)}} e^{\left[\frac{a(y_i)b(\theta_i) - c(\theta_i)}{\phi} \right]} \\ &= r(y, \phi) \frac{1}{e^{\frac{c(\theta_i)}{\phi}}} e^{\frac{a(y_i)b(\theta_i)}{\phi}} \\ &= r(y, \phi) s(\theta, \phi) e^{\frac{a(y_i)b(\theta_i)}{\phi}} \end{aligned} \quad (3.3)$$

$$\text{where } r(y, \phi) = \frac{1}{e^{d(y, \phi)}} \text{ and where } s(\theta, \phi) = \frac{1}{e^{\frac{c(\theta_i)}{\phi}}} \quad (3.4)$$

since the scale parameter ϕ remains the same, the distribution belongs to the exponential family if it can be written in the form

$$f(y; \theta) = r(y) s(\theta) e^{a(y)b(\theta)} \quad (3.5)$$

If $a(y) = y$, the distribution is in canonical form and $b(\theta)$ is called the natural parameter of the response distribution (De Jong & Heller, 2008). The specific elements of a GLM are (Covrig et al., 2015; Dobson & Barnett, 2008):

1. The random component given by the independent random variables Y_1, Y_2, \dots, Y_n not identically distributed. Note that the rv's \mathbf{Y}_i for the Oprisk data, indexed by the subscript i , have different expected values μ_i . Sometimes there may be only one observation y_i for each Y_i , but there may be several observations

y_{ij} ($j = 1, \dots, p_i$) for each \mathbf{Y}_i . The pdf or probability mass function of \mathbf{Y}_i is given in equation 3.5 for $f(y)$, which specifies that the distribution of the response is in the exponential family. The support set X of the rv Y_i is subset of \mathbf{N} of \mathbf{R} .

2. The second advance is the extension of computational methods to estimate the models systematic component, so called the "linear predictor" described in equation 3.1 built with $p + 1$ parameters $\beta = (\beta_0, \beta_1, \dots, \beta_p)$ and with p explanatory variables:

$$\eta_i = \beta_0 + \sum_{j=1}^p \beta_j x_{ij}, \quad i = 1, 2, \dots, n \quad (3.6)$$

3. The equation for η_i specifies to the situation that there is some non-linear function, a transformation of the mean, $g(\mu)$, that is linearly related to the explanatory variables contained on the r.h.s of equation 3.6, $\mathbf{X}_i^T \beta$, i.e.,

$$g(\mu_i) = \mathbf{X}_i^T \beta \quad (3.7)$$

The function $g(\mu_i)$ is called the link function.

Interpretation

Given a response variable y , for the initial formulation of glm's by Nelder & Wedderburn (1972), $b(\theta)$ determines the nature of the response distribution and the choice of link is suggested by the functional form of the relationship between the response and explanatory variables. In choosing these components extra steps are taken compared to ordinary regression modeling. Commonly used links functions are given in Table 3.1. Table 3.1 presents the various units produced for the various

Table 3.1: The generalized linear model link functions with their associated units of interpretation. Note: This list is not exhaustive and there are likely more GLMs that are used within prevention research.

Link Function	$g(\mu)$	Target variable Effect	Canonical link for
Identity	μ	Original Continuous Unit	normal
Log	$\ln \mu$	count	poisson
Logit	$\ln \frac{\mu}{1-\mu}$	Risk	binomial
Probit	$\phi^{-1}(\theta)$	Risk	binomial
Power	μ^p	Count	$\Gamma(p = -1)$
		Count	inverse Gaussian(p=-2)

GLM links.

Offsets

Modeling counts as realised operational hazard in an OpRisk group requires correction for the period in days d exposed to risk. If μ is the mean of the count y then the occurrence rate $\frac{\mu}{d}$ of interest and

$$g\left(\frac{\mu}{d}\right) = x^t \beta \quad (3.8)$$

When g is the log function, this becomes

$$\ln\left(\frac{\mu}{d}\right) = x^T \beta \quad \Rightarrow \quad \ln \mu = \ln d + x^T \beta \quad (3.9)$$

Where the variable d appears representing the risk exposure and $\ln d$ is called an “offset”. Equation 3.9 differs from the usual specification of the linear predictor due to the inclusion of the term $\ln d$. An offset is effectively another explanatory variable in the regression, with a β coefficient = 1. With the offset, y has expected value directly proportional to exposure:

$$\mu = de^{x^T \beta} \quad (3.10)$$

Offsets are used to correct for differing periods of observation (De Jong & Heller, 2008) i.e., in the opRisk dataset these are the times to detection (exposure) of the realised losses. The exposure measure is a known constant which is readily incorporated into the estimation procedure and is a quantity that is roughly proportional to the risk (Parodi, 2014) i.e., when the exposure (time to detection) doubles whilst everything else (e.g. interest on an interest rate swap) remains the same, the risk also doubles.

Generalized linear model for count data

Exponential family of distributions

The introduction to this chapter raised the question of increasing OpRisk hazards due to more complex transactions in which μ_i , the expected number of new cases on day t_i is modeled. The model assumes that if μ_i is the (rate) number of expected new OpRisk hazards can often increase exponentially through time. Hence, if μ_i is the expected number of new cases on day t_i , it increases according to:

$$\mu_i = \gamma \exp(\delta \tau_i)$$

where γ and δ are unknown parameters. Taking a log link turns the model into Generalised Linear Model (GLM) form so that:

$$\log(\mu_i) = \log(\gamma) + \delta \tau_i = \beta_0 + \tau_i \beta_1 \quad (3.11)$$

Where the LHS is the observed number of new cases over time $[\tau, \tau + 1]$, and the RHS is a linear in the parameters $\beta_0 = \log(\gamma)$ and $\beta_1 = \delta$. We define a binary response variable *LossIndicator*, which takes on the value 1 for realised losses, and the value 0 for pending losses, and/or near misses. The choice of the poisson distribution shows the number of relevant Oprisk event's incidence in a specified time interval $[\tau, \tau + 1]$ is probably a reasonable distribution to try. So the GLM for this situation uses a poisson response distribution, log link, and linear predictor $\beta_0 + t_i\beta_1$.

Many of the ideas and concepts of linear modelling carry over to generalized linear modelling, however the “generalized” term is used to refer to all linear models other than simple straight lines found in the “general” case. for example, the quadratic term $(\beta_2 t_i^2)$ could be added to the model, which usefully approximates other situations which may influence the counts adapted to the poisson case other than only the unchecked prevalence of Oprisk hazards. Amending the RHS of Equation 3.11 with the quadratic term so other situations other than the unrestricted spread of OpRisk hazards are represented yields:

$$\mu = d_i \exp(\beta_0 + \beta_1 \tau_i + \beta_2 \tau_i^2) \quad (3.12)$$

A poisson regression operational hazard model

The LHS of a GLM formula is the model's random component i.e., observations of the number of OpRisk transactions over the trading transaction period in an FI's portfolio; given by the independent random variables y_1, y_2, \dots, y_n , not i.i.d (Wood, 2017 & @covrig2015using). Y takes a (exponential) family argument, depending on parameters $\ln \lambda$, where λ which represents the average frequency of the OpRisk transactions. It is worth distinguishing between the response data y_i which

is an observation of Y .

The target variable (LossIndicator) which shows the number of relevant Oprisk event's incidence in a specified time interval τ and $\tau + 1$, justifying the choice of poisson distribution as a reasonable model for is count data. It's probability mass function (pdf) is:

$$Y \sim \text{poisson}(\lambda), \quad f(y; \lambda) = \frac{\lambda^y}{y!} \exp -y \quad (3.13)$$

where $y \in \mathbb{N}$, and $\lambda > 0$

the expectation and variance $E[Y] = \text{VaR}[Y] = \lambda^1$, are both equal to parameter λ simultaneously. The RHS of Equation 3.13 is the model's systematic component, and it specifies the linear predictor. It builds on equation 3.12 with $p + 1$ parameters, $\beta = (\beta_0 \dots, \beta_p)^t$, and p explanatory variables:

$$\nu = \beta_0 + \sum_{j=1}^p \beta_j x_{ij}, \quad \text{where } i = 1, \dots, n \quad (3.14)$$

If sample variables $Y_i \sim \text{Poisson}(\lambda_i)$, then $\mu = E[Y_i] = \lambda_i$; the link function between the random and systematic components, viz. a tranformation by the model by some function $g()$, which does not change features essential to to fitting, but rather a scaling in magnitude so that:

$$\begin{aligned} \nu_i &= g(\lambda_i) = \ln \lambda_i, & \text{that is} \\ \nu &= \beta_0 + \sum_{j=1}^p \beta_j x_{ij} \end{aligned} \quad (3.15)$$

so the mean frequency or otherwise the rate R , will be predicted by the model...

¹If you were to guess an independent Y_i from a random sample, the best guess is given by this expression

$$\begin{aligned}
\lambda_i &= d_i \exp(\beta_0 + \sum_{j=1}^p \beta_j x_{ij}) \quad \text{or} \\
\lambda_i &= d_i \cdot e^{\beta_0} \cdot e^{\beta_1 x_{i1}} \cdot e^{\beta_2 x_{i2}} \dots e^{\beta_p x_{ip}}
\end{aligned} \tag{3.16}$$

Where d_i represents the risk exposure for transaction i . Taking logs on both sides of equation 3.16, the regression model for the estimation of loss frequency is:

$$\ln \lambda_i = \ln d_i + \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} \tag{3.17}$$

where $\ln d_i$ is the natural log of risk exposure, called the “offset variable”.

The poisson distribution is restrictive when applied to approximate counts, due to the assumption made about it that the mean and variance of the number of events are equal. However, in models for count data where means are low so that the number of zeros and ones in the data is excessive are well adapted to the poisson case (Wood, 2017). These cases are characteristic of scenarios in OpRisk other than those modeling situations when the unchecked spreading of negligent behaviour may result in an operational hazard. For example, the negative binomial and/or quasipoisson regression models ascribe to data that exhibits *overdispersion*, wherein the variance is much larger than the mean for basic count data, therefore they have been eliminated in this paper.

Research Objective 1

To introduce the generalised additive model for location, scale and shape (GAMLSS) framework for OpRisk management, that captures exposures to forward-looking aspects in the OpRisk loss prediction problem, due to deep hierarchies in the features of covariates in the investment banking (IB) business environment, and

internal control risk factors (BEICF) thereof.

Exploratory data analysis

The main source of the analysis dataset is primary data, a collection of internal OpRisk losses for the period between 1 January 2013 and 31st March 2013 at an investment bank in SA. The method of data generation and collection is at the level of the individual trade deal, wherein deal information is drawn directly from the trade generation and and settlement system (TGSS) and edit detail from attribution reports generated in middle office profit & loss (MOPL). The raw source consists of two separate datasets on a trade-by-trade basis of daily frequencies (number of events) and associated loss severities.

The raw frequency data consists of 58,953 observations of 15 variables, within the dataset there are 50,437 unique trades. The raw severity data consists of 6,766 observations of 20 variables; within the severity dataset there are 2,537 unique trades. The intersection between the frequency and severity datasets consists of 2,330 individual transactions which represent realised losses, pending and/or near misses. This dataset is comprised of 3-month risk correction detail, in the interval between 01 January 2013 and 31 March 2013.

Two new variables are derived from the data; a target variable (LossIndicator) is a binary variable whereupon, a 1 signifies a realised loss, and 0 for those pending losses, or near misses. The *exposure* variable is computed by deducting the time between the trade amendment (UpdateTime) and the time when the trade was booked (TradeTime). It is a measure that is meant to be roughly proportional to the risk of the transaction or a group of transactions. The idea is that if the exposure (e.g. the duration of a trade, the number of allocation(trade splits), etc.) doubles whilst everything else (e.g. the rate, nominal of the splits, and others) remains the

Table 3.2: The contents of the traded transactions of the associated risk correction events.

Covariate	Storage	
	Levels	Type
Trade		numeric
UpdateTime		numeric
UpdatedDay		numeric
UpdatedTime		numeric
TradeTime		numeric
TradedDay		numeric
TradedTime		numeric
Desk	10	categorical
CapturedBy	5	categorical
TradeStatus	4	categorical
TraderId	7	categorical
Instrument	23	categorical
Reason	19	categorical
Loss		numeric
EventTypeCategoryLevel	5	categorical
BusinessLineLevel	8	categorical
LossIndicator	2	binary
exposure		numeric

same, then the risk also doubles.

In R, the GLM function works with two types of covariates/explanatory variables: numeric (continuous) and categorical (factor) variables as depicted in table 3.2. Multi-level categorical variables are recoded by building dummy variables corresponding to each level. This is achieved through an implemented algorithm in R, through a transformation as recommended for the estimation of the GLM, particularly in the estimation of the poisson regression model for count data.

The model revolves around the fact that for each categorical variable (covariate), previously transformed into a dummy variable, one must specify a reference category from which the corresponding observations under the same covariate are estimated and assigned a weight against in the model (Covrig et al., 2015). By default in the GLM, the first level of the categorical variable is taken as the refer-

ence level. As best practice, De Jong & Heller (2008), Frees & Sun (2010), Denuit, Maréchal, Pitrebois, & Walhin (2007), Cameron & Trivedi (2013) and others recommend that for each categorical variable one should specify the modal class as the reference level; as this variable corresponds to the level with the highest order of predictability, excluding the dummy variable corresponding to (weight coefficient = 0) the biggest absolute frequency.

Description of the dataset

In this section, section 3.9, the dataset called *OpRiskDataSet_exposure*, provides data on the increase in the numbers of operational events over a three month period, beginning 01 January 2013 to end of 20 March 2013. For each transaction, there is information about: trading risk exposure, trading characteristics, causal factor characteristics and their cost.

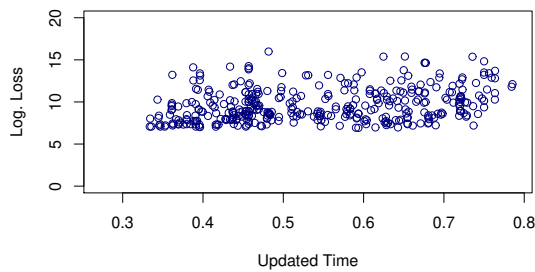
Characteristics of exposure

The exposure of risk of type i , d_i shows the daily duration, from when the trade was booked to the moment the operational risk event was observed and ended. This measure is defined this way when specifically applied to projecting the number of loss events (frequencies) and can be plotted as follows depicted in graphs depicted in Figure 9.2.

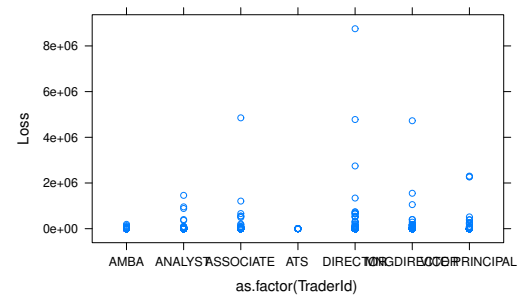
The variable follows a logistic trend on $[0, 1]$, implying an FI's operational risk portfolio rises like a sigmoid function throughout the period of observation, typically starting from 0, which then observes a plateau in growth. The average exposure is 389.99 or about 1 year.

Grid plots 9.2 portray the logistic function, together with a simple compari-

Intra-day Trend of Loss Severity

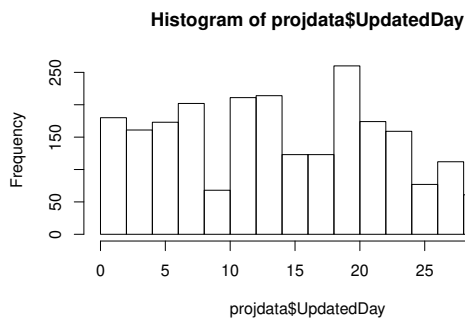


Trends of Loss Severities per Trader

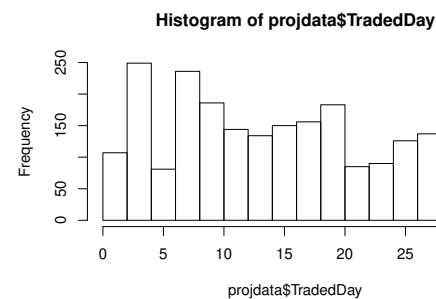


(a) Scatterplots

Loss per month



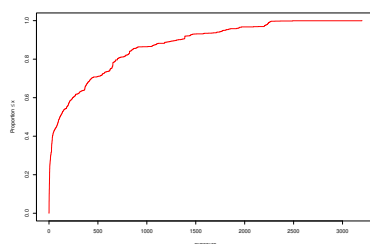
Trading frequency



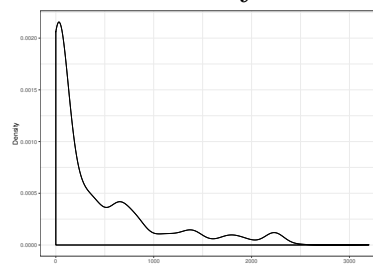
(b) Histograms

Figure 9.1: (a) Scatterplots of intra-day trend analysis for logs of severities of operational events and trends incident activity for identifying the role of the trader originating the incidents. (b) As for (a) but in the form of histograms showing the frequency distribution of the number daily operational incidents and the number of trades over a monthly period.

Distribution



Density



Digital Analysis

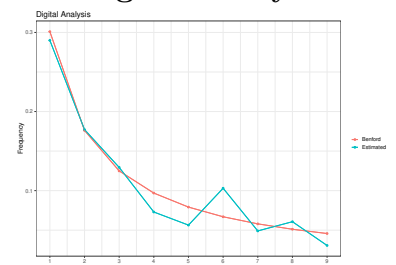


Figure 9.2: A simple comparison of the Sigmoidal like features of the fat-tailed, right skewed distribution for exposure, and first-digit frequency distribution from the exposure data with the expected distribution according to Benford's Law

son of first-digit frequency distribution analysis, according to Benford's Law, with exposure data distribution. The close fitting nature implies the data are uniformly distributed across several orders of magnitude, especially within the 1 year period.

Characteristics of the covariates

The characteristics of the operational risk portfolio are given by the following covariates: *UpdatedDay*, *UpdatedTime* - the day of the month and time of day the OpRisk incident occurs respectively; *TradedDay*, *TradedTime* - the day in the month and time of day the deal was originated respectively; The *LossIndicator* as indicated before is a binary variable consisting of two values: A 0, which indicates pending or near misses, and 1, if the incident results in a realised loss, meaning that there is significant p&L impact due to the OpRisk incident.

the *Desk* is the location in the portfolio tree the incident originated, it is a factor variable consisting of 10 categories; *CapturedBy*, the designated analyst who actions the incident, a factor variable consisting of 5 categories; *TraderId*, the trader who originates the deal, a factor variable with 7 categories; *TradeStatus*, the live status of the deal, a factor variable with 4 categories; *Instrument*, the type of deal, a factor variable with 23 categories; *Reason*, a description of the cause of the OpRisk incident, a factor variable with 19 levels; *EventTypeCategoryLevel*, 7 OpRisk event types as per Risk (2001), a factor variable with 5 categories; *BusinessLineLevel*, 8 OpRisk business lines as per Risk (2001), a factor variable with 8 categories.

The continuous numerical variable *Loss*, shows the financial impact (severity) of the OpRisk incident in Rands. For the most part (i.e. 96.1% of the time) OpRisk incidents result in pending losses and/or near misses, most realised losses (2.3%) lie within the [R200, 00, R300, 000] range. In the current portfolio there are also five

p&L impacts higher than **R2.5 million**.

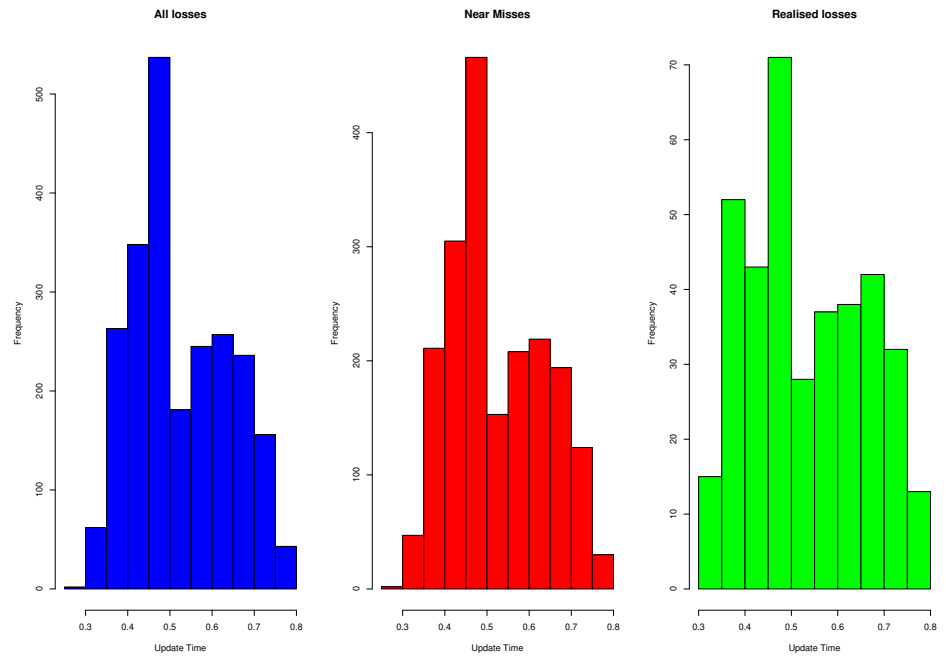
Characteristics of daily operational activity

The distribution of daily losses and/or pending/near misses by operational activities are represented in 9.3. Figure 9.3a shows that most operational events occur in times leading up to midday (i.e. 10:50AM to 11:50AM), the observed median is 11:39AM, and of these potential loss events, most realised losses occur closest to mid-day. The frequencies of the loss incidents in the analysed portfolio sharply decreases during the following period, i.e. from 12:10PM to 13:10PM, during which the least realised losses occur.

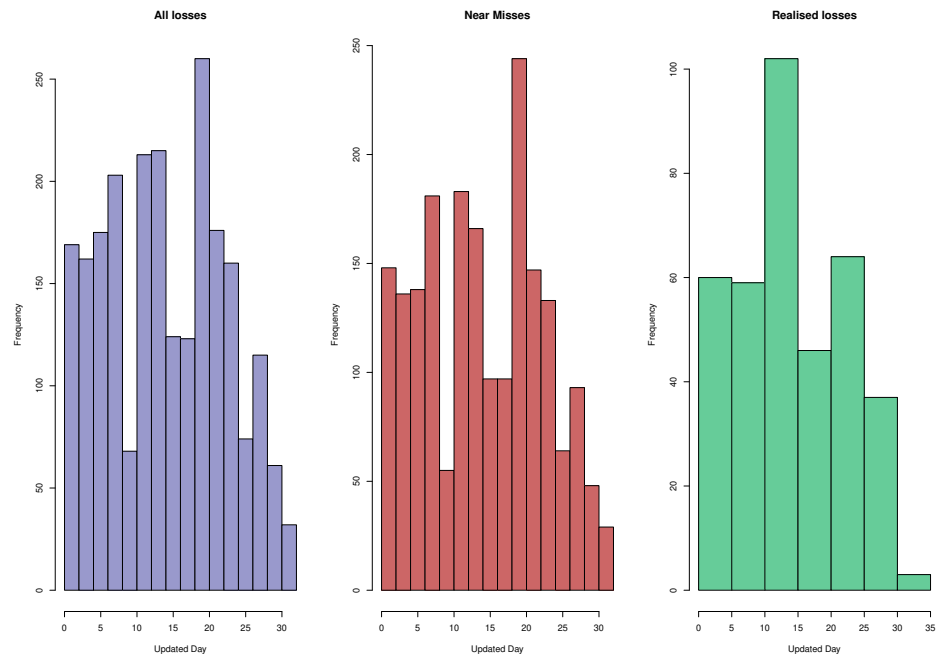
Figure 9.3b shows that operational activity increases in intensity in the days leading up to the middle of the month, i.e. 10th - 15th; the observed mean is 14.49 days, and of these potential loss events, realised losses especially impact on the portfolio during these days.

Similarly, the influence of trading desk's on the frequency of operational events can be analysed on the basis of the portfolio's bidimensional distribution by variables *Desk* and *LossIndicator*, which shows the proportions realised losses vs pending and/or near misses for each particular desk. The bidimensional distribution of *Desk* and *LossIndicator* is presented in a contingency table, Table 3.3, in which it's considered useful to calculate proportions for each desk category.

Thus, as illustrated in figure 9.5, from 23,5%; the highest proportion of realised losses per desk is the Money Market (MM) desk, the figures are decreasing, followed by Prime Services (22%); Bonds/Repos (21,5%); Equity (19,7%); Africa (16,9%); Commodities (13,8%); Rates (13,6%); Derivatives (10,5%); Structured Notes (SND) (8.6%), to the least proportion in the Management/Other, a category



(a) Frequency distributions of operational incidents by the time in the day



(b) Frequency distributions of operational incidents by the day in the month

Figure 9.3: The frequency distributions of All the losses, the realised losses, and pending/near misses of operational incidents by the day in the month when the incidents occurred

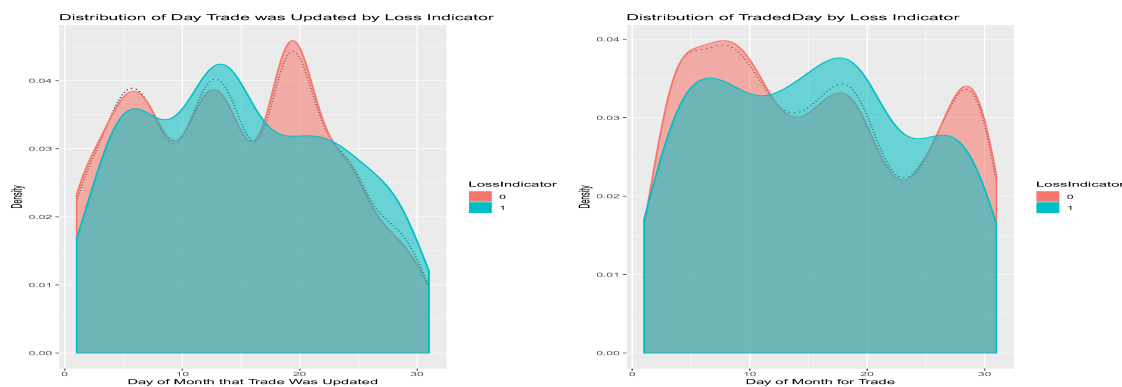


Figure 9.4: Density plots showing a comparison of realised vs pending losses and/near misses over a month for the day in the month the OpRisk incident was updated to the day in the month trades were traded/booked

Table 3.3: Occurrence of realised losses: proportions on desk categories

Desk	No. of transactions		
	no Loss	Loss	Total
Africa	49	10	59
Bonds/Repos	113	31	144
Commodities	282	45	327
Derivatives	205	24	229
Equity	269	66	335
Management/Other	41	2	43
Money Market	169	52	221
Prime Services	220	62	282
Rates	336	53	389
Structured Notes	275	26	301

where only 4,7% of operations activities were realised as losses.

This behaviour can be extended beyond the trading desk, as represented in Figure 9.6, a mosaic plot grid presenting the structure of the OpRisk portfolio by Instrument, TraderId, CapturedBy ² and the operational losses.

One can notice that the width of the bars corresponding to the different categories, i.e. Instrument, TraderId, CapturedBy, is given by their proportion in the

²i.e. the type of financial instrument, the trader who originated the incident on the deal, and the role of the technical support personnel who is involved in the query resolution.

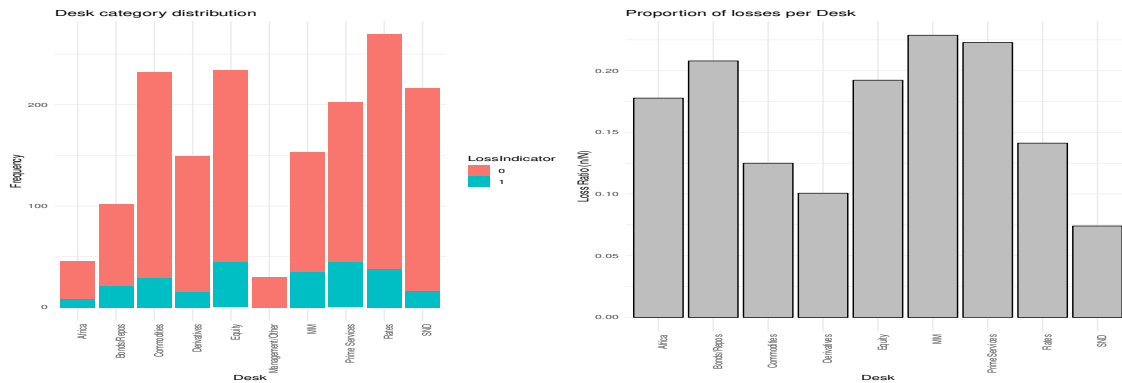


Figure 9.5: Histograms showing the proportions of realised losses vs all losses including pending and/or near misses by desk category

sample. In particular, for the category ‘at least one realised loss’, in the top right mosaic of Figure 9.6 portrays a increase in “riskiness” trending up from Associate to AMBA, Analyst, Vice Principal, Managing Director, Director, up to the risky ATS category, which are automated trading system generated trades.

Figure 9.6 bottom right mosaic plot for technical support personnel for the category ‘at least one realised loss’, portrays a downward trend, slowing in riskiness from Unauthorised User downward to Tech Support, Mid Office, Prod Controller down to the least risky Prod Accountant. This interpretation makes sense given unauthorised users are more likely to make impactful operational errors, technical support personnel would also be accountable for large impacts albiet for contrasting reasons, they are mandated to perform these deal adjustments which have unavoidable impacts associated with them, whereas the former group are unauthorised to perform adjustments therefore may lack the skill, or be criminally minded insiders acting on their own or in unison to enable their underhanded practices and intentions without raising any suspicion.

In another mosaic plot, Figure 9.7, the bidimensional distribution of transactions by trader and realised vs pending losses, conditional on the trade status is presented and analysed. Here, and in the contingency table, Table 3.6, we can clearly

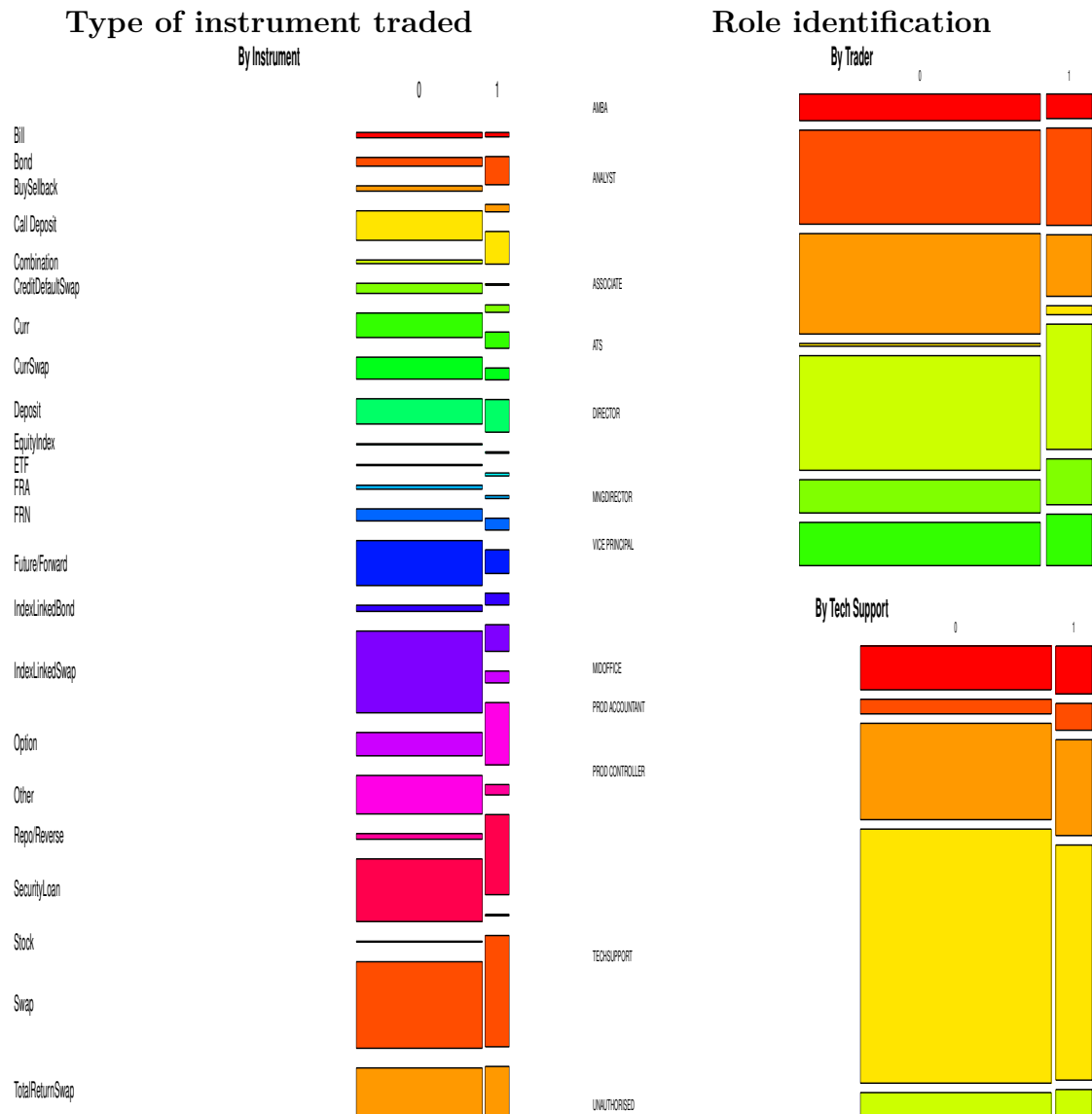


Figure 9.6: Mosaic grid plots for the bidimensional distribution by traded instrument, the trader originating the operational event, and by the technical support personnel involved in query resolution, against the dummy variable showing if a realised loss was reported.

Table 3.4: Summary statistics for all losses as per Instrument type

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Mean	23	34,603	46,007	306	7,697	44,157	192,513

Mosaic plot for trader identification and loss indicator, by trade status

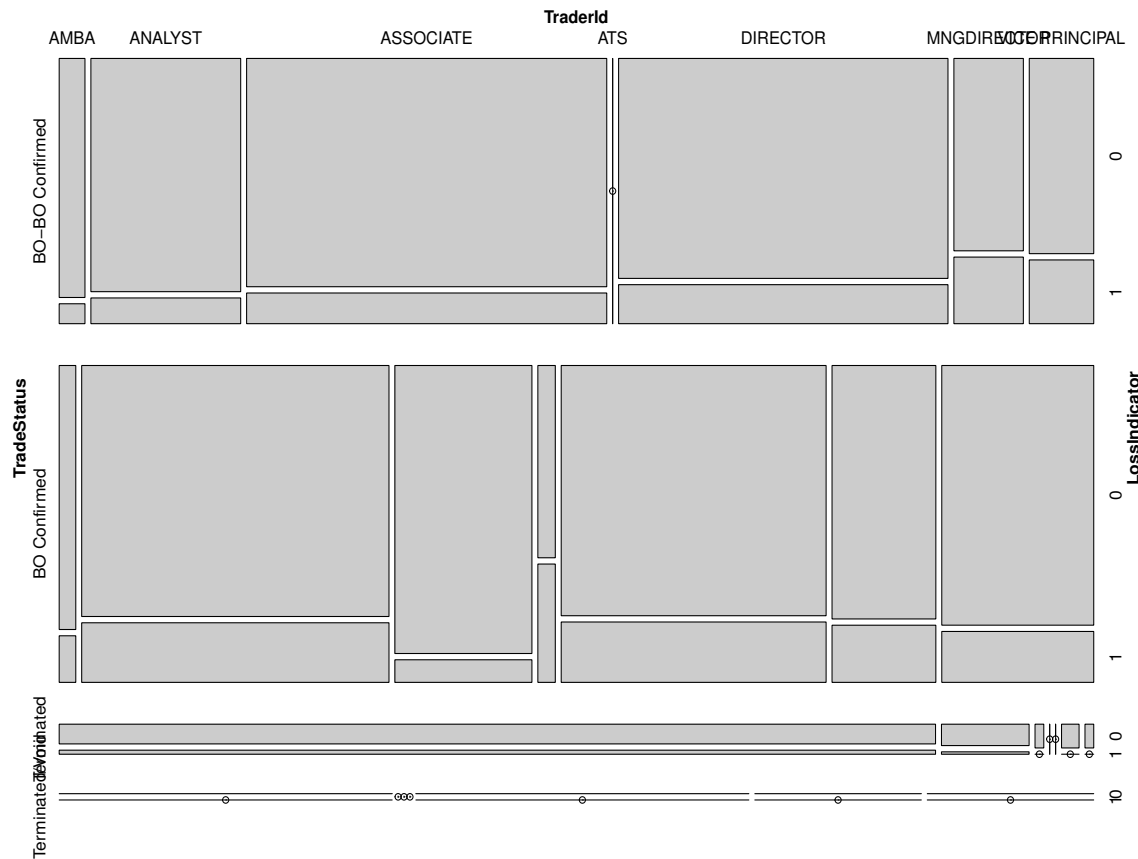


Figure 9.7: A mosaic plot representing the structure of the operational risk portfolio by trader identification (TraderId), the status of the trade (TradeStatus) and the number of realised losses vs pending or near misses

see the following trends: In BO-BO confirmed status - an increase in realised losses from the leftmost TraderID (i.e. AMBA) to right, and the opposite for transactions performed in BO Confirmed status (both with two exceptions). In particular, the biggest number of realised losses in both BO and BO-BO Confirmed statuses occur due to automated trading systems (ATS) who also give rise to the exceptions mentioned.

Table 3.6 and Figure 9.7 are obtained with the following commands:

Table 3.5 presents the most frequent category in the operational risk dataset for each possible covariate.

Crosstab of trader identification and loss indicator, by trade status

TradeStatus	Loss Indicator	Trader Identification						
		Amba	Analyst	Associate	ATS	Director	Mng Director	Vice Principal
BO-BO Confirmed	0	24	136	320	0	282	52	49
	1	2	15	43	0	50	18	16
BO Confirmed	0	17	299	153	13	257	102	153
	1	3	71	12	8	62	23	30
Terminated	0	83	9	1	0	0	2	1
	1	17	1	0	0	0	0	0
Terminated/Void	0	2	0	0	0	2	1	1
	1	0	0	0	0	0	0	0

Table 3.5: A contingency table showing the bidimensional distribution of transactions by trader identification vs realised and/or pending losses, conditional on the trade status

Modal classes for the categorical variables

Variable	Modal class or category	Name of modal class
Desk	Rates	DeskRates
CapturedBy	TECHSUPPORT	CapturedBy_TECHSUPPORT
TradeStatus	BO confirmed	TradeStatus_BO confirmed
TraderId	DIRECTOR	TraderId_DIRECTOR
Instrument	Swap	Instrument_Swap
Reason	Trade enrichment for system flow	Reason_Trade enrichment for system flow
EventTypeCategoryLevel	EL7	EventTypeCategoryLevel_EL7
BusinessLineLevel	BL2	BusinessLineLevel_BL2

Table 3.6: A contingency table showing the bidimensional distribution of transactions by trader identification vs realised and/or pending losses, conditional on the trade status

The estimation of some poisson regression generalised linear models (GLM's)

Section 3.3 introduced a GLM for the start of the expected number of operational events in the early stages. We aim to estimate the mean OpRisk frequency through a poisson classification model given by equation 3.13 using the `glm` function. The mean daily loss frequency in the risk correction statistics is estimated through the poisson regression model. Let us consider a model where the *LossIndicator* is the target variable: The following fits the model (the log link is canonical for the poisson distribution, and hence the R default) and checks it.

In calling the GLM we specify the target variable *LossIndicator*; the explanatory variables are composed of numeric, continuous and categorical variables. Where the variable in the argument of a GLM is categorical, one chose to specify the modal class as the reference level. A user defined function “getmode” has been created; it selects the modal observation in each factor, and the dataset is reordered using the *relevel* function in RStudio.

Other GLM arguments are: The afore-mentioned link function `poisson(link="log")`; a data frame containing the OpRisk dataset, `data=crs$training`; and the `offset=log(exposure)`, i.e. the variable representing a component known apriori, `coefficient= 1`, introduced in the linear predictor (Covrig et al., 2015).

Firstly, consider a GLM in which is introduced two explanatory variables, one numerical variable, *UpdatedTime*, and another categorical variable *Desk*. This will be our global model. We will use *LossesIndicator* as the target variable, while these two unique variables will be explanatory variables:

```
freqfit1 <- glm(LossesIndicator ~ TradedDay + Desk, data=crs$training,
               family=poisson(link = 'log'), offset = log(Exposure))
```


The output result of the estimation is presented below, where variables who were found to be significant predictors are indicated. The coefficients of the categorical variable *Desk* are reordered and weighted against the modal class: *DeskRates*. Interestingly the modal class does not show up in the results section (as the coefficient of the modal class = 0), given that the remaining classes are weighted against it.

```
##
## Call:
## glm(formula = LossesIndicator ~ TradedDay + Desk, family = poisson(link = "log")
##      data = crs$training, offset = log(Exposure))
##
## Deviance Residuals:
##      Min        1Q    Median        3Q        Max
## -2.9195   -0.5659   -0.2276   -0.0495    4.3643
##
## Coefficients:
##              Estimate Std. Error z value      Pr(>|z|)
## (Intercept)   -7.99905    0.19636 -40.736 < 0.0000000000000002 ***
## TradedDay     -0.01427    0.00667  -2.139    0.03242 *
## DeskAfrica     1.28441    0.41309   3.109    0.00188 **
## DeskBonds/Repos  1.86747    0.26354   7.086    0.000000000000138 ***
## DeskCommodities  0.72735    0.24727   2.941    0.00327 **
## DeskDerivatives -0.04525    0.27918  -0.162    0.87123
## DeskEquity     1.31836    0.22716   5.804    0.000000000648772 ***
## DeskManagement/Other -13.99193  286.05004  -0.049    0.96099
## DeskMM         0.21880    0.24801   0.882    0.37766
## DeskPrime Services  2.15462    0.22427   9.607 < 0.0000000000000002 ***
## DeskSND       -0.71920    0.28362  -2.536    0.01122 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##      Null deviance: 1943.5  on 1630  degrees of freedom
## Residual deviance: 1719.4  on 1620  degrees of freedom
## AIC: 2253.4
##
## Number of Fisher Scoring iterations: 15
```

Using this bivariate model, the estimated quarterly OpRisk (LossIndicators)

frequency of realised losses for each *Desk* category (excluding the insignificant ones) are:

- * $0,0012 = e^{-7.99905} \cdot e^{-0.01427} \cdot e^{1.28441}$, for the combination of the **UpdateTime** and **DeskAfrica** category, which implies that frequency of realised losses for this combination of predictor variables is $3.613(= \cdot e^{1.28441})$ fold (times) higher than the realised loss frequency of OpRisk causes in the reference desk category, viz. the **Rates** desk.
- * $0,0021 = e^{-7.99905} \cdot e^{-0.01427} \cdot e^{1.86747}$, for the combination of the **UpdateTime** and **DeskBonds/Repos** category, which implies that frequency of realised losses for this combination of predictor variables is $6,472(= \cdot e^{1.86747})$ fold higher than causes in the reference desk category.
- * $0,0007 = e^{-7.99905} \cdot e^{-0.01427} \cdot e^{0.72735}$, for the combination, which implies that frequency of realised losses for this combination of predictor variables is $2,070(= \cdot e^{0.72735})$ fold higher than the causes in the reference desk category.
- * $0,0012 = e^{-7.99905} \cdot e^{-0.01427} \cdot e^{1.31836}$, for the combination, which implies that frequency of realised losses for this combination of predictor variables is $3,737(= \cdot e^{1.31836})$ fold higher than the causes in the reference desk category.
- * $0,001373903 = e^{-7.99905} \cdot e^{-0.01427} \cdot e^{2.15462}$, for the combination with **DeskPrime Services**, an increase of $8,625(= \cdot e^{2.15472})$ fold times higher w.r.t the baseline (the **Rates** desk)
- * about $0.00000025 = e^{-7.99905} \cdot e^{-0.01427} \cdot e^{-0.71920}$ of the last desk category **DeskSND**, which means a decrease of about 50

The predicted mean frequency of realised losses for OpRisk incident i , for the model **freqfit1**, is given by:

$$\begin{aligned}\mu_i = & \text{exposure}_i \cdot e^{-7.99905 \cdot \text{Intercept}_i} \cdot e^{-0.01427 \cdot \text{UpdatedTime}_i} \cdot e^{1.28441 \cdot \text{DeskAfrica}_i} \\ & \cdot e^{1.86747 \cdot \text{DeskBonds/Repos}_i} \cdot e^{0.72735 \cdot \text{DeskCommodities}_i} \cdot e^{1.31836 \cdot \text{DeskEquity}_i} \\ & \cdot e^{2.15462 \cdot \text{DeskPrime Services}_i} \cdot e^{-0.71920 \cdot \text{DeskSND}_i}\end{aligned}\quad (3.18)$$

We now fit a more comprehensive model where we introduce more variables, in which show realised losses for quarterly OpRisk incidents for an all inclusive case. We will use “LossesIndicator” as the dependent variable, while the other variables will be predictor variables.

```
### Let us fit a GLM to our data. This will be our global model. We will use "LossesIndicator" as the dependent variable.
freqfit <- glm(LossesIndicator ~ UpdatedDay + UpdatedTime +
               TradedDay + TradedTime + Desk + CapturedBy +
               TradeStatus + TraderId + Instrument + Reason
               + EventTypeCategoryLevel1 + BusinessLineLevel1,
               data=crs$training, family=poisson(link = 'log'), offset = log(Exposure))
```

Which yields output (in summarised form):

Call:

```
glm(formula = LossesIndicator ~ UpdatedDay + UpdatedTime + TradedDay +
    TradedTime + Desk + CapturedBy + TradeStatus + TraderId +
    Instrument + Reason + EventTypeCategoryLevel1 + BusinessLineLevel1,
    family = poisson(link = "log"), data = crs$training, offset = log(Exposure))
```

Deviance Residuals:

	Min	1Q	Median	3Q	Max
	-3.7328	-0.3616	-0.1012	-0.0139	4.1107

Coefficients:

	Estimate	Std. Error	z value	(Intercept)	
(Intercept)	-8.601892	0.738478	-11.648	< 0.00000000000000002	***
UpdatedDay	-0.014075	0.010414	-1.352	0.17651	
UpdatedTime	0.105966	0.708733	0.150	0.88115	
TradedDay	-0.015601	0.008275	-1.885	0.05939	.
TradedTime	0.252615	0.782853	0.323	0.74693	
DeskAfrica	2.388334	0.575306	4.151	0.000033042768080	***
DeskBonds/Repos	2.975192	0.442633	6.722	0.000000000017977	***
DeskCommodities	1.142290	0.474629	2.407	0.01610	*
DeskDerivatives	0.952777	0.491440	1.939	0.05253	.
DeskEquity	1.745408	0.427535	4.082	0.000044556065633	***

DeskManagement/Other	-15.024612	620.154848	-0.024	0.98067
DeskMM	1.692119	0.583820	2.898	0.00375 **
DeskPrime Services	0.310749	1.303433	0.238	0.81156
DeskSND	1.100596	0.726644	1.515	0.12987
.
.
.
BusinessLineLevel1BL1	1.698196	0.729494	2.328	0.01992 *
BusinessLineLevel1BL3	-0.177178	0.652274	-0.272	0.78590
BusinessLineLevel1BL4	-1.547668	0.494473	-3.130	0.00175 **
BusinessLineLevel1BL5	-1.146241	0.501862	-2.284	0.02237 *
BusinessLineLevel1BL6	1.733747	1.354626	1.280	0.20059
BusinessLineLevel1BL7	1.593485	2.598998	0.613	0.53980
BusinessLineLevel1BL9	1.871917	1328.440227	0.001	0.99888

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

Null deviance: 1943.5 on 1630 degrees of freedom

Residual deviance: 1239.6 on 1553 degrees of freedom

AIC: 1907.6

Number of Fisher Scoring iterations: 16

Model selection and multimodel inference

The selection of the best-fit model from the list of possible combinations of predictor variables traditionally follows of a process removing/adding each variable progressively after each estimation, and propagating backward/forward, comparing goodness of fit tests at each stage. For example, if we compare the values of the Akaike information criteria (AIC) for the bivariate model **freqfit1** and the multivariate model **freqfit**, by AICs; we see that for the first (bivariate) model the AIC value is 2253.4 and 1907.6 for the second (multivariate) model, which suggests that the second model, **freqfit**, the model in which we considered an all inclusive list of 13 predictor variables is a better fit since there is a marked reduction/improvement in AIC magnitudes compared to the first value, hence **freqfit** is preferred over the

bivariate (first) model.

similarly, an estimation of the models by a comparison which enables the choice the most appropriate or “best” fit model, first through finding out its significance, viz. if the residual deviance and the corresponding number of degrees of freedom doesn’t have a value significantly bigger than 1: In the multivariate model $\text{freqfit } \frac{1239.6}{1553} = 0.8$, and then retaining the model with the smaller AIC value.

Burnham & Anderson (2002) introduction of an information-theoretic approach permits a data-based selection for the “best-fit” model in the analysis of the OpRisk dataset *OpRiskDataSet_exposure.csv*, and a ranking and weighting of what remains. This approach allows traditional (formal) statistical inference to be based on the selected “best-fit” model, which is now based on more than one model (multimodel inference). As a requirement the R package to load is the “**MuMIn**” Rstudio package.

Then, we use “dredge” function to generate models using combinations of the terms in the global model. The function will also calculate AICc values and rank models according to it. Note that AICc is AIC corrected for finite sample sizes. The process of analyzing data where the experimentalist has few or no a priori information, thus “all possible models” are considered by subjectively and iteratively searching the data for patterns and “significance”, is often called “data mining”, “data snooping” or the term “data dredging”.

The function “MuMIn::dredge” returns a list of 4097 models, which is every combination of predictor variable in the global model *freqfit*. Model number 894 is the best-fit: All predictor variables included in this model have a positive effect on the target variable except for the predictor *TrddD* (**TradedDay**) which has a negative effect on the likelihood of a realised loss (target variable *LossIndicator*) i.e., the

later in the month of the transaction, the less likely a loss is realised. Additionally, from the delta (=delta AIC) one cannot distinguish between models 894, 382, 1918 and 1406 since (using the common rule of thumb) they have $AIC < 2$.

Of the top seven models (listed below); 1918 & 2942 each hold nine; 894, 1406 & 1854 hold eight; 382 & 830 hold seven; and lastly 318 hold six predictor variables respectively. Where a variable doesn't have a value associated with it does not mean no effect, but rather that it was not included in the model. For example, model 894 returns a combination of the eight variables 1/2/3/4/5/6/7/8, corresponding to top most model in the following output predictor variables (abbreviated in the header row) below:

```
Global model call: glm(formula = LossesIndicator ~ UpdatedDay + UpdatedTime + TradedTime + Desk + CapturedBy + TradeStatus + TraderId + Instrument + Reason + EventTypeCategoryLevel1 + BusinessLineLevel1, family = poisson(link = "log"), data = crs$training, offset = log(Exposure))
```

Model selection table

	(Intrc)	BsLL1	Desk	ETCL1	Instr	Reasn	TrddD	TrdrI	TrdSt	UpdtD	UpdtT
894	-8.566	+	+	+	+	+	-0.014630000	+	+		
382	-8.627	+	+	+	+	+	-0.014730000	+			
1918	-8.362	+	+	+	+	+	-0.015200000	+	+	-0.012880000	
1406	-8.447	+	+	+	+	+	-0.015290000	+		-0.011540000	
830	-8.889	+	+	+	+	+		+	+		
318	-8.942	+	+	+	+	+		+			
1854	-8.705	+	+	+	+	+		+	+	-0.011920000	
2942	-8.730	+	+	+	+	+	-0.014640000	+	+		0.313300

Information from the AICc's values suggest, that of the top eight models have similar support, and their Akaike weights are not high relative to the $[0, 1]$ weight range: This is characteristic of the endemic nature of data dredging, as the literature suggests (Burnham & Anderson, 2002), and should generally be avoided to curb attendant inferential problems if a single model is chosen, e.g the risk of finding spurious effects, overfitting, etc. Burnham & Anderson (2002) advises that

model averaging is useful in finding a confirmatory result as estimates of precision should include model selection uncertainty. Even so, one can rule out many models on a priori grounds.

We now use “get.models” function to generate a list in which its objects are the fitted models. We will also use the “model.avg” function to do a model averaging based on AICc. Note that “subset=TRUE” will make the function calculate the average model (or mean model) using all models. However, we want to get only the models that have delta AICc < 2 ; we therefore use “subset=delta<2”

Now we have AICc values for our models and we have the average (mean) model.

Call:

```
model.avg(object = get.models(freqfits, subset = delta < 2))
```

Component model call:

```
glm(formula = LossesIndicator ~ <8 unique rhs>, family = poisson(link = "log"),
data = crs$training, offset = log(Exposure))
```

Component models:

	df	logLik	AICc	delta	weight
1/2/3/4/5/6/7/8	71	-879.52	1907.61	0.00	0.19
1/2/3/4/5/6/7	68	-882.87	1907.75	0.14	0.18
1/2/3/4/5/6/7/8/9	72	-878.68	1908.11	0.50	0.15
1/2/3/4/5/6/7/9	69	-882.17	1908.53	0.92	0.12
1/2/3/4/5/7/8	70	-881.13	1908.63	1.02	0.11
1/2/3/4/5/7	67	-884.50	1908.84	1.23	0.10
1/2/3/4/5/7/8/9	71	-880.41	1909.38	1.78	0.08
1/2/3/4/5/6/7/8/10	72	-879.41	1909.57	1.97	0.07

Term codes:

BusinessLineLevel1	Desk	EventTypeCategoryLevel1	Instrument	Reason
1	2	3	4	5
TradedDay	TraderId	TradeStatus	UpdatedDay	UpdateTime
6	7	8	9	10

Multimodel inference leads to more robust inferences, especially in the point of view that the selection of the model used to estimate the mean frequency must,

at the same time, serve the ultimate root cause analysis objective of OpRisk control, that decide calculating capital requirement, in OpVaR measures, taking into account as many characteristics of the trading OpRisk dataset as possible, as well considering how the variables interact with each other.

Yields a daily rate of $\lambda = 0.156958922$ or 0.1840831% per day.

```
library(e1071)

confusionMatrix(table(pred$Response, crs[["training"]][["LossesIndicator"]]))

## Confusion Matrix and Statistics
##
##
##           0      1
## 0 1142  132
## 1   233  124
##
##
##              Accuracy : 0.7762
##              95% CI : (0.7552, 0.7962)
##    No Information Rate : 0.843
##    P-Value [Acc > NIR] : 1
##
##
##              Kappa : 0.2714
##
##
##  Mcnemar's Test P-Value : 0.0000001657
##
##
##              Sensitivity : 0.8305
##              Specificity : 0.4844
##
##              Pos Pred Value : 0.8964
```



```
##          Neg Pred Value : 0.3473
##          Prevalence : 0.8430
##          Detection Rate : 0.7002
##    Detection Prevalence : 0.7811
##          Balanced Accuracy : 0.6575
##
##          'Positive' Class : 0
##
\textbf{
```

Modelling population size of the OpRisk events

}

We have gained initial insights through data exploration in Section 3.8 and then built models. The next critical step is to evaluate our model. For this we need to use a testing dataset whose function is to provide error estimates of the final result. The testing dataset is not used in building or even fine tuning the models that we build, for the sake of model building define a training dataset and a validation dataset to test different parameter settings or different choices of variables in the data mining part of the project.

We have a population of $K = 2330$ OpRisk events over the first quarter Q12013, and of these events we have a number $N = 371$ of realised losses. N is a discrete random variable modelled as a Poisson variable with rate λ . Each loss X_i is another random variable with an underlying severity distribution. How does the size K of the population enter the risk model?. It doesn't appear explicitly in the model (Parodi, 2014), however, it is taken into account during the creation of the model. Intuitively, the poisson rate λ is likely to be proportional to the current

$$\begin{aligned}
f(y|\mu, \sigma, \nu, \tau) &= (y^{(\nu-1)/\mu^{\nu}}) \cdot \frac{\tau}{\sigma} \cdot \frac{e^{(-0.5 \cdot |\frac{z}{c}|^{\tau})}}{(c \cdot 2(1 + \frac{1}{\tau}))} \cdot \Gamma(\frac{1}{\tau}) \\
\text{where } c &= [2^{\frac{-2}{\tau}} \cdot \frac{\Gamma(\frac{1}{\tau})}{\Gamma(\frac{3}{\tau})}]^{0.5}, \quad \text{where if } \nu \neq 0, \quad \text{then} \\
Z &= \frac{(\frac{y}{\mu})^{\nu} - 1}{\nu \cdot \sigma}, \quad \text{else } z = \frac{\log \frac{y}{\mu}}{\sigma}, \\
\text{for } y > 0, \quad \mu > 0, \sigma > 0, \nu &= (-\text{Inf}, +\text{Inf}) \quad \text{and } \tau > 0. \quad (3.20)
\end{aligned}$$

The BCPE adjusts the above density $f(y|\mu, \sigma, \nu, \tau)$, resulting from the condition $y > 0$. See Stasinopoulos, Rigby, Heller, Voudouris, & De Bastiani (2017) . We now consider a model where the *Loss* is the target variable: The following fits the model and checks it.

CHAPTER 4

Chapter 4's Title

CHAPTER 5

Chapter 5's Title

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