Assignment 8: Digital Fourier Transform

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The task

In this assignment, we learn to find frequency spectrums of different periodic functions.

Question 1

We'll first find the DFT of $y = \sin(5x)$. The expected spectrum is

$$Y(\omega) = \frac{1}{2j}[\delta(\omega - 1) - \delta(\omega + 1)]$$

We use 128 points between 0 and 2π. The obtained spectrum is as shown

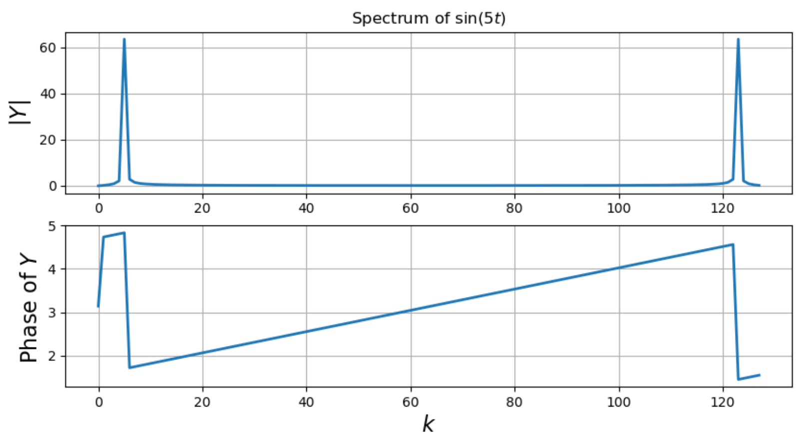

Figure 1: Spectrum obtained for $y = \sin(5x)$ on first trial (Question 1)

in Figure 1. We get spikes as expected, but not where we expected. The spikes have a height of 64, not 0.5. We should divide it by N to use it as a spectrum. We also have to correct the frequency axis. We shift the position in the frequency axis using the command `fftshift`. We also have to take 129 points and remove the last point to avoid considering both 0 and 2π. This spectrum we obtain is as shown in Figure 2. This is just as we expected.

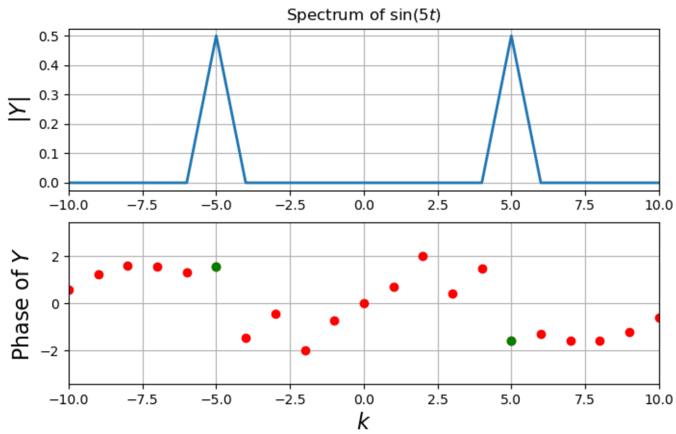

Figure 2: Spectrum obtained for $y = \sin(5x)$ (Question 1)

Now let's look at AM modulation. The function we analyse is

$$f(t) = (1 + 0.1\cos(t))\cos(10t)$$

Again we use 128 points like the previous function and plot the response as shown in Figure 3. We observe that this is not what we expected.

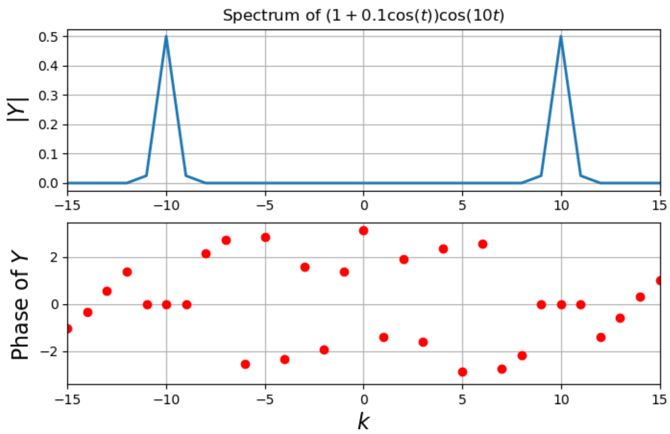

Figure 3: Spectrum obtained for $(1+0.1\cos(t))\cos(10t)$ on first trial (Question 1)

Hence, we increase the number of samples to 512, as well as stretch the time axis from -4π to 4π. The plot we obtain is as shown in Figure 4. This is like our expectations. Hence we proceed to the next question.

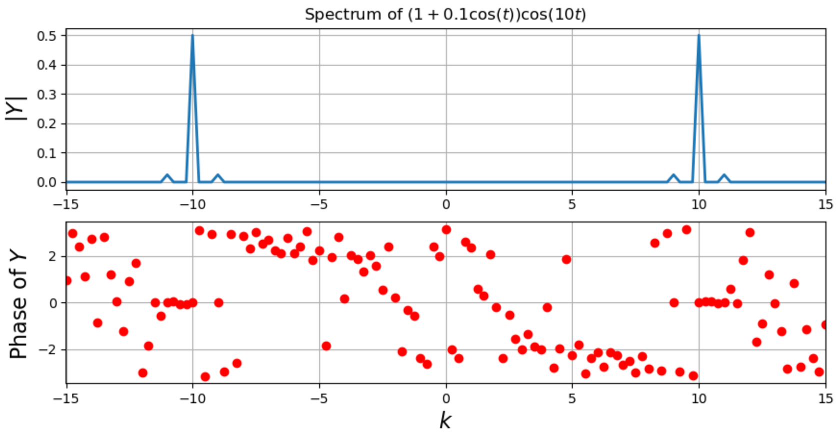

Figure 4: Spectrum obtained for $(1 + 0.1\cos(t))\cos(10t)$ (Question 1)

Question 2

We have to generate the spectrums of the following:

$$y_1(t) = \sin^3(t)$$

$$y_2(t) = \cos^3(t)$$

Using the same method that we used above, we get the spectrums to be as shown in Figure 5 and 6, with peaks at frequencies 1 and 3. This is what we expect because we know that both $\sin^3(t)$ and $\cos^3(t)$ has frequency components at 1 and 3 .

Question 3

Now we have to generate the spectrum of the following:

$$y(t) = \cos(20t + 5\cos(t))$$

The obtained plot with extended frequency axis is as shown in Figure 7. As expected, the spectrum centres around the frequencies 20 and -20. The rest of the spectrum around 20 (or -20) is due to the term $5\cos(t)$ inside the cosine.

Question 4

We try to get the spectrum of the following gaussian function:

$$f(t) = e^{-t^2/2}$$

On trying different ranges of time, we observe that for a smaller range, we don't get higher frequencies in the spectrum. As we increase the range, we

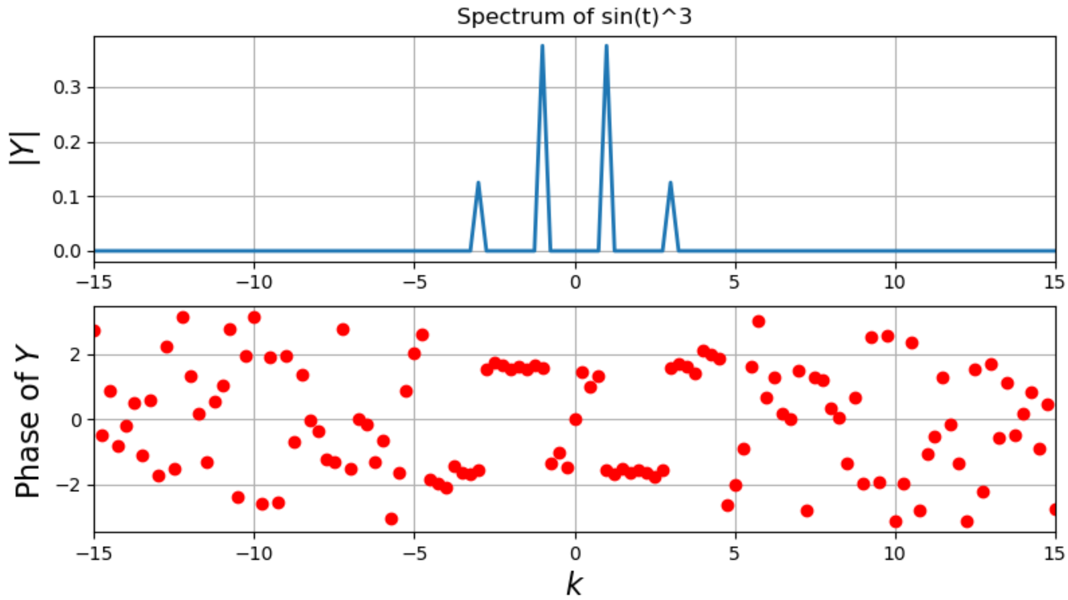

Figure 5: Spectrum obtained for $\sin^3(t)$ (Question 2)

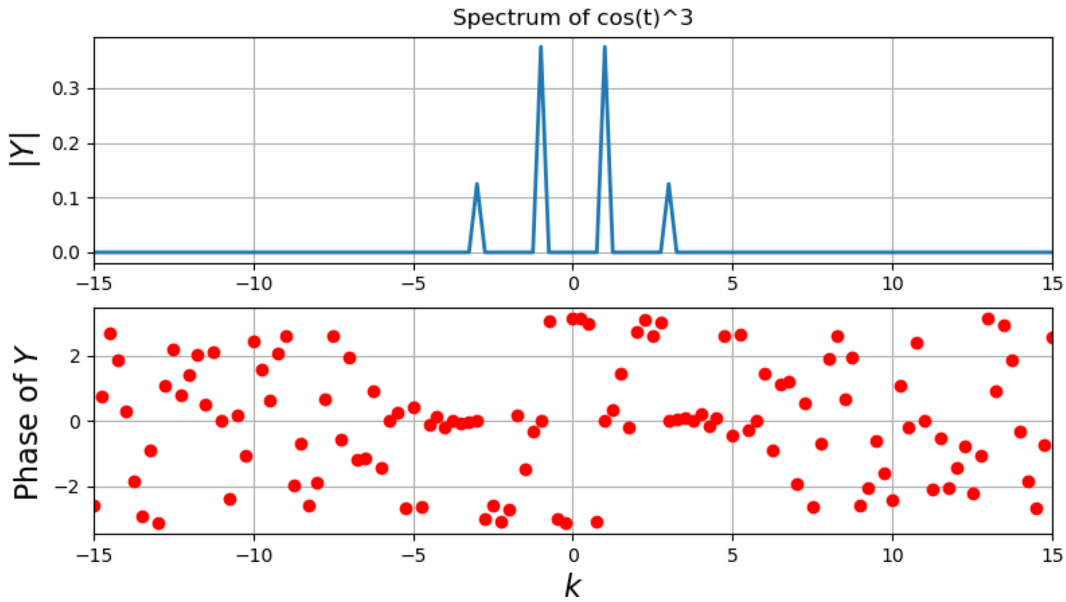

Figure 6: Spectrum obtained for $\cos^3(t)$ (Question 2)

get more accurate spectrums. When the range is increased beyond a limit, we tend to get a horizontal spectrum, which we know is obviously wrong. a

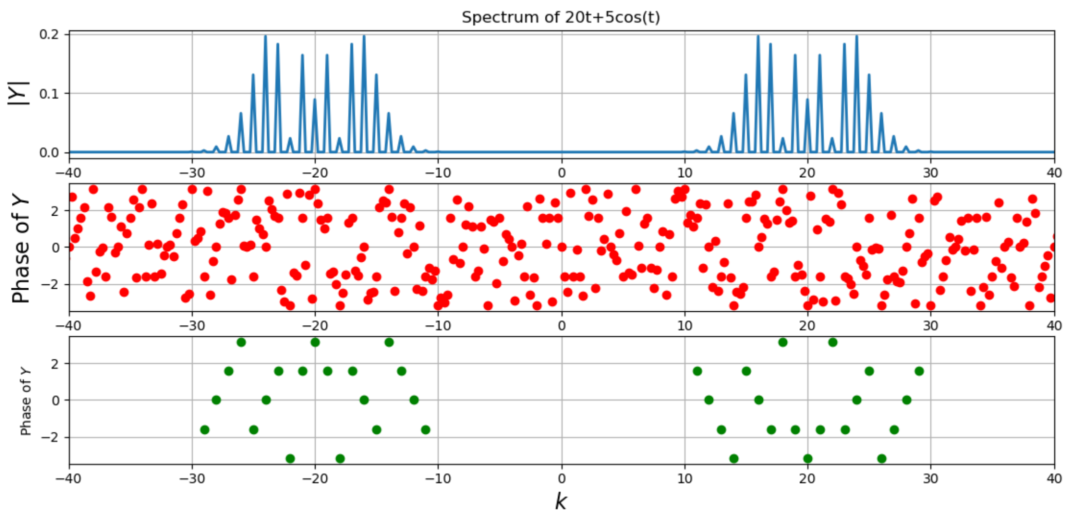

Figure 7: Spectrum obtained for $\cos(20t + 5\cos(t))$ (Question 3)

good spectrum obtained is as shown in Figure 8.

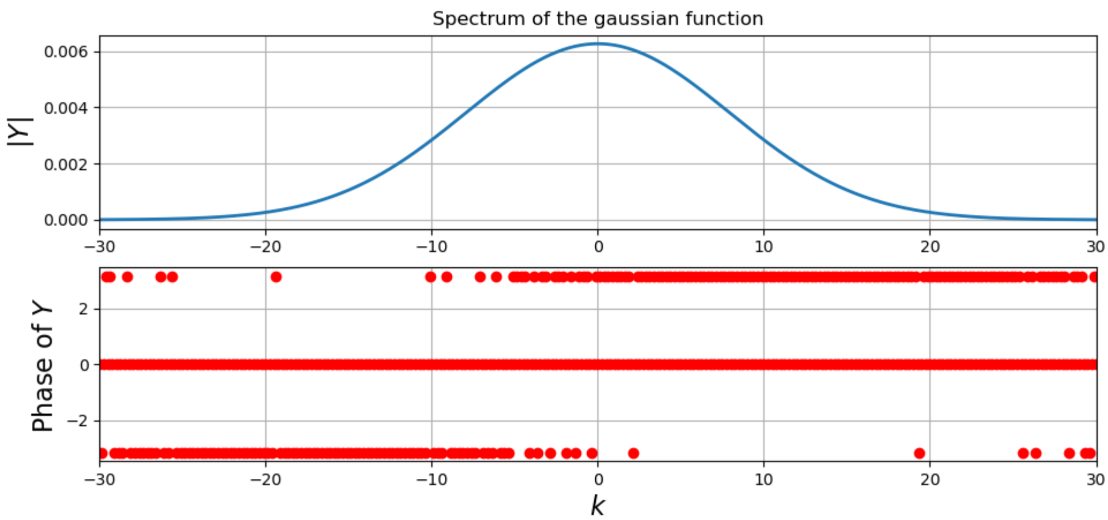

Figure 8: Spectrum obtained for $e^{-t^2/2}$ (Question 4)

Python code

The code is properly commented and completely vectorised.

Assignment 8 Code

```
#Assignment 8
import numpy as np
import matplotlib.pyplot as plt
import scipy.signal as sp
from scipy.linalg import lstsq
import scipy.integrate as spint
import mpl_toolkits.mplot3d.axes3d as p3
from scipy import ndimage
import sympy as spy

x=np.random.rand(100)
X=np.fft.fft(x)
y=np.fft.ifft(X)
print(np.abs(x-y).max())

x=np.linspace(0,2*np.pi,128)
y=np.sin(5*x)
Y=np.fft.fft(y)
plt.figure()
plt.subplot(2,1,1)
plt.plot(np.abs(Y),lw=2)
plt.ylabel(r"$|Y|$",size=16)
plt.title(r"Spectrum of $\sin(5t)$")
plt.grid(True)
plt.subplot(2,1,2)
plt.plot(np.unwrap(np.angle(Y)),lw=2)
plt.ylabel(r"Phase of $Y$",size=16)
plt.xlabel(r"$k$",size=16)
plt.grid(True)
# savefig("fig9-1.png")
plt.show()

x=np.linspace(0,2*np.pi,129)
x=x[:-1]
y=np.sin(5*x)
Y=np.fft.fftshift(np.fft.fft(y))/128.0
w = np.linspace(-64,64,129)
w=w[:-1]
plt.figure()
plt.subplot(2,1,1)
plt.plot(w,abs(Y),lw=2)
plt.xlim([-10,10])
```

```
plt.ylabel(r"$|Y|$",size=16)
plt.title(r"Spectrum of $\sin(5t)$")
plt.grid(True)
plt.subplot(2,1,2)
plt.plot(w,np.angle(Y),'ro',lw=2)
ii=np.where(np.abs(Y)>1e-3)
plt.plot(w[ii],np.angle(Y[ii]),'go',lw=2)
plt.xlim([-10,10])
plt.ylabel(r"Phase of $Y$",size=16)
plt.xlabel(r"$k$",size=16)
plt.grid(True)
# savefig("fig9-2.png")
plt.show()

# Now we look at AM modulation. The function we want to analyse is
# f(t)=(1+0.1 cos(t)) cos(10t)

t=np.linspace(0,2*np.pi,129)
t=t[:-1]
y=np.multiply((1+0.1*np.cos(t)),np.cos(10*t))
Y=np.fft.fftshift(np.fft.fft(y))/128.0
w = np.linspace(-64,64,129)
w=w[:-1]
plt.figure()
plt.subplot(2,1,1)
plt.plot(w,abs(Y),lw=2)
plt.xlim([-15,15])
plt.ylabel(r"$|Y|$",size=16)
plt.title(r"Spectrum of $\left(1+0.1\cos\left(t\right)\right)\cos\left(10
plt.grid(True)
plt.subplot(2,1,2)
plt.plot(w,np.angle(Y),'ro',lw=2)
# ii=np.where(np.abs(Y)>1e-3)
# plt.plot(w[ii],np.angle(Y[ii]),'go',lw=2)
plt.xlim([-15,15])
plt.ylabel(r"Phase of $Y$",size=16)
plt.xlabel(r"$k$",size=16)
plt.grid(True)
# savefig("fig9-2.png")
plt.show()

t=np.linspace(-4*np.pi,4*np.pi,513)
t=t[:-1]
y=np.multiply((1+0.1*np.cos(t)),np.cos(10*t))
```

```
Y=np.fft.fftshift(np.fft.fft(y))/512.0
w = np.linspace(-64,64,513)
w=w[:-1]
plt.figure()
plt.subplot(2,1,1)
plt.plot(w,abs(Y),lw=2)
plt.xlim([-15,15])
plt.ylabel(r"$|Y|$",size=16)
plt.title(r"Spectrum of $\left(1+0.1\cos\left(t\right)\right)\cos\left(10
plt.grid(True)
plt.subplot(2,1,2)
plt.plot(w,np.angle(Y),'ro',lw=2)
# ii=np.where(np.abs(Y)>1e-3)
# plt.plot(w[ii],np.angle(Y[ii]),'go',lw=2)
plt.xlim([-15,15])
plt.ylabel(r"Phase of $Y$",size=16)
plt.xlabel(r"$k$",size=16)
plt.grid(True)
# savefig("fig9-2.png")
plt.show()

#Question 2:

#sin(t)^3:
t=np.linspace(-4*np.pi,4*np.pi,513)
t=t[:-1]
y=np.multiply(np.multiply(np.sin(t),np.sin(t)),np.sin(t))
Y=np.fft.fftshift(np.fft.fft(y))/512.0
w = np.linspace(-64,64,513)
w=w[:-1]
plt.figure()
plt.subplot(2,1,1)
plt.plot(w,abs(Y),lw=2)
plt.xlim([-15,15])
plt.ylabel(r"$|Y|$",size=16)
plt.title(r"Spectrum of sin(t)^3")#$\left(1+0.1\cos\left(t\right)\right)\
plt.grid(True)
plt.subplot(2,1,2)
plt.plot(w,np.angle(Y),'ro',lw=2)
# ii=np.where(np.abs(Y)>1e-3)
# plt.plot(w[ii],np.angle(Y[ii]),'go',lw=2)
plt.xlim([-15,15])
plt.ylabel(r"Phase of $Y$",size=16)
plt.xlabel(r"$k$",size=16)
```

```
plt.grid(True)
# savefig("fig9-2.png")
plt.show()
#This is correct because we know that frequency components 1 and 3 exist

#cos(t)^3:
t=np.linspace(-4*np.pi,4*np.pi,513)
t=t[:-1]
y=np.multiply(np.multiply(np.cos(t),np.cos(t)),np.cos(t))
Y=np.fft.fftshift(np.fft.fft(y))/512.0
w = np.linspace(-64,64,513)
w=w[:-1]
plt.figure()
plt.subplot(2,1,1)
plt.plot(w,abs(Y),lw=2)
plt.xlim([-15,15])
plt.ylabel(r"$|Y|$",size=16)
plt.title(r"Spectrum of cos(t)^3")#$\left(1+0.1\cos\left(t\right)\right)\
plt.grid(True)
plt.subplot(2,1,2)
plt.plot(w,np.angle(Y),'ro',lw=2)
# ii=np.where(np.abs(Y)>1e-3)
# plt.plot(w[ii],np.angle(Y[ii]),'go',lw=2)
plt.xlim([-15,15])
plt.ylabel(r"Phase of $Y$",size=16)
plt.xlabel(r"$k$",size=16)
plt.grid(True)
# savefig("fig9-2.png")
plt.show()
#This is correct because we know that frequency components 1 and 3 exist

#Q3:

#cos(20t+5cos(t)):
t=np.linspace(-4*np.pi,4*np.pi,513)
t=t[:-1]
y=np.cos(20*t+5*np.cos(t))
Y=np.fft.fftshift(np.fft.fft(y))/512.0
w = np.linspace(-64,64,513)
w=w[:-1]
plt.figure()
plt.subplot(3,1,1)
plt.plot(w,abs(Y),lw=2)
plt.xlim([-40,40])
```

```
plt.ylabel(r"$|Y|$",size=16)
plt.title(r"Spectrum of 20t+5cos(t)")#$\left(1+0.1\cos\left(t\right)\righ
plt.grid(True)
plt.subplot(3,1,2)
plt.plot(w,np.angle(Y),'ro',lw=2)
# ii=np.where(np.abs(Y)>1e-3)
# plt.plot(w[ii],np.angle(Y[ii]),'go',lw=2)
plt.xlim([-40,40])
plt.ylabel(r"Phase of $Y$",size=16)
plt.xlabel(r"$k$",size=16)
plt.grid(True)
plt.subplot(3,1,3)
# plt.plot(w,np.angle(Y),'ro',lw=2)
ii=np.where(np.abs(Y)>1e-3)
plt.plot(w[ii],np.angle(Y[ii]),'go',lw=2)
plt.xlim([-40,40])
plt.ylabel(r"Phase of $Y$")#,size=16)
plt.xlabel(r"$k$",size=16)
plt.grid(True)
# savefig("fig9-2.png")
plt.show()

# What is happenning?

#Q4:
Time_start=-200
Time_end=200
N_pts=1024
t=np.linspace(Time_start,Time_end,N_pts+1)
t=t[:-1]
y=np.exp(-0.5*np.multiply(t,t))
Y=np.fft.fftshift(np.fft.fft(y))/float(N_pts)
w = np.linspace(-64,64,N_pts+1)
w=w[:-1]
plt.figure()
plt.subplot(2,1,1)
plt.plot(w,abs(Y),lw=2)
plt.xlim([-30,30])
plt.ylabel(r"$|Y|$",size=16)
plt.title(r"Spectrum of the gaussian function")#$\left(1+0.1\cos\left(t\r
plt.grid(True)
plt.subplot(2,1,2)
plt.plot(w,np.angle(Y),'ro',lw=1)
# ii=np.where(np.abs(Y)>1e-3)
```

```
# plt.plot(w[ii],np.angle(Y[ii]),'go',lw=2)
plt.xlim([-30,30])
plt.ylabel(r"Phase of $Y$",size=16)
plt.xlabel(r"$k$",size=16)
plt.grid(True)
# savefig("fig9-2.png")
plt.show()

# Time_start=-10, Time_end=10 doesn't give a smooth gaussian.
# Time_start=-20, Time_end=20 gives a smooth gaussian.
# As we increase time range, higher frequencies come in to the spectrum
# and the peak at highest frequency decreases.

################
# END OF CODE #
################
```