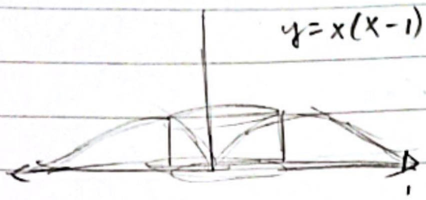
HW #2

Stewart 5.3 #1, 5, 9, 15, 37, 45

1)



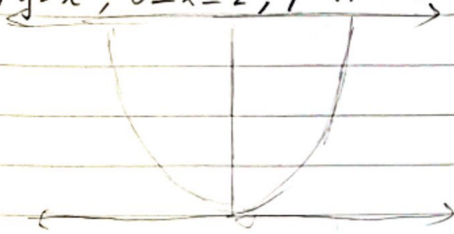
It is awkward to use the slicer method because we would have to subtract the center empty space, while shell method accounts for this.

circumference: $2\pi x$

height: $x(x-1)^2$

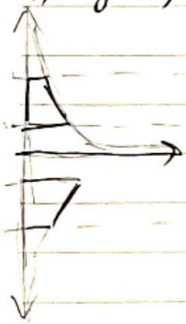
$$V = 2\pi \int_0^1 x(x(x-1)^2)\,dx =$$
$$2\pi \int_0^1 x(x(x^2-2x+1))\,dx =$$
$$2\pi \int_0^1 x^4 - 2x^3 + x^2\,dx =$$
$$2\pi \left[\tfrac{1}{5}x^5 - \tfrac{1}{2}x^4 + \tfrac{1}{3}x^3\right]_0^1 =$$
$$2\pi\left(\tfrac{1}{5} - \tfrac{1}{2} + \tfrac{1}{3}\right) = \tfrac{\pi}{15}$$

5) $y = x^2,\ 0 \le x \le 2,\ y = 4,\ x = 0$



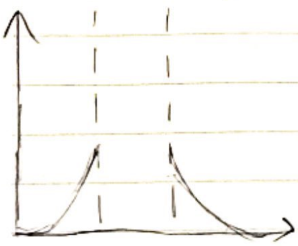
$$2\pi \int_0^2 x(x^2)\,dx = 2\pi\left[\tfrac{1}{4}x^4\right]_0^2 =$$
$$2\pi(4) = 8\pi$$

9) $xy = 1,\ x = 0,\ y = 1,\ y = 3$



$$2\pi \int_1^3 y\left(\tfrac{1}{y}\right)dy = 2\pi\left[y\right]_1^3 = 2\pi(3-1) = 4\pi$$

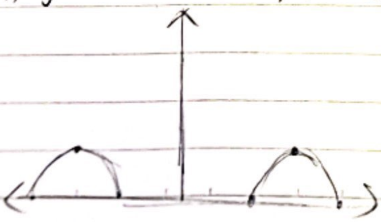
15) $y = x^4,\ y = 0,\ x = 1$, about $x = 2$



$$2\pi \int_0^1 x^4(2-x)\,dx = 2\pi \int_0^1 2x^4 - x^5\,dx = 2\pi\left[\tfrac{2}{5}x^5 - \tfrac{1}{6}x^6\right.$$
$$2\pi\left(\tfrac{2}{5} - \tfrac{1}{6}\right) = \tfrac{7}{15}\pi$$

37) $y = -x^2 + 6x - 8,\ y = 0$, about y-axis

Shell method



$$2\pi \int_2^4 x(-x^2+6x-8)\,dx =$$
$$2\pi \int_2^4 -x^3 + 6x^2 - 8x\,dx =$$
$$2\pi\left[-\tfrac{1}{4}x^4 + 2x^3 - 4x^2\right]_2^4 =$$
$$2\pi(-64+128-64+4-16+16) = 8\pi$$

45) A sphere of radius r

$y^2 + x^2 = r^2$

$4\pi \int_0^r x\sqrt{r^2 - x^2}\, dx =$

$y = \sqrt{r^2 - x^2}$

let $u = r^2 - x^2$

$4\pi \left[-\frac{1}{3} \sqrt[2]{(r^2 - x^2)^3} \right]_0^r$

$du = -2x\,dx$

$dx = \frac{du}{-2x}$

$-\frac{4\pi}{3} \left((r^2)^{3/2} - (r^2)^{3/2} - (r^2)^{3/2} - 0 \right)$

$\frac{x\sqrt{u}}{-2x} (du)$

$-\frac{4\pi}{3} \left(-r^3 \right) = \frac{4\pi r^3}{3}$

$-\frac{1}{2} u^{3/2}$

$-\frac{1}{3} u^{3/2}$ $\quad \frac{3}{2} \cdot \frac{1}{3} \quad \frac{1}{2}$