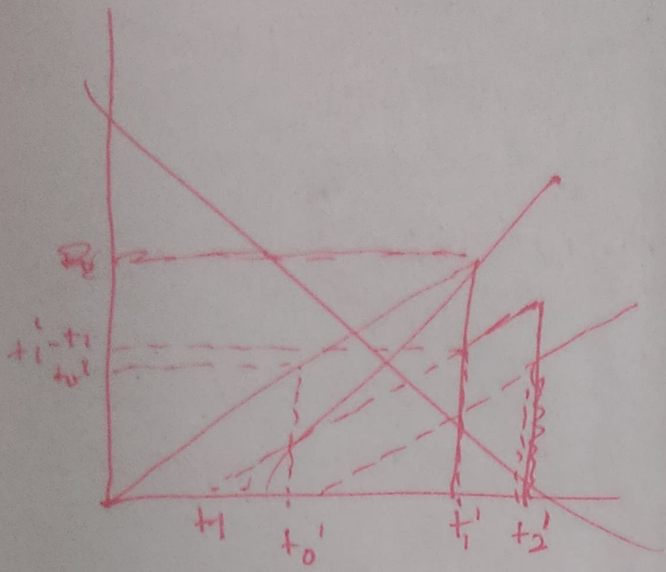
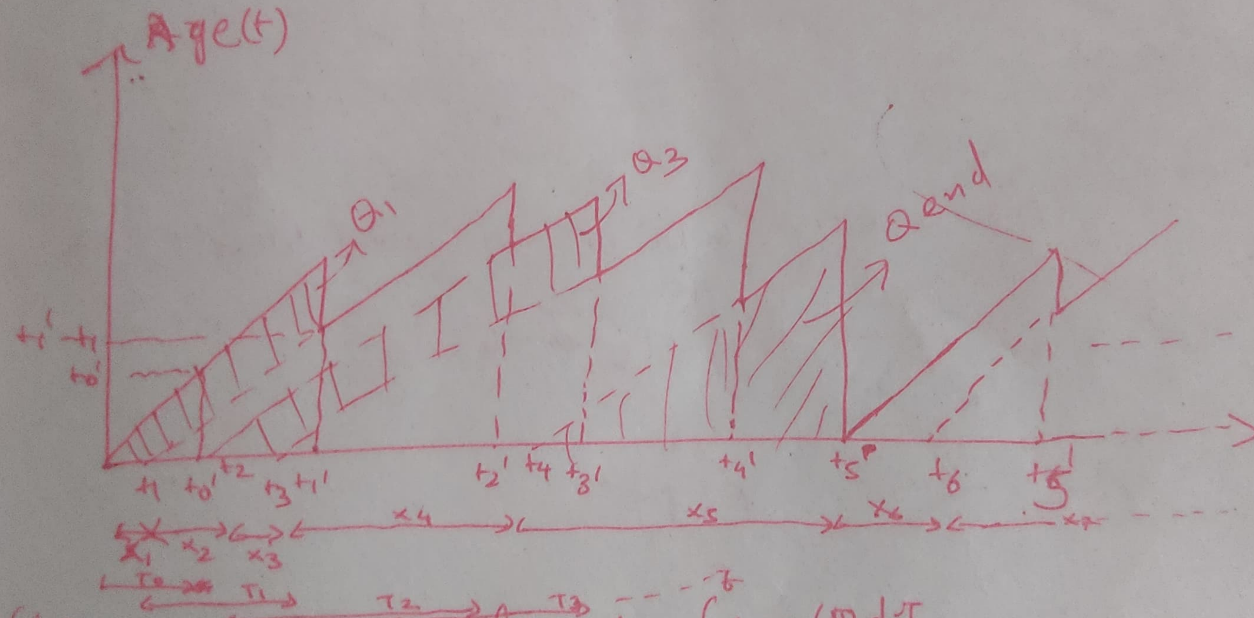
Comn : End Semester Exam

1) (a) Based on the Modified Age function, Here is the Sample Outcome :





(b) Time average age (Δ) $= \lim_{t \to \infty} \frac{1}{t} \int_0^t Age(\tau)\, d\tau$

$$\underbrace{\lim_{t\to\infty} \frac{1}{t} \sum_{i=1}^{N(t)} R_n}_{L_{N(t)}} \le \Delta \le \underbrace{\lim_{t\to\infty} \frac{1}{t} \sum_{i=1}^{N(t)+1} R_n}$$

$$= \lim_{t\to\infty} \frac{N(t)}{t} \frac{\sum_{i=1}^{N(t)} R_n}{N(t)}$$

$$= \frac{E[R_n]}{E(x^n)}$$

$$= \lim_{t\to\infty} \frac{N(t)+1}{t} \frac{\sum_{i=1}^{N(t)+1} R_n}{N(t)+1}$$

$$= \frac{E[R_n]}{E(x^n)}$$

By sandwich Theorem $\Rightarrow$ $\Delta = \dfrac{E[R_n]}{E[x^n]}$

WP

Using Wald's ~~Ird~~ Theorem

$E(X_J) = E[S_J] = E[\sum_{i=1}^{J} X_i] = E(X)E(J)$

$J \Rightarrow$ stoping Trial at which empty system is obser

Now $\int_{S_n^n}^{S_{n+1}^n} L(\tau)\,d\tau = \theta_1 + \theta_2 + \dots + \theta_{end} = \left(\sum_{i=N(S_n^{(n)})}^{N(S_{n+1}^n)-1} X_i T_i + \frac{X_i^2}{2}\right) + \theta_{en}$

$\theta_{end} = \cancel{\left(T_{N(S_{n+1}^n)} - \dots\right) X_{N(S_n^n - 1)}} + \frac{X_{N(S_{n+1}^n)}^2}{2}$

(A1) (6) continued

side Note
since this is an M/M/1 system, (memoryless arrivals and service)

clearly

~~E(J)~~

$$E\left[\sum_{i=1}^{J} x_i - \sum_{i=0}^{J-1} v_i\right] = \bar{x}$$

$$\therefore (\bar{x} - \bar{v}) E(J) = \bar{x}$$

$$\therefore E(J) = \frac{\bar{x}}{\bar{x} - \bar{v}}$$

and $E[x^n] = \frac{\bar{x}^2}{\bar{x} - \bar{v}}$

~~$E[R_n] \Rightarrow E[T_i x_i] = E[(W_i + S_i) x_i] = E[W_i x_i] + E[S_i] E[x_i]$~~

~~$E[W_i \mid x_i = x] = E[(T_i - x)^+ \mid x_i = x]$~~

$= E[(T_i - x)^+]$

$= E[(S_i + \bar{v} - x)^+]$

$E[x^n]$

$$\lim_{r \to \infty} \int_{S_n^r}^{S_{n+1}^r} \frac{L(\tau)}{r} d\tau = \frac{E[xT] + \frac{E[x^2]}{2}}{E(x)}$$

$$\therefore \Delta = \frac{\bar{x} - \bar{v}}{\bar{x}^2} \left(\frac{E[xT] + \frac{E[x^2]}{2}}{E(x)} \right)$$

for M/M/1, $\bar{x} = 1/\lambda$, $E(x^2) = 2/\lambda$

$$\lim_{t\to\infty} \int_{S_n^T}^{S_{n+1}} \frac{L(T)}{t}\,dt = \lim_{t\to\infty} \frac{1}{t} \sum_{i=N(S_n^T)}^{N(S_{n+1}^T)+1} X_i T_i + \frac{1}{t} \sum_{i=N(S_n^T)} \frac{X_i^2}{2}$$

$$\cancel{+\frac{1}{t}\left(S_n (S_{n+1}) \ldots n S_n^{T+1}\right)}$$

clearly $\lim_{t\to\infty} \frac{\left(N(S_{n+1}^T)+1 - N(S_n^T)\right)}{t}$

$$\cancel{E(R_n) \geq \frac{E}{E[X]}}$$

$$= E[X] = \bar{x}$$

$$\therefore E[R_n] = \frac{E(XT) + E[X^2]/2}{E[X]}$$

$$\therefore \lim_{t\to\infty} \Delta_T = \Delta = \left(\frac{\bar{x} - \bar{V}}{\bar{x}^2}\right) \times \frac{E[XT] + E(X^2)/2}{(\bar{x})}$$

~~for~~ we use $E(XT)$ result directly from paper for M/M/1 system as

$$E[XT] = \frac{1}{\mu^2} \frac{\rho}{1-\rho}$$

$$\cancel{= \frac{1}{\mu^2} \frac{\lambda/\mu}{(1-\lambda/\mu)}} =$$

where

$$\rho = \frac{\lambda}{\mu}$$

and $\bar{X} = \frac{1}{\lambda}$

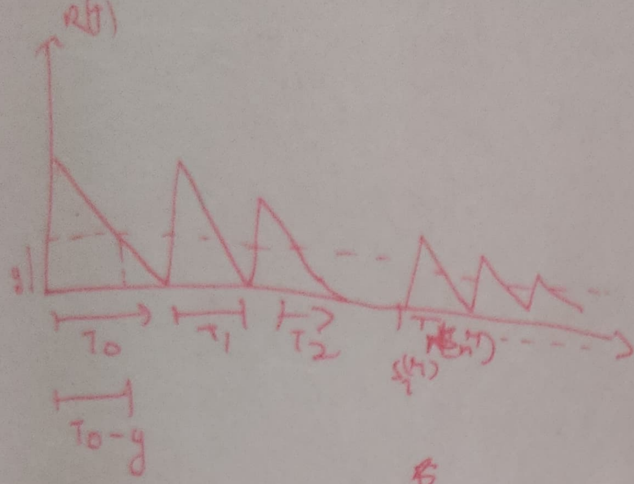
$$\bar{V} = \frac{1}{\mu}$$

$$\therefore \Delta = \frac{1/\lambda - 1/\mu}{\frac{1}{\lambda^3}} \left(\frac{\rho}{\mu^2(1-\rho)} + \frac{1}{\lambda^2}\right)$$

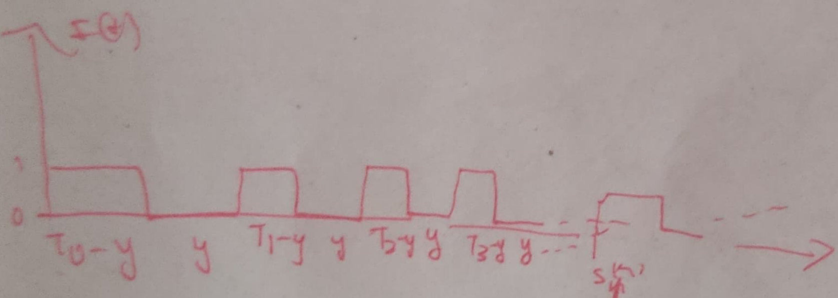
$$= (1-\rho)\left(\frac{\rho^3}{1-\rho} + 1\right) = \rho^3 + (1-\rho)$$

(i) we intend to find $\lim_{t\to\infty} \frac{1}{t} \int_0^\infty P(R(t) \geq y)\, dy$

If we try to formulate the Residual Time being greater than y, then we have



$$I\{R(t) \geq y\} = \begin{cases} 1 & R(t) > y \\ 0 & \text{otherwise} \end{cases}$$



∴ we have

$$\cancel{E(R_n) = E[X_n - y] = E[(T_n - y)\, I\{T_n - y \geq 1\}]}$$

$$\cancel{\sum \{E[T_n \mid T_n \geq y] - y\}\, P[T_n > y]}$$

$$E[R_n] = \sum_{i=N(S_n^{(n)})}^{N(S_{n+1}^{(n)})} E[(T_i - y)^+]$$

and understandable fuck ?

↳ st is possible

$$\lim_{t\to\infty} \frac{1}{t} \int_0^\infty P[T > y]\, dy = \frac{E(R_n)}{E[X_n]}$$

A2) Customer Service counting Process
~~Avg Th~~

$\downarrow$

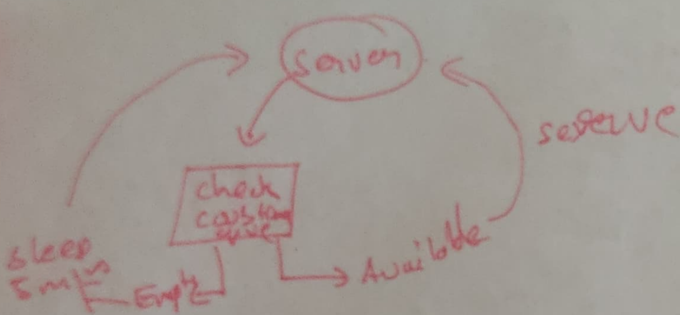
$C(t) \Rightarrow$ seq of $Y_1, Y_2, Y_3 \cdots$

↳ Interarrival Time $= u_i$'s iid $\sim$ Uniform

Arrival of customer in the system

↳ $A(t) \to S_1, S_2, \cdots S_n \cdots$

↳ Interarrival Times x_i's $\sim$ uniform



Let the The customer be waiting in the system ~~for~~ to be served by the server in queue ~~for~~ denoted by $W(t)$

$$E(W(t)) = E\left[\sum_{i=0}^{l} u_i\right] + E\left[\text{Time customer served in the server}\right]$$

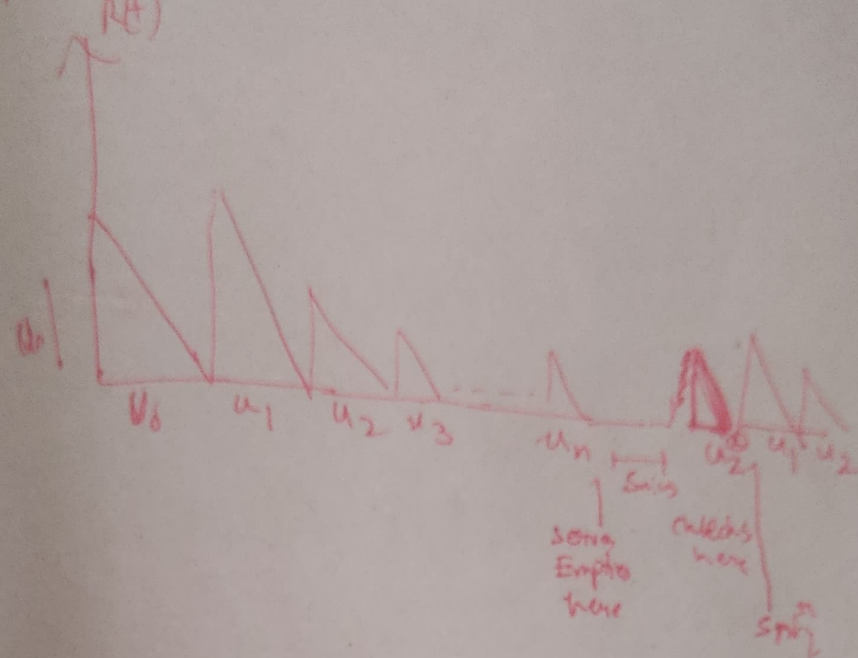
$$= E(l)\,E(u) + E(\;)$$

By Little's theorem

$$E(l) = \frac{E(W)}{E(X)} \quad \text{wP1}$$

$$\therefore E[W] = \frac{\overline{W}\, E(u)}{E(X)} + E(R(t)) \quad \text{wP1}$$

If we ~~see the~~ model $R(t)$ as a reward for CAT^{th} Residual times, we see a following Residual pattern



u_Z^* Represents time the packet waited when ~~sender~~ was in sleep $\sim$ uniform$(0,S)$

by using Reward-Renewal theorem

$$\cancel{E(R(t)) = \frac{E[R_n]}{E[X]}} =$$

$$\int_0^{S^T_{N^T(t)}} R(\tau)\,d\tau = \underbrace{\sum_{i=0}^{N^T(S_n^T(t))-1}}_{\text{By Renewal Reward Theorem}} \frac{u_i^2}{2} + \frac{u_Z^2}{2}$$

$$\le \sum_{i=0}^{N(t)} \frac{u_i^2}{2} + \frac{u_Z^2}{2}$$

Similarly

$$\int_0^{S^T(N^T(t))+1} R(\tau)\,d\tau = \left(\sum_{i=0}^{N^T(S_n^T(t)+1)} \frac{u_i^2}{2}\right) + \frac{u_Z^2}{2}$$

$$\therefore \lim_{t\to\infty} \int_0^t \frac{1}{t} R(\tau)\, d\tau = \lim_{t\to\infty} \frac{\sum_{i=1}^{N^\gamma(t)} R_i}{t} + \frac{\frac{1}{2}u_2^2}{N(t)}$$

$$= \left(\lim_{t\to\infty} \sum_{i=0}^{N^\gamma(t)} \frac{R_i}{t}\right) + \frac{E[u_2^2]}{2E[x]}$$

$$= \left(\lim_{t\to\infty} \sum_{i=0}^{N(t)} \frac{V_i^2}{2N(t)} \times \frac{N(t)}{t}\right) + \frac{E[u_2^2]}{2E[x]}$$

$$\lim_{t\to\infty} E[R(t)] = \qquad = \frac{E[u^2] + E[u_2^2]}{2E[x]}$$

w.p.1

$$\therefore E[W(t)] = \frac{E(u^2) + E(u_2^2)}{2E[x]} + \frac{E[u]\,\bar{w}^{q}}{E[x]}$$

expected unfinished work

$$E[\cancel{\text{tota}}\ T_i] = E[u] + E[W(t)]$$

$$E[u] = 7.5 \text{ mins} \qquad \left(\int_5^{10} \frac{x}{5}\, dx\right)$$

$$E[u^2] = \int_5^{10} \frac{x^2}{5}\, dx$$

$$= 58.33$$

$$E[u_2^2] = \int_0^5 \frac{x^2}{5}\, dx = 8.33$$

$\bar{w}^q$ is the sample-path average waiting delay.