Assignment-1(PCS)

27 January 2023 03:56 PM

Ans-1)

$$f(t) = A_1\cos(\omega_1 t + \theta_1) + A_2\cos(\omega_2 t + \theta_2)$$

when $\omega_1 = \omega_2 = \omega$

$$f(t) = A_1\cos(\underbrace{\omega t + \theta_1}_{\phi_1}) + A_2\cos(\overbrace{\omega t + \theta_2}^{\phi_2})$$

$$P_\infty = \lim_{T\to\infty}\left\{\frac{1}{2T}\int_{-T}^{T}(f(t))^2 dt\right\} = \lim_{T\to\infty}\left[\frac{A_1^2}{2T}\int_{-\omega T+\theta_1}^{\omega T+\theta_1}\cos^2\phi_1 \frac{d\phi_1}{\omega} + \frac{A_2^2}{2T}\int_{-\omega T+\theta_2}^{\omega T+\theta_2}\cos^2\phi_2 \frac{d\phi_2}{\omega} + \frac{A_1A_2}{2T}\int_{-T}^{T}2\cos(\phi_1)\cos(\phi_2)\,dt\right]$$

$$= \frac{A_1^2}{2}\lim_{T\to\infty}\left[\frac{1}{2T\omega}(\sin(\theta_1) - \sin(\theta_1) + 2\omega T)\right] + \frac{A_2^2}{2}\lim_{T\to\infty}\left[\frac{1}{2T\omega}(\sin(\theta_2) - \sin(\theta_2) + 2\omega T)\right]$$

$$+ A_1A_2\lim_{T\to\infty}\left[\frac{1}{2T}\int_{-T}^{T}(\cos(2\omega t + \theta_1 + \theta_2) + \cos(\theta_1 - \theta_2))dt\right]$$

$$= \frac{A_1^2}{2} + \frac{A_2^2}{2} + A_1A_2\left(0 + \cos(\theta_1 - \theta_2)\lim_{T\to\infty}\left[\frac{1}{2T}\times 2T\right]\right)$$

$$\Rightarrow \therefore \boxed{P_\infty = \frac{A_1^2}{2} + \frac{A_2^2}{2} + A_1A_2\cos(\theta_1 - \theta_2)}$$

Ans-2)

(i) $y(t) = \frac{1}{x(t)}\left(\frac{d\,x(t)}{dt}\right)^2$

$x_1(t) \to y_1(t)$

$x_2(t) \to y_2(t)$

$$a x_1(t) + b x_2(t) \Rightarrow y_L(t) = \frac{1}{a x_1(t) + b x_2(t)}\left(\frac{a\,d\,x_1(t)}{dt} + \frac{b\,d\,x_2(t)}{dt}\right)^2$$

$$\neq \frac{1 \times a^2}{a x_1(t)}\left(\frac{d(x_1(t))}{dt}\right)^2 + \frac{1\,b^2}{b x_2(t)}\left(\frac{d(x_2(t))}{dt}\right)^2$$

$$\neq a y_1(t) + b y_2(t)$$

$\therefore$ Not Linear

when

$x(t) \Rightarrow x(t-\tau)$

$\Downarrow$

$$y_L(t) = \frac{1}{x(t-\tau)}\left(\frac{d(x(t-\tau))}{dt}\right)^2$$

$$= y(t-\tau)$$

$\therefore$ Time Invarient

(ii) $y(t) = \int_{-\infty}^{t} x(\tau)\, e^{(-(t-\tau))}\, d\tau$

$$y(t) = \int_{-\infty} x(\tau)\, e \quad d\tau$$

$$\left.\begin{array}{l} x_1(t) \to y_1(t) \\ x_2(t) \to y_2(t) \end{array}\right\} \quad a x_1(t) + b x_2(t) \to y_L(t)$$

$$= \int_{-\infty}^{t} \left(a x_1(\tau) + b x_2(\tau)\right) e^{-(t-\tau)} d\tau$$

$$= a \int_{-\infty}^{t} x_1(\tau) e^{-(t-\tau)} d\tau + b \int_{-\infty}^{t} x_2(\tau) e^{-(t-\tau)} d\tau$$

$$= a y_1(t) + b y_2(t)$$

$\therefore$ Linear

$$x(t) \to x(t - t_0)$$

$$y_L(t) = \int_{-\infty}^{t} x(\tau - t_0)\, e^{-(t-\tau)} d\tau$$

$$y(t - t_0) = \int_{-\infty}^{t - t_0} x(\tau)\, e^{-(t - t_0 - \tau)} d\tau$$

$$= \int_{-\infty}^{t} x(k - t_0)\, e^{-(t-k)} dk \quad \left[\text{where } k = \tau + t_0\right]$$

$$= \int_{-\infty}^{t} x(\tau - t_0)\, e^{-(t-\tau)} d\tau \quad (\text{where } \tau = k)$$

$$= y_L(t)$$

$\therefore$ Time Invarient

(iii) $$y(t) = \sum_{n=-\infty}^{\infty} x[t]\, \delta(t - nT)$$

$$x_1(t) \to y_1(t)$$

$$x_2(t) \to y_2(t)$$

$$a x_1(t) + b x_2(t) \to y_L(t) = \sum_{n=-\infty}^{\infty} \left(a x_1(t) + b x_2(t)\right) \delta(t - nT)$$

$$= \sum_{n=-\infty}^{\infty} a x_1(t)\, \delta(t - nT) + \sum_{n=-\infty}^{\infty} b x_2(t)\, \delta(t - nT)$$

$$= a y_1(t) + b y_2(t)$$

$\therefore$ Linear

$$x(t) \to x(t - \tau)$$

$$y_L(t) = \sum_{n=-\infty}^{\infty} x(t - \tau)\, \delta(t - nT)$$

$$\neq \sum_{n=-\infty}^{\infty} x(t - \tau)\, \delta(t - \tau - nT)$$

$$\neq y(t-\tau)$$

$\therefore$ Time Varient

Ans-3)

Problem 3 For a sinusoidal input $s(t) = cos(2\pi f_0 t + \theta)$, prove that the response of an LTI system with real-valued impulse response $h(t)$ is given by

$$y(t) = (s * h)(t) = |H(f_0)| cos(2\pi f_0 t + \theta + \Phi)$$

where $|H(f_0)|$ is the magnitude of the transfer function at $f = f_0$, and Φ is the phase of the transfer function.

$$s(t) = \cos(2\pi f_0 t + \theta)$$

$$= \frac{1}{2}\left(e^{j(2\pi f_0 t + \theta)} + e^{-j(2\pi f_0 t + \theta)}\right)$$

$$= \frac{e^{j\theta}}{2} e^{j2\pi f_0 t} + \frac{e^{-j\theta}}{2} e^{-2\pi j f_0 t}$$

In F.S representation

with $a_1 = e^{j\theta}/2$

$a_{-1} = e^{-j\theta}/2$

Now $(h(t)) * (a_k e^{jk\omega_0 t})$

$$= a_k \int_{-\infty}^{\infty} h(\tau) e^{k\omega_0 j(t-\tau)} d\tau$$

$$= a_k e^{jk\omega_0 t} \int_{-\infty}^{\infty} h(\tau) e^{-k\omega_0 j\tau} d\tau$$

$$= a_k e^{jk\omega_0 t} H(jk\omega_0)$$

$$\therefore s(t) * h(t) = y(t) = \frac{e^{j\theta} H(j\omega_0) e^{j\omega_0 t}}{2} + \frac{e^{-j\theta}}{2} H(-j\omega_0) e^{-j\omega_0 t}$$

Now $H(j\omega_0) = \underbrace{|H(j\omega_0)|}_{\text{magnitude}} e^{j\phi}$

$\phi \rightarrow$ phase

or

$H(j\omega)$ $H(f_0) = |H(f_0)| e^{j\phi}$

and

$H(-j\omega)$ $H(-f_0) = |H(f_0)| e^{-j\phi}$

$$\therefore y(t) = \left[\frac{e^{j(\theta+\phi)} e^{j\omega_0 t}}{2} + \frac{e^{-j(\theta+\phi)} e^{-j\omega_0 t}}{2}\right] |H(f_0)|$$

$$\therefore y(t) = |H(f_0)| \cos(\omega_0 t + \theta + \phi)$$

where $\omega_0 = 2\pi f_0$

$\therefore \boxed{y(t) = |H(f_0)| \cos(\omega_0 t + \theta + \phi)}$

Answer-4 on Matlab and Reports