

Mathematical methods for physics and engineering
A comprehensive guide

The new edition of this highly acclaimed textbook contains several major additions, including more than four hundred new exercises (with hints and answers). To match the mathematical preparation of current senior college and university entrants, the authors have included a preliminary chapter covering areas such as polynomial equations, trigonometric identities, coordinate geometry, partial fractions, binomial expansions, induction, and the proof of necessary and sufficient conditions. Elsewhere, matrix decomposition, nearly singular matrices and non-square sets of linear equations are treated in detail. The presentation of probability has been reorganised and greatly extended, and includes all physically important distributions. New topics covered in a separate statistics chapter include estimator efficiency, distributions of samples, *t*- and *F*-tests for comparing means and variances, applications of the chi-squared distribution, and maximum-likelihood and least-squares fitting. In other chapters the following topics have been added: linear recurrence relations, curvature, envelopes, curve sketching, and more refined numerical methods.

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K. F. Riley, M. P. Hobson and S. J. Bence

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Contents

<i>Preface to the second edition</i>	xix
<i>Preface to the first edition</i>	xxi
1 Preliminary algebra	1
1.1 Simple functions and equations	1
<i>Polynomial equations; factorisation; properties of roots</i>	
1.2 Trigonometric identities	10
<i>Single angle; compound-angles; double- and half-angle identities</i>	
1.3 Coordinate geometry	15
1.4 Partial fractions	18
<i>Complications and special cases</i>	
1.5 Binomial expansion	25
1.6 Properties of binomial coefficients	27
1.7 Some particular methods of proof	30
<i>Proof by induction; proof by contradiction; necessary and sufficient conditions</i>	
1.8 Exercises	36
1.9 Hints and answers	39
2 Preliminary calculus	42
2.1 Differentiation	42
<i>Differentiation from first principles; products; the chain rule; quotients; implicit differentiation; logarithmic differentiation; Leibnitz' theorem; special points of a function; curvature; theorems of differentiation</i>	

CONTENTS

2.2	Integration	60
	<i>Integration from first principles; the inverse of differentiation; by inspection; sinusoidal functions; logarithmic integration; using partial fractions; substitution method; integration by parts; reduction formulae; infinite and improper integrals; plane polar coordinates; integral inequalities; applications of integration</i>	
2.3	Exercises	77
2.4	Hints and answers	82
3	Complex numbers and hyperbolic functions	86
3.1	The need for complex numbers	86
3.2	Manipulation of complex numbers	88
	<i>Addition and subtraction; modulus and argument; multiplication; complex conjugate; division</i>	
3.3	Polar representation of complex numbers	95
	<i>Multiplication and division in polar form</i>	
3.4	de Moivre's theorem	98
	<i>trigonometric identities; finding the nth roots of unity; solving polynomial equations</i>	
3.5	Complex logarithms and complex powers	102
3.6	Applications to differentiation and integration	104
3.7	Hyperbolic functions	105
	<i>Definitions; hyperbolic–trigonometric analogies; identities of hyperbolic functions; solving hyperbolic equations; inverses of hyperbolic functions; calculus of hyperbolic functions</i>	
3.8	Exercises	112
3.9	Hints and answers	116
4	Series and limits	118
4.1	Series	118
4.2	Summation of series	119
	<i>Arithmetic series; geometric series; arithmetico-geometric series; the difference method; series involving natural numbers; transformation of series</i>	
4.3	Convergence of infinite series	127
	<i>Absolute and conditional convergence; series containing only real positive terms; alternating series test</i>	
4.4	Operations with series	134
4.5	Power series	134
	<i>Convergence of power series; operations with power series</i>	
4.6	Taylor series	139
	<i>Taylor's theorem; approximation errors; standard Maclaurin series</i>	
4.7	Evaluation of limits	144

CONTENTS

4.8	Exercises	147
4.9	Hints and answers	152
5	Partial differentiation	154
5.1	Definition of the partial derivative	154
5.2	The total differential and total derivative	156
5.3	Exact and inexact differentials	158
5.4	Useful theorems of partial differentiation	160
5.5	The chain rule	160
5.6	Change of variables	161
5.7	Taylor's theorem for many-variable functions	163
5.8	Stationary values of many-variable functions	165
5.9	Stationary values under constraints	170
5.10	Envelopes	176
5.11	Thermodynamic relations	179
5.12	Differentiation of integrals	181
5.13	Exercises	182
5.14	Hints and answers	188
6	Multiple integrals	190
6.1	Double integrals	190
6.2	Triple integrals	193
6.3	Applications of multiple integrals	194
	<i>Areas and volumes; masses, centres of mass and centroids; Pappus' theorems; moments of inertia; mean values of functions</i>	
6.4	Change of variables in multiple integrals	202
	<i>Change of variables in double integrals; evaluation of the integral $I = \int_{-\infty}^{\infty} e^{-x^2} dx$; change of variables in triple integrals; general properties of Jacobians</i>	
6.5	Exercises	210
6.6	Hints and answers	214
7	Vector algebra	216
7.1	Scalars and vectors	216
7.2	Addition and subtraction of vectors	217
7.3	Multiplication by a scalar	218
7.4	Basis vectors and components	221
7.5	Magnitude of a vector	222
7.6	Multiplication of vectors	223
	<i>Scalar product; vector product; scalar triple product; vector triple product</i>	

CONTENTS

7.7	Equations of lines, planes and spheres	230
7.8	Using vectors to find distances <i>Point to line; point to plane; line to line; line to plane</i>	233
7.9	Reciprocal vectors	237
7.10	Exercises	238
7.11	Hints and answers	244
8	Matrices and vector spaces	246
8.1	Vector spaces <i>Basis vectors; inner product; some useful inequalities</i>	247
8.2	Linear operators	252
8.3	Matrices	254
8.4	Basic matrix algebra <i>Matrix addition; multiplication by a scalar; matrix multiplication</i>	255
8.5	Functions of matrices	260
8.6	The transpose of a matrix	260
8.7	The complex and Hermitian conjugates of a matrix	261
8.8	The trace of a matrix	263
8.9	The determinant of a matrix <i>Properties of determinants</i>	264
8.10	The inverse of a matrix	268
8.11	The rank of a matrix	272
8.12	Special types of square matrix <i>Diagonal; triangular; symmetric and antisymmetric; orthogonal; Hermitian and anti-Hermitian; unitary; normal</i>	273
8.13	Eigenvectors and eigenvalues <i>Of a normal matrix; of Hermitian and anti-Hermitian matrices; of a unitary matrix; of a general square matrix</i>	277
8.14	Determination of eigenvalues and eigenvectors <i>Degenerate eigenvalues</i>	285
8.15	Change of basis and similarity transformations	288
8.16	Diagonalisation of matrices	290
8.17	Quadratic and Hermitian forms <i>Stationary properties of the eigenvectors; quadratic surfaces</i>	293
8.18	Simultaneous linear equations <i>Range; null space; N simultaneous linear equations in N unknowns; singular value decomposition</i>	297
8.19	Exercises	312
8.20	Hints and answers	319
9	Normal modes	322
9.1	Typical oscillatory systems	323

CONTENTS

9.2	Symmetry and normal modes	328
9.3	Rayleigh–Ritz method	333
9.4	Exercises	335
9.5	Hints and answers	338
10	Vector calculus	340
10.1	Differentiation of vectors	340
	<i>Composite vector expressions; differential of a vector</i>	
10.2	Integration of vectors	345
10.3	Space curves	346
10.4	Vector functions of several arguments	350
10.5	Surfaces	351
10.6	Scalar and vector fields	353
10.7	Vector operators	353
	<i>Gradient of a scalar field; divergence of a vector field; curl of a vector field</i>	
10.8	Vector operator formulae	360
	<i>Vector operators acting on sums and products; combinations of grad, div and curl</i>	
10.9	Cylindrical and spherical polar coordinates	363
10.10	General curvilinear coordinates	370
10.11	Exercises	375
10.12	Hints and answers	381
11	Line, surface and volume integrals	383
11.1	Line integrals	383
	<i>Evaluating line integrals; physical examples; line integrals with respect to a scalar</i>	
11.2	Connectivity of regions	389
11.3	Green’s theorem in a plane	390
11.4	Conservative fields and potentials	393
11.5	Surface integrals	395
	<i>Evaluating surface integrals; vector areas of surfaces; physical examples</i>	
11.6	Volume integrals	402
	<i>Volumes of three-dimensional regions</i>	
11.7	Integral forms for grad, div and curl	404
11.8	Divergence theorem and related theorems	407
	<i>Green’s theorems; other related integral theorems; physical applications</i>	
11.9	Stokes’ theorem and related theorems	412
	<i>Related integral theorems; physical applications</i>	
11.10	Exercises	415
11.11	Hints and answers	420

CONTENTS

12 Fourier series	421
12.1 The Dirichlet conditions	421
12.2 The Fourier coefficients	423
12.3 Symmetry considerations	425
12.4 Discontinuous functions	426
12.5 Non-periodic functions	428
12.6 Integration and differentiation	430
12.7 Complex Fourier series	430
12.8 Parseval's theorem	432
12.9 Exercises	433
12.10 Hints and answers	437
13 Integral transforms	439
13.1 Fourier transforms	439
<i>The uncertainty principle; Fraunhofer diffraction; the Dirac δ-function; relation of the δ-function to Fourier transforms; properties of Fourier transforms; odd and even functions; convolution and deconvolution; correlation functions and energy spectra; Parseval's theorem; Fourier transforms in higher dimensions</i>	
13.2 Laplace transforms	459
<i>Laplace transforms of derivatives and integrals; other properties of Laplace transforms</i>	
13.3 Concluding remarks	465
13.4 Exercises	466
13.5 Hints and answers	472
14 First-order ordinary differential equations	474
14.1 General form of solution	475
14.2 First-degree first-order equations	476
<i>Separable-variable equations; exact equations; inexact equations, integrating factors; linear equations; homogeneous equations; isobaric equations; Bernoulli's equation; miscellaneous equations</i>	
14.3 Higher-degree first-order equations	486
<i>Equations soluble for p; for x; for y; Clairaut's equation</i>	
14.4 Exercises	490
14.5 Hints and answers	494
15 Higher-order ordinary differential equations	496
15.1 Linear equations with constant coefficients	498
<i>Finding the complementary function $y_c(x)$; finding the particular integral $y_p(x)$; constructing the general solution $y_c(x) + y_p(x)$; linear recurrence relations; Laplace transform method</i>	

CONTENTS

15.2	Linear equations with variable coefficients	509
	<i>The Legendre and Euler linear equations; exact equations; partially known complementary function; variation of parameters; Green's functions; canonical form for second-order equations</i>	
15.3	General ordinary differential equations	524
	<i>Dependent variable absent; independent variable absent; non-linear exact equations; isobaric or homogeneous equations; equations homogeneous in x or y alone; equations having $y = Ae^x$ as a solution</i>	
15.4	Exercises	529
15.5	Hints and answers	535
16	Series solutions of ordinary differential equations	537
16.1	Second-order linear ordinary differential equations	537
	<i>Ordinary and singular points</i>	
16.2	Series solutions about an ordinary point	541
16.3	Series solutions about a regular singular point	544
	<i>Distinct roots not differing by an integer; repeated root of the indicial equation; distinct roots differing by an integer</i>	
16.4	Obtaining a second solution	549
	<i>The Wronskian method; the derivative method; series form of the second solution</i>	
16.5	Polynomial solutions	554
16.6	Legendre's equation	555
	<i>General solution for integer ℓ; properties of Legendre polynomials</i>	
16.7	Bessel's equation	564
	<i>General solution for non-integer v; general solution for integer v; properties of Bessel functions</i>	
16.8	General remarks	575
16.9	Exercises	575
16.10	Hints and answers	579
17	Eigenfunction methods for differential equations	581
17.1	Sets of functions	583
	<i>Some useful inequalities</i>	
17.2	Adjoint and Hermitian operators	587

CONTENTS

17.3	The properties of Hermitian operators	588
	<i>Reality of the eigenvalues; orthogonality of the eigenfunctions; construction of real eigenfunctions</i>	
17.4	Sturm–Liouville equations	591
	<i>Valid boundary conditions; putting an equation into Sturm–Liouville form</i>	
17.5	Examples of Sturm–Liouville equations	593
	<i>Legendre's equation; the associated Legendre equation; Bessel's equation; the simple harmonic equation; Hermite's equation; Laguerre's equation; Chebyshev's equation</i>	
17.6	Superposition of eigenfunctions: Green's functions	597
17.7	A useful generalisation	601
17.8	Exercises	602
17.9	Hints and answers	606
18	Partial differential equations: general and particular solutions	608
18.1	Important partial differential equations	609
	<i>The wave equation; the diffusion equation; Laplace's equation; Poisson's equation; Schrödinger's equation</i>	
18.2	General form of solution	613
18.3	General and particular solutions	614
	<i>First-order equations; inhomogeneous equations and problems; second-order equations</i>	
18.4	The wave equation	626
18.5	The diffusion equation	628
18.6	Characteristics and the existence of solutions	632
	<i>First-order equations; second-order equations</i>	
18.7	Uniqueness of solutions	638
18.8	Exercises	640
18.9	Hints and answers	644
19	Partial differential equations: separation of variables and other methods	646
19.1	Separation of variables: the general method	646
19.2	Superposition of separated solutions	650
19.3	Separation of variables in polar coordinates	658
	<i>Laplace's equation in polar coordinates; spherical harmonics; other equations in polar coordinates; solution by expansion; separation of variables for inhomogeneous equations</i>	
19.4	Integral transform methods	681
19.5	Inhomogeneous problems – Green's functions	686
	<i>Similarities to Green's functions for ordinary differential equations; general boundary-value problems; Dirichlet problems; Neumann problems</i>	

CONTENTS

19.6 Exercises	702
19.7 Hints and answers	708
20 Complex variables	710
20.1 Functions of a complex variable	711
20.2 The Cauchy–Riemann relations	713
20.3 Power series in a complex variable	716
20.4 Some elementary functions	718
20.5 Multivalued functions and branch cuts	721
20.6 Singularities and zeroes of complex functions	723
20.7 Complex potentials	725
20.8 Conformal transformations	730
20.9 Applications of conformal transformations	735
20.10 Complex integrals	738
20.11 Cauchy’s theorem	742
20.12 Cauchy’s integral formula	745
20.13 Taylor and Laurent series	747
20.14 Residue theorem	752
20.15 Location of zeroes	754
20.16 Integrals of sinusoidal functions	758
20.17 Some infinite integrals	759
20.18 Integrals of multivalued functions	762
20.19 Summation of series	764
20.20 Inverse Laplace transform	765
20.21 Exercises	768
20.22 Hints and answers	773
21 Tensors	776
21.1 Some notation	777
21.2 Change of basis	778
21.3 Cartesian tensors	779
21.4 First- and zero-order Cartesian tensors	781
21.5 Second- and higher-order Cartesian tensors	784
21.6 The algebra of tensors	787
21.7 The quotient law	788
21.8 The tensors δ_{ij} and ϵ_{ijk}	790
21.9 Isotropic tensors	793
21.10 Improper rotations and pseudotensors	795
21.11 Dual tensors	798
21.12 Physical applications of tensors	799
21.13 Integral theorems for tensors	803
21.14 Non-Cartesian coordinates	804

CONTENTS

21.15 The metric tensor	806
21.16 General coordinate transformations and tensors	809
21.17 Relative tensors	812
21.18 Derivatives of basis vectors and Christoffel symbols	814
21.19 Covariant differentiation	817
21.20 Vector operators in tensor form	820
21.21 Absolute derivatives along curves	824
21.22 Geodesics	825
21.23 Exercises	826
21.24 Hints and answers	831
22 Calculus of variations	834
22.1 The Euler–Lagrange equation	835
22.2 Special cases	836
<i>F does not contain y explicitly; F does not contain x explicitly</i>	
22.3 Some extensions	840
<i>Several dependent variables; several independent variables; higher-order derivatives; variable end-points</i>	
22.4 Constrained variation	844
22.5 Physical variational principles	846
<i>Fermat's principle in optics; Hamilton's principle in mechanics</i>	
22.6 General eigenvalue problems	849
22.7 Estimation of eigenvalues and eigenfunctions	851
22.8 Adjustment of parameters	854
22.9 Exercises	856
22.10 Hints and answers	860
23 Integral equations	862
23.1 Obtaining an integral equation from a differential equation	862
23.2 Types of integral equation	863
23.3 Operator notation and the existence of solutions	864
23.4 Closed-form solutions	865
<i>Separable kernels; integral transform methods; differentiation</i>	
23.5 Neumann series	872
23.6 Fredholm theory	874
23.7 Schmidt–Hilbert theory	875
23.8 Exercises	878
23.9 Hints and answers	882
24 Group theory	883
24.1 Groups	883
<i>Definition of a group; examples of groups</i>	

CONTENTS

24.2	Finite groups	891
24.3	Non-Abelian groups	894
24.4	Permutation groups	898
24.5	Mappings between groups	901
24.6	Subgroups	903
24.7	Subdividing a group <i>Equivalence relations and classes; congruence and cosets; conjugates and classes</i>	905
24.8	Exercises	912
24.9	Hints and answers	915
25	Representation theory	918
25.1	Dipole moments of molecules	919
25.2	Choosing an appropriate formalism	920
25.3	Equivalent representations	926
25.4	Reducibility of a representation	928
25.5	The orthogonality theorem for irreducible representations	932
25.6	Characters <i>Orthogonality property of characters</i>	934
25.7	Counting irreps using characters <i>Summation rules for irreps</i>	937
25.8	Construction of a character table	942
25.9	Group nomenclature	944
25.10	Product representations	945
25.11	Physical applications of group theory <i>Bonding in molecules; matrix elements in quantum mechanics; degeneracy of normal modes; breaking of degeneracies</i>	947
25.12	Exercises	955
25.13	Hints and answers	959
26	Probability	961
26.1	Venn diagrams	961
26.2	Probability <i>Axioms and theorems; conditional probability; Bayes' theorem</i>	966
26.3	Permutations and combinations	975
26.4	Random variables and distributions <i>Discrete random variables; continuous random variables</i>	981
26.5	Properties of distributions <i>Mean; mode and median; variance and standard deviation; moments; central moments</i>	985
26.6	Functions of random variables	992

CONTENTS

26.7	Generating functions	999
	<i>Probability generating functions; moment generating functions; characteristic functions; cumulant generating functions</i>	
26.8	Important discrete distributions	1009
	<i>Binomial; geometric; negative binomial; hypergeometric; Poisson</i>	
26.9	Important continuous distributions	1021
	<i>Gaussian; log-normal; exponential; gamma; chi-squared; Cauchy; Breit-Wigner; uniform</i>	
26.10	The central limit theorem	1036
26.11	Joint distributions	1038
	<i>Discrete bivariate; continuous bivariate; marginal and conditional distributions</i>	
26.12	Properties of joint distributions	1041
	<i>Means; variances; covariance and correlation</i>	
26.13	Generating functions for joint distributions	1047
26.14	Transformation of variables in joint distributions	1048
26.15	Important joint distributions	1049
	<i>Multinomial; multivariate Gaussian</i>	
26.16	Exercises	1053
26.17	Hints and answers	1061
27	Statistics	1064
27.1	Experiments, samples and populations	1064
27.2	Sample statistics	1065
	<i>Averages; variance and standard deviation; moments; covariance and correlation</i>	
27.3	Estimators and sampling distributions	1072
	<i>Consistency, bias and efficiency; Fisher's inequality; standard errors; confidence limits</i>	
27.4	Some basic estimators	1086
	<i>Mean; variance; standard deviation; moments; covariance and correlation</i>	
27.5	Maximum-likelihood method	1097
	<i>ML estimator; transformation invariance and bias; efficiency; errors and confidence limits; Bayesian interpretation; large-N behaviour; extended ML method</i>	
27.6	The method of least squares	1113
	<i>Linear least squares; non-linear least squares</i>	
27.7	Hypothesis testing	1119
	<i>Simple and composite hypotheses; statistical tests; Neyman–Pearson; generalised likelihood-ratio; Student's t; Fisher's F; goodness of fit</i>	
27.8	Exercises	1140
27.9	Hints and answers	1145

CONTENTS

28	Numerical methods	1148
28.1	Algebraic and transcendental equations <i>Rearrangement of the equation; linear interpolation; binary chopping; Newton–Raphson method</i>	1149
28.2	Convergence of iteration schemes	1156
28.3	Simultaneous linear equations <i>Gaussian elimination; Gauss–Seidel iteration; tridiagonal matrices</i>	1158
28.4	Numerical integration <i>Trapezium rule; Simpson’s rule; Gaussian integration; Monte Carlo methods</i>	1164
28.5	Finite differences	1179
28.6	Differential equations <i>Difference equations; Taylor series solutions; prediction and correction; Runge–Kutta methods; isoclines</i>	1180
28.7	Higher-order equations	1188
28.8	Partial differential equations	1190
28.9	Exercises	1193
28.10	Hints and answers	1198
<i>Appendix Gamma, beta and error functions</i>		1201
A1.1	The gamma function	1201
A1.2	The beta function	1203
A1.3	The error function	1204
<i>Index</i>		1206

Preface to the second edition

Since the publication of the first edition of this book, both through teaching the material it covers and as a result of receiving helpful comments from colleagues, we have become aware of the desirability of changes in a number of areas. The most important of these is that the mathematical preparation of current senior college and university entrants is now less thorough than it used to be. To match this, we decided to include a preliminary chapter covering areas such as polynomial equations, trigonometric identities, coordinate geometry, partial fractions, binomial expansions, necessary and sufficient condition and proof by induction and contradiction.

Whilst the general level of what is included in this second edition has not been raised, some areas have been expanded to take in topics we now feel were not adequately covered in the first. In particular, increased attention has been given to non-square sets of simultaneous linear equations and their associated matrices. We hope that this more extended treatment, together with the inclusion of singular value matrix decomposition, will make the material of more practical use to engineering students. In the same spirit, an elementary treatment of linear recurrence relations has been included. The topic of normal modes has been given a small chapter of its own, though the links to matrices on the one hand, and to representation theory on the other, have not been lost.

Elsewhere, the presentation of probability and statistics has been reorganised to give the two aspects more nearly equal weights. The early part of the probability chapter has been rewritten in order to present a more coherent development based on Boolean algebra, the fundamental axioms of probability theory and the properties of intersections and unions. Whilst this is somewhat more formal than previously, we think that it has not reduced the accessibility of these topics and hope that it has increased it. The scope of the chapter has been somewhat extended to include all physically important distributions and an introduction to cumulants.

PREFACE TO THE SECOND EDITION

Statistics now occupies a substantial chapter of its own, one that includes systematic discussions of estimators and their efficiency, sample distributions and t - and F -tests for comparing means and variances. Other new topics are applications of the chi-squared distribution, maximum-likelihood parameter estimation and least-squares fitting. In other chapters we have added material on the following topics: curvature, envelopes, curve-sketching, more refined numerical methods for differential equations and the elements of integration using Monte Carlo techniques.

Over the last four years we have received somewhat mixed feedback about the number of exercises at the ends of the various chapters. After consideration, we decided to increase the number substantially, partly to correspond to the additional topics covered in the text but mainly to give both students and their teachers a wider choice. There are now nearly 800 such exercises, many with several parts. An even more vexed question has been whether to provide hints and answers to all the exercises or just to 'the odd-numbered' ones, as is the normal practice for textbooks in the United States, thus making the remainder more suitable for setting as homework. In the end, we decided that hints and outline solutions should be provided for all the exercises, in order to facilitate independent study while leaving the details of the calculation as a task for the student.

In conclusion, we hope that this edition will be thought by its users to be 'heading in the right direction' and would like to place on record our thanks to all who have helped to bring about the changes and adjustments. Naturally, those colleagues who have noted errors or ambiguities in the first edition and brought them to our attention figure high on the list, as do the staff at The Cambridge University Press. In particular, we are grateful to Dave Green for continued \LaTeX advice, Susan Parkinson for copy-editing the second edition with her usual keen eye for detail and flair for crafting coherent prose and Alison Woollatt for once again turning our basic \LaTeX into a beautifully typeset book. Our thanks go to all of them, though of course we accept full responsibility for any remaining errors or ambiguities, of which, as with any new publication, there are bound to be some.

On a more personal note, KFR again wishes to thank his wife Penny for her unwavering support, not only in his academic and tutorial work, but also in their joint efforts to convert time at the bridge table into 'green points' on their record. MPH is once more indebted to his wife, Becky, and his mother, Pat, for their tireless support and encouragement above and beyond the call of duty. MPH dedicates his contribution to this book to the memory of his father, Ronald Leonard Hobson, whose gentle kindness, patient understanding and unbreakable spirit made all things seem possible.

Ken Riley, Michael Hobson
Cambridge, 2002

Preface to the first edition

A knowledge of mathematical methods is important for an increasing number of university and college courses, particularly in physics, engineering and chemistry, but also in more general science. Students embarking on such courses come from diverse mathematical backgrounds, and their core knowledge varies considerably. We have therefore decided to write a textbook that assumes knowledge only of material that can be expected to be familiar to all the current generation of students starting physical science courses at university. In the United Kingdom this corresponds to the standard of Mathematics A-level, whereas in the United States the material assumed is that which would normally be covered at junior college.

Starting from this level, the first six chapters cover a collection of topics with which the reader may already be familiar, but which are here extended and applied to typical problems encountered by first-year university students. They are aimed at providing a common base of general techniques used in the development of the remaining chapters. Students who have had additional preparation, such as Further Mathematics at A-level, will find much of this material straightforward.

Following these opening chapters, the remainder of the book is intended to cover at least that mathematical material which an undergraduate in the physical sciences might encounter up to the end of his or her course. The book is also appropriate for those beginning graduate study with a mathematical content, and naturally much of the material forms parts of courses for mathematics students. Furthermore, the text should provide a useful reference for research workers.

The general aim of the book is to present a topic in three stages. The first stage is a qualitative introduction, wherever possible from a physical point of view. The second is a more formal presentation, although we have deliberately avoided strictly mathematical questions such as the existence of limits, uniform convergence, the interchanging of integration and summation orders, etc. on the

PREFACE TO THE FIRST EDITION

grounds that ‘this is the real world; it must behave reasonably’. Finally a worked example is presented, often drawn from familiar situations in physical science and engineering. These examples have generally been fully worked, since, in the authors’ experience, partially worked examples are unpopular with students. Only in a few cases, where trivial algebraic manipulation is involved, or where repetition of the main text would result, has an example been left as an exercise for the reader. Nevertheless, a number of exercises also appear at the end of each chapter, and these should give the reader ample opportunity to test his or her understanding. Hints and answers to these exercises are also provided.

With regard to the presentation of the mathematics, it has to be accepted that many equations (especially partial differential equations) can be written more compactly by using subscripts, e.g. u_{xy} for a second partial derivative, instead of the more familiar $\partial^2 u / \partial x \partial y$, and that this certainly saves typographical space. However, for many students, the labour of mentally unpacking such equations is sufficiently great that it is not possible to think of an equation’s physical interpretation at the same time. Consequently, wherever possible we have decided to write out such expressions in their more obvious but longer form.

During the writing of this book we have received much help and encouragement from various colleagues at the Cavendish Laboratory, Clare College, Trinity Hall and Peterhouse. In particular, we would like to thank Peter Scheuer, whose comments and general enthusiasm proved invaluable in the early stages. For reading sections of the manuscript, for pointing out misprints and for numerous useful comments, we thank many of our students and colleagues at the University of Cambridge. We are especially grateful to Chris Doran, John Huber, Garth Leder, Tom Körner and, not least, Mike Stobbs, who, sadly, died before the book was completed. We also extend our thanks to the University of Cambridge and the Cavendish teaching staff, whose examination questions and lecture hand-outs have collectively provided the basis for some of the examples included. Of course, any errors and ambiguities remaining are entirely the responsibility of the authors, and we would be most grateful to have them brought to our attention.

We are indebted to Dave Green for a great deal of advice concerning typesetting in L^AT_EX and to Andrew Lovatt for various other computing tips. Our thanks also go to Anja Visser and Graça Rocha for enduring many hours of (sometimes heated) debate. At Cambridge University Press, we are very grateful to our editor Adam Black for his help and patience and to Alison Woollatt for her expert typesetting of such a complicated text. We also thank our copy-editor Susan Parkinson for many useful suggestions that have undoubtedly improved the style of the book.

Finally, on a personal note, KFR wishes to thank his wife Penny, not only for a long and happy marriage, but also for her support and understanding during his recent illness – and when things have not gone too well at the bridge table! MPH is indebted both to Rebecca Morris and to his parents for their tireless

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K. F. Riley, M. P. Hobson and S. J. Bence

Frontmatter

[More information](#)

PREFACE TO THE FIRST EDITION

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