

ORBITAL MECHANICS FOR ENGINEERING STUDENTS

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C.1 **FUNCTION FILE** accel_3body.m

590

C.2 **SCRIPT FILE** threebody.m

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