Your Name: John Luke Penny
(This is an INDIVIDUAL assignment)

CSC 4512, Optimization Approaches in CS: Algorithms and Applications

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Louisiana State University
School of Electrical Engineering and Computer Science
Division of Computer Science and Engineering

Spring 2022 Semester

SUBMIT ALL PROJECTS Electronically via Moodle

A typical file name should have the following format:
"CSC4512_Spring2022_PROJ_2_YourFirstName_YourLastName.zip"
(Each submission involves more than one file; thus you will have to zip them into a single file.)

MAIN GOAL for PROJECT #2: Get familiarized with the process of developing algorithms for optimization problems.

Today's date: **Thursday,** **March 10, 2022**
Due date: **Thursday,** **March 31, 2022. By 10:00 PM of that day via MOODLE**
Maximum grade points = 100
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Recall that our TA is Augustine Orgah. His E-mail is: aorgah1@lsu.edu
CLEARLY EXPLAIN AND ORGANIZE YOUR ASNWERS! The TA may take points off otherwise. Your answers must be presented sequentially.

NOTE: Always observe the Policy Statement for this course regarding Cheating / Academic Misconduct as it is stated on page 4 of the syllabus and described in the first day of classes.

1. Problem Description

Suppose that given are pairs of positive numbers, say V_1 and V_2, where $V_1, V_2 \geq 0$. We want such pairs to be reciprocals of each other. That is, their product to be equal to 1. An example is the pair $V_1 = 2$ and $V_2 = 0.5$ (i.e., the pair {2, 0.5}). If this condition does not hold for a given pair, then we would like to **minimally adjust** it so the reciprocal condition will hold on the adjusted pair. Formulate this problem and then solve it by considering the following pairs of numbers as test problems (you may want to solve even more problems to gain additional insight):

Pair #1:	{2,	1}
Pair #2:	{0.2,	7}
Pair #3:	{3,	1/3}
Pair #4:	{8,	2}
Pair #5:	{1,	2}
Pair #6:	{2,	4}

Pair #7: {0.2 6}
Pair #8: {4, 0.2}

You have to formulate this problem as an optimization one. Note that the objective function and/or some or all of the constraint(s) may or may not be linear. Ideally, you need to develop an exact optimal approach or as a compromise a heuristic approach. In such case, you may want to provide some insight on how well your heuristic performs on random problems. You may also want to explore some theoretical aspects such as the following questions: Does this problem always have a solution? Can it have alternative optimal solutions? Can you guarantee if a solution is optimal? Do you need to write a computer program, or a theoretical approach may suffice to determine the optimal solution? and so on.

2. Deliverables

Make sure your deliverables include the following items (use separate files if needed):

1. Clearly explain what your solution approach is and how well it works. Consider answering the previous questions or even additional ones regarding any theoretical aspects of it.

2. If you write a computer program, then you need to provide the source code (in a different file) with adequate info on how one can run it and reproduce your results. Write plenty of comments to explain your code.

As a final note: You will need to spend time to understand the problem and then develop the solution approach(es). Thus, start ASAP as this project may take more time than what you may anticipate now.

Attach this form with your name filled in on front of your report

Optimization Project 2

John Luke
Renny

Optimize minimum adjustment needed to make a pair $\{X, Y\}$ the reciprocal of each other where $Y = 1/X$ + $X = 1/Y$. The adjustment is equal to the sum of the absolute value of the adjustments to $X + Y$, I.e. $Y + m = \frac{1}{X+n}$ + $X + n = \frac{1}{Y+m}$ and adjustment $= |n| + |m|$.

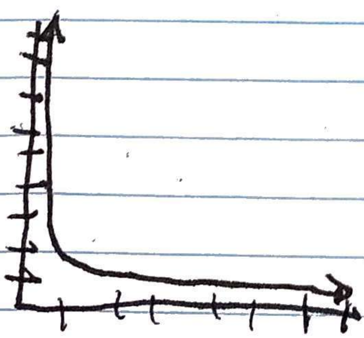
Hypothesis: To get the minimum adjustment, only the smallest number of the pair needs to be adjusted.

Rational: For the product of $(X+n)\cdot(Y+m)$ to be 1, either $(X+n) = (Y+m) = 1$, or $(X+n) > 1$ $(Y+m) < 1$ vice versa. Furthermore, once a number is < 1, any adjustment that is fractional will grow the reciprocal value quicker than changing values > 1.

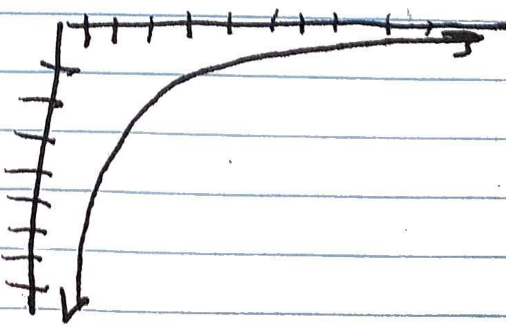
Function from rational $= 1/X = f$

$f' = -\frac{1}{x^2}$

graph of f



graph of f'



If the absolute value of f' is taken, it can be observed that as x approaches 0, the value grows larger. This shows that adjusting smaller numbers affects the reciprocal value greater. Due to this, it is more optimal to adjust one number, the smaller number, to the reciprocal. Again, this is due to the fact that to get the larger number to be the reciprocal, it will always have a larger adjustment than the smaller number.

Conjecture: To solve for minimal adjustment to reach the reciprocal, if pair is not already one, adjust the smallest value of the pair to reach reciprocity.

While this holds when at least 1 number is ≥ 1, if both are < 1, this algorithm doesn't hold.

ex. pair $\{.5, .5\}$

Conjecture

$\frac{1}{.5} = 2$

$2 - .5 = |1.5| =$ adjustment

$.5 \cdot (.5 + 1.5) = 1$

better solution

$(.5 + .5) \cdot (.5 + .5) = 1$

$|.5| + |.5| = 1 =$ adjustment

Using the rational of adjusting smaller numbers yeilds a greater result, it is more optimal to adjust both numbers to 1. Using this method, the maximum adjustment will be 2 when $x = y = 0$.

Interpreting program results:
The program tests 3 algorithms, adjusting the smallest, ~~adjust~~ adjust both to 1, and a hybrid. For a decision point, the hybrid determines which of the two algorithms to use of $x, y \leq 1$. If $x, y \leq 1$, the adjustment to 1 algorithm is used. Otherwise, the adjusting the smallest is used.

The results provided by the program shows that 1 is the proper break point and the hybrid algorithm is at the very least the most optimal of the 3. However, since each part of the hybrid is at least supported by conjecture to be optimal in the region it is used in.