

Task 1.2

1. Find the 20th term for each of the following arithmetic progressions:

a) 2, 5, 8, ...

4. Find the values of x so that $4x + 6$, $9x - 3$, $15x + 2$ will form the successive terms of an arithmetic progression.

5. How many terms are there in the sequence 11, 15, 19,..., 131?

6. Find the sum of the first 14 terms of the following series:

a) $7 + 12 + 17 + \dots$

Task 1.3

1. The first term of a geometric series is 6 and the common ratio is $\frac{3}{2}$. Write down the first five terms of the progression.

4. Find the sum to 10 terms of 1, 4, 16, 64, ...

6. If x , $x + 5$, $x + 11$ are in geometric progression, find the value of x and hence find T_6 .

Task 1.4

1. Find the limiting sum for each of the following geometric progressions:

a) $72 + 18 + 4\frac{1}{2} + \dots$

4. A ball dropped from a height of 15m and bounces up to $\frac{3}{5}$ of its height and continues bouncing rising $\frac{3}{5}$ of its height on each bounce until it eventually stops bouncing. What is the total distance through which the ball has travelled after being dropped?

6. The height of a 20 metre tree increased by 3 metres in one year. If in each succeeding year the growth is 75% of that in the previous year, *find the limiting height of the tree.*

Task 1.5 1. $\sum_{k=1}^{15} (6n - 1)$

Task 2.3

4. Nick is a real estate agent and earns a base salary of \$240 per week. As an incentive he is also paid a commission for the properties he sells as follows:

4½% commission for the first \$20,000

5% commission for the next \$45,000

6¼% commission on the amount in excess of \$65,000

Calculate Nick's income for the week if he sold a house for \$310,000.

Task 2.4

3. In one week, the hours worked by four different employees and their rates of pay are shown in the table on the following page. Calculate the weekly wage for each employee.

Employee	Pay rate per hour	Regular hours	Overtime		Weekly Wage
			Time-and-a-half	Double Time	
J. Jones	\$11.20	38	4	3	
M. Lee	\$12.60	38	6	-	
A. Wong	\$12.95	38	5	1	
E. Guo	\$14.84	38	2	-	

4. Carol worked for a successful nursery that rewarded each employee with a bonus equal to 6% of their annual income. During the year, Carol also used her four-week annual leave for a break and received a 16½% holiday loading for that period.

If Carol's regular fortnightly payments are \$1,436.78, calculate her total income for the year.

Task 2.5

1. Calculate the yearly tax payable for each of the following incomes:
 - a) \$12,572,
 - b) \$32,480 gross income with \$3,222 in allowable deductions,
 - c) gross pay of \$864.50 per week.

2. David is a real estate agent and is paid an annual salary of \$20,500. He is also paid a commission of 1½% on all sales he makes. If in one year his sales totalled \$2,890,425:
 - a) calculate his gross income for the year
 - b) calculate the tax payable if his allowable deductions amount to \$ 2,367.

3. Maxine earned a salary of \$43,562 in the past year as a teacher and also received \$190 each week (for 42 weeks) through her private tutoring. Her allowable deductions for the year amounted to \$3,120. If she pays 16% of her fortnightly salary in tax and has paid PAYE tax instalments throughout the year amounting to \$4,525, calculate:
- her gross income for the year,
 - her taxable income,
 - her refund or tax payable.

Task 3.1

$$\text{Yield} = \frac{\text{Return from money invested}}{\text{Amount of money invested}} \times 100\%$$

1. What dividend will Rod receive from 6,000 ordinary shares that have a face value of \$0.80 and a market value of \$1.15 if the dividend rate is 12½%?

2. Sarah buys 3,000 shares in Fresh OJ which have a face value of \$0.60 per share and a market value of \$1.40. Fresh OJ pays an 8% dividend to shareholders.

What is Sarah's yield on this investment?

4. Jay bought 4,500 shares in Phoney (a telephone company) at \$6.50 each. She purchased these shares through a broker and paid a brokerage fee of 6% of the share price per share. The face value of the shares was \$4.80.
- a) What was the total cost of the shares bought by Jay?
- b) Jay kept the shares for three years before selling them for \$9.20 each. During each of those three years, Phoney declared a 15% dividend on all shares. What was Jay's total dividend return on the shares?

5. Jack and Jill bought a house costing \$350,000. They paid a 37% deposit (down payment) and borrowed the remainder at a flat interest rate of $8\frac{1}{4}\%$ p.a. payable over 30 years. How much are their monthly instalments to pay off the loan?

Task 3.3

The compound interest formula:

$$A = P \left(1 + \frac{r}{100} \right)^{nt}$$

where: A = the future value of the investment

P = the principal amount invested

r = the interest rate adjusted to the interest periods

n = the number of compounding periods per year

t = the time in years

annually (yearly) $\Rightarrow n = 1$

biannually/semi-annually (every six months) $\Rightarrow n = 2$

quarterly (every three months) $\Rightarrow n = 4$

monthly (each month) $\Rightarrow n = 12$

weekly (each week) $\Rightarrow n = 52$

daily (each day) $\Rightarrow n = 365$

biennial (every 2 years) $\Rightarrow n = 0.5$

The depreciation formula:

$$A = P \left(1 - \frac{r}{100} \right)^n$$

where: A = the future depreciated value of the item

P = the initial value of the item

r = the interest rate adjusted to the interest periods

n = the number of depreciating periods per year

1. Calculate the compound interest on a principal of:
 - a) \$6,570 at 7% p.a. for 5 years compounded **annually**;
 - e) \$40,300 at 6¼% p.a. for 3 years compounded **quarterly**;
 - f) \$11,290 at 7% p.a. for 5.5 years compounded **monthly**.

2. Find the value after 6 years of:
 - a) A car costing \$32,000 depreciated at 18% p.a.
 - b) A town's population of 30,000 decreased at 3.7% p.a.
 - c) A mobile phone costing \$899 depreciated at 16.5% p.a.

Task 3.4

$$S = R \left[\frac{(1+i)^n - 1}{i} \right]$$

where: R = periodic payment of the annuity

$$i = \frac{r}{N} = \begin{cases} \text{the interest rate per interest period} \\ \text{(expressed as a decimal)} \end{cases}$$

N = the number of interest periods in 1 year

t = the term of the annuity

$n = Nt$ = the number of periodic payments to be made during the whole term.

- 10 / 16

Task 4.1

1. Consider the experiment where a single ticket is drawn from a box containing ten tickets numbered 1, 2, 3, ..., 10.
 - (a) List the sample space S for this experiment.
 - (b) List the subsets that correspond to the events:
 - (i) A: a number greater than 6
 - (ii) B: an odd number

2.
 - (a) Use a tree diagram to determine the sample space for the children in three-child families.
 - (b) List the subsets corresponding to the events:
 - (i) A: the youngest child is a boy
 - (ii) B: there is one girl
 - (iii) C: 'no boy has an older sister'
 - (iv) D: 'at least one boy'

Task4.2

1. A die is tossed. Find the probability of obtaining:

- (a) a 5
- (b) an even number
- (c) a number greater than 2
- (d) a number less than 1
- (e) a number greater than 0.

3. Suppose 60 overnight hotel guests are surveyed to determine what they ate for breakfast with the following results:

27 had eggs, 22 had bacon, 36 had toast,
19 had eggs and bacon, 16 had eggs and toast, 15 had bacon and toast,
12 had eggs, bacon and toast

A guest is selected at random. Find the probability that the guest had:

- (a) none of the three
- (b) toast but neither eggs nor bacon
- (c) eggs and bacon but not toast
- (d) bacon only.
- (e) Eggs or bacon
- (f) Eggs given that he had bacon

补充: Draw a Venn diagram for the following question:

In a group of 40 students, 20 are taking an English course, 18 are taking a mathematics course and 12 are taking both. If E denotes the event that a student is taking an English course and M denotes the event that a student is taking a mathematics course, determine

Task4.5

2. A bag has 2 green and 5 yellow marbles, and two marbles are drawn from the bag.
- (a) If the first marble is replaced before the second is drawn, find the probability of obtaining:
- (i) two green marbles
 - (ii) 1 green marble and 1 yellow
 - (iii) two marbles the same colour.
- (b) If the first marble is not replaced before the second is drawn, find the probability of obtaining:
- (i) two green marbles
 - (ii) 1 green marble and 1 yellow
 - (iii) two marbles the same colour.
3. Glen has a 75% chance of solving a problem while his friend Vicki has a 60% chance. Find the probability that:
- (a) only 1 solves the problem
 - (b) the problem is not solved.

Task 4.6

1. Two cards are drawn simultaneously from a standard pack of 52 playing cards. Use a tree diagram to find the probability that:
- (a) both are hearts
 - (b) only one is a heart.

补充练习:

There are 4 defective items in a box of 10 items. If two items are selected at random, find the probability of obtaining at least one defective item, assuming that:

- (a) the first item is replaced before the second is drawn
- (b) the first item is not replaced before the second is drawn

Task 4.7

1. The following table shows the political affiliation by gender of a sample of 250 men and women.

	men	women
Coalition	45	55
Labour	70	50
Other	15	15

A person is selected at random from this group.

- (a) Find the probability that the person:
- (i) is a woman given that the person supports Labour.
 - (ii) is a Coalition supporter given that the person is a male.
 - (iii) supports one of the other political parties.

Sequences and Series

Arithmetic Progressions

$$T_N = a + (N - 1)d$$

$$S_N = \frac{N}{2}(a + l) \text{ or } S_N = \frac{N}{2}(2a + (N - 1)d)$$

Geometric Progressions

$$T_N = ar^{N-1}$$

$$S_N = \frac{a(r^N - 1)}{r - 1} \text{ or } S_N = \frac{a(1 - r^N)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r}$$

Finance

$$\text{Yield} = \frac{\text{Return from money invested}}{\text{Amount of money invested}} \times 100 \%$$

Simple Interest

$$I = \frac{PRN}{100}$$

where I = interest earned.

P = principal amount invested

R = interest rate

N number of years

Compound Interest

$$A = P \left(1 + \frac{r}{100} \right)^n$$

where A = future value of the investment

P = principal amount invested

r = the interest rate adjusted to the interest periods

n = number of compounding periods

Depreciation

$$A = P \left(1 - \frac{r}{100} \right)^n$$

Future value (A) of an Annuity

$$A = R \left[\frac{(1 + r)^n - 1}{r} \right]$$

where R = periodic payment of the annuity

r = interest rate per interest period

n = number of periodic payments to be made

Present value (A) of an Annuity

$$A = R \left[\frac{1 - (1 + r)^{-n}}{r} \right]$$

where R = periodic payment of the annuity

r = interest rate per interest period

n = number of periodic payments to be paid

Sum of a Geometric Progression

$$S_N = \frac{a(r^N - 1)}{r - 1}$$