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Nonlinear local Lyapunov exponent and atmospheric predictability research

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Abstract Because atmosphere itself is a nonlinear system and there exist some problems using the linearized equations to study the initial error growth, in this paper we try to use the error nonlinear growth theory to discuss its evolution, based on which we first put forward a new concept: nonlinear local Lyapunov exponent. It is quite different from the classic Lyapunov exponent because it may characterize the finite time error local average growth and its value depends on the initial condition, initial error, variables, evolution time, temporal and spatial scales. Based on its definition and the atmospheric features, we provide a reasonable algorithm to the exponent for the experimental data, obtain the atmospheric initial error growth in finite time and gain the maximal prediction time. Lastly, taking 500 hPa height field as example, we discuss the application of the nonlinear local Lyapunov exponent in the study of atmospheric predictability and get some reliable results: atmospheric predictability has a distinct spatial structure. Overall, predictability shows a zonal distribution. Prediction time achieves the maximum over tropics, the second near the regions of Antarctic, it is also longer next to the Arctic and in subtropics and the mid-latitude the predictability is lowest. Particularly speaking, the average prediction time near the equation is 12 days and the maximum is located in the tropical Indian, Indonesia and the neighborhood, tropical eastern Pacific Ocean, on these regions the prediction time is about two weeks. Antarctic has a higher predictability than the neighboring latitudes and the prediction time is about 9 days. This feature is more obvious on Southern Hemispheric summer. In Arctic, the predictability is also higher than the one over mid-high latitudes but it is not pronounced as in Antarctic. Mid-high latitude of both Hemispheres (30°S—60°S, 30°—60°N) have the lowest predictability and the mean prediction time is just 3—4 d. In addition, predictability varies with the seasons. Most regions in the Northern Hemisphere, the predictability in winter is higher than that in summer, especially in the mid-high latitude: North Atlantic, North Pacific and Greenland Island. However in the Southern Hemisphere, near the Antarctic regions (60°S—90°S), the corresponding summer has higher predictability than its winter, while in other areas especially in the latitudes of 30°S—60°S, the prediction does not change obviously with the seasons and the average time is 3—5 d. Both the theoretical and data computation results show that nonlinear local Lyapunov exponent and the nonlinear local error growth really may measure the predictability of the atmospheric variables in different temporal and spatial scales.

Keywords: nonlinear, local, Lyapunov exponent, atmospheric predictability, maximal prediction time.

If neglecting the effects of some stochastic factors, we regard the atmospheric system as the deterministic nonlinear system and its development is determined by the physical laws, boundary condition and the initial condition. Numerical prediction is based on this idea: if given an accurate model and a perfect observation system, we can make an exact prediction for the future weather. However, initial small error is unavoidable and it will grow rapidly through nonlinear interactions in the atmospheric dynamics even if the model is somehow made perfect, all of which lead to the predictability studies.

At the first of the 1940s, Kolmogoroff stated that atmospheric initial errors would lead to different atmospheric states in a long time[1]. In 1957, Thompson[2] first put forward the problem of the atmospheric predictability. Later Lorenz[3] mentioned three methods that were used to study this problem. By now, most predictability studies are the model-based approach and estimating the prediction error is the main aim for the classic atmospheric predictability research[1,4−7]. Usually, given two very similar initial fields and integrating the same atmospheric model, we may obtain the initial error growth and according to the error doubling time, estimate the maximal prediction time. But the latest investigation shows that the prediction limitation achieved by this method depends on the calculation, computer accuracy and the numerical model itself[8−12], so the resulting prediction time is not the prediction limitation of the real atmosphere or climate, nor the exact measure of the model predictability. Dalcher and Kalnay[13] also pointed out that the estimated "doubling time of small error" was not a good measure of error growth. Because the true state of the atmosphere is not known due to uncertainties in the analysis and model initialization, it can not be measured directly; and the results gained by extrapolation are very sensitive to the parameterization in the empirical model of error growth. Arpe *et al.*[14] also referred that it was difficult to estimate small errors doubling time. However, error growth at the finite time is the best parameter because it is defined by the available (either model or observational) data.

Lyapunov exponents quantify the average exponential diverging (decaying) rate of the initial nearby orbits over the whole phase space for infinite time. In the chaotic systems theory, it is related to the mean growth rate of the initial error so in some researches, it is used to discuss the atmospheric predictability[15−17]. When there is at least one positive Lyapunov exponent, it means that initial nearby orbits dirvrge with the time and the attractor is the chaotic. Usually the inverse of the sum of all positive Lyapunov exponents or second order Renyi entropy is used to measure the predictability. In ergodic theory, Lyapunov exponents σ don't depend on the initial values and all orbits will lead to the same exponents, which mean that Lyapunov exponents characterize the global properties of the attractors[18]. But in fact the initial error growth is not the same everywhere[19−21] so if chaotic attractors' predictability is the function of time and space or we are interested in the short-term prediction, it is necessary to study the local dynamic properties of attractors. Later, some researchers[22−27] pointed out that local or limited time Lyapunov characteristic exponents may measure the local predictability and the definition is the average growth rate of the initial error in the finite time. Compared with the classic Lyapunov exponents, local Lyapunov exponents measure predictability in more effective way because they can exhibit the temporal and spatial structures, which can be used to detect the divergence (convergence) rate of the nearby orbits and to determine the regions with higher (lower) predictability. However the mentioned local Lyapunov exponents are essentially the same as Lyapunov exponents, both of them are assumed that the initial error is so small that its evolution satisfies the tangential linear equation, and they are obtained by solving the characteristic vectors and the characteristic values of the evolution matrix M. But with the error increasing, these conditions may not be satisfied; moreover the accuracy of the atmospheric variables is not infinitely small. So there are a lot of limitations using the linearized error growth equation to study the atmospheric predictability.

Because of the existing problems, in this paper we do not make linear approximation of the error growth equations, instead we use the initial error nonlinear growth equations to discuss the perturbation development, based on which we put forward a new concept: nonlinear local Lyapunov exponent by which to measure the initial error local nonlinear growth rate in finite time. Considering its definition and the atmospheric features, we provide the algorithm to the exponent for the experimental data and the method to estimate the maximal prediction time; lastly we compute the initial error locally average growth, determine the prediction time and discuss the variation of the global predictability with space and seasons.

1 Theoretical background

1.1 Classic Lyapunov exponent

The weather or climate system can be expressed by partial differential equations and by Galerkin method it becomes the nonlinear ordinary differential equations:

$$\frac{d}{dt}\boldsymbol{X}(t)=\boldsymbol{F}(\boldsymbol{X}(t)), \tag{1}$$

where $\boldsymbol{X}=(x_1, x_2, \cdots, x_n)^{\mathrm{T}}$. A solution of eq. (1) $\boldsymbol{X}(t)$ will be called a reference solution with its initial phase $\boldsymbol{X}_0=\boldsymbol{X}(0)$; Let vector $\delta\boldsymbol{X}_0$ be the infinitesimal initial perturbation supposed on $\boldsymbol{X}_0$ at the initial time, then after time t, $\boldsymbol{X}_0+\delta\boldsymbol{X}_0$ will develop into $\boldsymbol{X}(t)+\delta\boldsymbol{X}(t)$, which satisfy the equations:

$$\frac{d}{dt}(\boldsymbol{X}(t)+\delta\boldsymbol{X}(t))=\boldsymbol{F}(\boldsymbol{X}(t)+\delta\boldsymbol{X}(t)). \tag{2}$$

(2)−(1):

$$\frac{d}{dt}\delta\boldsymbol{X}(t)=\boldsymbol{F}(\boldsymbol{X}(t)+\delta\boldsymbol{X}(t))-\boldsymbol{F}(\boldsymbol{X}(t)). \tag{3}$$

Expanding to the first order in $\boldsymbol{X}(t)$, error evolution obeys the tangent linear equations:

$$\frac{d}{dt}\delta\boldsymbol{X}=\boldsymbol{G}\delta\boldsymbol{X}, \tag{4}$$

where $\boldsymbol{G}$ is the Jacobian matrix,

$$G_{i,j}=\frac{\partial F_i}{\partial X_j}. \tag{5}$$

The solution of eq. (4) is written as

$$\delta\boldsymbol{X}(t)=\boldsymbol{M}(t,t_0)\delta\boldsymbol{X}(t_0), \tag{6}$$

where the matrix $\boldsymbol{M}(t, t_0)$ is called the evolution matrix and it depends on the time. Eq. (4) means that the growth of the initial infinitesimal error is related to the characteristic values of the corresponding Jacobian matrix $G_{N\times N}$, but Lyapunov exponents do not equal the characteristic values of the Jacobian matrix and they are derived by the evolution matrix $\boldsymbol{M}(t, t_0)$, the relation between $\boldsymbol{G}$ and $\boldsymbol{M}(t, t_0)$ is

$$\boldsymbol{M}(t,t_0)=\mathrm{e}^{\int_{t_0}^{t}\boldsymbol{G}(t')dt'}. \tag{7}$$

Oseledec[28] proved that for almost every initial point $\boldsymbol{X}(t_0)$, the following limitation are always satisfied:

$$\boldsymbol{M}_\infty=\lim_{t\to\infty}(\boldsymbol{M}\times\boldsymbol{M}^*)^{1/2t}, \tag{8}$$

where $\boldsymbol{M}^*$ is the adjoint of $\boldsymbol{M}$, meanwhile, there exists an orthonormal set of vectors $\boldsymbol{f}_i$ such that

$$\sigma_i=\lim_{t\to\infty}\frac{1}{t}\ln\left|\boldsymbol{M}(t,t_0)\boldsymbol{f}_i(t_0)\right|. \tag{9}$$

σ_i is a characteristic value of $\boldsymbol{M}$ and $\boldsymbol{f}_i$ must be an associated characteristic vector. It is defined as Lyapunov exponent; n dimensional phase space has n characteristic values and they make up the Lyapunov spectrum. They mean the initial infinitesimal errors average exponential growth rate along the characteristic vectors $\boldsymbol{f}_i$ in the phase space. The largest Lyapunov exponent is the growth rate of initial perturbation along the most rapidly growing direction. Lyapunov exponents do not depend on the initial value and they characterize the global properties of the attractors in dynamical system.

The above derivation is assumed that the magnitude of initial small errors is small enough so that linearity assumptions holds, but as well known with the error's increasing nonlinear role becomes more and more important to the error growth, thus the linear approximation is not applicable. Some papers discussed in detail about the limitation of using the linearized model to study the error development[29–31]. In this paper, we keep the nonlinear term and define a new dynamic diagnostics: nonlinear local Lyapunov exponent, which can be used to measure the error nonlinear local growth in finite time.

1.2 Nonlinear local Lyapunov exponent

Let $\boldsymbol{x} = (x_1, x_2, \cdots\cdots, x_n)^{\mathrm{T}}$ be the variables participating in the dynamics, the vector $\boldsymbol{x}$ and its perturbation vector $\boldsymbol{x}+\boldsymbol{\delta}$ satisfy the following equations:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{F}(\boldsymbol{x},t), \quad (10)$$

$$\frac{d(\boldsymbol{x}+\boldsymbol{\delta})}{dt} = \boldsymbol{F}(\boldsymbol{x}+\boldsymbol{\delta},t), \quad (11)$$

where $\boldsymbol{\delta} = (\delta_1, \delta_2, \cdots, \delta_n)^T$ is the perturbation vector. Then the error evolution laws are

$$\frac{d\boldsymbol{\delta}}{dt} = \tilde{\boldsymbol{F}}(\boldsymbol{x},\boldsymbol{\delta},t)\boldsymbol{\delta}, \quad (12)$$

where $\tilde{\boldsymbol{F}}(\boldsymbol{x},\boldsymbol{\delta},t) = [\boldsymbol{F}(\boldsymbol{x}+\boldsymbol{\delta},t) - \boldsymbol{F}(\boldsymbol{x},t)]/\boldsymbol{\delta}$ are typically nonlinear error growth functions. The solution for eq. (12) is written as

$$\boldsymbol{\delta}(t) = \eta(\boldsymbol{x},\boldsymbol{\delta},t)\boldsymbol{\delta}(0), \quad (13)$$

where $\boldsymbol{\eta}(\boldsymbol{x}, \boldsymbol{\delta}, \boldsymbol{t})$ are the nonlinear error propagator operators, which govern the growth of initial errors arising from uncertainties in initial conditions.

Compared with the linear operators $\boldsymbol{M}(t, t_0)$ in eq. (6), nonlinear propagator operators $\boldsymbol{\eta}(\boldsymbol{x}, \boldsymbol{\delta}, \boldsymbol{t})$ are essentially different. The latter depends on not only time t, but also the variables $\boldsymbol{x}$ and the initial errors $\boldsymbol{\delta}$, which is the special property of the nonlinear theory. For variable x_i, we define the nonlinear local Lyapunov exponent:

$$\lambda(x_i,\delta_i,t) = \frac{1}{t}\ln\frac{|\delta_i(t)|}{|\delta_i(0)|}. \quad (14)$$

By eq. (14), we know in the limited time t, initial error of the variable x_i develops with the average rate $\mathrm{e}^{\lambda(x_i,\delta_i,t)}$; the exponent λ depends on the initial value, time interval, and initial error so it describes the local properties of attractors. Positive nonlinear local Lyapunov exponent indicates that the distance between two neighboring orbits will be amplified and the correlation between the two nearby trajectories is lost. The larger the exponent is, the faster the error grows and so the lower the predictability is.

From the above, we know that there is essential difference between the classic Lyapunov exponent and nonlinear local Lyapunov exponent. First, they have the different precondition; the former is based on the tangent linear error growth equations while the latter includes the nonlinear role in the equation. Second, the former is to discuss the average expansion rate of all initial errors in the infinite long time, according to the erogic theory its value is independent on the initials, so the error growth given by the Lyapunov exponent is uniform over the whole state space. In addition, it just measures the mean growth rate along the characteristic vector $\boldsymbol{f}_i$, but can not measure the expansion along any variable direction. For example, the largest Lyapunov exponent just measures the mean growth rate along the most rapidly growth direction. However, the nonlinear local Lyapunov exponent may discuss the average growth rate of the initial error in the prescribed time along any variable direction and its value depends on the initial value, initial error, variables, time interval, temporal and spatial scale, which characterizes the local properties of the attractors. So for a complicated nonlinear atmospheric system, nonlinear local Lyapunov exponent may study the predictability of all variables in the different time and spaces. In view of the theoretical analysis, it may better measure the initial error nonlinear evolution. Nonlinear Lyapunov exponent in this paper is also different from the local Lyapunov exponent[22−27] mentioned before, the latter is essentially the same as the Lyapunov exponent; both are based on the linearized error growth equation. However the real atmospheric initial error is not infinitesimal, so it has much limitation using the linear theory to discuss the predictability.

2 Data, computation method and the comparison to the classic method

2.1 Data

The data in this paper is the NCEP reanalysis data: 1958—1997 500 hPa daily 4-time geopotential height, 2.5° lat. by 2.5° lon. resolution, 144×73 grid points in global.

2.2 Introduction of the calculation of nonlinear local Lyapunov exponent

(1) Given a time series $x(t)$, taking $t_0 = 1$, $x(t_0)$ as the initial point, during the same time $[t_0-\eta, t_0+\eta]$ in 40 years, we find the nearest neighbor to the initial point and regard it as the smallest initial error of $x(t_0)$, it is

denoted $x'(t_0)$. By doing this we guarantee two states have the same dynamical characteristics. The distance between two points is

$$L(t_0) = |x'(t_0) - x(t_0)| . \quad (15)$$

This step is important and quite different from the Wolf[32] algorithm. Classical Lyapunov exponent is unrelated to initial errors δ, so it is reasonable to find the nearest neighbor to the initial point in the whole phase space as the perturbed initial value; but for nonlinear local Lyapunov exponent, it depends on δ so it must be careful to find a meaningful nearest point. Two points are required not only to have the sufficiently small distance, but also the almost same dynamic characteristics. As well known, there exists a distinctive annual periodic in the weather system, so for an initial point $x(t_0)$ in spring, its nearest neighbor in distance may be $x(t)$, where t is some day in autumn, but such two points have the different dynamic characteristics and their evolution tendency are opposite. Fig. 2 is the initial error time evolution which is obtained by Wolf methods; it shows that error is always increasing which is not identical to the real world; it implies that this method has some limitation. The method in this paper has the atmospherics dynamic background and may be used to compute the real initial error growth. Moreover, our approach avoids the problems in the reconstructed phase space method[33].

(2) After the evolution time T, the initial point $x(t_0)$ evolves into $x(t_0+T)$ along the orbit, and $x'(t_0)$ develops into $x'(t_0+T)$. The initial length will become the new length:

$$L'(t_0 + T) = |x'(t_0 + T) - x(t_0 + T)| . \quad (16)$$

The average growth rate of the initial error in the time T is

$$\lambda_{T1} = \frac{1}{T} \ln \frac{L'(t_0 + T)}{L(t_0)} . \quad (17)$$

(3) Taking $x(t_0+1)$ as the initial point, repeating processes (1) and (2), we can get λ_{T2} which means its average error growth rate in time interval T.

(4) This process is repeated until the last point of $\{x_i\}$, for every point we obtain its error average growth rate λ_{TK} in the time T. Then we take the mean value of λ_{TK} as the approximation of nonlinear local Lyapunov exponent in the time step T.

$$\lambda(T) = \frac{1}{T} \sum_{K=1}^{N} \lambda_{TK} . \quad (18)$$

(5) $T = T+1$, repeating processes (1)—(4), we get the relationship between $\lambda(T)$ and T.

(6) From formulas (17) and (18) we get the relative errors mean growth in time T,

$$Err(T) = \lambda(T) \times T = \frac{1}{N} \sum_{K=1}^{N} \ln \frac{L'(t_K)}{L(t_0)} . \quad (19)$$

By investing the relative errors growth, we define the time when errors reach the saturating level as the maximal prediction time.

Fig. 1 shows the nonlinear local Lyapunov exponent against evolution time, the data is the zonally averaged time series in four latitudes: 0, 30°N, 60°N, 90°N.

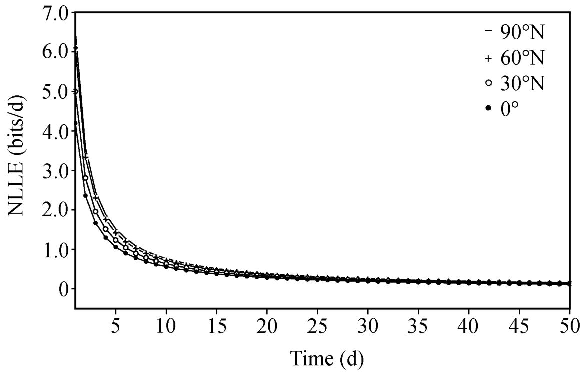

Fig. 1. Time evolution of the nonlinear local Lyapunov exponent (NLLE: nonlinear local Lyapunov exponent).

From Fig. 1, we know that at any time, nonlinear local Lyapunov exponent is positive and it decreases monotonously, which means that atmospheric initial error increases with time but due to the nonlinear interaction its growth rate decreases. It implies that for real atmospheres, initial error cannot be infinitesimal and its growth satisfies the nonlinear laws, so there are some problems using the linear theory to study the atmospheric error growth. Fig. 1 also shows that in the same time intervals, initial error grows slowest in the equator, the second slower in 30°N, 60°N and in 90°N error grows fastest. The distribution of the initial minimum error in four latitudes shows that: error is smallest in the equator and it becomes larger as the latitude increases, which means that the larger the initial error is, the faster the growth rate becomes.

To further demonstrate the attribution of nonlinear term in the error growth equation, we compare the different results obtained by two theories and methods.

2.3 Results comparing between two theories and the determination of maximal prediction time

Figs. 2 and 3 show the initial relative errors time evolution computed by the linear theory method (Wolf, *et al*., 1985) and the nonlinear theory introduced in this paper, the data are the zonally averaged time series in four latitudes. In addition, according to the saturation error we determine the maximal prediction

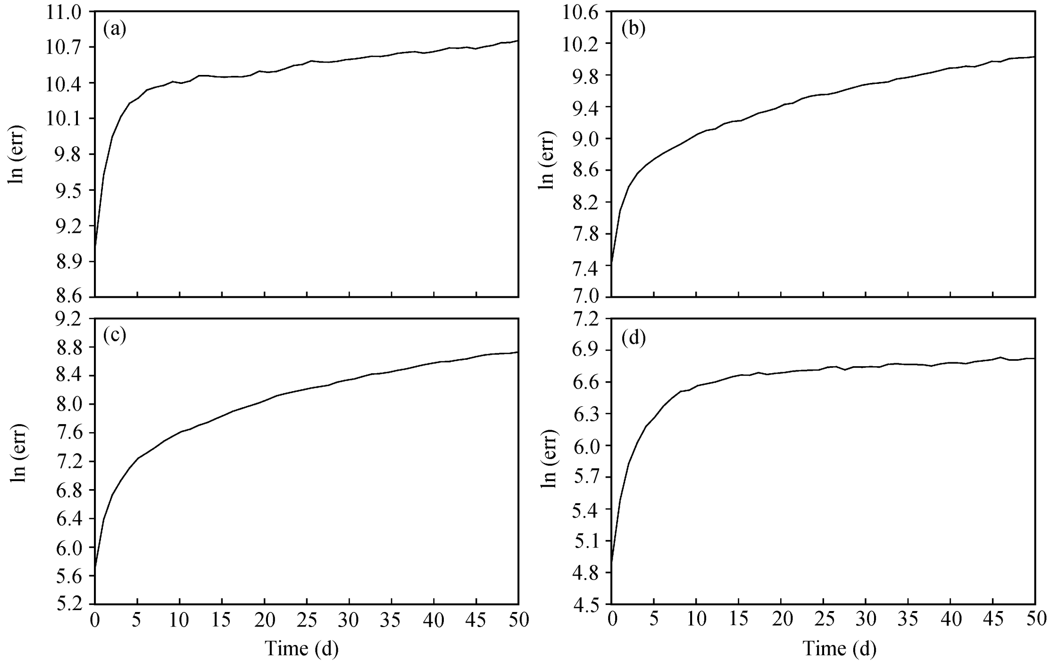

Fig. 2. Initial relative errors (Err) linear grow with the time. (a), (b), (c), (d) are the computation results in four latitudes: 90ºN, 60ºN, 30ºN, 0º , respectively. The data are the zonally averaged time series.

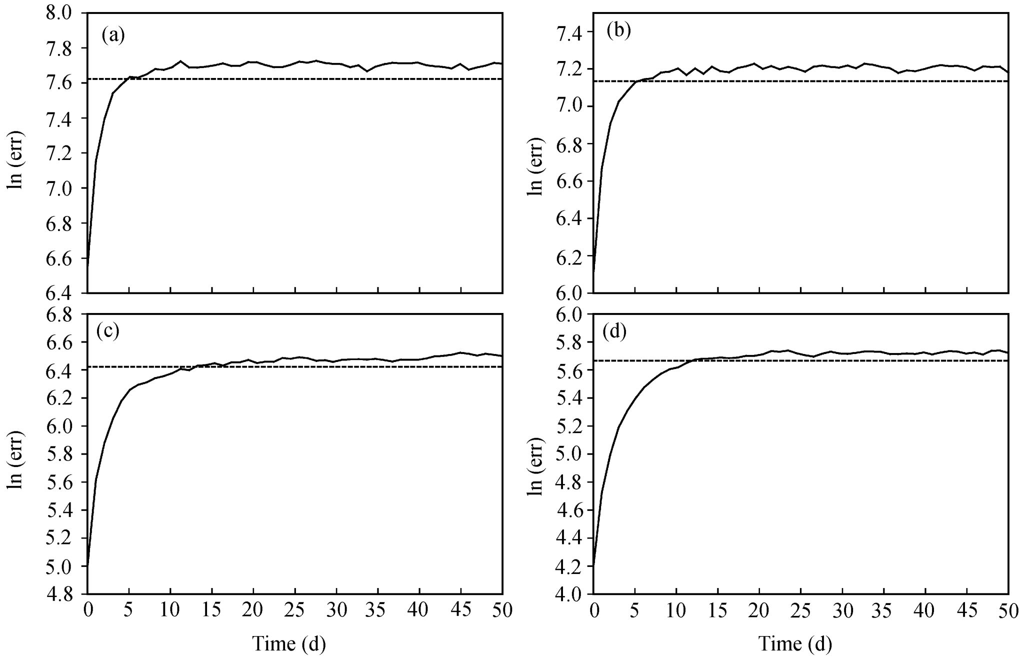

Fig. 3. Same as in Fig. 2 but for the initial error nonlinear growth. The 99% of estimated saturation value (averaged error between randomly chosen atmospheric sates) is also shown.

time, which is another advantage above the previous predictability theoretical research.

From Figs. 2 and 3 we know that the results computed by two theories are totally different. At first, they show the similar tends although their values are different, which means that when initial error is very small or during the initial growth period, error linear growth theory is still applicable; however as time increases, the linear theory result shows that error is always increasing monotonously, while according to nonlinear theory, initial error will reach the saturated value in some period and oscillate around it. The difference indicates that when errors increase to some value, linearized error growth theory is not applicable anymore and the nonlinear theory is required to answer the questions regarding the error growth and the gain in predictability. Saturation relative error (random error) means that the system enters the stochastic state, at which the initial information in the system is lost and the prediction is meaningless, so the associated time is the maximal prediction time. The limit of predictability was defined as the intersection between the curve representing the error average growing and the level line of 99% of the saturation error. The saturation error is estimated: by Fig. 3 we know after 15 days, the relative error growth is almost zero, so the estimation of the saturated error is defined by the geometrical average of the relative errors after 15 days, the reason to choose a limit value slightly below the saturation level is to reduce the effect of sampling fluctuations. For zonally averaged height field in four latitudes, the maximal prediction times are 7 days, 6.5 days, 11 days and 13.5 days respectively. These results are identical to the well-known results, which means that the nonlinear error growth theory can describe the real atmospheric initial errors evolution, compared to the linear theory; it has more advantages.

3 The application of the nonlinear local Lyapunov exponent in the atmospheric predictability

In the above, we use zonally averaged geopotential height time series to compute the nonlinear local Lyapunov exponent, the relative error time evolution and define the time when error reaches the saturated level as the maximal prediction time. The results show that exponent in this paper may better measure the evolution rate of the atmospheric initial perturbation. Next we compute the initial error growth of the 500 hPa height in different space, seasons and further obtain the temporal and spatial distribution of the global atmospheric predictability.

3.1 The globally spatial distribution of the atmospheric predictability

For the time series of all the grids in the global, we compute the nonlinear local Lyapunov exponent and the relative error growth, determine the maximal prediction time and obtain its spatial distribution, as shown in Fig. 4.

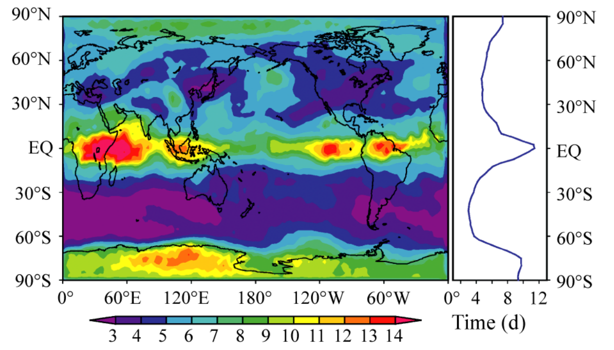

Fig. 4. Panel on the left is the global distribution of the maximal prediction time in the whole year and on the right is the latitudinal distribution of the zonal mean prediction time.

Fig. 4 shows that atmospheric predictability has a distinct spatial structure. Roughly speaking, it has a zonal distribution, the highest predictability exists in the tropics, the second is over the Antarctic; the predictability is also higher in the Arctic and over the subtropics and the mid-high latitudes the predictability is lowest. In particular, near the equator the average prediction time is 12 days, main regions of high predictability are the tropical Indian Ocean, Indonesia and the neighboring regions, the tropical Eastern Pacific and the prediction time is about two weeks. Interestingly, the predictability over the Antarctic is enhanced and the time is about 9 days, it is particularly evident during its austral summer (Fig. 6), which is perhaps related to the small high-frequency variability that is probably linked to the low baroclinicity and the low wind speeds in summer. Using the numerical model, Reichler and Roads[34], Kumar *et al.*[35], and Bacmeister *et al.*[36] also confirmed that there is a higher predictability near the neighbor of the Antarctic. Trenberth[37] obtained the similar conclusion by the study of the experimental data. These verify the results in this paper in all aspects. Moreover, in this paper we may estimate the maximal prediction time, which shows that this method has more advantages. The predictability over the Arctic is also higher than the mid-high latitudes, but the feature is less pronounced than the one near the Antarctic. In subtropics and mid latitudes (30°—60°S and 30°—60°N) regions, predictability is lowest and the average time is just 3—4 days. Predictability also shows the zonal asymmetry, for example, the North Atlantic and the middle of North Pacific have a higher predictability than other regions in the same latitude.

3.2 The latitudinal distribution of the maximal prediction time

Due to the difference of the spatial scale, zonally averaged fields may have different predictability from the original fields. In order to verify it, we compute the nonlinear local Lyapunov exponent according to the zonal mean time series and also obtain the maximal prediction time in all latitudes (see Fig. 5).

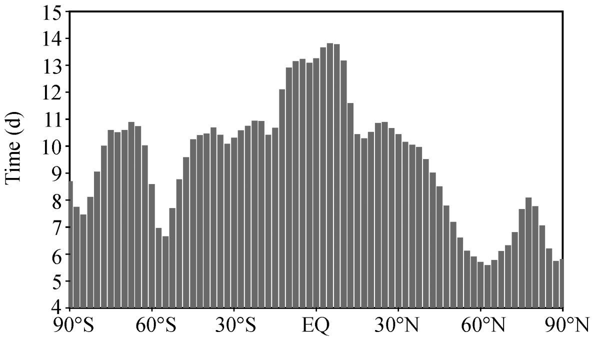

Fig. 5. Prediction time distributions for zonally averaged height.

Note that the prediction time of 500 hPa zonally averaged height shows an almost symmetric distribution in the Northern and Southern Hemispheres, which is that from the tropics to the mid-high latitudes, the prediction time decreases to the minimum in the regions of 50°—60°, while it tends to increase near the Antarctic and Arctic. The predictability over the tropics (10°S—10°N) is highest and the average prediction time is two weeks; in the latitudes of 15°—45°, the prediction time is about 10—11 days; while in 50°—60°, it is just 5—6 days. The decrement in the mid latitude of the Northern Hemisphere is much faster than the same latitudes of the Southern Hemisphere. Next to the polar, the predictability is enhanced particularly in 65°S—80°S, which has 8—9 days predictability. In latitudes of 70°—85°N, the predictability is also higher than the neighboring and the prediction time is about one week, but its enhancement is not as remarkable as the case in the Antarctic. All of these results are similar to Fig. 4, the only exception is that in the areas of 15°—45° of the two Hemispheres, the zonally averaged field seems to have the same prediction time, while for the daily original field, the predictability is weakened as the latitudes increase. Moreover, Fig. 5 shows that the zonally averaged field has a higher predictability than the daily original field, which means that predictability is indeed related to the spatial scale and the larger the scale is, the higher the predictability becomes.

3.3 Prediction time global distribution in winter and summer

The predictability changes in different seasons. In order to explore seasonal difference in the magnitude and spatial characteristics of the global atmospheric predictability, we compute the maximal prediction time in winter according to daily relative error nonlinear growth on November—February; similarly according to the daily relative error growth on June—August, we obtain the summer prediction time. Results are shown in Figs. 6 and 7.

In either winter or summer, atmospheric predict-

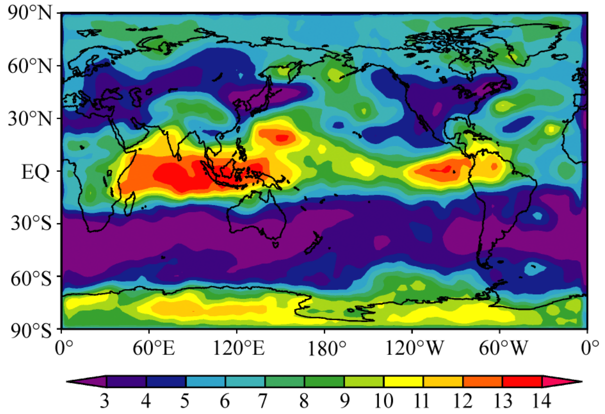

Fig. 6. Similar to Fig. 4 but for the Northern Hemisphere winter (DJF).

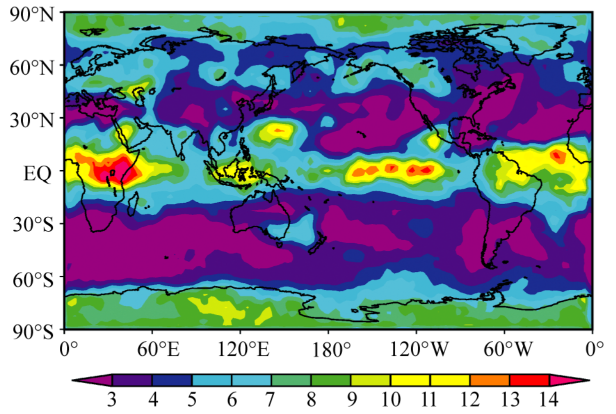

Fig. 7. Similar to Fig. 4 but for the Northern Hemisphere (NH) summer (JJA).

ability also shows the distinctive zonal distribution, similar to the average case in whole year. But comparing Fig. 6 to Fig. 7 we know that the distributions of the prediction time are quite different in the two contrasting seasons. In the Northern Hemispheric winter, the higher predictability regions are located in the tropical Indian Ocean, Indonesia and the neighboring regions, the tropical eastern Pacific Ocean; while in the Northern Hemispheric summer, they are located in Africa, the tropical Atlantic and the tropical mid-eastern pacific. During both seasons, there exist higher predictability regions near the Antarctic, but this feature is more distinct in the Southern Hemispheric summer. For the most regions in mid-high latitudes of the Northern Hemisphere (30°N — 60°N), the predictability is higher in winter than in summer, especially in the North Pacific (near the American coast), North Atlantic and other regions. While in Western Europe, the predictability in summer is higher than in winter. In the mid- high latitudes of the Southern Hemisphere (30°S—60°S), the prediction time in most areas changes little, it is about 3—5 days.

4 Summary

Estimating the growth of small initial errors in finite time is the main aim and method to study the atmospheric predictability. Because atmosphere itself is a nonlinear system and there exist some problems using the linearized equations to study the initial error growth, so in this paper based on the nonlinear error growth equation we first put forward a new concept: nonlinear local Lyapunov exponent. It is the locally average nonlinear divergence rate of the initial error in finite time. From the theoretical analysis, we know that the exponent has both the mathematical and physical bases and compared to the classical Lyapunov exponents, it may better quantify the time evolution of the real atmospheric initial observational error. Considering its definition and the atmospheric dynamics, we provide the reasonable algorithm to the exponent for observational data. By the computation results contrasting, we discuss in detail the difference between the linear theory and nonlinear theory and focus on the role of the nonlinear term to the error growth. Results show that the nonlinear local Lyapunov exponent really may better characterize the real atmospheric initial error nonlinear growth rate. Next, we define the time when error reaches the 99% saturated value as the maximal prediction time, which is the essential difference from most classic predictability researches. Lastly, we discuss the application of the nonlinear local Lyapunov exponent in the study of the atmospheric predictability and obtain spatial distribution and seasonal difference of the global atmospheric predictability. The main conclusions are: atmospheric predictability has a distinct spatial structure. Roughly speaking, it shows a remarkable zonal distribution. The highest predictability is located over Tropics and the Antarctic, near the Arctic it is relatively larger but in the subtropical and the mid-latitude of the two Hemispheres the predictability is lowest. In particular, near the equator the average prediction time is 12 days, main regions of high predictability are tropical Indian Ocean, Indonesia and the neighboring regions, tropical Eastern Pacific and the time is about two weeks. Interestingly, over the Antarctic atmosphere has a higher predictability of 9 days than mid-high latitude; this feature is particularly evident during its austral summer. Near the Arctic, the predictability is also relatively higher than the neighboring but the feature is less pronounced than the case near the Antarctic. In addition, predictability changes in different seasons. For most regions in the Northern Hemisphere, the predictability is higher in winter than in summer, especially in the mid-high latitude, North Atlantic, North Pacific, Greenland and other regions; in the Southern Hemisphere, over the Antarctic regions the predictability is higher in the associated summer than in winter, other regions especially in 30°S—60°S, it hardly changes and the time is about 3—5 days.

Besides 500 hPa height, we also discuss by this exponent the predictability of 850 hPa temperature field, obtain some reasonable results. All of these show that the nonlinear local Lyapunov exponent really may be used to explore the predictability of all variables in the different temporal and spatial regions. With the further research, we believe that the nonlinear local Lyapunov exponent has a better application in the atmospheric and oceanic theoretical research.

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