# An actuator space optimal kinematic path tracking framework for tendon-driven continuum robots: Theory, algorithm and validation

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This repository implements the algorithms presented in our article. If you enjoy this repository and use it, please cite our paper.

Our previous work (titled: An Efficient Multi-solution Solver for the Inverse Kinematics of 3-Section Constant-Curvature Robots) appears in proceedings of Robotics: Science and Systems 2023.

```
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```
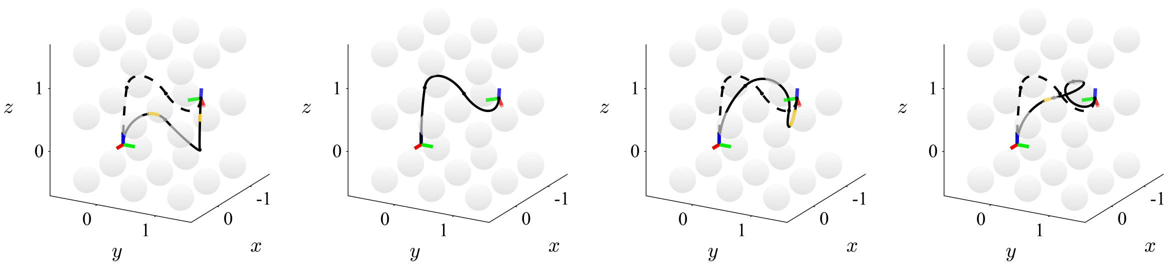
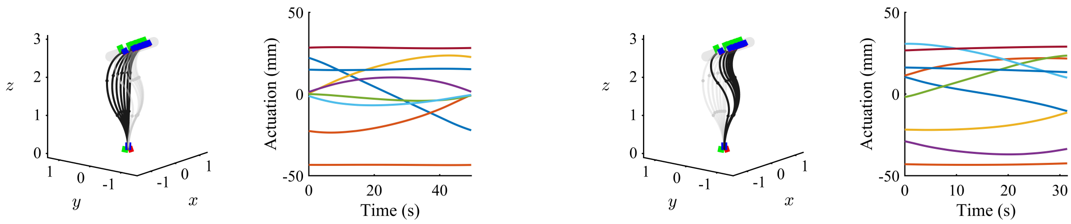

## Demo

**(main_demo.m)** Results of multiple solutions obtained by our algorithm.



**(main_demo2.m)** Results of tracking a straight line path in two different configurations obtained by our algorithm.



# Package overview

**Demo (2 files)**

- main_demo.m
- main_demo2.m

**Solver (7 files)**
public:

- micsolver.m
- micsolverd.m

private:

- rho.m
- soln2xi.m
- get_err.m
- solve_r1.m
- solve_r2.m

**Planner (2 files)**
public:

- dp.m
- allocate_time.m

**Numerical methods (5 files)**
public:

- revise_grad.m
- revise_dls.m
- revise_newton.m

private:

- revise_plot.m
- jacobian3cc.m

**Quaternion operations (3 files)**

- up_plus.m
- up_oplus.m
- up_star.m

**Lie algebra operations (4 files)**

- up_hat.m

- up_vee.m
- exphat.m
- veelog.m

**Conversions (6 files)**

- arc2q.m
- q2arc.m
- arc2xi.m
- xi2arc.m
- xi2len.m
- q2rot.m
- rot2q.m

**Other tools (6 files)**

- circles3.m
- circles3c.m
- frame.m
- get_end.m
- collision_indicator.m
- collision_marker.m

# Functions

## Solver

### micsolver.m

`MICSOLVER` Multi-solution solver for the inverse kinematics of 3-section constant-curvature robots.

`[NOS, NOI] = MICSOLVER(L1, L2, L3, Q, R, TOL, IS_A_SOL)` returns the result of solving the 3-section inverse kinematics problem. The function uses preset resolutions or numerical methods to address the inverse kinematics problem. The function exits after one solution is found.

*Input parameters*
`L1, L2, L3` section length
`Q, R` desired end rotation and translation
`TOL` error tolerance
`IS_A_SOL` function handle
It is used to judge if the given parameter is a solution to the inverse kinematics problem. The function has two inputs `(ERR, XI)` and one output in boolean type.

*Output parameters*
`NOS` number of solutions
`NOI` number of iterations in numerical correction

*Example*

```
L1 = 1; L2 = 1; L3 = 1;
xi = arc2xi(L1, L2, L3, pi.*[1,2,1,2,1,2].*rand(1, 6));
T = get_end(L1, L2, L3, xi);
q = rot2q(T(1:3, 1:3));
r = T(1:3, 4);
tol = 1e-2; fun = @(e, x) e < tol;
tic;
[nos, ~] = micsolver(L1, L2, L3, q, r, tol, fun);
rt = toc*1000;
if nos
    fprintf('A solution is found in %.2f ms.\n', rt);
end
```

### micsolverd.m

`MICSOLVERD` Multi-solution solver (debug) for the inverse kinematics of 3-section constant-curvature robots.

`[SOL, NOS, NOI] = MICSOLVERD(L1, L2, L3, Q, R, PAR, NOC, TOL, MSTEP, IS_A_SOL, PLOT_EC, PLOT_IT)` returns the result of solving the 3-section inverse kinematics problem.

*Input parameters*
`L1, L2, L3` section length
`Q, R` desired end rotation and translation
`PAR` length of the partition
Scalar. The interval $[0, 1)$ is partitioned into subintervals with length `PAR`. A smaller `PAR` means finer resolution. We recommend `PAR = 0.03` for better efficiency and `PAR = 0.01` for more solutions.
`NOC` number of corrections
A 1-by-2 array. The model parameters $\hat{\boldsymbol{r}}*3$ and hat $\hat{\boldsymbol{r}}*1$ are corrected for `NOC(1)` and `NOC(2)` times after the approximation, respectively. Large `NOC` provides closer initial value and more computations. We recommend `NOC = [1, 2]` for better efficiency and `NOC = [5, 5]` for better estimation.
`TOL` error tolerance
`MSTEP` allowed maximum steps of iterations
`IS_A_SOL` function handle
This function is used to judge if the given parameter is a solution to the inverse kinematics problem. The function has two inputs `(ERR, XI)` and one output in boolean type.
`PLOT_EC` set `'plot'` to visualise the traversal
`PLOT_IT` set `'plot'` to visualise the numerical correction

*Output parameters*
`SOL` solutions
A 6-by-`NOS` array. Each column is the overall exponential coordinate `XI`.
`NOS` number of solutions
`NOI` number of iterations in numerical correction

*Example*

```
L1 = 1; L2 = 1; L3 = 1;
alpha = 15*pi/16; omega = [0.48; sqrt(3)/10; -0.86];
q = [cos(alpha/2); sin(alpha/2)*omega];
r = [-0.4; 1.1; 0.8];
tol = 1e-2; fun = @(e, x) e < tol;
[sol, ~, ~] = micsolverd(L1, L2, L3, q, r, ...
                         0.01, [5, 5], tol, 10, fun, ...
                         'plot', 'plot');
for eta = 1: size(sol, 2)
    fh = figure();
    circles3(fh, L1, L2, L3, sol(:, eta), 'k-');
    view(119, 20);
end
```

### rho.m

`RHO` Computes the linear distance between two ends of a circular arc.

This is a private function of our solver.

### soln2xi.m

`SOLN2XI` Converts the output of our solver to an exponential coordinate.

This is a private function of our solver.

### get_err.m

`GET_ERR` Computes the error between desired and current end pose.

This is a private function of our solver.

### solve_r1.m

`SOLVE_R1` Computes the model parameter of the 1st section.

This is a private function of our solver.

### solve_r2.m

`SOLVE_R2` Computes the model parameter of the 2nd section using rotational and translational constraints.

This is a private function of our solver.

## Planner

### dp.m

`DP` Finds the shortest path in a graph.

`[PATH, COST] = DP(XISC, LOSSFCN)` returns the paths and corresponding costs using the Dijkstra's algorithm. The cell array `XISC` defines the vertices. The function handle `LOSSFCN` defines the weight of two adjacent edges. The output `PATH` and `COST` are cell arrays.

### allocate_time.m

`ALLOCATE_TIME` allocates optimal time to a given sequence of concatenated parameters, considering the actuator velocity constraints.

`TS = ALLOCATE_TIME(XIS)` returns the time array `TS`.

## Numerical methods

### revise_*.m

`REVISE_*` Correct the initial value with a numerical method.

`[XI_STAR, ERR, K] = REVISE_*(L1, L2, L3, Q, R, XI, MSTEP, TOL, TYPE)` returns the result of numerical correction.

*Methods*
`grad` gradient method
`dls` damped least square method
`newton` Newton-Raphson method

*Input parameters*
`L1, L2, L3` section length
`Q, R` desired end rotation and translation
`XI` initial value
`MSTEP` allowed maximum steps of iterations
`TOL` error tolerance
`TYPE` set `'plot'` to visualise the numerical correction

*Output parameters*
`XI_STAR` final value
`ERR` final error
`K` steps of iterations

*Example*

```
L1 = 1; L2 = 1; L3 = 1;
alpha = 15*pi/16; omega = [0.48; sqrt(3)/10; -0.86];
q = [cos(alpha/2); sin(alpha/2)*omega];
r = [-0.4; 1.1; 0.8];
xi_0 = arc2xi(L1, L2, L3, pi.*[1, 2, 1, 2, 1, 2].*rand(1, 6));
[xi, err, noi] = revise_grad(L1, L2, L3, q, r, xi_0, 2000, 1e-2, 'plot');
[xi, err, noi] = revise_dls(L1, L2, L3, q, r, xi_0, 2000, 1e-2, 'plot');
[xi, err, noi] = revise_newton(L1, L2, L3, q, r, xi_0, 200, 1e-2, 'plot');
```

### revise_plot.m

`REVISE_PLOT` Visualises the numerical correction.

This is a private function of numerical methods.

### jacobian3cc.m

`JACOBIAN3CC` Computes the Jacobian matrix when the forward kinematics is expressed by the product of exponentials formula.

`J = JACOBIAN3CC(L1, L2, L3, XI)` returns the 6-by-6 Jacobian matrix.

## Quaternion operations

### up_plus.m

`UP_PLUS` Computes the matrix for left multiplications of quaternions.

`Q_UP_PLUS = UP_PLUS(Q)` returns the left multiplication matrix of `Q`.

### up_oplus.m

`UP_OPLUS` Computes the matrix for right multiplications of quaternions.

`Q_UP_OPLUS = UP_OPLUS(Q)` returns the right multiplication matrix of `Q`.

### up_star.m

`UP_STAR` Computes the quaternion conjugation.

`Q_UP_STAR = UP_STAR(Q)` returns the conjugation of `Q`.

## Lie algebra operations

### up_hat.m

`UP_HAT` Computes the Lie algebra of a vector.

`M = UP_HAT(V)` is an element of $\mathsf{so}3$ or $\mathsf{se}3$, where `V` is an element of $\mathbb{R}^3$ or $\mathbb{R}^6$, respectively.

### up_vee.m

`UP_VEE` Computes the vector of a Lie algebra.

`V = UP_VEE(M)` is an element of $\mathbb{R}^3$ or $\mathbb{R}^6$, where `M` is an element of $\mathsf{so}*3$ or $\mathsf{se}*3$, respectively.

### exphat.m

`EXPHAT` Composition of the hat map and the matrix exponential.

`M = EXPHAT(V)` is a matrix in $\mathsf{SO}*3$ or $\mathsf{SE}*3$ and is computed using Rodrigues' formula. The vector `V` is in $\mathbb{R}^3$ or $\mathbb{R}^4$. The hat map sends `V` to an element of $\mathsf{so}*3$ or $\mathsf{se}*3$.

### veelog.m

`VEELOG` Composition of the matrix logarithm and the vee map.

`V = VEELOG(M)` is a vector in $\mathbb{R}^3$ or $\mathbb{R}^4$ and is computed using Rodrigues' formula. The matrix `M` is in $\mathsf{SO}*3$ or $\mathsf{SE}*3$. The vee map sends an element of $\mathsf{so}*3$ or $\mathsf{se}*3$ to a vector.

## Conversions

### arc2q.m

`ARC2Q` Converts arc parameters to a quaternion.

`Q = ARC2Q(KAPPA, PHI, L)` computes the quaternion `Q` representing the end rotation of a 1-section constant-curvature robot with curvature `KAPPA`, bending angle `PHI`, and section length `L`.

*Example*

```
kappa = 2*pi/3;
phi = pi/3;
q = arc2q(kappa, phi, 1);
```

### q2arc.m

`Q2ARC` Converts a quaternion to arc parameters.

`ARC = Q2ARC(Q, L)` computes the arc parameters of a 1-section constant-curvature robot, including the curvature `KAPPA` and bending angle `PHI`. The section length is `L`. The quaternion `Q` represents the end rotation.

*Example*

```
q = [1/2; -3/4; sqrt(3)/4; 0];
arc = q2arc(q, 1);
```

### arc2xi.m

`ARC2XI` Converts the arc parameters of 3 sections to the exponential coordinate.

`XI = ARC2XI(L1, L2, L3, ARC)` computes the exponential coordinate `XI` of a 3-section constant-curvature robot. The section lengths are `L1, L2` and `L3`, respectively. The parameter `ARC` is an array containing curvatures and bending angles of each section.

*Example*

```
k1 = 4*sqrt(5)/5; p1 = atan(2);
k2 = sqrt(37)/5; p2 = -pi+atan(6);
k3 = sqrt(10)/5; p3 = -atan(3);
xi = arc2xi(1, 1, 1, [k1, p1, k2, p2, k3, p3]);
```

### xi2arc.m

`XI2ARC` Converts the exponential coordinate to the arc parameters of 3 sections.

`ARC = XI2ARC(L1, L2, L3, XI)` computes the arc parameter `ARC` of a 3-section constant-curvature robot. The section lengths are `L1, L2` and `L3`, respectively. The parameter `XI` is the overall exponential coordinate.

*Example*

```
xi = [-1.6; 0.8; 1.2; -0.2; 0.6; 0.2];
arc = xi2arc(1, 1, 1, xi);
```

### xi2len.m

`XI2LEN` Converts the concatenated parameter to the actuator lengths.

`LEN = XI2LEN(XI)` computes the actuator lengths `LEN` of a 3-section constant-curvature robot. The concatenated parameter `XI` is defined in our article.

*Example*

```
xi = [-1.6; 0.8; 1.2; -0.2; 0.6; 0.2];
len = xi2len(xi);
```

### q2rot.m

`Q2ROT` Converts a quaternion to a rotation matrix.

`R = Q2ROT(Q)` returns the rotation matrix that is equivalent to the quaternion.

### rot2q.m

`ROT2Q` Converts a rotation matrix to a quaternion.

`Q = ROT2Q(R)` returns the quaternion that is equivalent to the rotation matrix.

## Other tools

### circles3.m

`CIRCLES3` Visualises the 3-section constant-curvature robot with given model parameters.

`CIRCLES3(FH, L1, L2, L3, XI, TYPE)` displays the plot in target figure `FH`. The robot is described by the section lengths `L1, L2, L3` and the overall exponential coordinate `XI`. Line colours and styles are specified in the character string `TYPE`.

*Example*

```
L1 = 1; L2 = 1; L3 = 1;
xi_1 = [-1.60; 0.08; 1.20; -0.20; 0.60; 0.20];
circles3(1, L1, L2, L3, xi_1, 'k--');
xi_2 = [-0.39; 0.48; -1.13; 0.47; 1.79; -0.17];
circles3(1, L1, L2, L3, xi_2, 'k-');
view(75, 9);
```

### circles3c.m

`CIRCLES3` Visualises the 3-section constant-curvature robot with given model parameters and a specified colour.

`CIRCLES3C(FH, L1, L2, L3, XI, TYPE, COLOUR)` displays the plot in target figure `FH`. The robot is described by the section lengths `L1, L2, L3` and the overall exponential coordinate `XI`. Line styles are specified in the character string `TYPE`, line colours are specified in the triple `COLOUR`.

*Example*

```
L1 = 1; L2 = 1; L3 = 1;
xi_1 = [-1.60; 0.08; 1.20; -0.20; 0.60; 0.20];
circles3c(1, L1, L2, L3, xi_1, '--', [0.1, 0.1, 0.1]);
xi_2 = [-0.39; 0.48; -1.13; 0.47; 1.79; -0.17];
circles3c(1, L1, L2, L3, xi_2, '-', [0.2, 0.2, 0.2]);
view(75, 9);
```

### frame.m

`FRAME` draws a coordinate frame with specified position (and orientation) and transparency.

`FRAME(FH, A, ALPHA)` displays the plot in target figure `FH`. The position (and orientation) is specified by `A` and the transparency is specified by `ALPHA`.

*Example*

```
frame(1, [eye(3), zeros(3, 1); 0, 0, 0, 1], 0.3);
```

### get_end.m

`GET_END` Computes the end pose of a 3-section constant-curvature robot.

`T = GET_END(L1, L2, L3, XI)` returns the 4-by-4 matrix of end pose.

*Example*

```
xi = [-1.6; 0.8; 1.2; -0.2; 0.6; 0.2];
T = get_end(1, 1, 1, xi);
```

### collision_indicator.m

`COLLISION_INDICATOR` Computes the minimal distance between the sample points and the spherical obstacles.

`COLLIDE = COLLISION_INDICATOR(L1, L2, L3, XI, RO, ROR, SMP)` returns the minimal distance `COLLIDE`. If `COLLIDE > 0`, then no collision occurs. If `COLLIDE < 0`, then the robot collides with obstacles. The robot is described by the section lengths `L1, L2, L3` and the overall exponential coordinate `XI`. The obstacles are spheres centring at `RO` with radius `ROR`. The sample points are distributed uniformly along the robot curve with the number `SMP`.

*Example*

```
L1 = 1; L2 = 1; L3 = 1;
xi = [-1.6; 0.8; 1.2; -0.2; 0.6; 0.2];
ro = [0.8; 0.7; 0.6];
ror = 0.4;
collide = collision_indicator(L1, L2, L3, xi, ro, ror, 10);
```

### collision_marker.m

`COLLISION_MARKER` Visualises the collision part in yellow.

`COLLISION_MARKER(L1, L2, L3, XI, RO, ROR)` draws the part of the robot that is collided with obstacles. The robot is described by the section lengths `L1, L2, L3` and the overall exponential coordinate `XI`. The obstacles are spheres centring at `RO` with radius `ROR`.

*Example*

```
L1 = 1; L2 = 1; L3 = 1;
xi = [-1.6; 0.8; 1.2; -0.2; 0.6; 0.2];
ro = [0.8; 0.7; 0.6];
ror = 0.4;
circles3(1, L1, L2, L3, xi, 'k-');
collision_marker(L1, L2, L3, xi, ro, ror);
view(75, 9);
```