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Series 8

1. See files `MonteCarloEuler.m` and `MonteCarloEulerBSCall.m`.

For a fixed $M \in \mathbb{N}$, an error equilibration is achieved by setting $\Delta t \approx M^{-1/2}$.

```
Weak convergence rate of the MonteCarloEuler scheme w . r . t . to step size delta: 0.98672

Weak convergence rate of the MonteCarloEuler scheme w . r . t . to number of samples M: -0.49336
```

Abbildung 1: The RMSE converges with order $\mathcal{O}(\Delta t + M^{-1/2})$ for the Monte Carlo-Euler method.

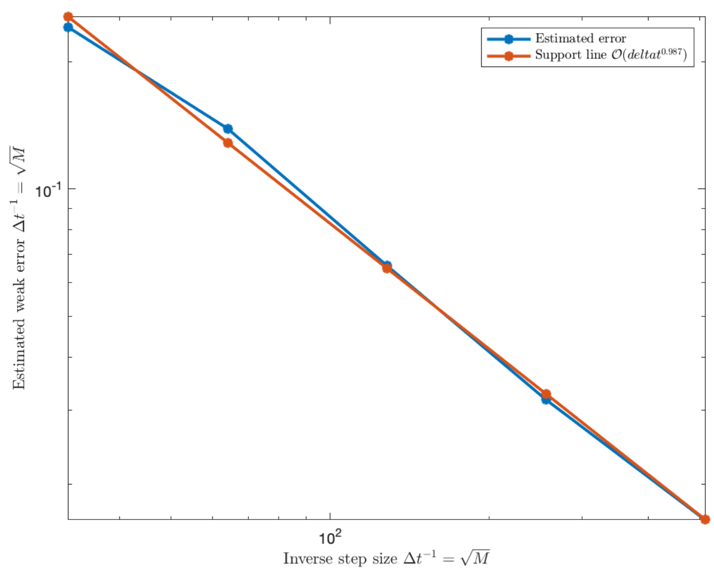

Abbildung 2: Due to the error balancing $\Delta t \approx M^{-1/2}$, the overall RMSE is of order $\mathcal{O}(\Delta t)$.

2. **a)** See file `MonteCarloEulerHestonCall1.m`.

```
>> MonteCarloEulerHestonCall1
Weak convergence rate of the MonteCarloEuler scheme w . r . t . to step size delta: 0.9155
Weak convergence rate of the MonteCarloEuler scheme w . r . t . to number of samples M: -0.45775
```

Abbildung 3: We use the error balancing $\Delta t \approx M^{-1/2}$ for the truncated Euler scheme. The RMSE converges again roughly with order $\mathcal{O}(\Delta t + M^{-1/2})$, hence at the same rate as for the call price in the Black-Scholes model.

b) See file `MonteCarloEulerHestonCall2.m`.

```
>> MonteCarloEulerHestonCall2
Weak convergence rate of the MonteCarloEuler scheme w . r . t . to step size delta: 0.46462
Weak convergence rate of the MonteCarloEuler scheme w . r . t . to number of samples M: -0.23231
```

Abbildung 4: Using $\Delta t \approx M^{-1/2}$ as before, we obtain a slower error decay of order $\mathcal{O}(\Delta t^{1/2}) = \mathcal{O}(M^{-1/4})$.

```
>> MonteCarloEulerHestonCall2
Weak convergence rate of the MonteCarloEuler scheme w . r . t . to step size delta: 0.49444
Weak convergence rate of the MonteCarloEuler scheme w . r . t . to number of samples M: -0.49444
```

Abbildung 5: If we change to $\Delta t \approx M^{-1}$, the overall error still decays with the rate $\mathcal{O}(\Delta t^{1/2})$. Since $\Delta t = M^{-1}$ the statistical error still decays as $\mathcal{O}(M^{-1/2})$ as in part a), whereas the discretization error is now only of order $\mathcal{O}(\Delta t^{1/2})$.

> *Remark:* The worse rate of $\mathcal{O}(\Delta t^{1/2})$ is due to the different parameter setting in the Heston model. It now holds that $\sigma_{\mathrm{v}}^2 > 2ab$, thus the *Feller condition* is violated and V hits zero with a positive probability. Therefore, the Euler discretization frequently produces negative values that have to be truncated. This introduces an additional bias in the scheme. It can be shown that the truncated Euler scheme for the CIR process converges weakly with rate $\mathcal{O}(\Delta t)$ if the *Feller condition* $\sigma_{\mathrm{v}}^2 > 2ab$ holds as in part a).

3. See file `MCERichardsonExtrapolation.m`.

```
>> MCERichardsonExtrapolation
Weak convergence rate of the MonteCarloEuler scheme w . r . t . to step size delta: 2.206
Weak convergence rate of the MonteCarloEuler scheme w . r . t . to number of samples M: -0.5515
```

Webpage: `https://moodle-app2.let.ethz.ch/course/view.php?id=17423`
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