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# The Extended Linear-Drift Model of Memristor and Its Piecewise Linear Approximation

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**Abstract:** Memristor is introduced as the fourth basic circuit element. Memristor exhibits great potential for numerous applications, such as emulating synapse, while the mathematical model of the memristor is still an open subject. In the linear-drift model, the boundary condition of the device is not considered. This paper proposes an extended linear-drift model of the memristor. The extended linear-drift model keeps the linear characteristic and simplicity of the linear-drift model and considers the boundary condition of the device. A piecewise linear approximation model of the extended linear-drift model is given. Both models are suitable for describing the memristor.

**Key words:** memristor; mathematical model; piecewise linear; hysteresis

## 1 Introduction

In Ref. [1], memristor is introduced as the fourth basic circuit element, which is as basic as the resistor, the capacitor, and the inductor. The memristor is characterized by a relationship between the flux $\varphi$ and the charge $q$. More specifically, the relationship between current $i$ and voltage $v$ of a memristor is defined by

$$v(t) = M(q(t))i(t) \tag{1}$$

where

$$M(q(t)) = \mathrm{d}\varphi(q)/\mathrm{d}q \tag{2}$$

In this memristor definition, $\varphi(q)$ describes the relationship between the flux and the charge of the memristor, and the charge $q(t)$ is the state variable. $M(q(t))$ has the unit of resistance, and is called the memristance (resistance of memristor)[2, 3]. At a certain time $t_0$, the memristance $M(q(t_0)) = v(t_0)/i(t_0)$ of a memristor is determined by $q(t_0) = \int_{-\infty}^{t_0} i(\tau)\mathrm{d}\tau$, i.e., the memristor remembers the amount of the charge passed through it. More detailed character of the memristor can be seen in Ref. [1]. Different memristors can be distinguished from each other by the specific forms of memristance $M(q(t))$. Several studies have been conducted based on the memristor definition (Eq. (1)), for example, SPICE modeling of the memristor[3], the memristor oscillator[4], and memristor implementation of the IDS method[5].

In Refs. [2, 6], the concept of memristor is generalized to memristive systems and circuit elements with memory such as memcapacitors and meminductors. A memristive system can be described by

$$\dot{x} = f(x, i, t) \tag{3a}$$

$$v = R(x, i, t)i \tag{3b}$$

where $x$ is the state variable of the memristive system. The memristor is a special case of the memristive system. For a memristor shown in Eq. (1), letting $x = q$ , $\dot{x} = \dot{q} = i$ and $R(x, i, t) = M(q(t))$, it is not hard to see that the system also fits the memristive system description (Eqs. (3)). A general memristive system may not be transformed into the memristor

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definition. Therefore, studies and methods based on the memristor definition may not be easily generalized to memristive systems.

In Ref. [7], a physical model of the memristor is proposed. The physical model describes a two-terminal electrical device. The device is a thin semiconductor film, whose thickness is $D$, sandwiched between two metal contacts. The semiconductor film has two regions: one region with a high dopant concentration and low resistance $R_{\mathrm{ON}}$, and the other region has a low concentration of dopant with a considerably higher resistance $R_{\mathrm{OFF}}$. In the remainder of this paper, we refer the device as the HP device. The HP device has many applications. Define the thickness of the region with low resistance $R_{\mathrm{ON}}$ as $w(t) \in [0, D]$. According to Ref. [7], for the simplest case of ohmic electronic conduction and linear ionic drift in a uniform field with average ion mobility $\mu_v$, the device can be described with the linear-drift model:

$$v(t) = \left(R_{\mathrm{ON}}\frac{w(t)}{D} + R_{\mathrm{OFF}}\left(1 - \frac{w(t)}{D}\right)\right) i(t) \quad (4a)$$

$$\frac{\mathrm{d}w(t)}{\mathrm{d}t} = \mu_v \frac{R_{\mathrm{ON}}}{D} i(t) \quad (4b)$$

As shown in Ref. [7], the state variable of Eqs. (4) is $w(t)$, unlike $q(t)$ in the memristor definition, but from Eqs. (4), the following relation of $w(t)$ and $q(t)$ can be obtained by the equation

$$w(t) = \mu_v \frac{R_{\mathrm{ON}}}{D} q(t) + w(0) \quad (5)$$

where $w(0)$ is the initial thickness of the region with low resistance $R_{\mathrm{ON}}$. Then the linear-drift model of the HP device can be equivalently described under the memristor definition, where memristance is a linear function of $q(t)$ as follows:

$$v(t) = M(q(t))i(t) \quad (6a)$$

$$M(q(t)) = M_0 - K_M \Delta R q(t) \quad (6b)$$

In Eq. (6b), $K_M = \mu_v \frac{R_{\mathrm{ON}}}{D^2}$, $\Delta R = R_{\mathrm{OFF}} - R_{\mathrm{ON}}$ and the initial memristance $M_0 = R_{\mathrm{ON}}\frac{w(0)}{D} + R_{\mathrm{OFF}}\left(1 - \frac{w(0)}{D}\right)$.

According to Refs. [7, 8], Eq. (5) is only valid for values of $w(t)$ in the interval $[0, D]$, corresponding to $q(t) \in [q_{\min}, q_{\max}]$, $q_{\min} = -(w(0)D)/(\mu_v R_{\mathrm{ON}})$, $q_{\max} = ((D - w(0))D)/(\mu_v R_{\mathrm{ON}})$. Boundary condition is not considered in the linear-drift model when $w$ reaches zero or $D$. Then for simulation and analysis based on Eqs. (4) or (6), the parameters of applied voltage or current must be properly chosen so that they will not cause $w(t)$ overflows $[0, D]$. For example, if a positive current $i(t)$ is applied for a sufficiently long period of time, so that the $q(t)$ exceeds $q_{\max}$. Then the $w(t)$ calculated by Eq. (5) will be larger than $D$ and the memristance calculated by Eq. (6b) will be smaller than $R_{\mathrm{ON}}$, which is contradictory with the definition of $w(t)$ and the physical properties of the HP device.

These limitations of the linear-drift model show that the boundary condition of the device when $w$ is close to zero or $D$ must be considered in order to give a proper mathematical model of the HP device. We propose an extended linear-drift model of the HP device. The extended linear-drift model preserves the linear characteristic of the linear-drift model (Eqs. (6)) while $q(t)$ varies between $[q_{\min}, q_{\max}]$, and deals with the boundary effects by extending the definition of the function relationship of $w(t)$ and $q(t)$ for $q(t) \notin [q_{\min}, q_{\max}]$. Therefore the extended linear-drift model is valid for arbitrary applied voltage or current, while the applied voltage or current must be properly chosen for the linear-drift model. The extended linear-drift model naturally has an explicit memristance expression in the memristor definition. The simulation result of the extended linear-drift model is consistent with the results given in Ref. [7], as will be shown in Section 2.

According to the definition, the memristor is characterized by the relationship between the flux $\varphi$ and the charge $q$. From the extended linear-drift model which naturally has an explicit memristance expression, we calculate the corresponding $\varphi - q$ curve. Compare this $\varphi - q$ curve with a existing piecewise linear $\varphi - q$ curve which is assumed to characterize a memristor[1,4], we found that the existing piecewise linear model may not properly approximate the $\varphi - q$ curve derived from the extended linear-drift model of the HP device. We propose a suitable piecewise linear approximation of the HP device based on the extended linear-drift model. The simulation result shows that our piecewise linear model is a good approximation. Our piecewise approximation linear model is simpler and retains the main characteristics of the HP device. With the piecewise linear approximation model, piecewise linear techniques could be used to design and analyze physical systems consisting of memristor and other circuit elements.

## 2 The Extended Linear-Drift Model

The "pinched hysteresis loop" is one of the important characters of the memristor[2,7,9]. Figure 1 shows the

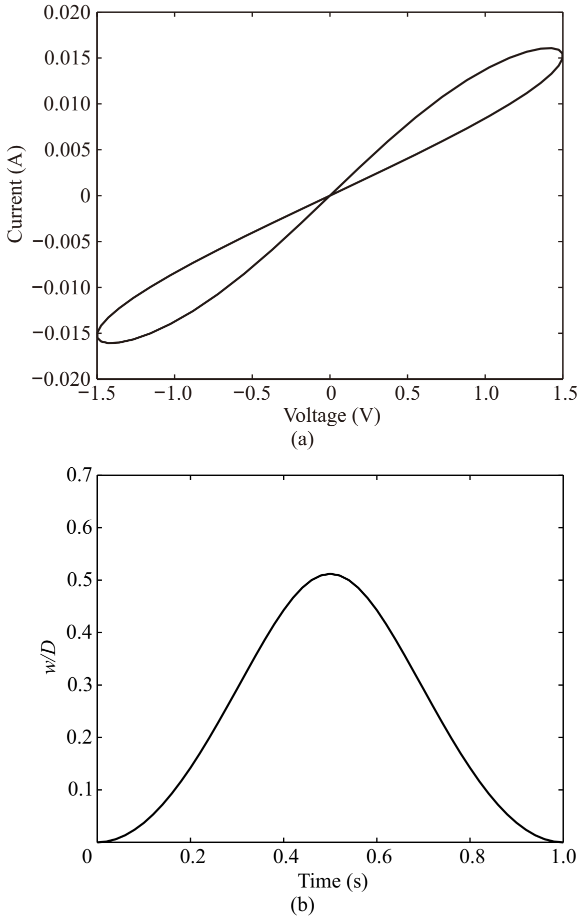


**Fig. 1 Simulation results of the linear-drift model. Typical pinched hysteresis loop is observed in the $i-v$ curve. $R_{ON} = 1\,\Omega$, $R_{OFF} = 125\,\Omega$, $D = 10\,$nm, $\mu_v = 10^{-10}\,$cm$^2$s$^{-1}$V$^{-1}$, $w(0) = 0$. Input $v(t) = 1.5\sin(2\pi t)$.**

pinched hysteresis loop in the $i - v$ curve of the linear-drift model. Note that the parameters of applied voltage are carefully chosen so that during the simulation $q(t) \in [q_{\min}, q_{\max}]$ and $w(t) \in [0, D]$ is satisfied.

Noting the aforementioned limitations of the linear-drift model, here we propose the extended linear-drift model of the HP device. Specifically,

$$v(t) = M_E(q(t))i(t) \tag{7a}$$

$$M_E(q(t)) = \begin{cases} R_{OFF}, & q(t) \leqslant q_{\min}; \\ M_0 - K_M \Delta R q(t), & q_{\min} < q(t) < q_{\max}; \\ R_{ON}, & q(t) \geqslant q_{\max} \end{cases} \tag{7b}$$

Then the relation of $w(t)$ and $q(t)$ in the extended linear-drift model is

$$w(t) = \begin{cases} 0, & q(t) \leqslant q_{\min}; \\ \mu_v \dfrac{R_{ON}}{D} q(t) + w(0), & q_{\min} < q(t) < q_{\max}; \\ 1, & q(t) \geqslant q_{\max} \end{cases} \tag{8}$$

The extended linear-drift model (Eqs. (7)) keeps the linear characteristic of the linear-drift model for $q_{\min} < q(t) < q_{\max}$. And when $q(t)$ reaches $q_{\min}$ or $q_{\max}$, corresponding to $w(t)$ reaches boundary 0 or $D$, the memristance $M_E(q)$ remains $R_{OFF}$ or $R_{ON}$ respectively. When the boundary is reached, the extended linear-drift model can still "remember" the charge passing through by calculating $q(t)$. This is consistent with the fact that for a general memristor, the memristance is determined by the charge $q(t)$.

The extended linear-drift model naturally has an explicit memristance expression. The memristance $M_E(q)$ is plotted in Fig. 2. Then using this formulation, the extended linear-drift model can be directly applied to existing studies, methods, and applications based on the memristor definition. Particularly, the extended linear-drift model can be a good complement for studies using the linear-drift model (Eqs. (4) or (6)), such as Refs. [3, 5, 10]. The extended linear-drift model is exactly the same with the linear-drift model for $q_{\min} < q(t) < q_{\max}$. The pinched hysteresis loop is the key feature of memristor[2,7]. The simulation is executed using the extended linear-drift model, as shown in Fig. 3. The pinched hysteresis loop is observed, which is consistent with the results given in Ref. [7].

Another existing way to deal with the boundary condition is to use window function[7,8]. In the window function nonlinear model, corresponding to nonlinear drift when $w$ is close to 0 or $D$, a window function

$$F_p(w) = 1 - \left(\frac{2w}{D} - 1\right)^{2p} \tag{9}$$

is multiplied to the right-hand side of Eq. (4b), and leads to

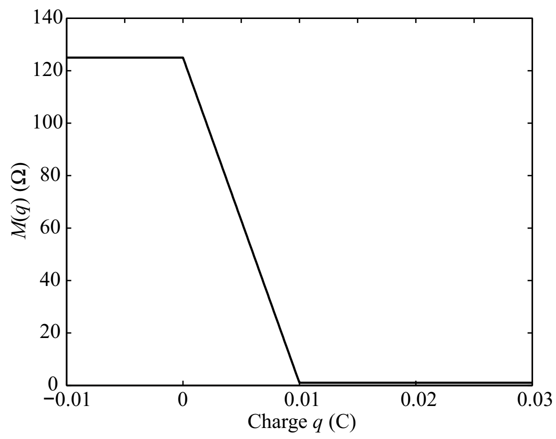


**Fig. 2 Memristance of the extended linear-drift model. $R_{ON} = 1\,\Omega$, $R_{OFF} = 125\,\Omega$, $D = 10\,$nm, $\mu_v = 10^{-10}\,$cm$^2$s$^{-1}$ V$^{-1}$, $w(0) = 0$, corresponding to $q_{\min} = 0\,$C and $q_{\max} = 0.01\,$C.**

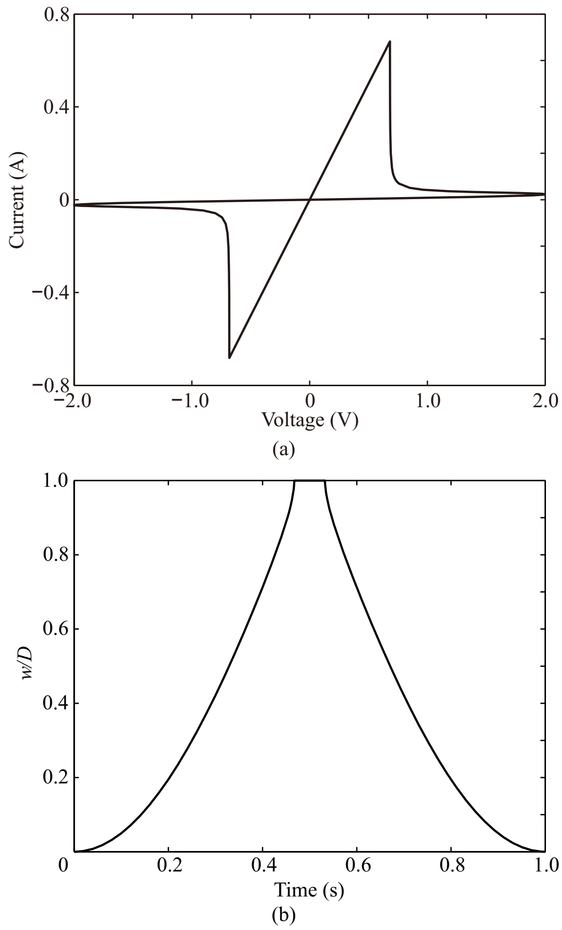


**Fig. 3 Simulation results of the extended linear-drift model. The amplitude of the input is enlarged so that the boundary of $w(t)$ is reached. $R_{ON}=1\,\Omega$, $R_{OFF}=125\,\Omega$, $D=10\,$nm, $\mu_v=10^{-10}\,$cm$^2$s$^{-1}$V$^{-1}$, $w(0)=0$. Input $v(t)=2\sin(2\pi t)$.**

$$v(t)=\left(R_{ON}\frac{w(t)}{D}+R_{OFF}\left(1-\frac{w(t)}{D}\right)\right)i(t) \quad (10a)$$

$$\frac{dw(t)}{dt}=\mu_v\frac{R_{ON}}{D}F_p(w)i(t) \quad (10b)$$

In $F_p(w)$, parameter $p$ is a positive integer. As $w(t)$ is close to 0 or $D$, the window function $F_p(w)$ in Eq. (10b) will tend to 0. Therefore according to Eq. (10b), $dw(t)/dt$ will also tend to 0, which makes $w(t)$ stay between $[0, D]$. Then there is no limits on applied voltage or current signal in simulation.

In practical numerical simulation, since for $w(t)=0$ or $D$, $F_p(w)=0$, which implies $dw(t)/dt=0$. So once $w(t)$ reaches 0 or $D$ in the numerical simulation, $w(t)$ will not change to a different value no matter what voltage or current is applied to Eqs. (10). This problem is reported in Refs. [11-13], and is called as the backing problem[12]. Note that in the simulation, the extended linear-drift model can be started from $M_0=R_{OFF}$ or $R_{ON}$, corresponding to $w(t)=0$ or $D$, and will not be troubled by the backing problem when $M_E(q)$ reaches $R_{OFF}$ or $R_{ON}$.

For an arbitrary $p$ ($p>1$), no explicit relation $M(q)$ between the memristance and the charge has been got from the window function nonlinear model. Therefore the window function nonlinear model cannot be directly applied to existing studies, methods, and applications based on the memristor definition. Next we will show one of the advantages of the explicit memristance expression. Given an input signal $i(t)$ with an explicit expression of the charge $q(t)=\int_{-\infty}^{t} i(\tau)d\tau$, the explicit expression of the output signal can be directly written accroding to Eq. (1) with an explicit memristance expression. For example, given input signal $i(t)=A\sin(\omega t)$, then $q(t)=\int_{0}^{t} i(\tau)d\tau=A(1-\cos(\omega t))/\omega$. For a explicit memristance expression such as Eqs. (7), we can get the explicit expression of the output signal $v(t)=M_E(q(t))i(t)=AM_E(A(1-\cos(\omega t))/\omega)\sin(\omega t)$. As a comparison, for memristive system such as the window function nonlinear model, without knowing the explicit memristance expression in the memristor definition, the output signal has to be obtained through numerical simulation. Simulation may be time consuming and simulation errors need to be considered, when comparing it with obtaining the exact explicit expression of the output signal. Another problem with the window function nonlinear model is how to select a proper parameter $p$, since different $p$ will lead to different characteristics of the system[8,14].

## 3 Piecewise Linear Approximation of the Extended Linear-Drift Model

So far, the extended linear-drift model (Eqs. (7)) has been given. In the memristor definition, memristance $M(q(t))=d\varphi(q)/dq$, therefore $\varphi(q)$ relationship is an important and basic way to characterize a memristor[1]. By integrating Eq. (7b), we can get the $\varphi-q$ curve of the extended linear-drift model as follows (we assume $\varphi|_{t=0}=0$ and $q|_{t=0}=0$):

$$\varphi_E(q)=\int M_E(q(t))=$$

$$\begin{cases} R_{OFF}q(t)+C_{OFF}, & q(t)\leqslant q_{min}; \\ M_0q(t)-\frac{1}{2}K_M\Delta Rq^2(t), & q_{min}<q(t)<q_{max}; \\ R_{ON}q(t)+C_{ON}, & q(t)\geqslant q_{max} \end{cases} \quad (11)$$

where constant $C_{\text{OFF}} = M_0 q_{\min} - \frac{1}{2} K_M \Delta R q_{\min}^2 - R_{\text{OFF}} q_{\min}$, $C_{\text{ON}} = M_0 q_{\max} - \frac{1}{2} K_M \Delta R q_{\max}^2 - R_{\text{ON}} q_{\max}$. The illustration of $\varphi_{\text{E}}(q)$ can be seen in Fig. 4

Note that $\varphi_{\text{E}}(q)$ is a nonlinear function, therefore system consisting of such a memristor is also a nonlinear system. To model and analyze a nonlinear system, piecewise linear function is a powerful tool. A piecewise linear function equals an affine function in each subregion ("piece") of the domain. Through piecewise linear identification, one nonlinear system can be approximated by a set of linear systems with the corresponding subregions. Then one may divide and conquer the original nonlinear problem by applying linear techniques to each of the subregion separately. There are many algorithm specially designed for piecewise linear systems, see Refs. [15,16] for example.

The piecewise linear function has already been used in memristor studies. In Refs. [1, 4], the following piecewise linear $\varphi - q$ curve is assumed to characterize a memristor:

$$\varphi_{\text{C}}(q) = bq + 0.5(a - b)(|q + 1| - |q - 1|) \quad (12)$$

where $a$ and $b$ are the parameters of the model. With $a = b$, the memristor characterized by Eq. (12) behaves like a linear resistor with resistance $a$. So we can suppose $a \neq b$ for general cases[1,4]. In the following of this paper, the above piecewise linear model is referred to as the existing piecewise linear model. The illustration of $\varphi_{\text{C}}(q)$ can be seen in Fig. 5.

Compare $\varphi_{\text{C}}(q)$ (Fig. 5) with $\varphi_{\text{E}}(q)$ (Fig. 4), it is not hard to see $\varphi_{\text{C}}(q)$ may not approximate $\varphi_{\text{E}}(q)$

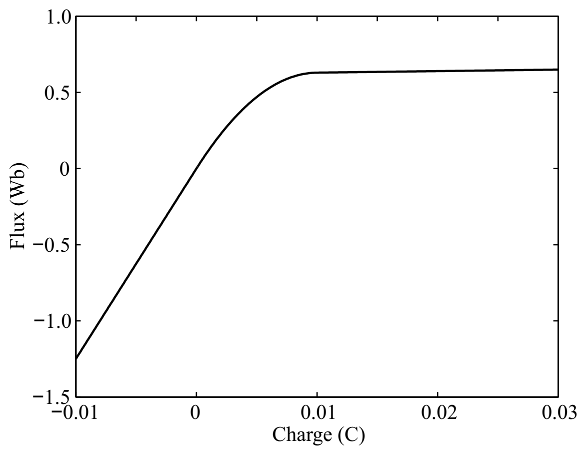


**Fig. 4 Illustration of $\varphi_{\text{E}}(q)$. $\varphi_{\text{E}}(q)$ is calculated from the extended linear-drift model of the HP device. Initial memristance $M_0 = R_{\text{OFF}}$, corresponding to $w(0) = 0$.**

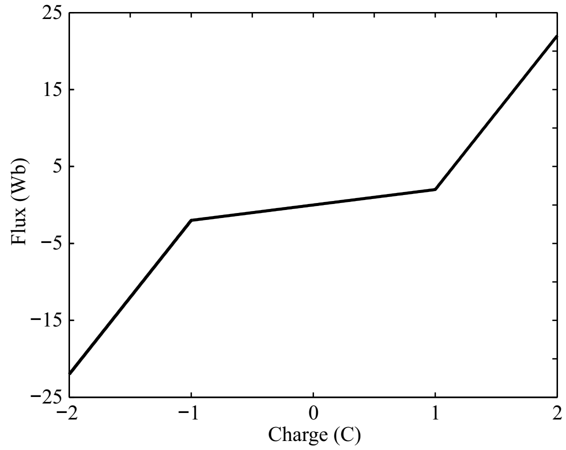


**Fig. 5 Illustration of $\varphi_{\text{C}}(q)$ of the existing piecewise linear model.**

well because $\varphi_{\text{E}}(q)$ does not have symmetry as $\varphi_{\text{C}}(q)$. According to physical model of HP memristor given in Ref. [7], the memristance of the physical model is a monotonic function of $q$. This can be seen from the memristance expression of both the linear-drift model (Eq. (6b)) and the extended linear-drift model (Eq. (7b)). But for a system characterized by $\varphi_{\text{C}}(q)$, both increasing or decreasing $q$ may cause the memristance value change from $a$ to $b$, i.e., the memristance of the existing piecewise linear model is not a monotonic function of $q$, which is not consistent with the HP device. Furthermore, while $a$ and $b$ can be arbitrary positive number, existing piecewise linear model $\varphi_{\text{C}}(q)$ may not approximate $\varphi_{\text{E}}(q)$ and other mathematical models (such as the window function nonlinear model) of the HP device well.

Considering that there are advantages of piecewise linear systems over general nonlinear systems in modeling and further analysis, we managed to give a proper piecewise linear approximation of the HP device based on the extended linear-drift model, in place of the existing one (Eq. (12)). More specifically, we propose a suitable piecewise linear function to approximate $\varphi_{\text{E}}(q)$. The nonlinear part of $\varphi_{\text{E}}(q)$ is $M_0 q(t) - \frac{1}{2} K_M \Delta R q^2(t)$, for $q_{\min} < q(t) < q_{\max}$. And for $q(t) \leqslant q_{\min}$ or $q(t) \geqslant q_{\max}$, $\varphi_{\text{E}}(q)$ is a linear function of $q$. Consequently, by maintaining the linear parts and omitting the nonlinear part, we believe the following piecewise linear function

$$\hat{\varphi}_{\text{E}}(q) = \min\{R_{\text{OFF}} q(t) + C_{\text{OFF}}, R_{\text{ON}} q(t) + C_{\text{ON}}\} \quad (13)$$

is a good piecewise linear approximation of $\varphi_{\text{E}}(q)$. Then we can use it as the piecewise linear model of the HP device.

Figure 6 shows the $\varphi - q$ curve of our piecewise linear model (Eq. (13)), i.e., $\hat{\varphi}_{\mathrm{E}}(q)$. The main difference between $\varphi_{\mathrm{E}}(q)$ and $\hat{\varphi}_{\mathrm{E}}(q)$ happens between $q_{\min} < q(t) < q_{\max}$, while $\varphi_{\mathrm{E}}(q)$ and $\hat{\varphi}_{\mathrm{E}}(q)$ are exactly the same when $q(t) \leqslant q_{\min}$ or $q(t) \geqslant q_{\max}$. Our piecewise linear model is more simple than the nonlinear model, and keeps the major ON and OFF features of the HP device. The piecewise linear model still fits the memristor definition. With our piecewise linear model to describe the HP device, piecewise linear techniques could be directly used to design and analyze physical systems consisting of the HP device.

Figure 7 shows the $i - v$ curve of the piecewise linear model (Eq. (13)) under the same simulation condition which is used in Fig. 3. The pinched hysteresis loop in the $i - v$ curve of the piecewise linear model is qualitatively similar to the one of the extended linear-drift model. The piecewise linear model (Eq. (13)) may be quite useful in cases where the ON and OFF states of the device are major concerns, such as logic operations[17] and nonvolatile memory[9]. In such cases, using the piecewise linear model $\hat{\varphi}_{\mathrm{E}}(q)$ in place of nonlinear model $\varphi_{\mathrm{E}}(q)$ for describing the device may simplify the problem while maintaining acceptable accuracy.

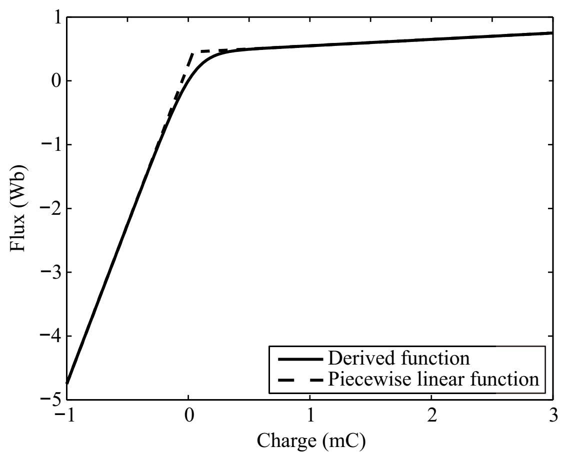


**Fig. 6 Illustration of the derived function $\varphi_{\mathrm{E}}(q)$ and its piecewise linear approximation $\hat{\varphi}_{\mathrm{E}}(q)$.**

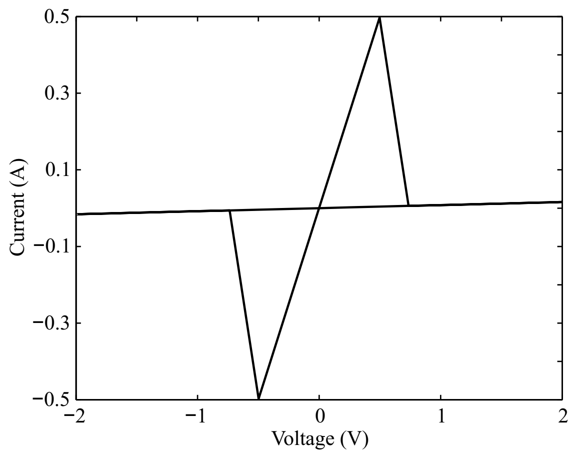


**Fig. 7 $i - v$ curves of the piecewise linear model $\hat{\varphi}_{\mathrm{E}}(q)$. The simulation parameters are the same with the simulation in Fig. 3.**

## 4 Conclusions

Memristors exhibit great potential in numerous applications as the fourth basic circuit element. Besides logic operations[17], nonvolatile memory[9], and memristor oscillator[4], memristors can also be widely used in neuromorphic devices and learning networks[14]. For any of these applications, a suitable mathematical model of the memristor is necessary.

To describe the HP device, the extended linear-drift model is a proper choice, as there are limitations of the linear-drift model caused by the boundary condition of the device. The boundary condition was taken into consideration in the extended linear-drift model. The extended linear-drift model naturally has an explicit memristance expression in the memristor definition, while the window function models do not have such an explicit memristance expression. This makes the extended linear-drift model a good mathematical model of the HP device, and can be easily applied to existing studies, methods, and applications involving memristor/HP device.

The piecewise linear technique may be a good way to deal with the nonlinearity of $\varphi_{\mathrm{E}}(q)$ derived from the extended linear-drift model. It is important to note that the existing piecewise linear model may not approximate the HP device well. A proper piecewise linear model is raised, by approximating the $\varphi_{\mathrm{E}}(q)$ with piecewise linear function. Then piecewise linear techniques can be effectively applied to model and analyze a system that involves a memristor/HP device. The piecewise linear model is less complicated than the nonlinear model, which may make further simulation and analysis easier.

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