Review and Simulation of Memristors using MATLAB and PSpice

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Abstract— **This paper concentrates on the memristor and discusses its properties. The HP model for the memristor is likewise talked about, and its working is examined through simulations. Potential applications of the memristor are studied about at length. Conclusions are drawn based on the study and simulations and the scope for future developments are contemplated.**

Index Terms— *memory resistor, non-volatile memory, crossbar array, pinched hysteresis loop, nanometer scale*

I. Introduction

In 1971, Leon Chua hypothesized the fourth fundamental circuit component while attempting to build up a missing constitutive relationship between the electrical charge and the magnetic flux to complete the symmetry. Chua named this speculative nonlinear gadget, memristor (memory + resistor) since it showed the hysteresis property of the-then ferromagnetic center memory furthermore the dissipative qualities of a resistor. Plainly, in such gadgets, the nonlinear resistance can be remembered inconclusively by controlling the stream of the electrical charge or the magnetic flux. The memristor was proposed to have a memresistance, M and a function defined as $d\varphi = Mdq$.

II. MEMRISTOR THEORY

A. *Memristor*

It is defined as a two-terminal circuit element in which the flux between the two terminals is a function of the measure of electric charge that has passed through the device. Memristor is not an energy- storage element. [8]

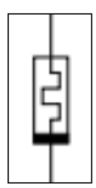

FIG 1:MEMRISTOR

B. Memristance

Memristance is a property of the memristor [7]. If charge flows in one direction through a circuit, the resistance of that component of the circuit will increase, and if charge flows in the opposite direction, the resistance will decrease. If the flow of charge is stopped by turning off the applied voltage, the component will 'remember' the last resistance that it had, and when the flow of charge starts again, the resistance of the circuit will be what it was when it was last active.

C. Analogy of a Memristor

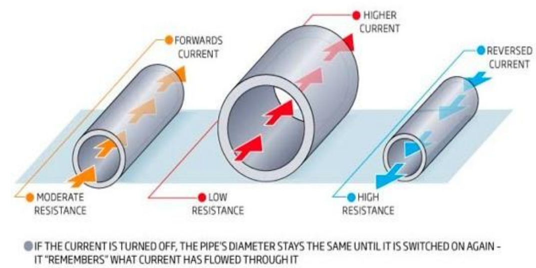

FIG 2: ANALOGY OF A MEMRISTOR(Adapted from [3])

An analogy for a memristor is a different kind of pipe—whose diameter expands or shrinks depending on the amount and direction of the water flow through it. If the pipe has a larger diameter, the flow of water through the pipe is faster, just like more current flows through resistor with a small value of resistance. The diameter of the pipe decreases when the water flows in the opposite direction, thus slowing down the water flow. If no water is let into the pipe, the pipe will retain its most recent diameter until the water is turned back on. Thus, the pipe remembers the amount of the water that has flowed through it.

III. PROPERTIES OF A MEMRISTOR

A. φ-q Curve

- Slope of the φ-q curve = M(q)
- $M(q) \geq 0$
- p(i)= $M(q)(i(t)2) \geq 0$

Therefore, from the above conditions, the φ-q curve is monotonically increasing.

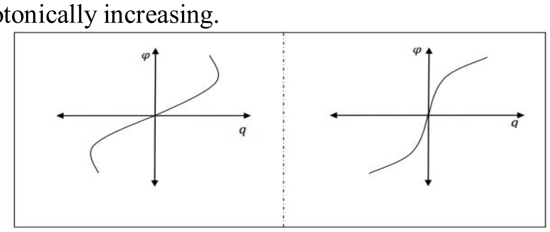

FIG 3: φ-Q CURVE OF A MEMRISTOR

B. Current-Voltage Curve

The memristor has a pinched hysteresis loop current—voltage characteristic. For a memristor excited by a periodic signal, when the voltage is zero, the current is also zero and vice versa. Thus, both voltage and current have identical zero-

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crossing. If any device has a current–voltage hysteresis curve, then it is either a memristor or a memristive device. [7] Pinched hysteresis loop shrinks with the increase in the excitation frequency.

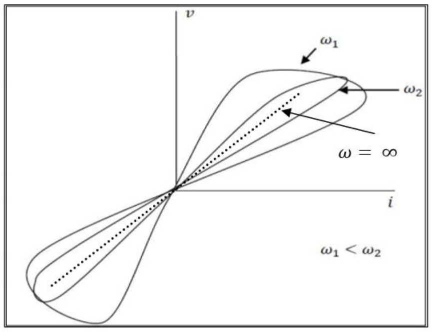

FIG 4: Hysteris Loop of a Memristive System

C. Memristive Systems

The fundamental memristive system theorem states that 'Any system that exhibits a pinched hysteresis loop on its current-voltage characteristics when driven by a dc or a sinusoidal signal is termed as a memristive system.' [7]

IV. IMPLEMENTATION OF MEMRISTORS BY HP LABS

37 years later, electronics have gotten small enough to reveal the secrets of that fourth element. On April 30, 2008 a team at HP Labs announced the development of a switching memristor based on a thin film of titanium dioxide, having approximately linear charge-resistance.

A. Importance of Crossbar Array

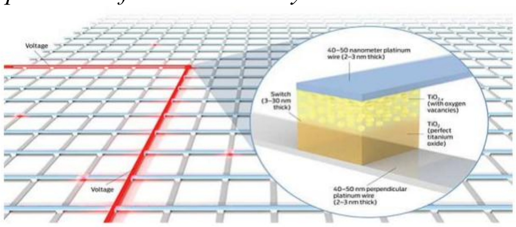

FIG 5: Crossbar Array Configuration (Adapted From [3])

The Crossbar Array is an array of perpendicular wires. Anywhere two perpendicular wires cross, there is a switch connecting them. In order to connect any horizontal wire to a vertical wire at any orientation on the grid, the switch sandwiched between those two wires must be closed. The general idea was to enable communication by opening and closing these switches by applying voltages to the ends of the wires. By this description, it is understood that a crossbar array is basically a storage system, with an open switch representing a zero and a closed switch representing a one. [3]

Crossbar Arrays use redundancy, ensuring that any defect present in the device can be avoided. Also, because of their simplicity, crossbar arrays have a much higher density of switches than a comparable integrated circuit based on transistors.

B. Development of Titanium Dioxide

The reason why Titanium dioxide was chosen was due to the equations of drift of oxygen vacancies in TiO_2 and their influence on the electronic conduction in the material which were identical with the equivalent circuit model.[2]

The Crossbar Array was used for the implementation of the memristor. But, one of the major setbacks was the small off-to-on resistance ratio of the switches. Hence, it was calculated that to get a high-performance memory, the switches were to be made with a resistance ratio of at least 1000 to 1. In other words, in its off state, a switch had to be 1000 times as resistive as it was in its on state.

In order to achieve this, **scanning tunneling microscopy (STM)** was used. A tunneling microscope creates atomic-resolution images by checking a sharp needle over a surface and measuring the electric current that streams between the molecules at the tip of the needle and the surface the needle is testing. The general dependable guideline in STM is that moving that tip 0.1 nm more like a surface expands the tunneling current by one order of magnitude. Similar mechanism was required by which the distance between two wires in the crossbar could be changed by 0.3 nm. This would then yield a proportion of 1000:1 electrical switching ratio that was required. [2]

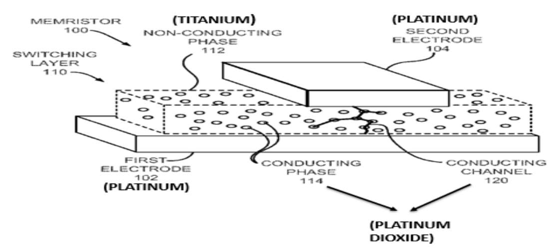

FIG 6: Patent of the model developed

To adhere to the constraints of a material that could change its physical dimensions in the above manner, the device was built like a tiny sandwich. Two platinum electrodes (the intersecting wires of the crossbar junction) functioned as the "bread" on either end of the device. The bottom surface of the platinum wire was oxidized to make an extremely thin layer of platinum dioxide, which is highly conducting. Next, a dense film of specially designed switching molecules of one molecule thick was created. Over this "monolayer" a 2 to 3 nm layer of titanium metal was deposited, which bonds strongly to the molecules and were intended to glue them together. The final layer was the top platinum electrode. [2]

Going by Chua's graphs, it was assumed that memristance had something to do with the switches. By 2004, a gadget was built that could rip the tiny devices open which gave way to do some forensics. When ripped apart, the little sandwiches separated at their molecule layer.

Under the molecular layer, instead of platinum dioxide, there was only pure platinum. Above the molecular layer, instead of titanium, an unexpected and unusual layer of titanium dioxide was found. The titanium had sucked the oxygen right out of the platinum dioxide. The oxygen atoms had migrated through

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the molecules and been consumed by the titanium. But, it was seen that the switching molecules were completely unaffected by this motion—they were intact and well ordered, which established that they had an important function in the device.
The titanium dioxide found was not just regular titanium dioxide. It had split itself up into two chemically different layers. Adjacent to the molecules, the oxide was stoichiometric TiO_2, having a ratio of exactly 2 to 1. But closer to the top platinum electrode, the titanium dioxide was missing a small amount of its oxygen, between 2 and 3 percent, called oxygen-deficient titanium dioxide TiO_{2-x}.[2]

C. Working Of A Memristor

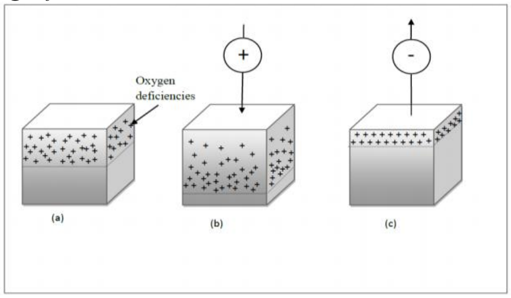

FIG 7: Working Of Memristor(Adapted from [7])

The TiO_2 is electrically insulating but the TiO_{2-x} is conductive, because its oxygen vacancies are donors of electrons, which makes the vacancies themselves positively charged so that they can be pushed up and down at will in the titanium dioxide material.

A positive voltage applied to the switch repelled the positive oxygen deficiencies in the metallic upper TiO_{2-x} layer, sending them into the insulating TiO_2 layer below. That caused the boundary between the two materials to move down, increasing the conductivity of the entire switch. The more positive voltage is applied, the more conductive the switch becomes.

A negative voltage on the switch attracted the positively charged oxygen particles, pulling them out of the TiO_2. This reaction caused the switch as a whole to become more resistive. The more negative voltage is applied, the less conductive and more resistive the switch becomes.

What made this switch memristive was that when the voltage was turned off, the oxygen bubbles did not migrate. They stayed where they are, which meant that the boundary between the two titanium dioxide layers was frozen. That is how the memristor 'remembers' how much voltage was last applied.

As charge flows through the device, the resistance drops, and the current increases more rapidly with increasing voltage until the maximum is reached. Therefore, the results obtained by HP labs researcher's depicts the properties of the ideal memristor proposed by Chua.

D. LINEAR DRIFT MODEL[6]

The simple mathematical model of the HP memristor is given by:

$$M(q) = R_{OFF}\left(1 - \frac{R_{ON}}{\beta} q(t)\right) \quad (11)$$

Where $\beta = \frac{D^2}{\mu_D}$ has the dimensions of magnetic flux. μ_D is the average drift velocity and has the units cm^2/sV; D is the thickness of titanium-dioxide film; R_{ON} and R_{OFF} are 'on-state' and 'off- state' resistances; and q(t) is the total charge passing through the memristor device.
Assume a uniform electric field across the device. Therefore, there is a linear relationship between drift-diffusion velocity and the net electric field. The state equation can be written as

$$\frac{1}{D}\frac{dw(t)}{dt} = \frac{R_{ON}}{\beta} i(t) \quad (12)$$

On integrating (12), we get

$$\frac{w(t)}{D} = \frac{w(t_o)}{D} + \frac{R_{ON}}{\beta} q(t) \quad (13)$$

Where $w(t_o)$ is the initial length of w. The speed of drift under a uniform electric field across the device is then given by $v_D = \frac{dw(t)}{dt}$

In an uniform field $D=v_D t$. In this case, $Q_D=it$, defines the amount of charge required to move the boundary from $w(t_o)$, where $w \to 0$, to distance $w(t_D)$, where $w \to D$. Therefore, $Q_D = \frac{\beta}{R_{ON}}$. Thus,

$$\frac{w(t)}{D} = \frac{w(t_o)}{D} + \frac{q(t)}{Q_D} \quad (15)$$

If $(t) = \frac{w(t)}{D}$, then eqn (15) can be written as,

$$x(t) = x(t_o) + \frac{q(t)}{Q_D} \quad (16)$$

The amount of charge that is passed through the channel over the required charge for a conductive channel is given as $\frac{q(t)}{Q_D}$.
Using the equations in (10),

$$v(t) = \left(R_{ON}\frac{w(t)}{D} + R_{OFF}\left(1 - \frac{w(t)}{D}\right)\right) i(t) \quad (17)$$

Substituting $x(t) = \frac{w(t)}{D}$ in Eqn (17),

$$v(t) = \left(R_{ON}x(t) + R_{OFF}\left(1 - x(t)\right)\right) i(t) \quad (18)$$

If we assume that the initial charge $q(t_o) = 0$, then $w(t) = w(t_o) \neq 0$, and

$$M_O = R_{ON}\left(x(t_o) + r\left(1 - x(t_o)\right)\right) \quad (19)$$

Where $r = \frac{R_{OFF}}{R_{ON}}$ and M_o is the memristance value at t_o. Thus the memristance at time t is given by

$$M(q) = M_O - \Delta R\left(\frac{q(t)}{Q_D}\right) \quad (20)$$

Where $\Delta R = R_{OFF} - R_{ON}$. When $R_{OFF} \gg R_{ON}, M_O \approx R_{OFF}$. Substituting eqn(20) into $v(t) = M(q)i(t)$, when $i(t) = \frac{dq(t)}{dt}$, we get,

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$$v(t) = \left(M_O - \Delta R\left(\frac{q(t)}{Q_D}\right)\right)\frac{dq(t)}{dt} \tag{21}$$

Since $M(q) = \frac{d\varphi(q)}{dq}$, the solution is,

$$q(t) = \frac{Q_D M_O}{\Delta R}(1 \pm \sqrt{1 - \frac{2\Delta R}{Q_D M_O^2}\varphi(t)} \tag{22}$$

For $\Delta R \approx M_O \approx R_{OFF}$, Eqn(22) becomes,

$$q(t) = Q_D\left(1 - \sqrt{1 - \frac{2}{Q_D R_{OFF}}\varphi(t)}\right) \tag{23}$$

If $Q_D = \frac{D^2}{\mu_D R_{ON}}$, Then the internal state of the memristor is,

$$x(t) = 1 - \sqrt{1 - \frac{2\mu_D}{rD^2}\,\varphi(t)} \tag{24}$$

The current-voltage relationship in this case is,

$$i(t) = \frac{v(t)}{R_{OFF}\sqrt{\left(1 - \frac{\mu_D}{rD^2}\varphi(t)\right)}} \tag{25}$$

Eqn(25) shows the inverse-square relation between memristance and TiO_2 thickness D. Thus, for smaller values of D, the memristance shows improved characteristics.

V. SIMULATIONS

A. SIMULATIONS USING MATLAB

For this simulation, the width D of the TiO_2 film is considered to be 10 nm and the dopant mobility $\mu_D = 10^{-10}$ $cm^2\ s^{-1}\ V^{-1}$. The values assumed are —R_{ON} =100Ω, R_{OFF} =16KΩ. The simulation results are shown below.

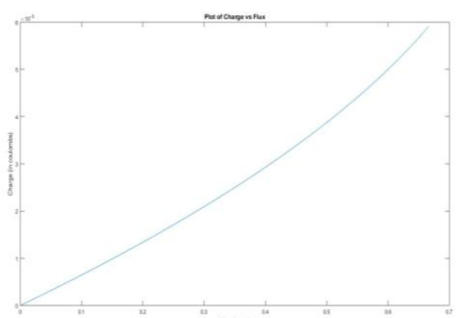

FIG 8: Plot Of Charge Vs Flux

From the graph, it is seen that the plot of Charge vs Flux is almost linear, in line with the conditions postulated by Chua.

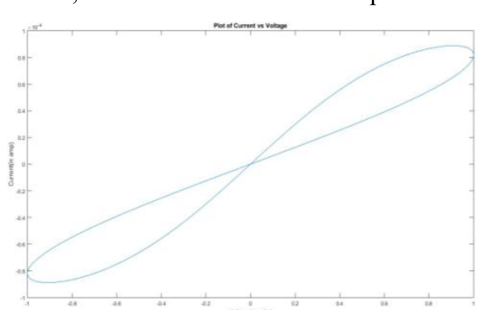

FIG 9: Plot of Current vs Voltage for w=3rad/s

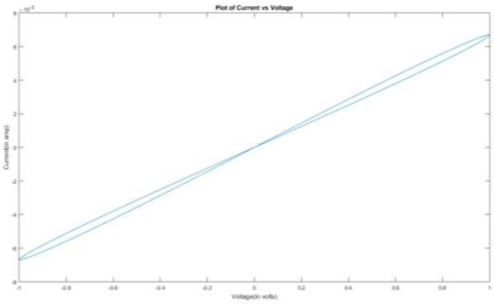

FIG 10: Plot of Current vs Voltage for w=8rad/s

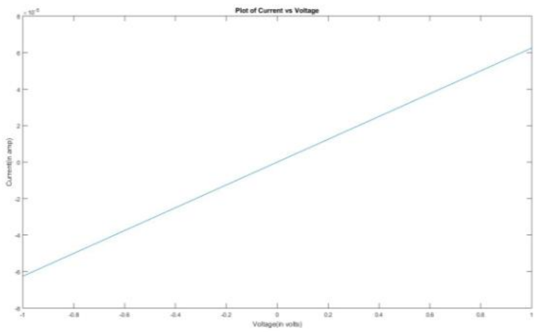

FIG 11: Plot Of Current Vs Voltage For w =∞

The current-voltage plot of the memristor is a pinched hysteresis loop as was observed by Prof Chua. It is seen that as the excitation frequency is increased, the curve shrinks and when w approaches infinity, the curve becomes a straight line showing that it behaves like a resistor.

B. SIMULATIONS USING PSPICE

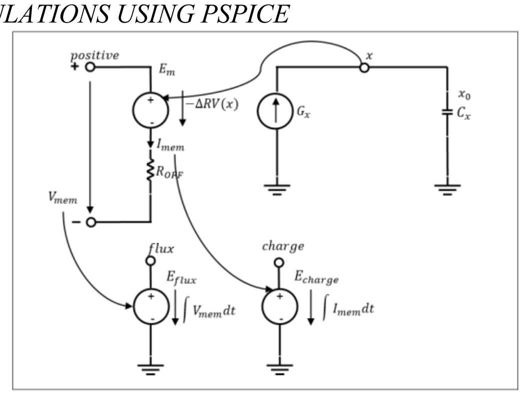

FIG 12: Pspice Model Used(Adapted from [7])

In the above circuit, V_{mem} is the input voltage and I_{mem} is modeled to be the current through the memristor. The flux is calculated by integrating the voltage V_{mem} and the charge is calculated by integrating the current I_{mem}. E_m is the voltage source whose terminal voltage is controlled according to the formula $-x\Delta R$. G_x is a current source whose current is controlled according to the equation I_{mem} f(V(x)) where V(x) is the voltage across the capacitor C_x and it models the normalized width of the doped layer.

The relation between memristor current and voltage is modeled as,

$$R_{MEM}(x) = R_{OFF} - x\Delta R \tag{26}$$

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Where

$$\Delta R = R_{OFF} - R_{ON} \qquad (27)$$

R_{OFF} is the resistor in series voltage source whose terminal voltage is controlled by the formula, $-x\Delta R$. The voltage V(x) across the capacitor models the normalized width x of the doped layer. The initial state of x is modeled by the initial voltage of the capacitor. The flux is calculated by the time-integral of voltage, and the charge is calculated by the time-integral of current. [7]

For this simulation, the width D of the TiO_2 film is considered to be 10 nm and the dopant mobility $\mu_D = 10^{-10}$ cm^2 s^{-1} V^{-1}. The values assumed are —R_{ON} =1KΩ, R_{OFF} =100KΩ. The initial resistance required to model the initial conditions is taken to be 80KΩ. The simulation results are shown below.

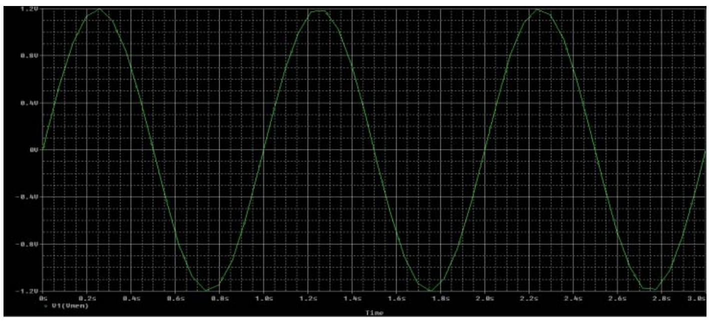

FIG 13: An input voltage of $\boldsymbol{v_o sin w_o t}$ is applied to the memristor

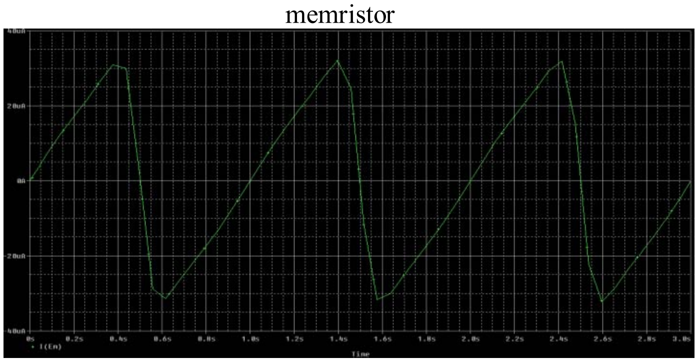

FIG 14: Waveform of Current I_{mem} through the Memristor

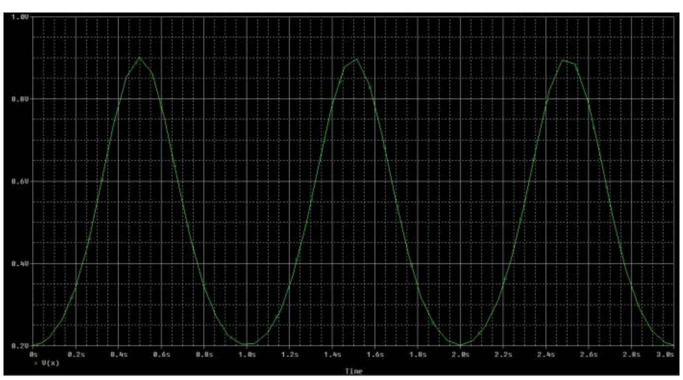

FIG 15: Voltage across the Capacitor C_x

FIG 16: Plot of Charge vs Flux

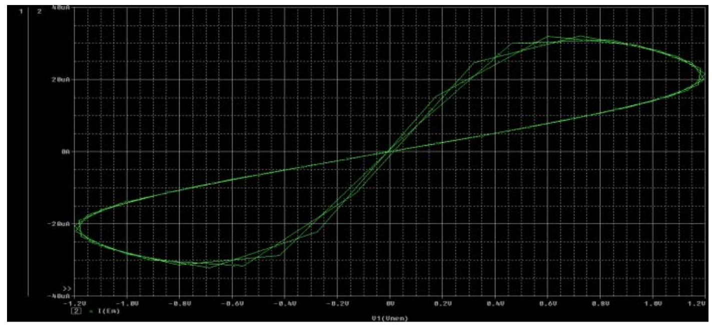

FIG 17: Plot of Current vs Voltage for Freq=1Hz

FIG 18: Plot of Current vs Voltage for Freq=2Hz

VI. POTENTIAL APPLICATIONS OF A MEMRISTOR

A. Logical Implementation Using Memristors [4]

Each logic gate is operated applying a single voltage which is divided between the memristors according to their logic values, and so switches them or not. Due to this fact the output memristor must be initialized prior to the operation.

Eg: OR Gate

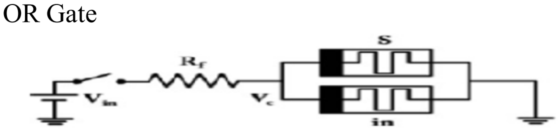

FIG 19: NOT Gate

This gate consists of an input and output memristor, and a regular resistor is used for voltage division. The output memristor is initialized to '0' prior to calculation. The constrains on the input voltage are:

$$\left(\frac{1}{2} + \frac{R_f}{R_{off}}\right) i_{th} < V_{in} < (1 + \frac{R_f}{R_{on}}) i_{th}$$

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B. Memory and Storage

Memristors can be used as non-volatile memory, permitting more prominent data density than hard drives. The memristor based crossbar latch memory prototyped by HP can fit 100 gigabits within a square centimeter. [8] HP also claims that memristor memory can handle up to 1,000,000 read/write cycles before degradation, as compared to flash at 100,000 cycles. In addition, memristors also consume less power. [7] In memristor memories, the reading operation is performed by applying a voltage lesser than the threshold value.

C. Instant-On Computers [3]

The fundamental property of a memristor is that its ability to retain the resistance it had when it was last on even when the voltage is turned off. This property makes memristors very useful in computer memory. The ability to always remember resistance values implies that it could be used as a nonvolatile memory. If a laptop contained memory built using memristors, removing the battery from the computer would not lose any data. When the battery is returned, the computer would turn on exactly where the user left off. Hence, memristors can remove lengthy reboot or unintentional loss of memory due to a power failure.

D. Resistive RAM [5]

ReRAMs or Resistance Switching Memories (also referred to as memristor memories) is based on the two logical values, “1” and “0” where they are corresponding to low resistance state (ON mode) and high resistance state (OFF mode) respectively. Several materials were used as transition metal oxides in such structures. The unipolar and bipolar switching mechanisms were used to achieve this. The state of the device can be changed by applying a potential across the device. For bipolar switching devices, in order to switch back to its previous state, a negative voltage should be applied. In case of unipolar switching, the switching voltage should be higher than two threshold voltages to change the state of the device.

E. Hybrid Chip [5]

HP used crossbar arrays in a hybrid chip, that contained a combination of both transistors and memristors. A single layer of CMOS transistors can actually drive several memristor layers in an efficient way. The whole structure consumes less power which results in less heat generation. There exists more logic gates per unit area of the chip due to the higher density of the crossbars. The switches can be switched in nanoseconds, while the energy necessary for a single switch is in the orders of pico joule. Such structures allow for high functionality and performance while keeping up with the Moore's law.

F. Brain Circuits

The neural network in the human mind has the ability to create strong or weak connections. Nonlinear dependencies caused in the function of a neuron is dependent on the action potentials that it receives. The only physical device that could imitate these nonlinearities is the memristor.

Earlier this month, the researchers at University of Southampton, UK, and members of the EU-funded RAMP project have demonstrated how memristors could aid the development of more precise and affordable neuroprosthetics and bioelectric medicines. Monitoring neuronal cell activity is fundamental to neuroscience and in this sense, they perform a role akin to biological synapses and possess the intrinsic ability to simultaneously carry out computational tasks and store information at aggressively downscaled volumes and power dissipation.[9]

VII. CONCLUSION

This paper gives a detailed analysis of a fourth fundamental concept called as a memristor. The properties postulated by Chua back in the 1970s were proved by the experiments conducted by the HP labs. Since the memristor is a property that is more dominant in the nanometer scale, the device took time to fabricate and was fabricated by the HP labs in 2008. Simulations done have shown that the memristor equations derived by the researchers at HP labs is in line with that of Chua’s theory of memristors. After the device has been fabricated, numerous applications were found for the same using the exclusive properties of the memristor.

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