

# Existence and Uniqueness

Consider the augmented matrix:

$$\left( A \mid \vec{b} \right) = \left( \begin{array}{ccccc|c} 1 & 3 & 0 & 7 & 0 & 4 \\ 0 & 0 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 6 \end{array} \right)$$

**The leading one's are in first, third, and fifth columns.**

- The pivot columns (leading ones) of  $A$  are the first, third, and fifth columns
- The corresponding variables of the system  $A\mathbf{x} = \mathbf{b}$  are  $x_1, x_3$ , and  $x_5$ . Variables that correspond to a pivot are basic variables.
- Variables that are not basic are free variables. They can take any value.
- The free variables are  $x_2$  and  $x_4$ . Any choice of the free variables leads to a solution of the system.

**Unique solution** exists if and only if there are no free variables.

A linear system is **consistent** if it has at least one solution.

## Note

If you see something like this ( $b$  is a non-zero number):

$$[0 \ 0 \ 0 \ \dots \ 0 \mid b]$$

~~RUN~~ you **don't** have a [Consistent System](#).