Homogeneous systems

Linear systems of the form $A\mathbf{x} = \mathbf{0}$ are homogeneous.
Linear systems of the form $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} \neq \mathbf{0}$, are inhomogeneous.

Here the trivial/nontrivial refers to $\mathbf{x}$.

If $\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ it is trivial solution.

Observations

$A\vec{x} = \vec{0}$ has a nontrivial solution

$\iff$ there is a free variable

$\iff$ A has a column with no pivot.