Matrix Multiplication

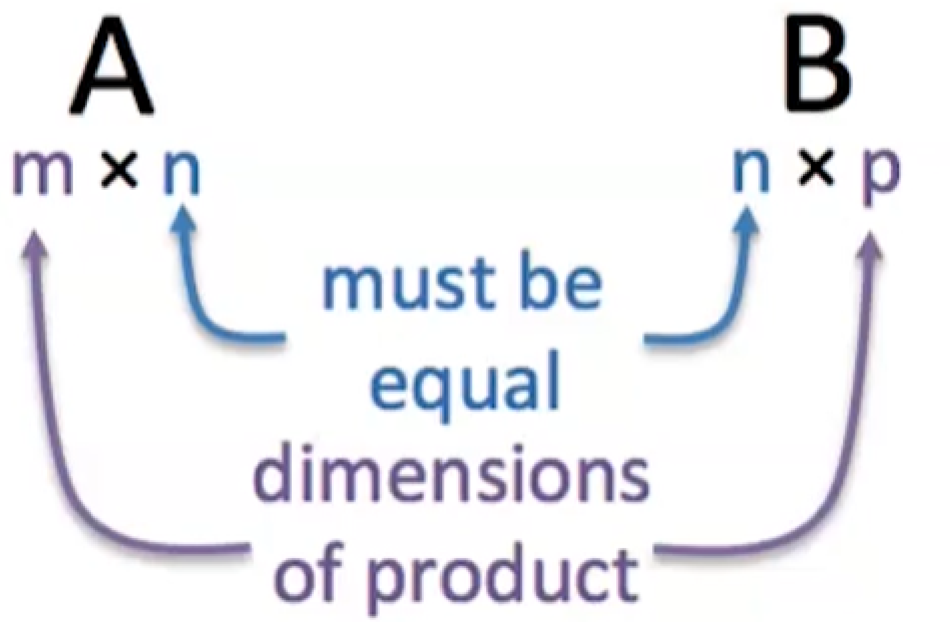
#matrix_multiplication

Let A be a $m \times n$ matrix, and B be a $n \times p$ matrix. The product is AB an $m \times p$ matrix, equal to:

$$AB = A\left(\mathbf{b_1} \;\ldots\; \mathbf{b_p}\right) = \left(A\mathbf{b_1} \;\ldots\; A\mathbf{b_p}\right)$$

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 3 & 4 & 0 \end{bmatrix} \\ &= \left(\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right) \\ &= \begin{bmatrix} 4 & 0 & 0 \\ 5 & 4 & 0 \end{bmatrix} \end{aligned}$$

(Note: This is very similar to Matrix Vector Product)



Let A, B, C be matrices of the sizes needed for the matrix multiplication to be defined, and A is a $m \times n$ matrix.

1. (Associative) $(AB)C = A(BC)$
2. (Left Distributive) $A(B + C) = AB + AC$
3. (Right Distributive) $(A + B)C = AC + AC$
4. (Identity for matrix multiplication) $I_m A = AI_n$

Warnings:

1. (non-commutative) In general, $AB \neq BA$.
2. (non-cancellation) $AB = AC$ does not mean $B = C$.
3. (Zero divisors) $AB = 0$ does not mean that either $A = 0$ or $B = 0$.