Existence and Uniqueness

Consider the augmented matrix:

$$\left(A \mid \vec{b} \right) = \begin{pmatrix} 1 & 3 & 0 & 7 & 0 & 4 \\ 0 & 0 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 6 \end{pmatrix}$$

The leading one's are in first, third, and fifth columns.

- The pivot columns (leading ones) of A are the first, third, and fifth columns
- The corresponding variables of the system $A\mathbf{x} = \mathbf{b}$. are $x_1, x_2,$ and x_5 Variables that correspond to a pivot are basic variables.
- Variables that are not basic are free variables. They can take any value.
- The free variables are x_2 and x_4. Any choice of the free variables leads to a solution of the system.

Unique solution exists if and only if there are no free variables.

A linear system is **consistent** if it has at least one solution.

Note

If you see something like this (b is a non-zero number):

$$\begin{pmatrix} 0 & 0 & 0 & \ldots & 0 \mid b \end{pmatrix}$$

~~RUN~~ you **don't** have a Consistent System.