

Linear Transformations

$$T : R^n \rightarrow R^m \text{ is linear if } \begin{cases} T(u + v) &= T(u) + T(v) \\ T(cv) &= cT(v) \end{cases}$$

Create a 2×2 matrix A that applies a linear transformation that rotates by an angle θ

$$\begin{aligned} A &= [\vec{a}_1, \vec{a}_2] \\ T(\vec{e}_1) &= \vec{a}_1 \\ T(\vec{e}_1) &= \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} \\ \vec{a}_1 &= \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} \\ T(\vec{e}_2) &= \vec{a}_2 \\ T(\vec{e}_2) &= \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix} \\ \vec{a}_2 &= \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix} \end{aligned}$$

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$