Consistent Systems

Augmented Matrices

This:

$$\begin{cases} x_1 - 2x_2 + x_3 = 0 \\ 2x_2 - 8x_3 = 7 \end{cases}$$

Becomes:

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 7 \end{array}\right]$$

A linear system is **consistent** if it has at least one solution.
Two matrices are **row equivalent** if a sequence of row operations transforms one matrix into the other.

Suppose

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

A and B are row equivalent
A and C are not row equivalent

Are the augmented matrices $(A \,|\, \vec{b})$ and $(C \,|\, \vec{b})$ consistent?

$$[A \mid \mathbf{b}] = \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & 1 \end{array}\right]$$

$$[A \mid \mathbf{b}] = \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 1 \end{array}\right]$$

So $[A \mid \mathbf{b}]$ are consistent.
And $[C \mid \mathbf{b}]$ are not consistent.