Leontif Input-Output Model

#Leontif

A Leontif Input-Output Model is a model that describes a economy whos' parts need resources to provide resources.

C is the consumption matrix.
Entries of C are $C_{i,j}$, with $C_{i,j} \in [0,1]$, and
$C\vec{x} =$ units consumed
$\vec{x} - C\vec{x} =$ units left after internal consumption

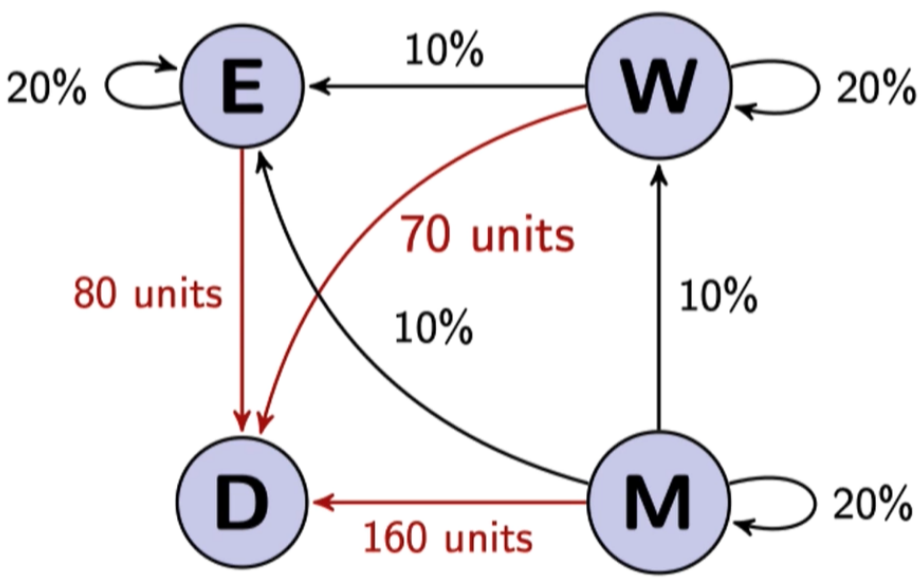
$\vec{d}$ is the external demand.
$\vec{x}$ is the produced quantity.

So for an economy we can derive the formula,

$$\color{green}{\text{Total production}} - \color{blue}{\text{Total internal usage}} = \color{red}{\text{Total external demand}}$$

$$\color{green}{\vec{x}} - \color{blue}{C\vec{x}} = \color{red}{\vec{d}}$$

Supposes we have a economy that looks like so,



E, W, M all produce something but D only consumes.

So we know that D is our external demand

Our internal usage usages are all the black arrows in the diagram.

So we need to find total production

First define $\vec{x} = \begin{bmatrix} x_e \\ x_w \\ x_m \end{bmatrix}$

We can construct $C\vec{x}$

$$C\vec{x} = x_e \begin{bmatrix} 20\% \\ 10\% \\ 10\% \end{bmatrix} + x_w \begin{bmatrix} 0\% \\ 20\% \\ 10\% \end{bmatrix} + x_m \begin{bmatrix} 0\% \\ 0\% \\ 20\% \end{bmatrix}$$

$$C\vec{x} = x_e \begin{bmatrix} .2 \\ .1 \\ .1 \end{bmatrix} + x_w \begin{bmatrix} 0 \\ .2 \\ .1 \end{bmatrix} + x_m \begin{bmatrix} 0 \\ 0 \\ .2 \end{bmatrix}$$

$$C = \begin{bmatrix} .2 & 0 & 0 \\ .1 & .2 & 0 \\ .1 & .1 & .2 \end{bmatrix}$$

We can construct $\text{external demand}, \vec{d}.$

$$\begin{bmatrix} 80 \\ 70 \\ 160 \end{bmatrix}$$

Now we can put this in our formula to get $\vec{x}$

$$\vec{x} - C\vec{x} = \vec{d}$$

$$(I - C)\vec{x} = \vec{d}$$

$$\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} .2 & 0 & 0 \\ .1 & .2 & 0 \\ .1 & .1 & .2 \end{bmatrix} \right) \vec{x} = \begin{bmatrix} 80 \\ 70 \\ 160 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} 100 \\ 100 \\ 225 \end{bmatrix}$$