

Homogeneous systems

Linear systems of the form $A\mathbf{x} = \mathbf{0}$ are homogeneous.

Linear systems of the form $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} \neq \mathbf{0}$, are inhomogeneous.

Here the **trivial/nontrivial** refers to \mathbf{x} .

If $\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ it is **trivial** solution. Otherwise it is **nontrivial**

Observations

$A\vec{x} = \vec{0}$ has a nontrivial solution

\iff there is a free variable

$\iff A$ has a column with no pivot.

Example

Identify the free variables, and the solution set, of the system.

$$x_1 + 3x_2 + x_3 = 0$$

$$2x_1 - x_2 - 5x_3 = 0$$

$$x_1 - 2x_3 = 0$$

In [Echelon Form](#):

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x_1 + -2x_3 = 0$$

$$x_2 + x_3 = 0$$

$$0 = 0$$

$$x_1 = 2x_3$$

$$x_2 = -x_3$$

$$x_3 = x_3$$

$$\text{Solution set} = \mathbf{x} = \begin{bmatrix} 2x_3 \\ -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$