

Span

Given vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_p \in \mathbb{R}^n$, and scalars c_1, c_2, \dots, c_p . The set of all linear combinations of $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_p$ is called the **span** of $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_p$

Is \vec{y} in the span of vectors \vec{v}_1 and \vec{v}_2 ?

$$\vec{v}_1 = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 2 \\ 5 \\ 6 \end{pmatrix}, \text{ and } \vec{y} = \begin{pmatrix} 7 \\ 4 \\ 15 \end{pmatrix}.$$

$$c_1 \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ -2c_1 \\ -3c_1 \end{bmatrix} + \begin{bmatrix} 2c_2 \\ 5c_2 \\ 6c_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 15 \end{bmatrix}$$

$$\left(\begin{array}{cc|c} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -3 & 6 & 15 \end{array} \right)$$

$$\left(\begin{array}{cc|c} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -1 & 2 & 5 \end{array} \right)$$

$$\left(\begin{array}{cc|c} 2 & 0 & 2 \\ 0 & 1 & -6 \\ -1 & 2 & 5 \end{array} \right)$$

$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -6 \\ -1 & 2 & 5 \end{array} \right)$$

$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -6 \\ 0 & 0 & 18 \end{array} \right)$$

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$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{array} \right)$$

So y is not in $\text{span} \{ \vec{v}_1, \vec{v}_2 \}$