Partitioned Matrices

Imagine a matrix A,

$$A = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 6 \\ 7 & 9 & 1 & 6 \end{bmatrix}$$

Partitioned it could look like this,

$$\begin{aligned} A &= \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 5 \\ 6 \\ 6 \end{bmatrix} \\ \begin{bmatrix} 7 & 9 & 1 \end{bmatrix} & \begin{bmatrix} 6 \end{bmatrix} \end{bmatrix} \\ &= \begin{bmatrix} I_3 & U \\ V & X \end{bmatrix} \end{aligned}$$

We can even perform matrix multiplication,

$$\begin{aligned} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} I_2 & X \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix} \\ &= I_2 U + XY \\ &= \begin{bmatrix} 2 & -1 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -1 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$