Span

Given vectors $\mathbf{v_1}, \mathbf{v_2}, \dots, \mathbf{v_p} \in \mathbb{R}^n$. The set of all linear combinations of $\mathbf{v_1}, \mathbf{v_2}, \dots, \mathbf{v_p}$ is called the **span** of $\mathbf{v_1}, \mathbf{v_2}, \dots, \mathbf{v_p}$.

Thus, $\text{span}\{\mathbf{v_1}, \mathbf{v_2}, \dots, \mathbf{v_p}\}$ can be represented like this:

$$c_1\mathbf{v_1} + c_2\mathbf{v_2} + \cdots + c_p\mathbf{v_p}$$

Where $c_1, c_2, \dots, c_p$ are scalars.

Is $\vec{y}$ in the span of vectors $\vec{v}_1$ and $\vec{v}_2$?

$$\vec{v}_1 = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 2 \\ 5 \\ 6 \end{pmatrix}, \text{ and } \vec{y} = \begin{pmatrix} 7 \\ 4 \\ 15 \end{pmatrix}.$$

$$c_1 \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ -2c_1 \\ -3c_1 \end{bmatrix} + \begin{bmatrix} 2c_2 \\ 5c_2 \\ 6c_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 15 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -3 & 6 & 15 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -1 & 2 & 5 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ 0 & 1 & -6 \\ -1 & 2 & 5 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ 0 & 1 & -6 \\ 0 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 19 \\ 0 & 1 & -6 \\ 0 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 19 \\ 0 & 1 & -6 \\ 0 & 0 & 9 \end{array}\right]$$

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$$\left[\begin{array}{cc|c} 1 & 0 & 19 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{array}\right]$$

So y is not in span $\{\mathbf{v_1}, \mathbf{v_2}\}$

Look at Row Operations