Inverse of a Matrix

 $A \in \mathbb{R}^{n \times n}$ is invertible if there is a $C \in \mathbb{R}^{n \times n}$ so that:

$$AC = CA = I_n$$

If so we write, $C = A^{-1}$

A has an inverse if and only if there is a pivot on every row and column.

For a 2×2

Compute

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Use it to solve a Linear systems

$$Aec{x}=b \ A^{-1}Aec{x}=A^{-1}b \ Iec{x}=A^{-1}b \ ec{x}=A^{-1}b$$

Computing A^{-1}

- 1. Row reduce the augmented matrix $(A \mid I_n)$ to RREF.
- 2. If the reduction is in the form, $(I_n \mid B)$ then A is invertible and $B = A^{-1}$. Else, A is not invertible.

For example to find
$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$$
:

$$=egin{bmatrix} 0 & 1 & 2 & 1 & 0 & 0 \ 1 & 0 & 3 & 0 & 1 & 0 \ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \ = egin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \ = (I_3 \mid A^{-1}) \ A^{-1} = egin{bmatrix} 0 & 1 & -3 \ 1 & 0 & -2 \ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Why does this work?

Using <u>elementary matrices</u> we can see that when we row reduce the augmented matrix $(A \mid I_n)$ to <u>RREF</u>, we are simple applying row operation, in other words we are simple applying <u>transformations</u> using elementary matrices.

So if,

$$egin{aligned} (E_k\cdots E_3E_2E_1)A&=I_n\ E_k\cdots E_3E_2E_1&=A^{-1}\ ext{as},\ A^{-1}A&=I_n \end{aligned}$$

Some properties:

$$(A^{-1})^{-1} = A$$

 $(AB)^{-1} = B^{-1}A^{-1}$
 $(A^T)^{-1} = (A^{-1})^T$