Linear Combinations

Given vectors $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p} \in \mathbb{R}^n$, and scalars $c_1, c_2, \ldots, c_p$, the vector $\mathbf{y}$ where

$$\mathbf{y} = c_1\mathbf{v_1} + c_2\mathbf{v_2} + \ldots + c_p\mathbf{v_p}$$

is called a linear combination of $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p}$ with the weights $c_1, c_2, \ldots, c_p$.

$$c_1 - c_2 = 1$$
$$c_1 + c_2 = 3$$
$$0c_1 + 0c_2 = 1$$

We need $c_1 \text{ and } c_2$ such that:

$$c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

So,

$$\begin{bmatrix} c_1 \\ c_1 \end{bmatrix} + \begin{bmatrix} -c_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

So finally,

$$c_1 - c_2 = 1$$
$$c_1 + c_2 = 3$$

Same question, can $\mathbf{y}$ be represented as a linear combination of $\mathbf{v_1} \text{ and } \mathbf{v_2}$:

$$c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$$

$$
\begin{aligned}
c_1 - c_2 &= 1 \\
c_1 + c_2 &= 3 \\
0c_1 + 0c_2 &= 1
\end{aligned}
$$

Thus, the system is **inconsistent** (no consistent).