Linear Independence

Given a set if vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$

They are **linearly independent** if:

$$c_1\mathbf{v}_1, c_2\mathbf{v}_2, \ldots, c_k\mathbf{v}_k = \mathbf{0}$$

Only has trivial solution. Else they are **linearly dependent**
On the flip side, if:

$$c_1\mathbf{v}_1, c_2\mathbf{v}_2, \ldots, c_k\mathbf{v}_k = \mathbf{0}$$

And a nontrivial solution ($c_1, c_2, \ldots, c_k \neq \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$) solution exists, they are **linearly dependent**.

What is the smallest number of vectors needed in a parametric solution to a linear system?

Vectors are dependent if:

- One of the vectors is a 0 vector.
- They are in each others Span ($cv_1 = v_2 \text{When } c \neq \mathbf{0}$)

Theorems

- **More Vectors Than Elements**: The vectors must be **linearly dependent** as column cannot be pivotal so there are free variables.
- **Set Contains Zero Vector**: If one of the vectors is a **0** vectors the system is **linearly dependent**