Parametric Vector Form

In general, suppose the free variables for $A\mathbf{x} = \mathbf{0}$ are $x_k, \ldots, x_n$. Then all the solutions to $A\mathbf{x} = \mathbf{0}$ can be written as:

$$\mathbf{x} = x_k\mathbf{v}_k + x_{k+1}\mathbf{v}_{k+1} + \cdots + x_n\mathbf{v}_n$$

For some $\mathbf{v}_k, \ldots \mathrm{v}_n$.
This is **Parametric Vector Form**.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 4 \\ 2x_1 - x_2 - 5x_3 &= 1 \\ x_1 - 2x_3 &= 1 \end{aligned}$$

In RREF:

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{aligned} x_1 &= 1 + 2x_3 \\ x_2 &= 1 - x_3 \\ x_3 &= x_3 \end{aligned}$$

$$\mathbf{x} = \begin{bmatrix} 1 + 2x_3 \\ 1 - x_3 \\ 0 + x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2x_3 \\ -x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$\text{if, } \mathbf{x} = \begin{bmatrix} 1 + 2x_3 \\ 1 - x_3 \\ 0 + x_3 \end{bmatrix}$$

$$\text{in parametric vector form, } \mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$