

Parametric Vector Form

In general, suppose the free variables for $A\mathbf{x} = \mathbf{0}$ are x_k, \dots, x_n . Then all the solutions to $A\mathbf{x} = \mathbf{0}$ can be written as:

$$\mathbf{x} = x_k \mathbf{v}_k + x_{k+1} \mathbf{v}_{k+1} + \cdots + x_n \mathbf{v}_n$$

For some $\mathbf{v}_k, \dots, \mathbf{v}_n$.

This is **Parametric Vector Form**.

$$x_1 + 3x_2 + x_3 = 4$$

$$2x_1 - x_2 - 5x_3 = 1$$

$$x_1 - 2x_3 = 1$$

In [RREF](#):

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x_1 = 1 + 2x_3$$

$$x_2 = 1 - x_3$$

$$x_3 = x_3$$

$$\mathbf{x} = \begin{bmatrix} 1 + 2x_3 \\ 1 - x_3 \\ 0 + x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2x_3 \\ -x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$\text{if, } \mathbf{x} = \begin{bmatrix} 1 + 2x_3 \\ 1 - x_3 \\ 0 + x_3 \end{bmatrix}$$

$$\text{in parametric vector form, } \mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$