

Existence and Uniqueness

Consider the augmented matrix:

$$\left(A \mid \vec{b} \right) = \left(\begin{array}{ccccc|c} 1 & 3 & 0 & 7 & 0 & 4 \\ 0 & 0 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 6 \end{array} \right)$$

The leading one's are in first, third, and fifth columns.

- The pivot columns (leading ones) of A are the first, third, and fifth columns
- The corresponding variables of the system $A\vec{x} = \vec{b}$ are x_1, x_3 , and x_5 . Variables that correspond to a pivot are basic variables.
- Variables that are not basic are free variables. They can take any value.
- The free variables are x_2 and x_4 . Any choice of the free variables leads to a solution of the system.

Unique solution exists if and only if there are no free variables.

A linear system is **consistent** if it has at least one solution.

Note

If you see something like this:

$$(0 \ 0 \ 0 \ \dots \ 0 \mid 1)$$

~~Run~~ you **don't** have a Consistent System.