

Non-constant heat transfer equation

$$\frac{\partial u}{\partial t} = \gamma \frac{\partial^2 u}{\partial x^2} \quad (\gamma > 0)$$

Finite difference solutions for initial value problems. Solution

domain $(x, t) \in [0, 1] \times [0, \infty]$, The initial value conditions and boundary conditions are

$$\begin{cases} u(x, 0) = f(x) \\ u(0, t) = a(t) \\ u(1, t) = b(t) \end{cases}$$

where $\gamma = 1$. The disposal boundary conditions are taken as

$$F(x) = \begin{cases} 0 & 0 < x < 0.3 \\ 1 & 0.3 \leq x \leq 0.7 \\ -\frac{10}{3}x + \frac{10}{3} & 0.7 \leq x \leq 1.0 \end{cases}$$

Take the number of grid points $M_x = 100$, and ask to calculate the data at

$t = 0.01, 0.1, 1, 10$. The time step in the calculation is taken as

$$\sigma = \gamma \Delta t / (\Delta x^2) = 0.1, 0.5, 1.$$

Differential formats: FTCS format, BTCS format