

An approach to the cross product challenge

1. The cross-product in analytic form

Let (x_1, y_1, z_1) be the coordinates of \vec{v}_1 and (x_2, y_2, z_2) be the coordinates of \vec{v}_2 . Their analytic expressions are as follows:

$$\begin{aligned}\vec{v}_1 &= x_1 \cdot \vec{i} + y_1 \cdot \vec{j} + z_1 \cdot \vec{k} \\ \vec{v}_2 &= x_2 \cdot \vec{i} + y_2 \cdot \vec{j} + z_2 \cdot \vec{k}\end{aligned}$$

The only thing left to do now is to also write their cross-product in terms of its coordinates in the space.

$$\vec{v}_1 \times \vec{v}_2 = (x_1 \cdot \vec{i} + y_1 \cdot \vec{j} + z_1 \cdot \vec{k}) \times (x_2 \cdot \vec{i} + y_2 \cdot \vec{j} + z_2 \cdot \vec{k})$$

Keeping in mind that

$$\vec{i} \times \vec{j} = \vec{k}, \quad \vec{i} \times \vec{k} = -\vec{j}, \quad \vec{j} \times \vec{i} = -\vec{k}, \quad \vec{j} \times \vec{k} = \vec{i}, \quad \vec{k} \times \vec{i} = \vec{j}, \quad \vec{k} \times \vec{j} = -\vec{i}$$

After the necessary rearrangements and calculations:

$$\vec{v}_1 \times \vec{v}_2 = (y_1 z_2 - z_1 y_2) \cdot \vec{i} + (z_1 x_2 - x_1 z_2) \cdot \vec{j} + (x_1 y_2 - y_1 x_2) \cdot \vec{k}$$

2. The close relationship with matrix determinants

There's an interesting thing to note here:

$$x_1 y_2 - y_1 x_2 = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}, \quad z_1 x_2 - x_1 z_2 = \begin{vmatrix} z_1 & x_1 \\ z_2 & x_2 \end{vmatrix}, \quad y_1 z_2 - z_1 y_2 = \begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix}$$

Where we use the notation $|\cdot|$ for matrix determinant. Notice the *beautiful* rotational symmetry?

3. Jelly code explanation

Well... not much to explain here. It just generates the matrix:

$$\begin{pmatrix} x_1 & y_1 & z_1 & x_1 \\ x_2 & y_2 & z_2 & x_2 \end{pmatrix}$$

And for each pair of neighbouring matrices, it computes the determinant of the matrix formed by joining the two.

(*): Everything is better in *L^AT_EX* and I have to develop my skills with it