Proof that no prime powers are correct

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> **Definition.** A positive integer n is called **correct** if and only if the sum of the squares of its positive divisors is equal to $(n+3)^2$.

Proposition: No integer of the form p^m is correct, where $m \in \mathbb{N}$ and p is prime.

Proof: First, keep in mind that

$$(n+3)^2 = (p^m+3)^2 = p^{2m} + 6p^m + 9 \tag{1}$$

Then, the sum S of its positive divisors is given by

$$S = \sum_{d \in D^+_{p^m}} d^2$$

But

$$D^+_{p^m} = \{0 \le k \le m \mid p^k\}$$

And therefore

$$S = \sum_{k=0}^{m} (p^k)^2 = \sum_{k=0}^{m} p^{2k} \tag{2}$$

Now let's suppose that p^m is correct. In this case, from $(1), (2)$ it follows that

$$p^{2m} + 6p^m + 9 = \sum_{k=0}^{m} p^{2k}$$

Subtracting p^{2m} from both sides, we get

$$6p^m + 9 = \sum_{k=0}^{m-1} p^{2k}$$

$\blacksquare$