An approach to the cross product challenge

1. The cross-product in analytic form

Let (x_1, y_1, z_1) be the coordinates of $\vec{v_1}$ and $(x_2, y_2, z_2$ be the coordinates of $\vec{v_2}$. Their analytic expressions are as follows:

$$\begin{aligned}\vec{\boldsymbol{v_1}} &= x_1 \cdot \vec{\boldsymbol{i}} + y_1 \cdot \vec{\boldsymbol{j}} + z_1 \cdot \vec{\boldsymbol{k}} \\ \vec{\boldsymbol{v_2}} &= x_2 \cdot \vec{\boldsymbol{i}} + y_2 \cdot \vec{\boldsymbol{j}} + z_2 \cdot \vec{\boldsymbol{k}}\end{aligned}$$

The only thing left to do now is to also write their cross-product in terms of its coordinates in the space.

$$\vec{\boldsymbol{v_1}} \times \vec{\boldsymbol{v_2}} = \left(x_1 \cdot \vec{\boldsymbol{i}} + y_1 \cdot \vec{\boldsymbol{j}} + z_1 \cdot \vec{\boldsymbol{k}}\right) \times \left(x_2 \cdot \vec{\boldsymbol{i}} + y_2 \cdot \vec{\boldsymbol{j}} + z_2 \cdot \vec{\boldsymbol{k}}\right)$$

Keeping in mind that

$$\vec{\boldsymbol{i}} \times \vec{\boldsymbol{j}} = \vec{\boldsymbol{k}},\ \vec{\boldsymbol{i}} \times \vec{\boldsymbol{k}} = -\vec{\boldsymbol{j}},\ \vec{\boldsymbol{j}} \times \vec{\boldsymbol{i}} = -\vec{\boldsymbol{k}},\ \vec{\boldsymbol{j}} \times \vec{\boldsymbol{k}} = \vec{\boldsymbol{i}},\ \vec{\boldsymbol{k}} \times \vec{\boldsymbol{i}} = \vec{\boldsymbol{j}},\ \vec{\boldsymbol{k}} \times \vec{\boldsymbol{j}} = -\vec{\boldsymbol{i}}$$

After the necessary rearrangements and calculations:

$$\vec{\boldsymbol{v_1}} \times \vec{\boldsymbol{v_2}} = (y_1 z_2 - z_1 y_2) \cdot \vec{\boldsymbol{i}} + (z_1 x_2 - x_1 z_2) \cdot \vec{\boldsymbol{j}} + (x_1 y_2 - y_1 x_2) \cdot \vec{\boldsymbol{k}}$$

2. The close relationship with matrix determinants

There's an interesting thing to note here:

$$x_1 y_2 - y_1 x_2 = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}, \quad z_1 x_2 - x_1 z_2 = \begin{vmatrix} z_1 & x_1 \\ z_2 & x_2 \end{vmatrix}, \quad y_1 z_2 - z_1 y_2 = \begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix}$$

Where we use the notation $|\cdot|$ for matrix determinant. Notice the *beautiful* rotational symmetry?

3. Jelly code explanation

Well... not much to explain here. It just generates the matrix:

$$\begin{pmatrix} x_1 & y_1 & z_1 & x_1 \\ x_2 & y_2 & z_2 & x_2 \end{pmatrix}$$

And for each pair of neighbouring matrices, it computes the determinant of the matrix formed by joining the two.

(*): Everything is better in LaTeX and I have to develop my skills with it