圆形势阱

无量纲化

其中 a 是格点间距。带“ ' ”是无量纲量

紧束缚近似

$$H = \sum_i \epsilon_i a_i^\dagger a_i - t \sum_{i,j} a_i^\dagger a_j$$

其中$\epsilon_i = 4t + U_i, t = \frac{\hbar^2}{2ma^2}$。

- $k = \frac{\sqrt{2mE}}{\hbar} = \sqrt{E'}\frac{\sqrt{2mt}}{\hbar} = \frac{\sqrt{E'}}{a} \to k' = \sqrt{E'}$
- $x = x'a$
- $k = \frac{k'}{a}$

色散等值线

由于

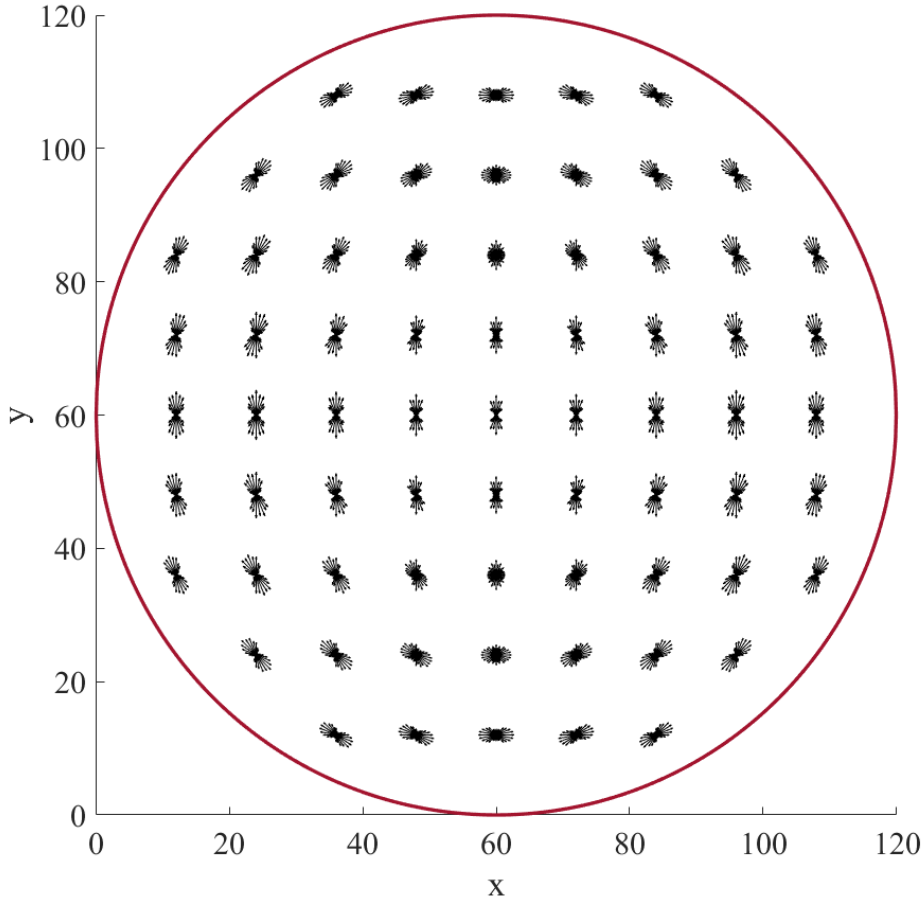
$$\begin{aligned}
\mathrm{Hu}(\psi, \mathbf{r}_0, \mathbf{k}_0) &= \int \psi^*(\mathbf{r}) \mathrm{e}^{-\frac{(\mathbf{r}-\mathbf{r}_0)^2}{4\sigma^2} + \mathrm{i}\mathbf{k}_0\cdot\mathbf{r}}\,\mathrm{d}\mathbf{r} \\
&= \int \psi(\mathbf{r})_{probality}\, \mathrm{e}^{-\mathrm{i}\mathbf{k}(\mathbf{r})\cdot\mathbf{r}} \mathrm{e}^{-\frac{(\mathbf{r}-\mathbf{r}_0)^2}{4\sigma^2} + \mathrm{i}\mathbf{k}_0\cdot\mathbf{r}}\,\mathrm{d}\mathbf{r} \\
&= \int \psi(\mathbf{r'})_{probality}\, \mathrm{e}^{-\frac{(\mathbf{r'}-\mathbf{r'}_0)^2}{4\sigma'^2} + \mathrm{i}[\mathbf{k'}_0 - \mathbf{k'}(\mathbf{r'})]\cdot\mathbf{r'}}\,\mathrm{d}\mathbf{r'}
\end{aligned}$$

因此为了使得测试波矢$\mathbf{k'}_0$能检测出$\mathbf{k'}$，需要让k'_0与k'在同一条色散等值线上，即$k'_0 \approx \sqrt{E'}$

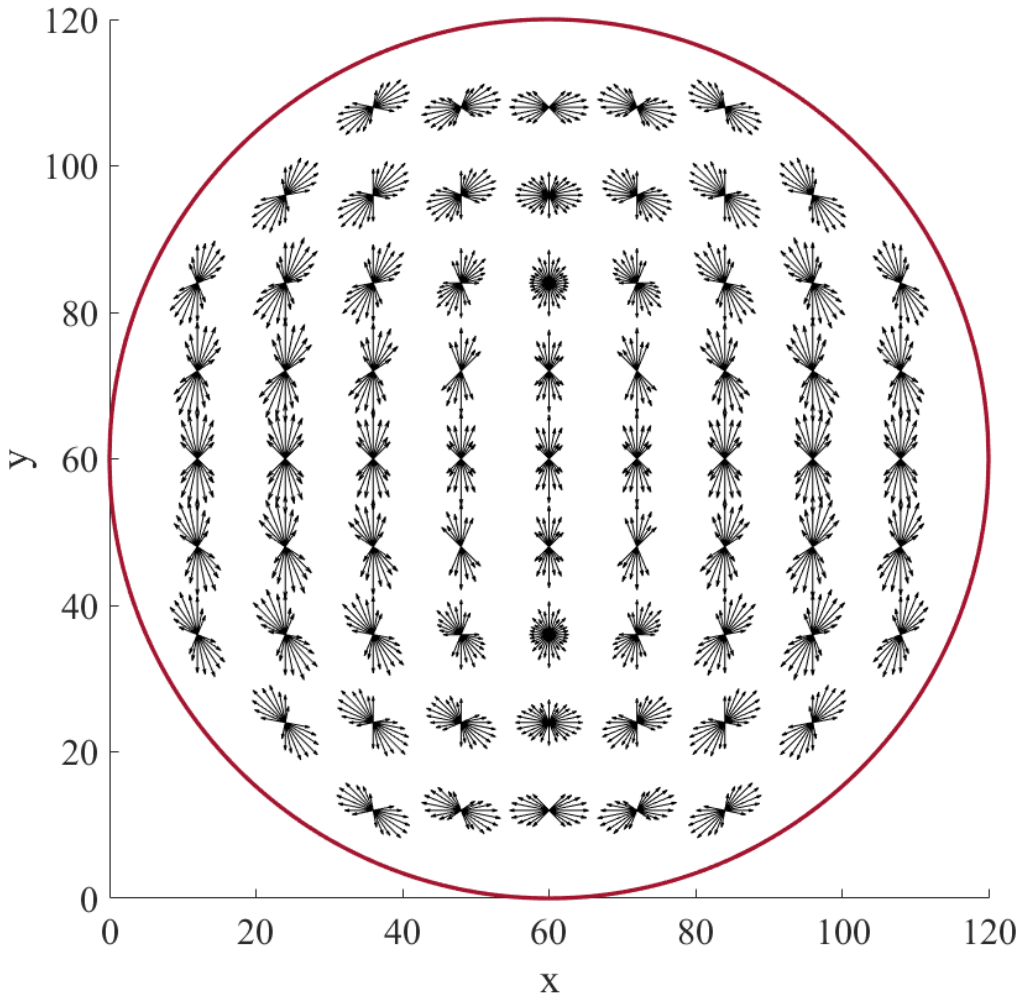
σ对Husimi图的影响

$\sigma = 25$

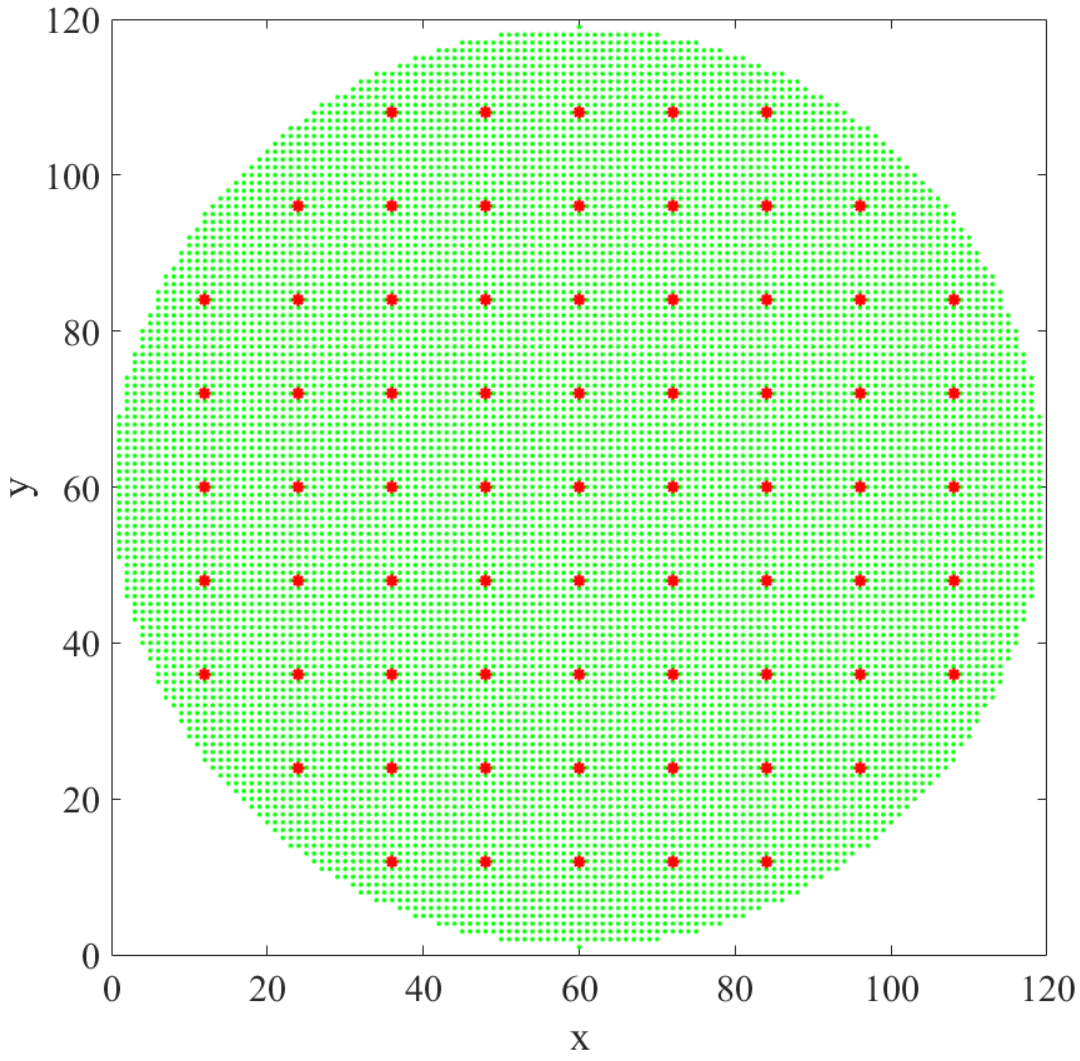
- raw Husimi Map



- processed Husimi Map

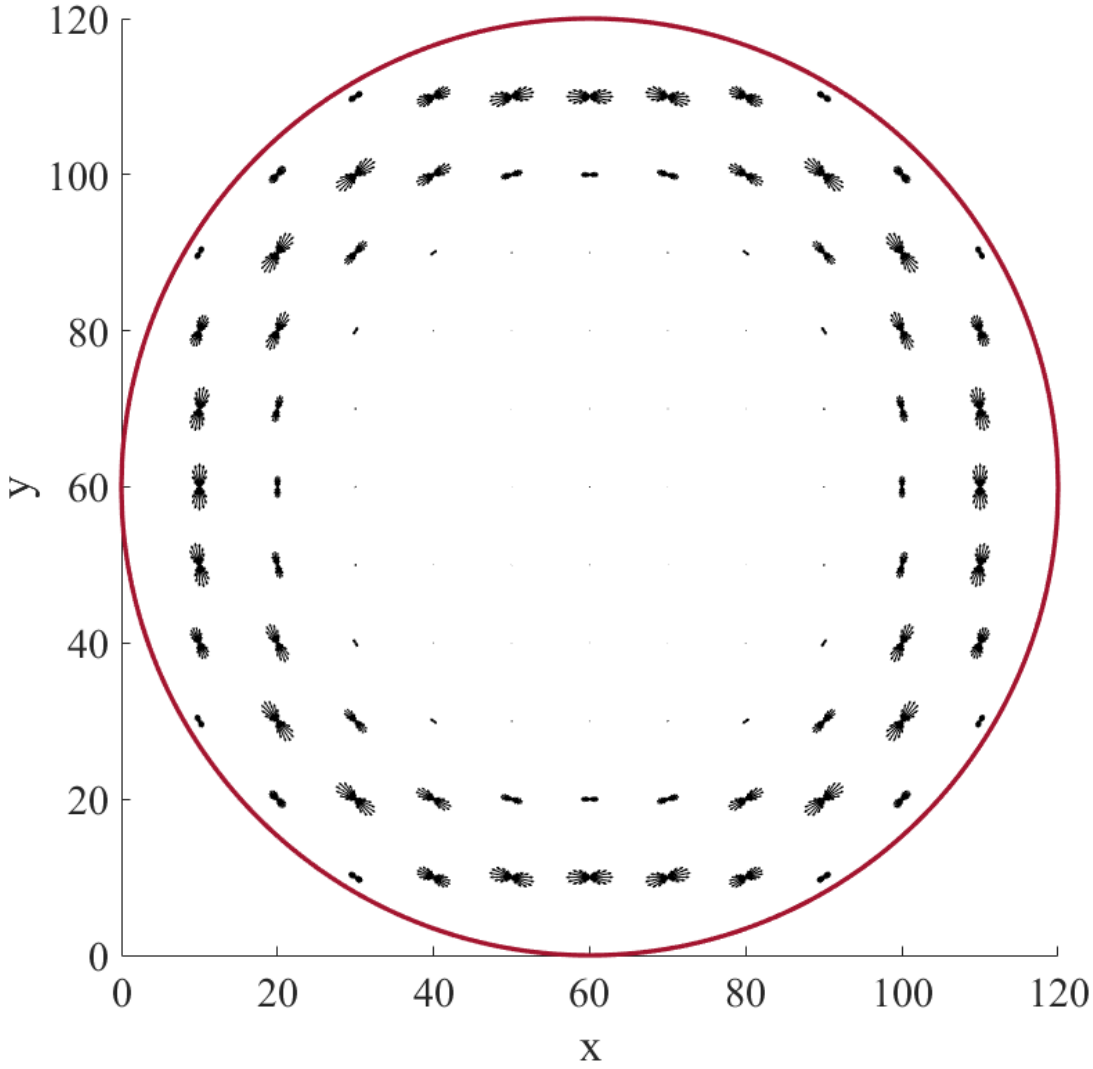


- sample points

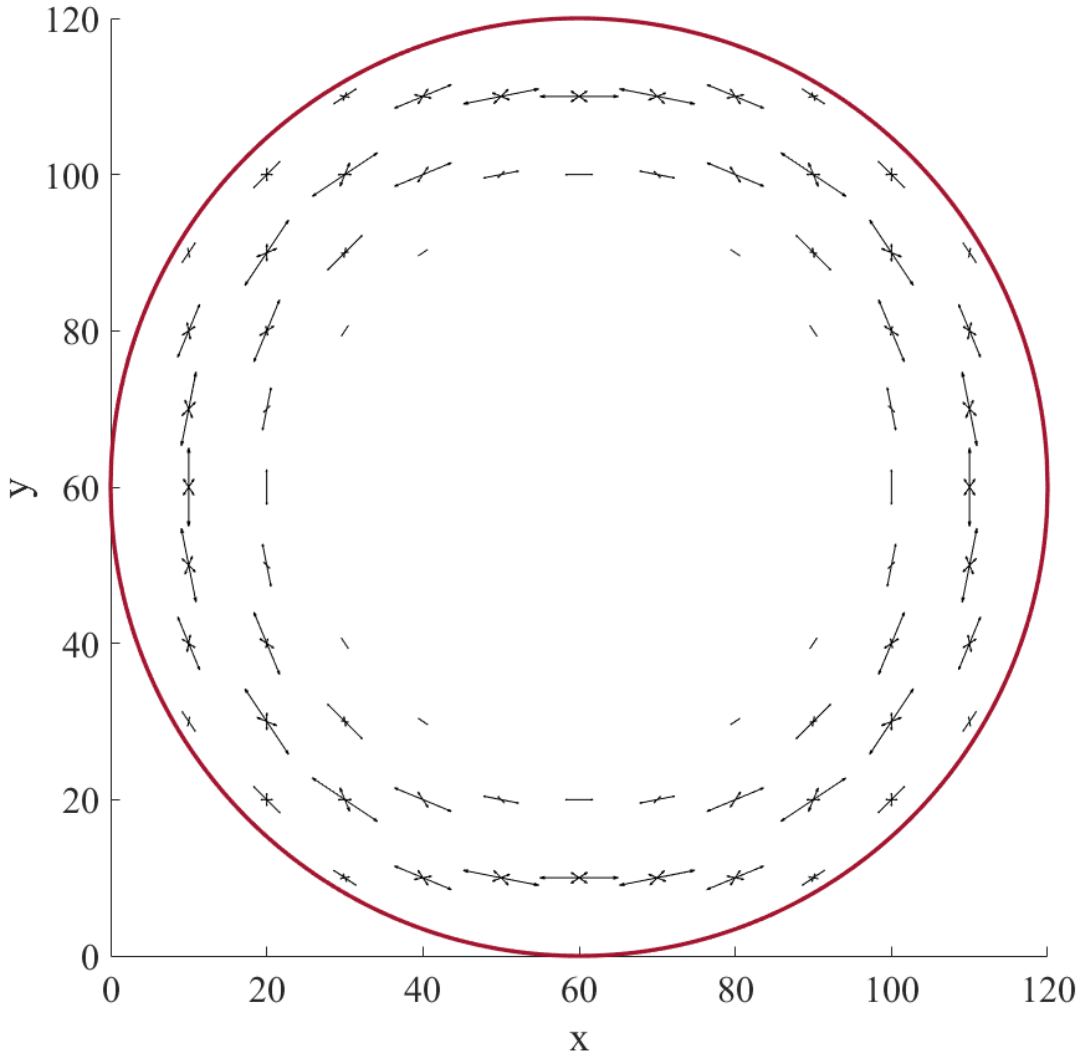


$\sigma = 5$

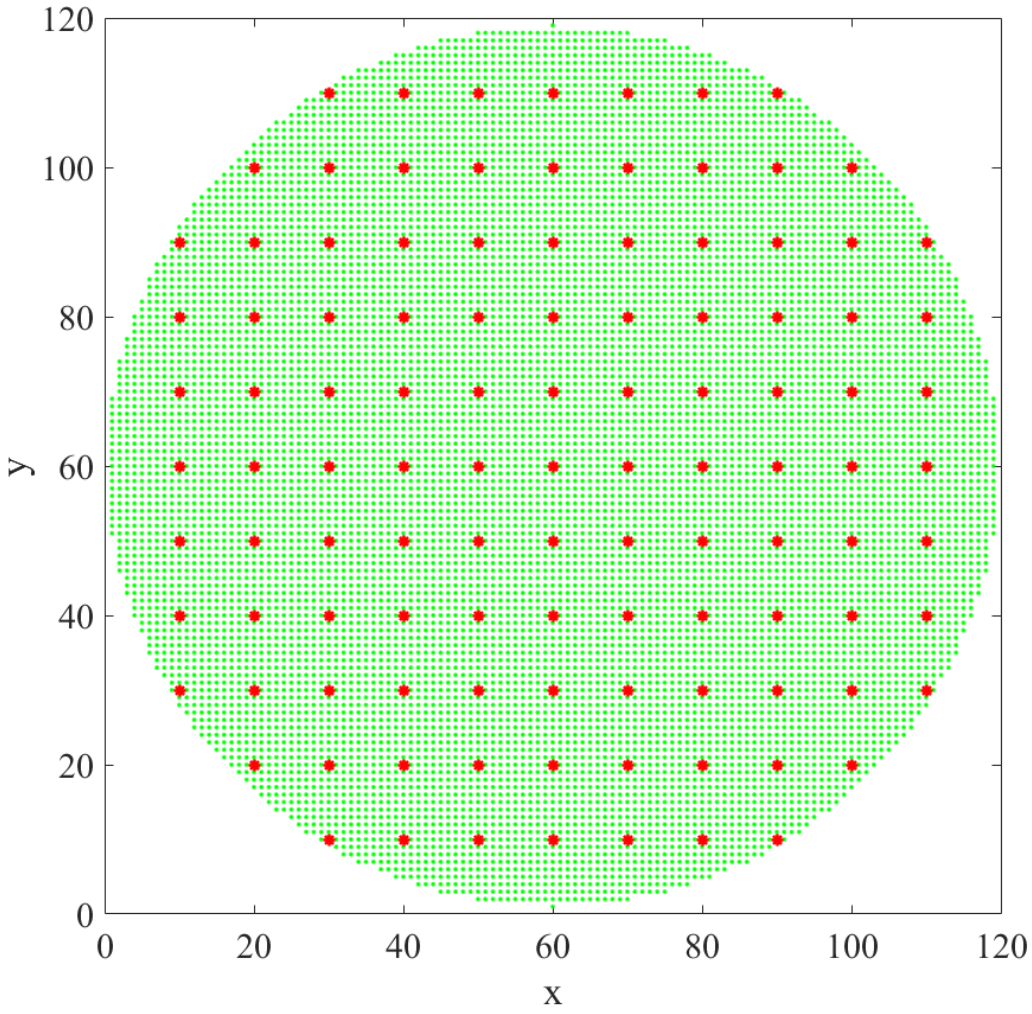
- raw Husimi Map



- processed Husimi Map

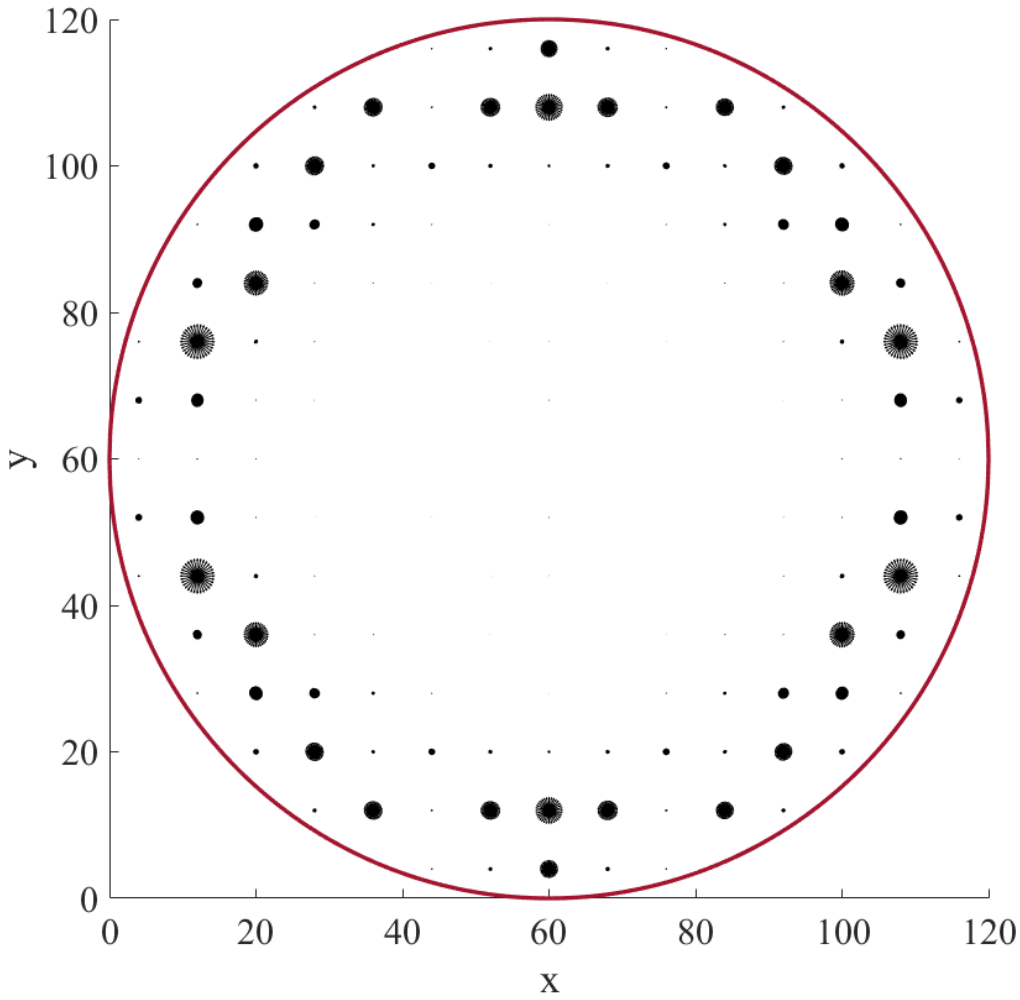


- sample points

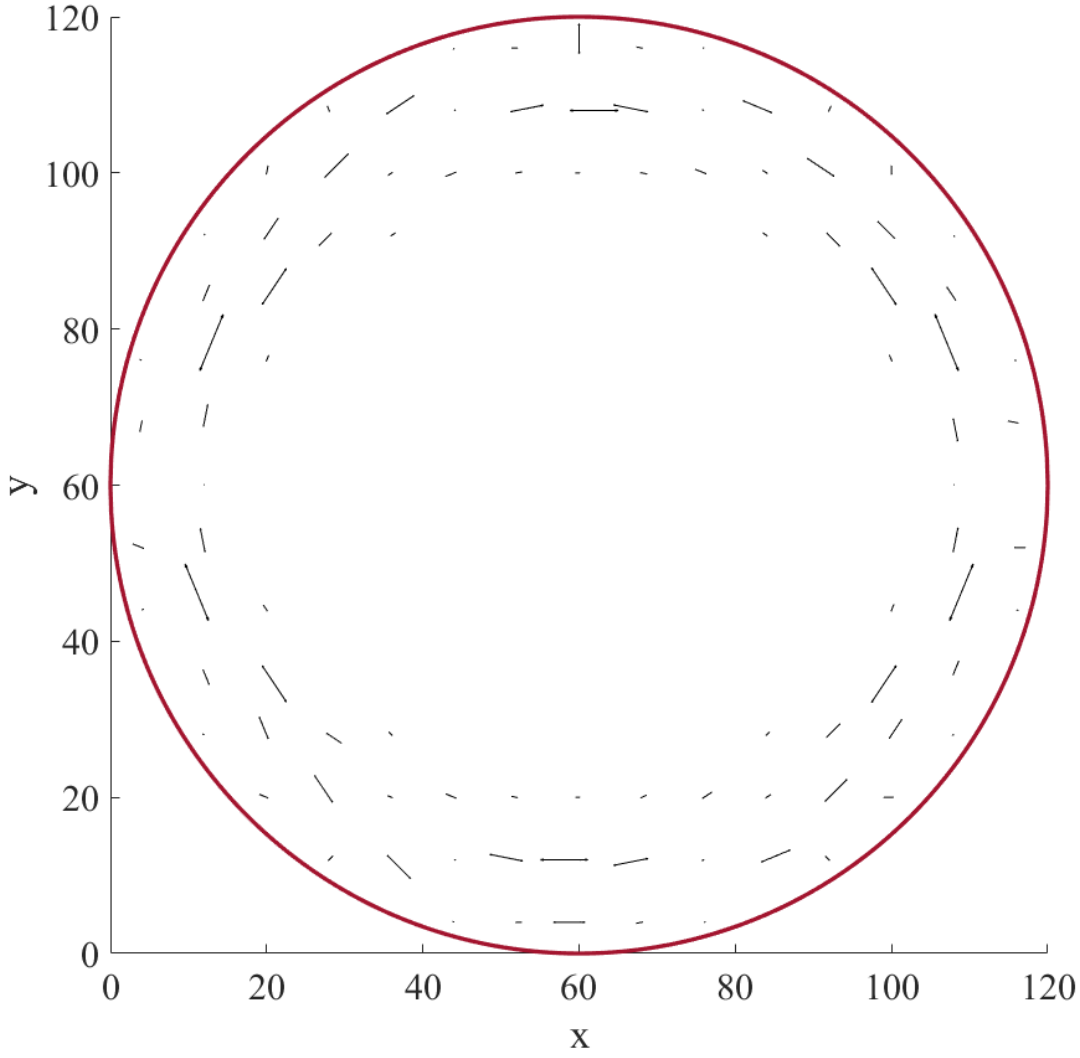


$\sigma = 1$

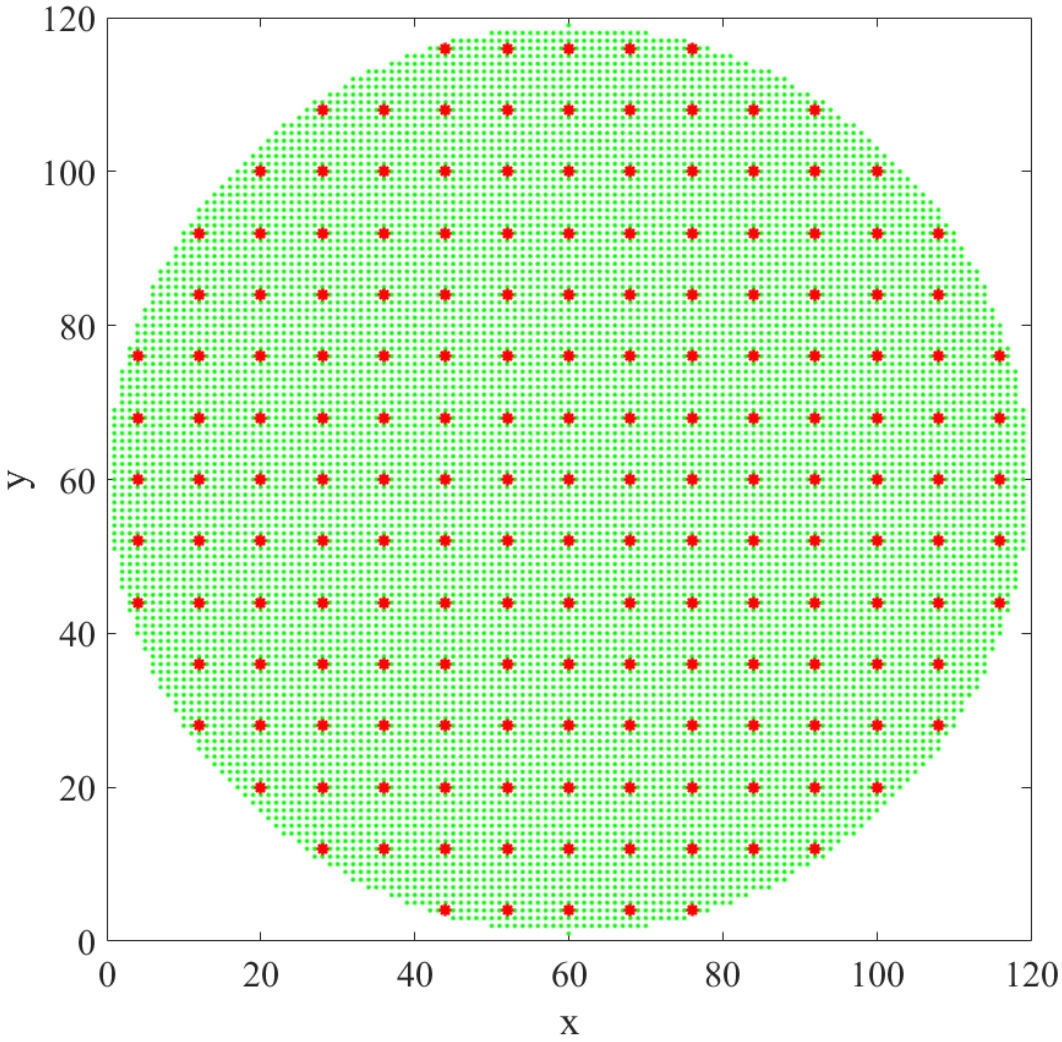
- raw Husimi Map



- processed Husimi Map



- sample points

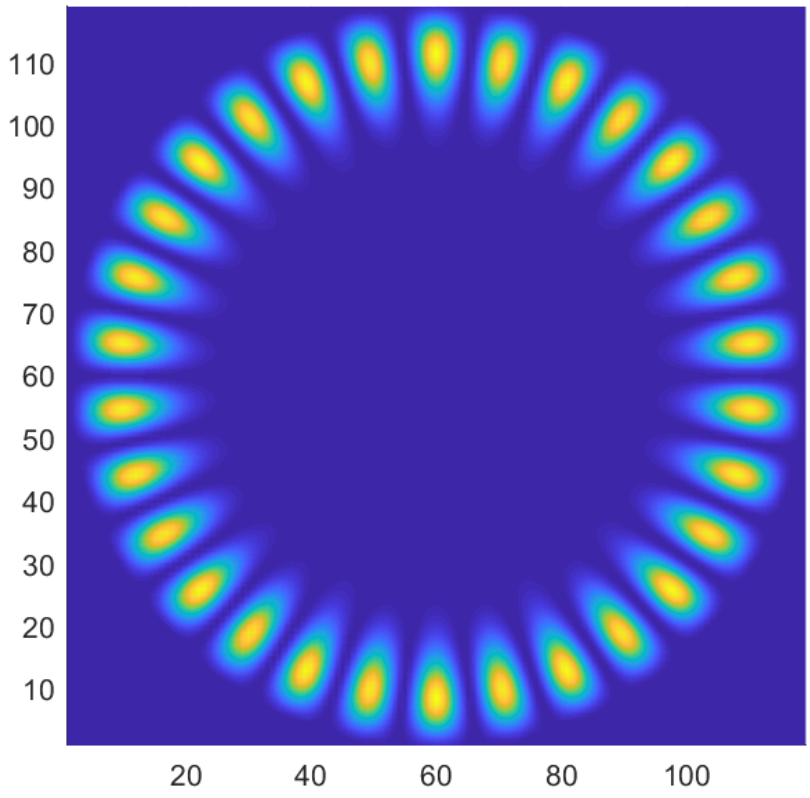


从上面的图中可以看出，如果sigma取得过大（Δk较小），位形空间的不确定度就很大，需要的有效积分区域就越多，进而导致Husimi流的不准确。如果sigma取得过小（Δk较大），动量空间的不确定度就很大，需要的有效积分区域就越小，进而导致Husimi流的不准确。因此，需要选择一个折中的σ，使得两面都兼顾到。造成这一切的原因是不确定关系:$\Delta x \Delta k = \frac{1}{2}$

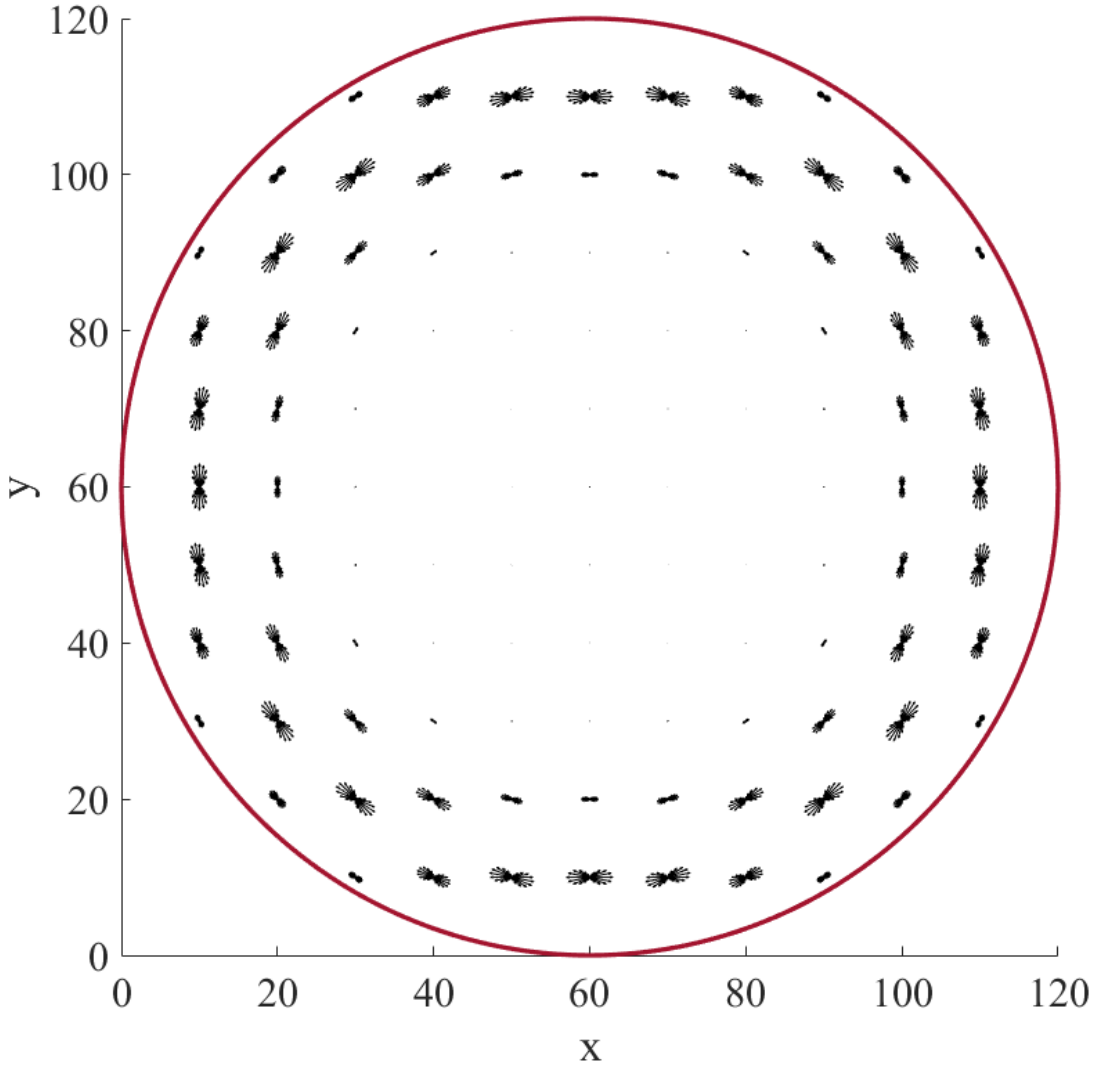
“重复”的结果

$n = 0$

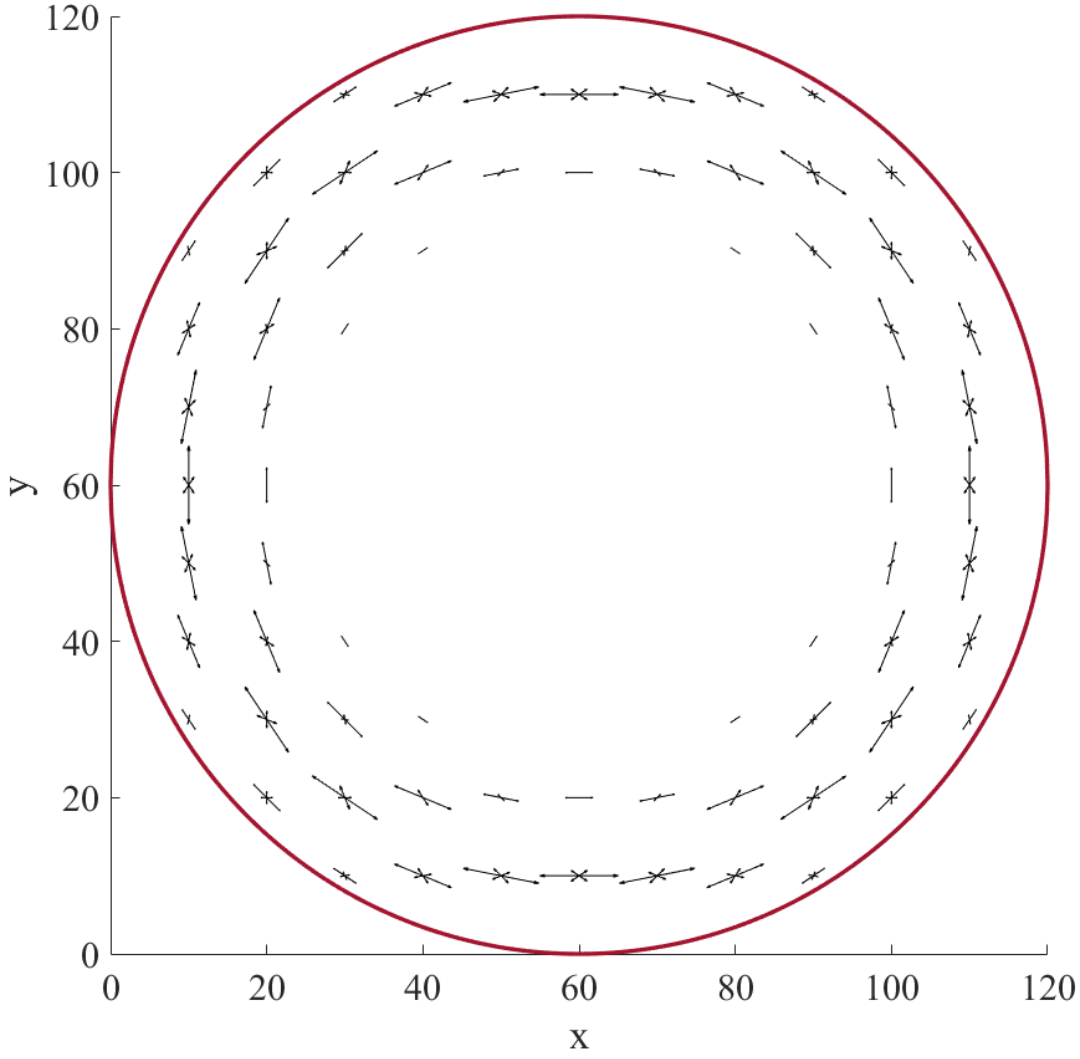
- wave function



- raw Husimi map

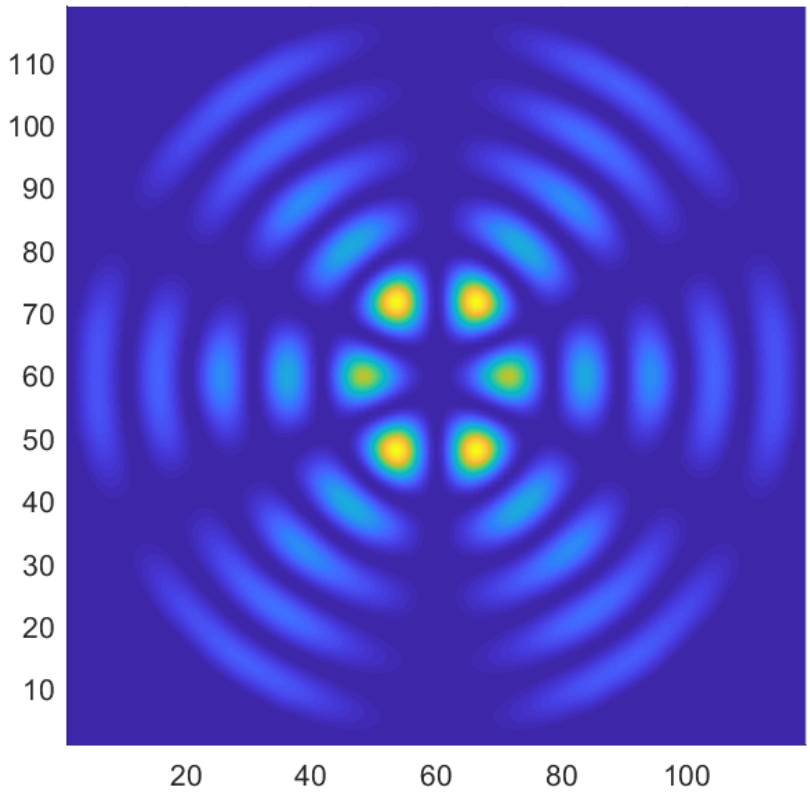


- processed Husimi map

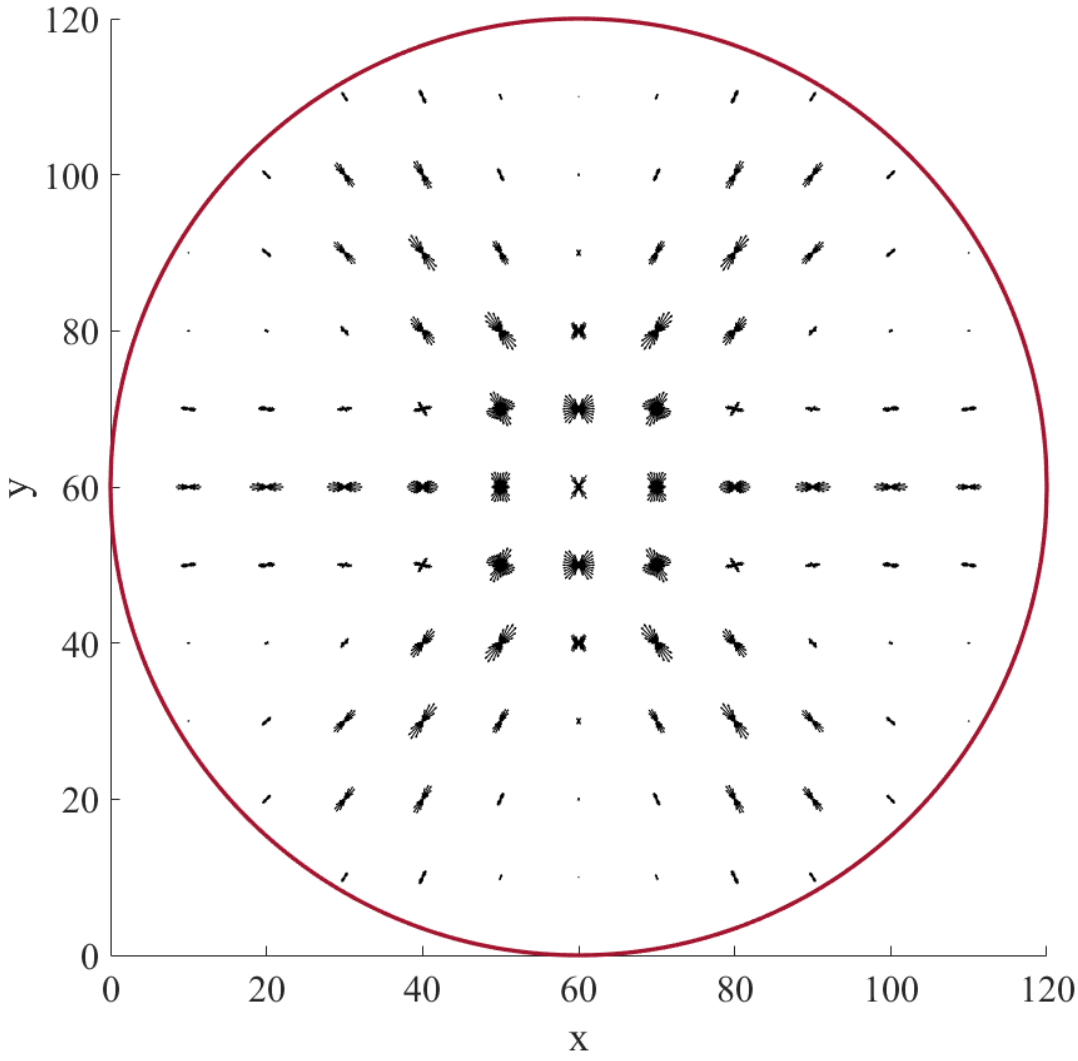


$n \gg m$

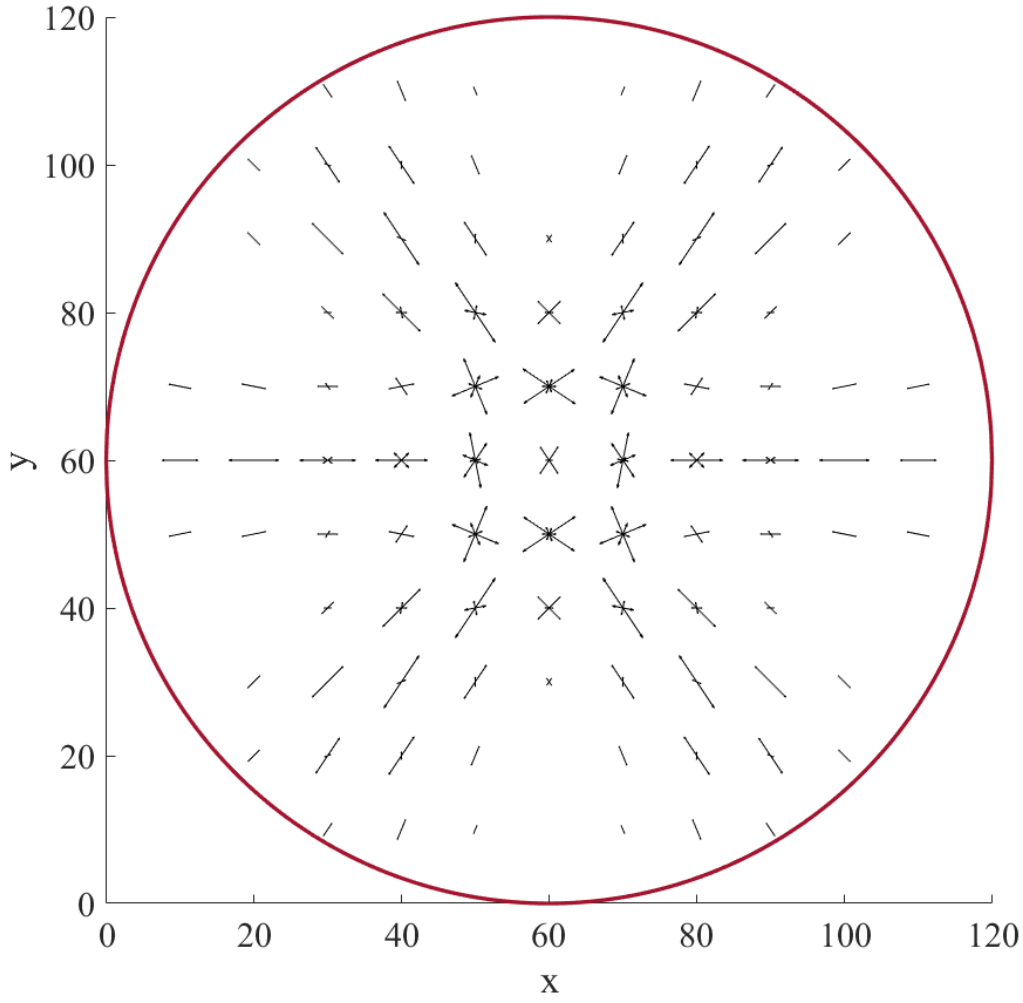
- wave function



- raw Husimi map

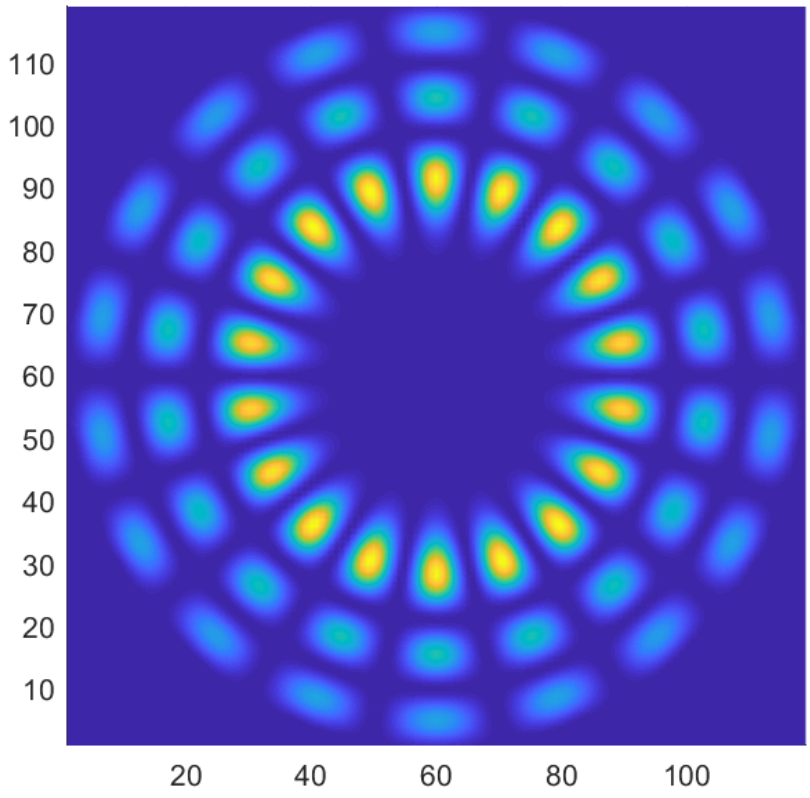


- processed Husimi map

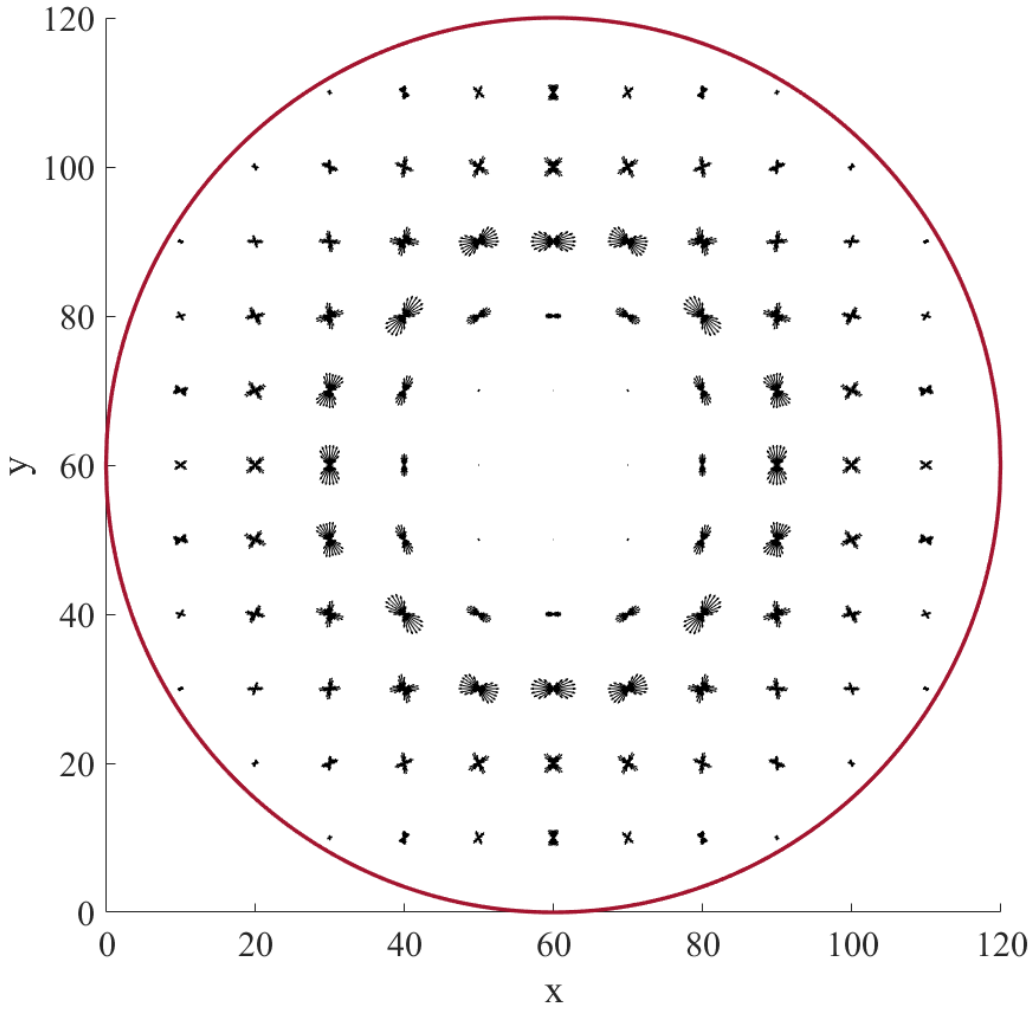


$n \approx m$

- wave function



- raw Husimi map



- processed Husimi map