

Algorithms

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1 Introduction

This paper introduces various algorithms used in our research.

2 Asynchronous Baseline Algorithm

Below is a table of symbols used in the algorithms:

Symbol	Description
J	Number of classes
T	Global rounds
E	Edge rounds
K	Local epochs
L	Number of edge servers
B	The buffer of the cloud server with a static length
N^l	Number of clients in the l -th edge server
$D_{i,j}$	A subset of the local dataset D_i of the i -th client, containing training instances of class j .
\mathcal{N}_j^l	Number of clients in edge l containing class j that have participated in aggregation
$\mathcal{N}_j^{l,\text{old}}$	Last number of clients in edge l containing class j that have participated in aggregation
S^l	Set of clients participating in training in the l -th edge server
\bar{C}_j	Aggregated prototype of class j in the cloud edge server
C_j^l	Aggregated prototype of class j from the l -th edge server
$C_j^{l,\text{old}}$	Last version of aggregated prototype of class j from the l -th edge server stored in the cloud server
$c_{i,j}^l$	Aggregated prototype of class j from the client i in the l -th edge server
$c_{i,j}^{l,\text{old}}$	Last version of the aggregated prototype of class j from client i in the l -th edge server stored in the edge server

Table 1: Symbol Table

The following algorithm demonstrates how to calculate the factorial of a number.

Algorithm 1 Hierarchical Federated Prototype Learning -Part 1

```

1: procedure CLOUD SERVER EXECUTES
2:   Initialize weights for clients with heterogeneous models.
3:   All edge servers execute in parallel.
4:   for  $t = 1, \dots, T$  do
5:     Clear the buffer  $B$ 
6:     while  $B$  is not full do ▷ Async process
7:       Receive a triple  $(C^l, \mathcal{N}^l, \mathcal{X}^l)$  from one edge server.
8:       Populate  $B$  with the received triple.
9:     end while
10:     $\bar{C}, G \leftarrow \text{CloudUpdate}(B)$ 
11:    Send  $\bar{C}, G$  to edge servers participating in the current global aggregation.
12:    These edge servers re-execute.
13:  end for
14: end procedure
15: procedure EDGE SERVER EXECUTES
16:   Receive  $\bar{C}, G$  from the cloud server
17:   Choose a set of clients  $S^l$  to train in parallel.
18:   for  $e = 1, \dots, E$  do ▷ E now is static 1
19:     Send  $\bar{C}, G$  to client  $i \in S^l$ 
20:     for each client  $i$  in parallel do
21:        $(c_i^l, \mathcal{X}_i^l) \leftarrow \text{ClientUpdate}(i, \bar{C}, G)$ 
22:     end for ▷ Wait for all clients
23:      $(C^l, \mathcal{N}^l, \mathcal{X}^l) \leftarrow \text{EdgeAggregate}(\{(c_i^l, \mathcal{X}_i^l)\}_{i \in S^l})$ 
24:      $\bar{C} \leftarrow \text{EdgeUpdate}(\bar{C}, C^l)$  ▷ not used now
25:   end for
26:   Send a triple  $(C^l, \mathcal{N}^l, \mathcal{X}^l)$  to the cloud server
27: end procedure

```

Algorithm 2 Hierarchical Federated Prototype Learning -Part 2

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1: procedure CLOUDUPDATE(B)
2:   for  $j = 1, \dots, J$  do
3:      $\hat{C}_j \leftarrow \sum_{l=1}^L \mathcal{N}_j^{l, \text{prev}} \cdot \bar{C}_j$   $\triangleright$  Extend the aggregated prototypes  $\bar{C}$ 
4:     for  $(C^l, \mathcal{N}^l) \in B$  do
5:        $\hat{C}_j \leftarrow \hat{C}_j + \mathcal{N}_j^l \cdot C_j^l$ 
6:       if  $C_j^{l, \text{prev}}$  is not empty then
7:          $\hat{C}_j \leftarrow \hat{C}_j - \mathcal{N}_j^{l, \text{prev}} \cdot C_j^{l, \text{prev}}$ 
8:       end if
9:        $\mathcal{N}_j^{l, \text{prev}} \leftarrow \mathcal{N}_j^l$ 
10:    end for
11:     $\bar{C}_j \leftarrow \frac{\hat{C}_j}{\sum_{l=1}^L \mathcal{N}_j^{l, \text{prev}}}$ 
12:  end for
13:   $C^{l, \text{prev}} \leftarrow C^l$  for  $l \in B$ 
14:   $\mathcal{X}^{l, \text{cloud}} \leftarrow \mathcal{X}^l$  for  $l \in B$ 
15:   $\mathcal{X} \leftarrow \bigcup_{l \in L} \mathcal{X}^{l, \text{cloud}}$ 
16:  Train the global classifier  $G$  by using  $\mathcal{X}$ .
17:  return  $\bar{C}, G$ 
18: end procedure
19: procedure EDGEAGGREGATE( $l, \{(c_i^l, \mathcal{X}_i^l)\}_{i \in S^l}$ )
20:   for  $j = 1, \dots, J$  do
21:      $\hat{C}_j^l \leftarrow \mathcal{N}_j^l \cdot C_j^l$ 
22:     for each  $c_i^l$  do
23:        $\hat{C}_j^l \leftarrow \hat{C}_j^l + c_{i,j}^l$ 
24:       if  $c_{i,j}^{l, \text{prev}}$  is not empty then
25:          $\hat{C}_j^l \leftarrow \hat{C}_j^l - c_{i,j}^{l, \text{prev}}$ 
26:       else
27:          $\mathcal{N}_j^l \leftarrow \mathcal{N}_j^l + 1$ 
28:       end if
29:     end for
30:      $C_j^l \leftarrow \frac{\hat{C}_j^l}{\mathcal{N}_j^l}$ 
31:   end for
32:    $c_i^{l, \text{prev}} \leftarrow c_i^l$  for  $i \in S^l$ 
33:    $\mathcal{X}_i^{l, \text{edge}} \leftarrow \mathcal{X}_i^l$  for  $i \in S^l$ 
34:    $\mathcal{X}^l \leftarrow \bigcup_{i \in N^l} \mathcal{X}_i^{l, \text{edge}}$ 
35:   return  $(C^l, \mathcal{N}^l, \mathcal{X}^l)$ 
36: end procedure
37: procedure CLIENTUPDATE( $i, \bar{C}, G$ )
38:   Receive  $\bar{C}, G$  from the edge server
39:   for  $k = 1, \dots, K$  do
40:     DVFS to be implemented...
41:     for batch  $(x, y) \in D_i$  do
42:       Compute client prototypes by Eq.?.
43:       Compute loss by Eq.? using client prototypes and the global
44:       classifier  $G$ .
45:       Update client model according to the loss.
46:     end for
47:     Store the features and labels of  $D_i$  in  $\mathcal{X}_i^l$ .
48:   end for
49:   return  $(c_i^l, \mathcal{X}_i^l)$ 
50: end procedure

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3 Proposed Algorithm