

# Semi-Asynchronous Energy-Efficient Federated Prototype Learning for Client-Edge-Cloud Architectures

*Abstract—*

**Index Terms—**Federated Prototype Learning, Hierarchical Architecture, Heterogeneous Models, Asynchronous Communication, Energy Efficiency

## I. INTRODUCTION

## II. RELATED WORK

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### A. Proposed Algorithm New Version

TABLE I  
SYMBOL TABLE

Symbol	Description
$J$	Number of classes
$T$	Global communication rounds
$E$	Edge communication rounds
$K$	Local train epochs
$L$	Number of edge servers
$B$	The buffer of the cloud server with a static length
$N^l$	The number of clients on the $l$ -th edge server
$\mathcal{N}_j^l$	The number of instances belonging to class $j$ on edge $l$ that have already participated in computation
$\mathcal{N}_j^{l, \text{prev}}$	Previous number of instances belonging to class $j$ on edge that have already participated in computation $l$
$\mathcal{N}_j^{l, i}$	The number of instances belonging to class $j$ on client $i$ on edge $l$
$S^l$	Set of clients participating in training in the $l$ -th edge server
$\bar{C}_j$	Aggregated prototype of class $j$ in the cloud edge server
$C_j^l$	Aggregated prototype of class $j$ from the $l$ -th edge server
$C_j^{l, \text{prev}}$	Previous version of aggregated prototype of class $j$ from the $l$ -th edge server in the cloud server
$c_{i,j}^l$	Aggregated prototype of class $j$ from the client $i$ in the $l$ -th edge server
$c_{i,j}^{l, \text{prev}}$	Previous version of the aggregated prototype of class $j$ from client $i$ in the $l$ -th edge server in the edge server
$D_{i,j}^l$	A subset of the local dataset $D_i^l$ of the $i$ -th client in the $l$ -th edge server, containing training instances of class $j$ .
$\mathcal{X}_i^l$	The feature and label set of the $i$ -th client in the $l$ -th edge server, containing all features of $D_i$ .
$G^l$	The global classifier in the $l$ -th edge server.

The following algorithm demonstrates how to calculate the factorial of a number.

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### Algorithm 1 Hierarchical Federated Prototype Learning -Part 1

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1: procedure CLOUD SERVER EXECUTES
2:   Initialize weights for clients with heterogeneous mod-
   els.
3:   All edge servers execute in parallel.
4:   for  $t = 1, \dots, T$  do
5:     Clear the buffer  $B$ 
6:     while  $B$  is not full do ▷ Async process
7:       Receive a triple  $(C^l, \mathcal{N}^l)$  from one edge server.
8:       Populate  $B$  with the received triple.
9:     end while
10:     $\bar{C}, G \leftarrow \text{CloudUpdate}(B)$ 
11:    Send  $\bar{C}, G$  to edge servers participating in the
    current global aggregation.
12:    These edge servers re-execute.
13:  end for
14: end procedure
15: procedure EDGE SERVER EXECUTES
16:   Receive  $\bar{C}, G$  from the cloud server
17:   Choose a set of clients  $S^l$  to train in parallel.
18:   for  $e = 1, \dots, E$  do ▷ E now is static 1
19:     Send  $\bar{C}, G$  to client  $i \in S^l$ 
20:     for each client  $i$  in parallel do
21:        $c_i^l \leftarrow \text{ClientUpdate}(i, \bar{C}, G)$ 
22:     end for ▷ Wait for all clients
23:      $(C^l, \mathcal{N}^l) \leftarrow \text{EdgeAggregate}(\{c_i^l\}_{i \in S^l})$ 
24:      $\bar{C} \leftarrow \text{EdgeUpdate}(\bar{C}, C^l)$  ▷ not used now
25:   end for
26:   Send a triple  $(C^l, \mathcal{N}^l)$  to the cloud server
27: end procedure

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**Algorithm 2** Hierarchical Federated Prototype Learning -Part 2
 

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1: procedure CLOUDUPDATE(B)
2:   for  $j = 1, \dots, J$  do
3:      $\hat{C}_j \leftarrow \sum_{l=1}^L \mathcal{N}_j^{l,\text{prev}} \cdot \bar{C}_j \triangleright$  Extend the aggregated
       prototypes  $\bar{C}$ 
4:     for  $(C^l, \mathcal{N}^l) \in B$  do
5:        $\hat{C}_j \leftarrow \hat{C}_j + \mathcal{N}_j^l \cdot C_j^l$ 
6:       if  $C_j^{l,\text{prev}}$  is not empty then
7:          $\hat{C}_j \leftarrow \hat{C}_j - \mathcal{N}_j^{l,\text{prev}} \cdot C_j^{l,\text{prev}}$ 
8:       end if
9:        $\mathcal{N}_j^{l,\text{prev}} \leftarrow \mathcal{N}_j^l$ 
10:    end for
11:     $\bar{C}_j \leftarrow \frac{\hat{C}_j}{\sum_{l=1}^L \mathcal{N}_j^{l,\text{prev}}}$ 
12:  end for
13:   $C^{l,\text{prev}} \leftarrow C^l$  for  $l \in B$ 
14:  for  $j = 1, \dots, J$  do
15:     $\mu_j \leftarrow \bar{C}_j$ 
16:     $\Sigma_j = \frac{1}{N_j-1} \sum_{l=1}^L \mathcal{N}_j^{l,\text{prev}} (C^{l,\text{prev}} - \mu)(C^{l,\text{prev}} - \mu)^\top$ 
17:  end for
18:  Generate samples  $X \sim \mathcal{N}(\mu, \Sigma)$  using the multivariate
  normal distribution
19:  Use the generated samples  $X$  to train the global
  classifier  $G$ 
20:  return  $\bar{C}, G$ 
21: end procedure
22: procedure EDGEAGGREGATE( $l, \{c_i^l\}_{i \in S^l}$ )
23:   for  $j = 1, \dots, J$  do
24:      $\hat{C}_j^l \leftarrow \mathcal{N}_j^l \cdot C_j^l$ 
25:     for each  $c_i^l$  do
26:        $\hat{C}_j^l \leftarrow \hat{C}_j^l + \mathcal{N}_j^{l,i} \cdot c_{i,j}^l$ 
27:       if  $c_{i,j}^{l,\text{prev}}$  is not empty then
28:          $\hat{C}_j^l \leftarrow \hat{C}_j^l - \mathcal{N}_j^{l,i} \cdot c_{i,j}^{l,\text{prev}}$ 
29:       else
30:          $\mathcal{N}_j^l \leftarrow \mathcal{N}_j^l + \mathcal{N}_j^{l,i}$ 
31:       end if
32:     end for
33:      $\bar{C}_j^l \leftarrow \frac{\hat{C}_j^l}{\mathcal{N}_j^l}$ 
34:   end for
35:    $c_i^{l,\text{prev}} \leftarrow c_i^l$  for  $i \in S^l$ 
36:   return  $(C^l, \mathcal{N}^l)$ 
37: end procedure
38: procedure CLIENTUPDATE( $i, \bar{C}, G$ )
39:   Receive  $\bar{C}, G$  from the edge server
40:   for  $k = 1, \dots, K$  do
41:     DVFS to be implemented...
42:     for batch  $(x, y) \in D_i$  do
43:       Compute client prototypes by Eq.?.
44:       Compute loss by Eq.? using client prototypes
       and the global classifier  $G$ .
45:       Update client model according to the loss.
46:     end for
47:   end for
48:   return  $c_i^l$ 
49: end procedure

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V. EXPERIMENTS

VI. CONCLUSION