Faculty of Engineering and Technology
Electrical and Computer Engineering Department

Communication Systems
ENEE2312
Task1
Suggested questions, HW1

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Problem#1: The impulse response of a linear time-invariant system is given by:

$h(t) = e^{(-2piBt)}u(t)$

a. Is this system causal? Explain

b. Is this system stable? Explain

c. Find ∫0-5 $h(t) \cdot s(t-1)$ dt

a) The impulse response h(t), is indeed causal. Let me explain why:

In the given impulse response:

- e^(-2piBt) is a decaying exponential term.
- u(t)$u(t)$ is the unit step function, which is zero for $t < 0$ and one for $t \geq 0$.

Since the impulse response is zero for negative times and only involves terms that decay with increasing time, the system described by this impulse response is causal. The output at any given time is determined only by the past and present values of the input, satisfying the causality condition.

b) For a system to be considered stable, its impulse response should have a bounded response as time t goes to infinity. In the case of the given impulse response. based on the provided impulse response, the system is stable. The exponential decay ensures that the system response remains bounded as time progresses.

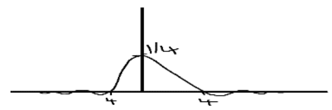
c)

$$\text{Remember} \int_{t_1}^{t_2} \delta(t-t_0)\, x(t)\, dt = \overbrace{\begin{cases} \overset{1}{x(t_0)} & t_1 < t_0 < t_2 \\ 0 & \text{o.w} \end{cases}}^{1 \cdot 2}$$

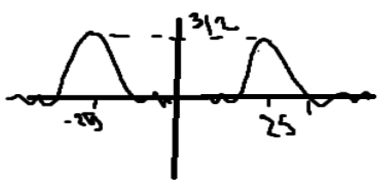
Hence, expiration = h(1) = e^(-2piB)u(1) = e^(-2piB) @ t>1

Problem #2: Evaluate and plot the spectrum of the following signals

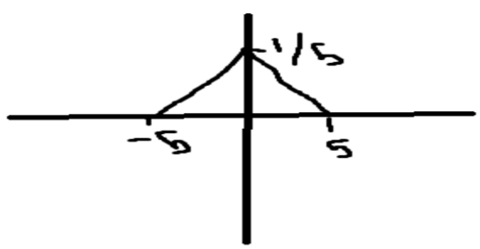
a) x1(t) = Λ(4t)
x1(f) = ¼ sinc^2(f/4)



b) x2(t) = Λ(t/3)cos(5opit)
x2(f) = 3/2(sinc^2(3(f-25))+ sinc^2(3(f+25)))

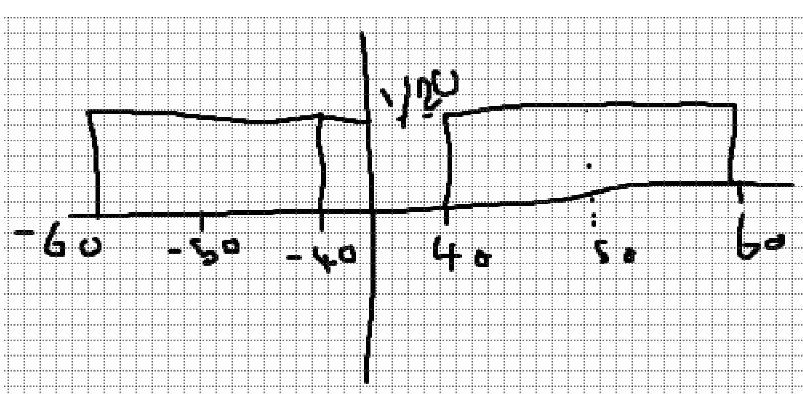


c) x3(t) = sinc^2(5t)
x3(f) = 1/5 Λ(-f/5)



d) x4(t) = sinc(10t)cos(100pit)

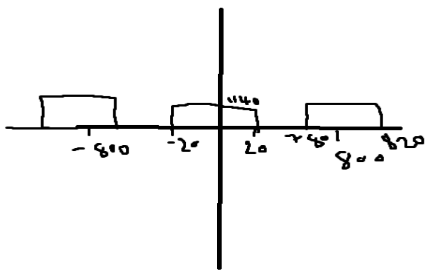
x4(f) = 1/20(pi(-1/10(f-50)) +pi(1/10 (f+50)))



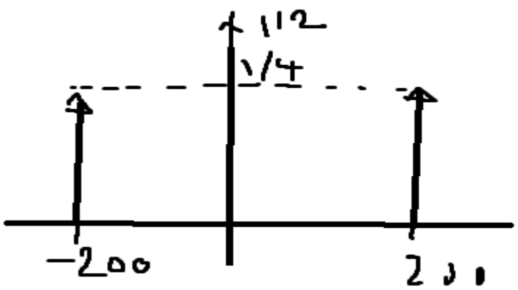
e) $X5(t) = \Pi(20t)cos^2(800pi\ t)$

$X5(t) = 0.5\ \Pi(20t) + 0.5\ \Pi(20t)cos(1600pi\ t)$

$X5(f) = 1/40\ sinc(f/20) + 1/80\ sinc(1/20(f-80)) + 1/80\ sinc(1/20(f+80))$



f) $X6(t) = cos^2(200\ pi\ t)$
$X6(t) = \frac{1}{2} + \frac{1}{2} cos(400\ pi\ t)$
$X6(f) = \frac{1}{2} s(f) + \frac{1}{4}(s(f-200) + s(f-200))$



Problem #3: Evaluate the bandwidth of the following base band signals

a) $X1(t) = \Lambda(4t)$
$X1(f) = \frac{1}{4} sinc^2(f/4)$
BW = 4 Hz

b) $X2(t) = \Pi(t/10)$
$X2(f) = 10\ sinc(10f)$
BW = 1/10 Hz

c) $X3(t) = cos(40\ pi\ t) + cos(60\ pi\ t) + cos(100\ pi\ t)$
$X3(f) = \frac{1}{2} (S(f-20) + S(f+20) + S(f-30) + S(f+30) + S(f-50) + S(f+50))$

BW = 50 – 20 = 30 Hz

Problem #4: Evaluate the bandwidth of the following pass band signals

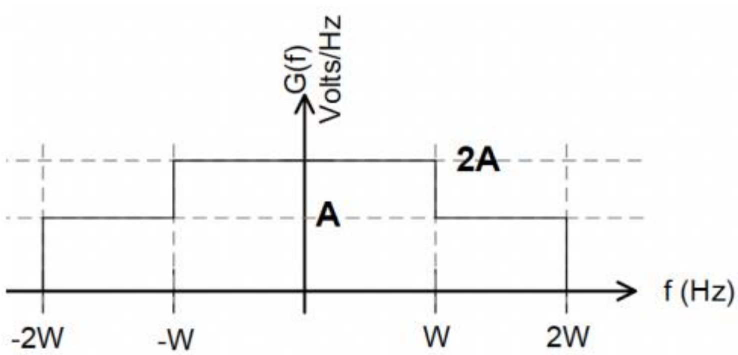
a) $X1(t) = \text{sinc}^2(t/10)\cos(100 \pi t)$
   $X1(f) = 5(\Lambda(10(f-50)) + \Lambda(10(f+50)))$

   BW = 50 Hz
b) $X2(t) = = \text{sinc}(t/10)\cos(100 \pi t)$
   $X2(f) = 5(\Pi(10(f-50)) + \Pi(10(f+50)))$

   BW = 50 Hz

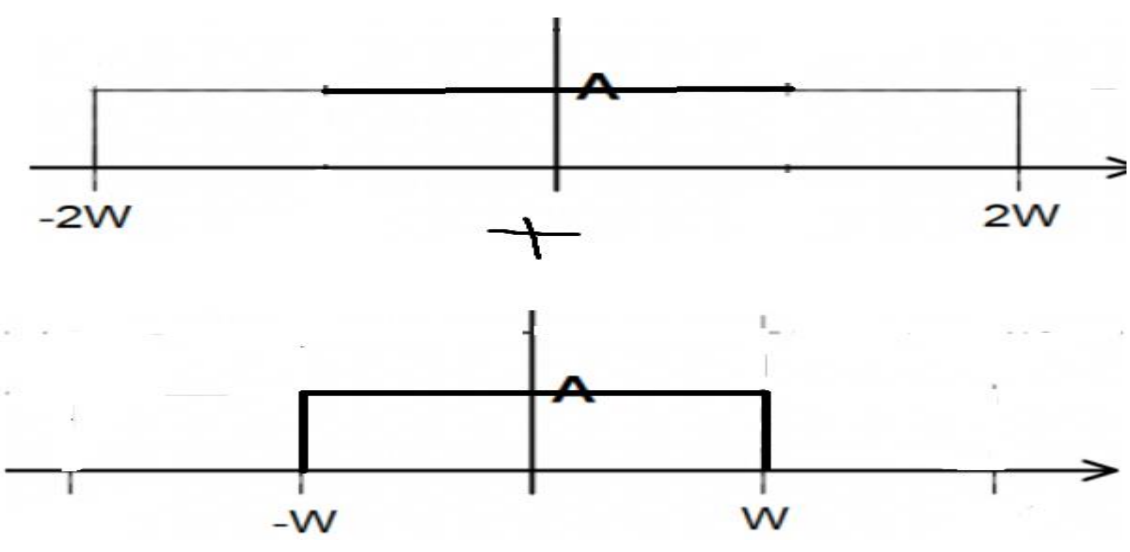
c) $X3(t) = \cos(100 \pi t)\cos(300 \pi t)$
   $X3(f) = ¼ (S(f + 100) + S(f - 100) + S(f + 200) + S(f - 200))$
   BW = 200

Problem #5: The Fourier transform G(f) of a signal g(t) is given as



a) Express the Fourier transform G(f) in terms of singularity functions

$G(f) = A\,\Pi(f/4w) + A\,\Pi(f/2w)$



b) Based on result obtained in part a, find the signal g(t)

g (t) = 4wA sinc(4wt) + 2wA sinc(2wt)

c) Find the absolute bandwidth of g(t)

BW = 1/2w – 1/4w = 1/2w Hz

d) Find the energy in g(t)

$E=\int_{-\infty}^{\infty}|G(f)|^{2}df=\int_{-2W}^{2W}|\ A\,\Pi(f/4w)+A\,\Pi(f/2w)\ |^{2}df = 16\ (A^{2})\ W$

e) If g(t) is passed through an ideal low pass filter with bandwidth 3w/2

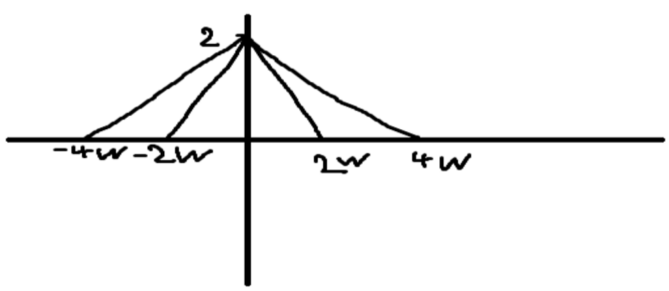
I. Evaluate and plot the output of the filter

$h(t)=2w\,\mathrm{sinc}(2wt)$ → filter impulse response

assume y(t) is the ouput → $y(t)=g(t)*h(t)=\int_{-\infty}^{\infty}g(\tau)\cdot h(t-\tau)d\tau$

$y(t)=\int_{-\infty}^{\infty}[4wA\,\mathrm{sinc}(4w\tau)+2wA\,\mathrm{sinc}(2w\tau)]\cdot[2w\,\mathrm{sinc}(2w(t-\tau))]d\tau$

$y(t)=2A\,\mathrm{rect}(t/4w)+2A\,\mathrm{rect}(t/2w)$



II. Find the energy in the signal at the filter output

$E_{y}=\int_{-\infty}^{\infty}|y(t)|^{2}dt=\int_{-\infty}^{\infty}|2A\,\mathrm{rect}(4wt)+2A\,\mathrm{rect}(2wt)|^{2}dt$

$=E_{y}=(18A^{2})/W$

Problem#6: Consider the periodic signal Xp(t) shown in Fig.2

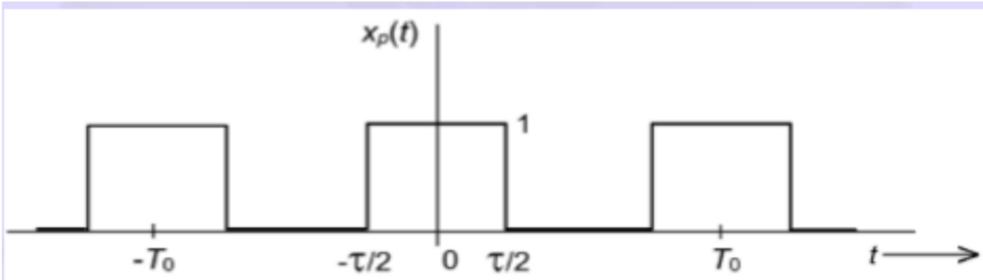


Fig.2

a) Express the signal Xp(t) in terms of pulse function

   Xp(t) = $\sum$ n=-∞ - ∞ (Π(t-nT0/ τ))

b) Specify the types of symmetry

   Even symmetry

c) Evaluate the duty cycle of the periodic signal Xp(t)

   Q = Th/Tt * 100% = ½ * 100% = 50%

d) Evaluate and plot the spectrum of Periodic signal Xp(t)

   @ n = 0

   Xp(f) = τ sinc(τf)

   Hence,

   Xp(f) = $\sum$ n=-∞ - ∞ (τ sinc(f-nτ))

e) Evaluate the Complex Exponential Fourier Series,Xn.

   Xn = 1/T =$\int$−∞ - ∞ (X(t) exp(-j n W0 t) dt)

   Xn = 1/ (2j pi n)

f) Evaluate the Trigonometric Fourier Series.

   an = 2 RE{Xn} = 0

   bn = -2 Im{Xn} = { -1/ (2 pi n) @ n > 0 ; 1/ (2 pi n) @ n < 0) }

   a0 = 1/T0 =$\int$−∞ - ∞ (X(t) dt) = 1/T0 $\int$− τ − τ (1 dt) = (1/T0) (τ) = τ/T0

g) Using Parseval's theorem to evaluate the average power of the periodic signal Xp(t)

   Pav = |X0|^2 + 2 $\sum$ n=-∞ - ∞ (|Xn|^2) = ∞ since periodic

Problem #7: Consider the differential equation of the RC circuit shown in Fig. 3 is
RC dy(t)/dt + y(t) = x(t)

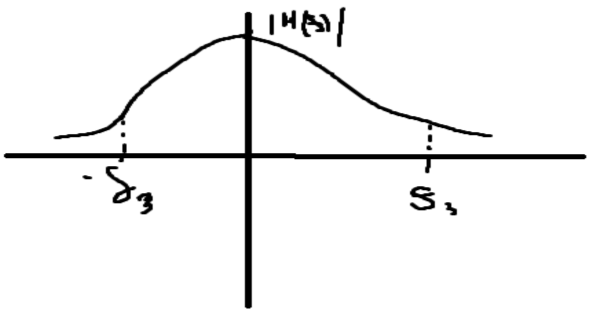
a) Evaluate the Frequency response of the system, H(f)
R C j 2 pi f + y(f) = x(f) → H(f) = y(f)/x(f) →H(f) = 1/(1 + j 2 pi f R C)

b) Using inverse Fourier Transform to evaluate the impulse response, h(t)

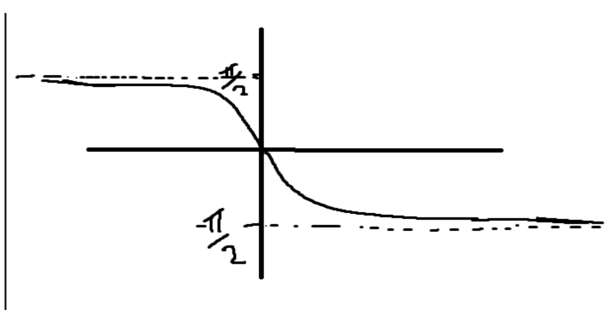
h(t) = F^-1(H(f)) →h(t) = 1/RC exp(-t/RC) u(t)

c) Evaluate and plot the amplitude and phase of the frequency response of the system, H(f)

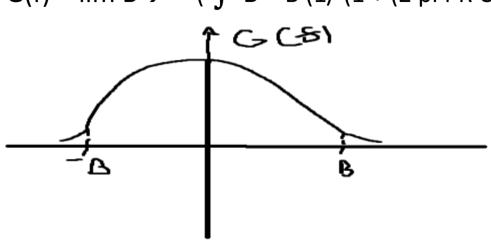
|H(f)| = 1/(1 + (2 pi f R C)^2)^1/2



phase(H(f)) = -tan^-1(2 pi f R C) = tan-1 (f/f3)



d) Evaluate and plot the energy spectral density of the system.
G(f) = |H(f)|^2 = 1/ (1 + (2 f pi R C)^2)
G(f) = lim B→ ∞(∫–B – B (1/ (1 + (2 pi f R C)^2)))



e) Evaluate the 3-dB bandwidth of the system

BW = 1/ (2 pi R C)

f) Based on result obtained in part e. Specify the values of R, and C elements to cover signals in bandwidth 5kHz.

Assuming C = 1 u f

R = 1 / (2 pi BW c)

➔ R = 100 ohm

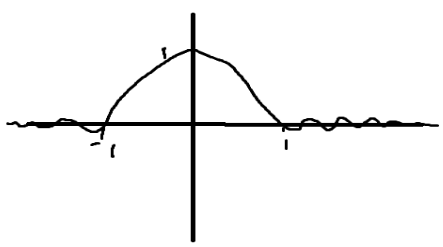
Problem #8: Given the signals:

X(t) = Π(t – 0.5) and

h (t) = $\sum$ n=-∞ - ∞ (S(t – 2n))

a) Evaluate and plot the spectrum of x(t), and h(t)

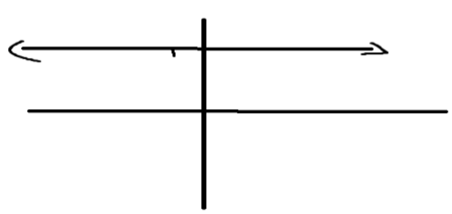
X(f) = sinc(f) exp(-j 2 pi f 0.5)



@ n = 0

h(t) = S(t)

h(f) = f S(f) = 1



b) Evaluate the complex exponential Fourier series of signal h(t)

Xn = 1/T0 $\int_{-\infty - \infty}$ (X(t) exp(- j n W0 t) dt)

= S(f – 2n)/ (j 2 pi n)

c) Evaluate and plot the spectrum of the signal y(t) = x(t) $*$ h(t)
Since h(t) is the train of delta

y(t) = $\sum_{n=-\infty - \infty}$ Π(t – 0.5 – 2n)

d) Based on result obtained in b. Evaluate the signal y(t) in time domain

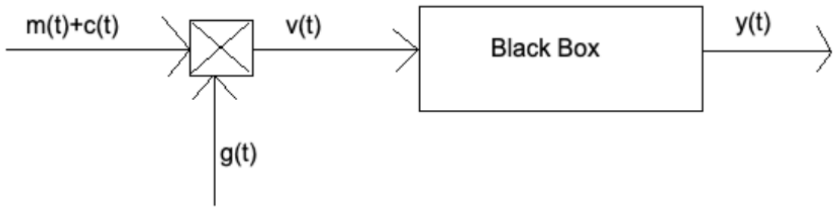
y(t) = $\sum_{n=-\infty - \infty}$ (Xn exp(j2 pi n f0 t)) =
$\sum_{n=-\infty - \infty}$ (0.5 * exp(-j 2 pi n) * exp(j 2 pi n f0 t))
= $\sum_{n=-\infty - \infty}$ 0.5
= 0.5

➔ **y(t) = 0.5**

Problem #9: Consider the following modulator system shown in Fig.4. Assume the signals

$$c(t) = A_c \cos(2\pi f_c t), \text{ and } g(t) = \begin{cases} 1, & -\frac{T_c}{4} \leq t < \frac{T_c}{4} \\ 0, & \frac{T_c}{4} \leq t < \frac{3T_c}{4} \end{cases}$$

Where $g(t + T_c) = g(t)$



a) Represent g(t) in terms of the trigonometric Fourier Series signals

a0 = 1/T0 =∫−∞ - ∞ (X(t) dt) = 1/T0 (∫−Tc/4 – Tc/4 (1 dt) + ∫Tc/4 – 3Tc/4 (0 dt))

= t |-Tc/4 → Tc/4 = 2 * Tc/4 = Tc/2

Hence, a0 = (Tc/(2T0))

an = 2/T0 ∫−Tc/4 – Tc/4 (1 cos(n W0 t)df) = 1/(pi n) sin(n W0 Tc/4)

bn = 0 since the function is even

b. Implement a suitable system inside black box to get a signal $y(t) = [1 + K_a\, m(t)] \cos(2\pi f_c t)$. Specify the value of constant k_a

Amplitude Modulator (Multiplier): Multiply the message signal $m(t)$ by the carrier signal $c(t)$.

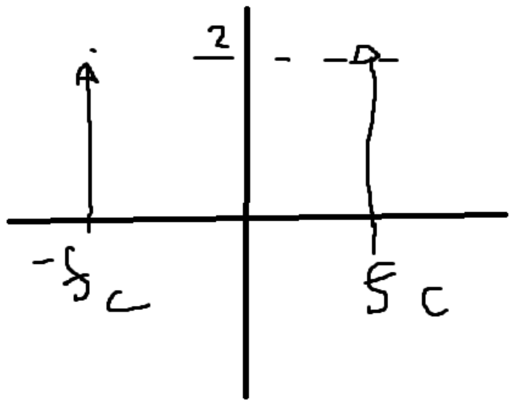
$u(t)=m(t)\cdot c(t)$

Adder: Add 1 to the result of the multiplication.

$V(t)=u(t)+1$

Ka = 1/Ac

c) Plot the spectrum of the signal y(t)



d) Evaluate the average power of the signal y(t)

Pav = (X^2)/2

➔ **Pav = (| 1 + Ka m(t) |^2)/ ((1 + 1)^2)/2**

➔ **= 2 watt**