

## Flexible Strategy Use in Young Children's Tic-Tac-Toe

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In domains with multiple competing goals, people face a basic challenge: How to make their strategy use flexible enough to deal with shifting circumstances without losing track of their overall objectives. This article examines how young children meet this challenge in one such domain, tic-tac-toe. Experiment 1 provides an overview of development in the area; it indicates that children's tic-tac-toe strategies are rule based and that new rules are added one at a time. Experiment 2 demonstrates that even young children flexibly tailor their strategy use to meet shifting circumstances. Experiment 3 indicates that these adaptations are accomplished through a process of goal-based resource allocation, whereby children focus their cognitive resources on applying rules most consistent with their current primary goal. A computer simulation specifies how this process works and demonstrates its sufficiency for producing behavior much like that of the children. The findings are discussed as part of a broader framework of mechanisms for generating problem-solving approaches.

Individual children usually are depicted as using a single strategy to perform a given task. However, studies in such diverse domains as arithmetic, language development, causal reasoning, problem solving, and serial recall have shown that, in fact, they often use multiple strategies (Bisanz & LeFevre, 1990; Geary & Burlingham-DuBree, 1989; Goldman, Pellegrino, & Mertz, 1988; Kuczaj, 1977; Maratsos, 1983; McGilly & Siegler, 1989; Shultz, Fisher, Pratt, & Rulf, 1986; Siegler & Shrager, 1984). This use of

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multiple approaches is not just a curious, incidental feature of their performance; it helps them adapt to varying demands of problems and situations. For example, even 4- and 5-year-olds choose in an adaptive way whether to solve an arithmetic problem by stating a retrieved answer or by using a backup strategy such as counting their fingers. They usually state retrieved answers on easy problems, where retrieval yields accurate as well as fast performance, and rely predominantly on the slower backup strategies on difficult problems, where such approaches are necessary for accurate performance.

Although these studies have advanced our knowledge of how children choose strategies, they generally have been limited to tasks with a single, unchanging goal: to add the numbers, to remember the list, to identify the cause, to choose the word, and so on. These types of tasks, however, represent only a subset of the problems that people must solve. We also engage in tasks where problem solving entails multiple goals, and where changing problems and situations create competition among the goals.

Domains with multiple, potentially conflicting goals are often those where an individual's actions are contingent on the actions of other people: driving in traffic, speculating on the stock exchange, or playing chess, for example. As in domains in which strategy choices have been studied previously, people operating in these domains might adopt singular goals such as getting home as quickly as possible, maximizing the expected value of investments, or not losing the game. But, unlike previously studied domains, these task environments are not solely under the individual's control. The shortest route may be clogged with other drivers. The most promising stocks may have been bid up so high that they carry unacceptable risk. Even a strong chess opponent may blunder, thus creating opportunities to win rather than simply not losing. In these situations, pursuing an unchanging goal, regardless of the actions of others, may at best make it difficult to achieve the goal, and at worse may result in delays, bankruptcy, or missed opportunities.

Adaptive strategy use in such domains requires that individuals change goals and strategies flexibly enough to adjust to shifting circumstances. This flexibility cannot be unlimited, however. Along with adjusting to changing circumstances, people must also avoid losing track of their overall objectives. If a commuter's goal is to get home as quickly as possible and he or she comes upon a parkway choked with traffic, the commuter may exit at the first opportunity and take secondary roads home. This may be an adaptive response to the immediate obstacle of the traffic jam, but may not be adaptive in terms of the main goal of getting home as soon as possible. The secondary roads may be so much slower that the trip will take longer than if the commuter had stayed on the congested parkway. In such situations, rather than allowing the immediate need for flexibility to dominate, adaptive strategy use requires people to balance flexible responding to immediate circumstances with stable pursuit of long-term goals.

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Our purpose in this study is to extend research on children's strategy selection to domains with multiple, shifting, and potentially conflicting goals. The domain that we examine is children's tic-tac-toe. We are specifically interested in how children make their strategy use flexible enough to adapt to changing situations yet stable enough to satisfy longer term goals.

### A GAME OF COMPETING GOALS

Tic-tac-toe is a simple children's game in which two players take turns drawing tokens (X's or O's) on a  $3 \times 3$  grid. Winning involves a player placing three tokens in a row, column, or diagonal. The activity is related to many three-in-a-row children's games worldwide and is an indirect descendent of games played by children in ancient Egypt (Zaslavsky, 1982).<sup>1</sup>

Tic-tac-toe is of interest because it is an elegant example of a domain where an individual's goals potentially conflict. As with many two-person games, the competing goals are those of winning and not losing. Because focusing on one can lead to neglect of the other, the tension between these goals is high, and the need to simultaneously satisfy both is paramount.

Adults find this potential conflict trivial. The optimal tic-tac-toe strategy is quite simple. It insures a draw, while at the same time maximizing the chance that an opponent's mistake could yield a win. To the extent that players follow such an expert strategy, they minimize the need to respond to situational variation because they are assured of at least a draw regardless of their opponent's skill.

Observing children playing tic-tac-toe, however, quickly demonstrates that the seamless and stable goal integration adults take for granted was not always so seamless or stable. Consider the behavior of one 5-year-old girl, a pilot subject in Experiment 2, who played a series of games against a computer program.

The girl began the first game by stating that she was trying to win. Diligently attempting to put three X's in a row, she failed to notice the program's progress towards a win, missed a blocking opportunity, and lost the first game. She exclaimed her displeasure and added, "Next game, I'm not going to let that computer beat me!" In the next game she went on the defensive, explaining all of her moves in terms of thwarting her opponent. Two examples of her explanations were, "If it goes down the middle, I'll block it like this," and "I'll stop it here, so it can't win." This defensive stance allowed the girl to play the program to a draw. Satisfied, she switched back to an offensive

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<sup>1</sup> According to the *Oxford English Dictionary* (1989), the name tic-tac-toe (originally tick-tack-toe) first appeared in the mid 1800s as the name French schoolchildren gave to a game in which they dropped pencils onto a slate. 'Tick-tack' is supposed to be the sound of one pencil dropping.

mode for the next game, predicting, "I'm going to find a way to beat the computer this time." Instead, neglecting defense once more, she lost. She responded to this loss by trying not to lose the next game. This alternation between offensive and defensive focus continued for most of the 16-game session.

The girl's losses were not caused by her not knowing how to block; when operating with a defensive focus, she blocked quite consistently. Nor did the neglect come from indifference to losing; she was visibly perturbed when the computer won. Instead, the missed blocks seemed the consequence of adopting an offensive goal, concentrating on opportunities to win, and failing to use her blocking skill. Although her example was particularly dramatic, it was not isolated. Other pilot subjects showed evidence of focusing on a single goal and suffering similar consequences.

Children's difficulty in simultaneously satisfying offensive and defensive goals demands that they act strategically to take advantage of transitory factors such as the skill of their opponent, whether winning or not losing is more important to them in the particular situation, and whether they move first or second in the particular game. They cannot afford the expert's mundane stability because they do not know any approach that guarantees an acceptable outcome. In short, children's limited skill is what makes tic-tac-toe a game for them.

We conducted three experiments to determine how children adapt their strategy use in tic-tac-toe to meet competing goals. In Experiment 1, we explored the developmental sequence of tic-tac-toe strategies. In Experiment 2, we investigated children's flexibility in modifying strategies in response to different goals and situational demands. In Experiment 3, we distinguished between two mechanisms children might use to achieve this flexibility. Finally, we wrote a computer simulation to provide a detailed demonstration of how the mechanism suggested by the results of Experiment 3 simultaneously generates flexibility and adherence to long-term goals.

### EXPERIMENT 1

Two previous studies have examined children's tic-tac-toe. Sutton-Smith and Roberts (1967) found improvements with age in skill at the game, especially between third and sixth grades. They interpreted the improvement in terms of development of general strategic competence. Spitz and Winters (1977) found that the tic-tac-toe playing of retarded children lagged behind that of mental age peers of typical IQ. They concluded that mental age assessments do not fully capture the differences in problem solving that separate retarded and average IQ groups. Neither of these investigations, however, analyzed children's strategies in much detail. Therefore, the main goal of Experiment 1 was to identify the developmental sequence of strategies leading to mastery of tic-tac-toe.

Tic-tac-toe strategies have two components: the set of rules that direct action and the method used to select which rule to apply.<sup>2</sup> Table 1 describes, within a production system framework, the rules postulated to comprise an expert's knowledge of tic-tac-toe. It is an elaboration of Newell and Simon's (1972) model. The one important difference between it and Newell and Simon's formulation is that the current model includes a Block Fork Rule, which together with the other rules, enables it to win or draw in all cases.

The rules in Table 1 represent all of the knowledge needed to play tic-tac-toe flawlessly. However, they yield flawless performance only when paired with an effective means for selecting which rule to apply. Some of this selection is accomplished in matching the condition sides of rules to the working memory representation of the current state of the board. If a rule's conditions are not met, it does not enter into the competition for application on that cycle. But, as illustrated in Figure 1, the state of a tic-tac-toe board often matches the conditions of more than one rule. Such situations necessitate the second component of a tic-tac-toe strategy: a *conflict resolution procedure* for choosing among the satisfied rules.

The method of conflict resolution used in Newell and Simon's (1972) tic-tac-toe program, and the one adopted here, is *rule ordering*. This is a simple yet powerful procedure in which rules are ordered into a hierarchy and then considered in that hierarchical order. The first rule whose conditions are matched fires.

The success of rule ordering as a conflict resolution method depends on the order of rules within the hierarchy. To achieve expert tic-tac-toe play, the hierarchy must integrate rules that respond to offensive and defensive goals. One way to achieve this integration is to construct the hierarchy according to the depth into the game at which each rule pays off. Consider the first four rules in Table 1. Each rule achieves its payoff one move deeper into the game than the previous rule. First, the player evaluates the conditions of Win, a rule with an immediate pay-off. If a winning move is possible, then it is made. If not, the player evaluates the next rule, Block, which has its effect of preventing a loss one move after it is made. If a block is not possible, the player then seeks to use the Fork Rule, which results in a win two moves after it is made. If no fork can be created, the player attempts to use the Block Fork rule, which precludes the opponent from setting up a fork on the opponent's next move, thus avoiding a potential loss three moves in the future. Rather than focusing exclusively on offensive or defensive

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<sup>2</sup> Currently, there is little consensus about the most useful definition of "strategy." For example, in a recent volume on children's strategy use (Bjorklund, 1990), some contributors defined only conscious, planful processes as strategic; others defined strategies as only those processes that streamline performance; still others considered any goal-directed, nonobligatory cognitive activity—conscious or unconscious, efficient or inefficient—as a strategy. In the absence of a compelling reason to adopt one of the more restrictive definitions, we have adopted the final, most inclusive, meaning of "strategy."

TABLE 1  
Model of Expert Performance

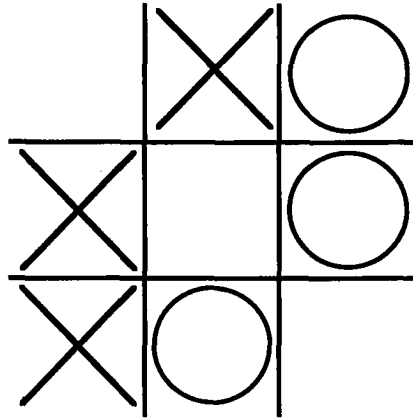
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Win	<p><i>If there is a row, column, or diagonal with two of my pieces and a blank space,</i>  <i>Then play the blank space (thus winning the game).</i></p>
Block	<p><i>If there is a row, column, or diagonal with two of my opponent's pieces and a blank space,</i>  <i>Then play the blank space (thus blocking a potential win for my opponent).</i></p>
Fork	<p><i>If there are two intersecting rows, columns, or diagonals with one of my pieces and two blanks, and</i>  <i>If the intersecting space is empty,</i>  <i>Then move to the intersecting space (thus creating two ways to win on my next turn).</i></p>
Block Fork	<p><i>If there are two intersecting rows, columns, or diagonals with one of my opponent's pieces and two blanks, and</i>  <i>If the intersecting space is empty,</i>  <i>Then</i>  <p style="padding-left: 40px;"><i>If there is an empty location that creates a two-in-a-row for me (thus forcing my opponent to block rather than fork),</i>  <i>Then move to the location.</i>  <i>Else move to the intersection space (thus occupying the location that my opponent could use to fork).</i></p> </p>
Play Center	<p><i>If the center is blank,</i>  <i>Then play the center.</i></p>
Play Opposite Corner	<p><i>If my opponent is in a corner, and</i>  <i>If the opposite corner is empty,</i>  <i>Then play the opposite corner.</i></p>
Play Empty Corner	<p><i>If there is an empty corner,</i>  <i>Then move to an empty corner.</i></p>
Play Empty Side	<p><i>If there is an empty side,</i>  <i>Then move to an empty side.</i></p>

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goals, this depth-based hierarchy alternates between considering rules oriented towards offensive and defensive goals, thus integrating them.

We hypothesize that expert tic-tac-toe players use such a depth-based conflict resolution method to decide among applicable rules. It seems to be the only simple conflict resolution method that invariably leads to the optimal move. The Appendix indicates that a computer simulation that utilizes this conflict resolution method produces perfect performance, far better than simulations that use five alternative schemes for resolving conflicts among



**Figure 1.** It is X's turn in this partially played tic-tac-toe game. The configuration matches the conditions of four different rules: Win (a move to the top right); Block (a move to the bottom left); Play Center (a move to the middle); and Play Empty Corner (a move to either empty corner). The problem faced by the X's is to choose which rule to apply and how to apply it.

applicable rules. Thus, the Table 1 rules and the rule-ordering conflict resolution method constituted our model of the end state of tic-tac-toe knowledge.

In Experiment 1, we examined how children progress toward this end state. In particular, we used a rule-assessment approach to examine acquisition of the four tic-tac-toe rules specifically aimed at winning or not losing: Win, Block, Fork, and Block Fork. If tic-tac-toe strategies reflect a collection of independent rules, as depicted in Table 1, we would expect the strategies to develop one rule at a time. We also would expect that rules requiring less extensive search of potential future configurations should be acquired before rules requiring more extensive search. One reason for this prediction is that more extensive searches place greater demands on working memory. Another reason is that the rules that require deeper search often demand knowledge of the rules that require less search. For example, setting up a fork presupposes that you can fire the Win Rule on your next turn to complete the forking maneuver. Thus, we expected children to acquire rules in the order: Win, Block, Fork, Block Fork.

## Method

**Participants.** Participants were 17 Carnegie Mellon undergraduates and three groups of children from a suburban Pittsburgh public school: 20 kindergartners ( $M = 73$  months); 20 first graders ( $M = 85$  months); and 20 third graders ( $M = 108$  months). An additional 6 kindergartners and 3 first graders failed to show any knowledge of tic-tac-toe in a practice game; they were excluded from the remainder of the experiment.

**Materials.** Subjects were presented partially played games of tic-tac-toe and asked to make the next move. Four types of games were constructed: games in which the best available move would produce a win, a block, a fork, or blocking of a potential fork, respectively. Two versions of each of the four types of games were constructed, with the correct move in one instance being to a corner and in the other to a side (Figure 2). Each of these eight versions was presented in each of the four possible 90° rotations, resulting in a total of 32 games. Games were presented on paper and were ordered so that a game of each type was in each successive set of four.

**Procedure.** Subjects participated individually in two sessions, playing 16 games of one-move tic-tac-toe in each. They were told they would be playing a game like tic-tac-toe except that some X's and O's were already filled in; their job was to make the next move for the X's. Following a practice problem, 16 one-move games were presented. After each, the experimenter asked the subject to explain why she had chosen that move. Several days later, subjects had a second 16-game session. Each session lasted approximately 20 min and was videotaped.


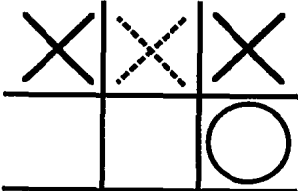

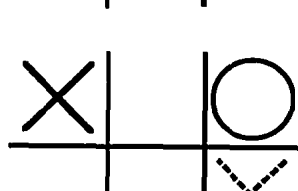

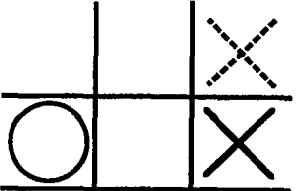
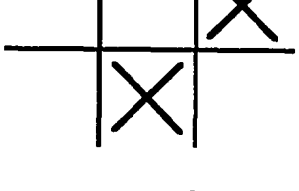
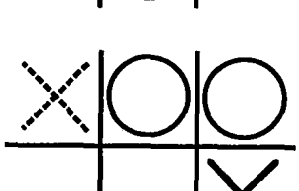
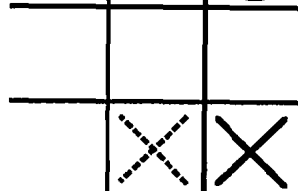

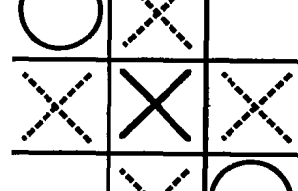

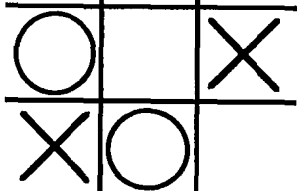
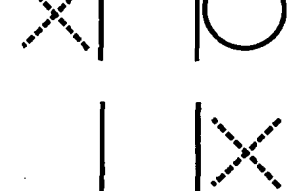
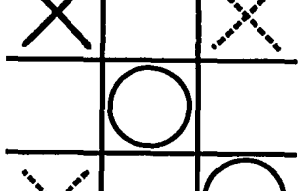

### Results and Discussion

We first present summary data on the frequency with which different-age subjects used each tic-tac-toe rule. Then we examine the sequence in which the rules were acquired. Because subjects could make a correct move without using the rule of interest (e.g., by guessing), they were scored as having used a rule only when they (a) made the move the rule predicted, and (b) explained the move in accord with the rule. Reliability for scoring of the verbalizations was checked by having a second rater score 100 trials from randomly selected subjects. Agreement between the two raters was 95%.

Table 2 indicates the percentage of subjects of each age who used each rule at least once. As shown, most kindergartners knew how to win, and almost half could block. Every first grader could win, almost all could block, and a few could set up forks. All third graders could win and block, and most could set up forks. Undergraduates were the best tic-tac-toe players, but were not perfect. Almost 20% showed no evidence of being able to set up forks, and 35% showed no evidence of knowing how to block potential forks.

Table 2 strongly suggests that the rules emerged one-by-one, rather than multiple rules typically emerging together. The kindergartners' performance showed that acquiring the Win Rule did not imply acquiring any other rule. The first graders' data indicated that acquiring the Block Rule did not imply acquiring the Fork or the Block Fork Rules. The third graders' data demonstrated that acquiring the Fork Rule did not imply acquiring the Block Fork Rule.



Win	Block	Fork	Block Fork
			
			
			
			

**Figure 2.** The 8 one-move games in one of their four rotations. Correct moves are indicated by the dashed X's (multiple X's indicate multiple correct moves). See Table 1 for explanations of the rules underlying correct moves.

TABLE 2  
Percentage of Subjects of Each Age Who Made at Least One Correct Move  
and Provided a Consistent Explanation for Each Tic-Tac-Toe Rule

Group	Win	Type of Game		
		Block	Fork	Block Fork
Kindergartners	95	45	0	0
First graders	100	80	10	0
Third graders	100	100	65	0
Adults	100	100	82	65

We had predicted that subjects would acquire the rules in an order that paralleled the depth-based hierarchy experts use for selecting individual moves. This implied that subjects should not use a rule without already knowing those above it in the hierarchy. For example, subjects should not know how to block before knowing how to win.

To test this prediction, we tabulated the number of subjects whose rule use conformed to the hypothesized developmental sequence. We adopted the strict criterion that to be counted as conforming to the sequence, subjects must have full knowledge of all rules above the lowest rule in the hierarchy about which they showed any knowledge. Subjects were classified as having: (a) full knowledge of a rule, if they used it in at least seven of the eight games where it was optimal; (b) some knowledge of a rule, if they used it in one to six of the eight games where it was optimal; and (c) no knowledge of a rule, if they never used it.

Consistent with our prediction, 83% of subjects (72–90% of each age group) gave evidence of acquiring the four rules in the order of the proposed hierarchy (Table 3), never using rules lower in the hierarchy without showing full knowledge of all higher rules. Only 3% of subjects showed full knowledge of a rule lower in the hierarchy before full knowledge of a rule higher in it. (The remaining 14% of subjects showed partial knowledge of the two rules that they used that were lowest in the hierarchy, which seemed neither supportive of, nor in conflict with, the proposed sequence.)

Table 3 also indicates that subjects often used their most advanced rules sporadically. Half used their most advanced rule in one to six of the eight trials. All levels of inconsistent use were observed fairly frequently. Aggregating across age groups, 23% of these partial uses of a subject's most advanced rule involved one use (in the eight trials), 19% two uses, 9% three, 21% four, 9% five, and 19% six. This inconsistent use of the most advanced rule was as characteristic of adults as of children; 50% of adults and 50% of children who fit the proposed rule hierarchy used their most advanced rule inconsistently (Table 3).

In summary, the results of Experiment 1 indicated that tic-tac-toe strategy development involves one-by-one acquisition of a hierarchy of rules. The

TABLE 3  
 Percentage of Subjects Whose Rule Use Was Consistent  
 With Each State Within the Predicted Developmental Sequence

Predicted Orderings	Kinder- garten	First Grade	Third Grade	Adults
1) Some Win	10	5	—	—
1) Win	45	15	—	—
1) Win				
2) Some Block	20	30	5	—
1) Win				
2) Block	5	35	25	12
1) Win				
2) Block				
3) Some Fork	—	5	55	6
1) Win				
2) Block				
3) Fork	—	—	5	18
1) Win				
2) Block				
3) Fork				
4) Some Block Fork	—	—	—	18
1) Win				
2) Block				
3) Fork				
4) Block Fork	—	—	—	18
Total % Fitting Predictions	80	90	90	72

order of acquisition generally conforms to a depth-based sequence: first winning, then blocking, then forking, then blocking potential forks. Rules are often used sporadically at first, but become stable with increased knowledge of the domain.

## EXPERIMENT 2

Experiment 2 was conducted to explore systematically the phenomenon we observed in the aforementioned pilot subject: that young children sometimes focus on offensive goals and other times on defensive goals, with corresponding decrements in attention to the other type of goal. As noted earlier, advanced players have no need to shift their focus in this way. The expert strategy integrates the subgoals of winning and not losing to always identify the best move. But what about novices? Experiment 1 indicated that 5- and 6-year-olds possess only the beginnings of the rule hierarchy. Many applied even the most basic rules inconsistently. Until they develop a complete rule

hierarchy that they systematically search, children cannot be assured of winning or drawing. Instead, they may note situational variation, focus on a particular goal, and adapt their choices among rules to meet that goal.

In Experiment 2, we examined two variables that might affect children's goals and thus their tic-tac-toe playing. One variable, arising from the nature of the game, was which player moves first. Whoever moves first in tic-tac-toe always makes as many moves as, or more moves than, the opponent. Consequently, that player often encounters more opportunities to win. For the same reason, whoever moves second usually encounters more situations that demand a block. If children respond adaptively to these pressures, they should focus on offensive goals when they move first and defensive goals when they move second. This analysis led us to predict that children would respond correctly to a higher percentage of opportunities to win when they went first and to a higher percentage of potential blocks when they went second.

We also attempted to manipulate children's goals experimentally, through instructions and rewards that encouraged either an offensive or a defensive focus. This manipulation was intended to determine whether 5- and 6-year-olds could flexibly adapt their strategies to pressures separate from those imposed by the inherent structure of the game. In everyday games of tic-tac-toe, such adaptations are important in helping children adjust to transitory factors, such as the skill level of particular opponents or developments within a particular game. Again, we predicted that children's rule use would be differentially affected by the goals they were encouraged to adopt. Encouragement to focus on offense should lead to detection of a higher percentage of potential wins, and encouragement to focus on defense should lead to detection of a higher percentage of potential blocks.

A further aim of Experiment 2 was to study children's tic-tac-toe strategy use in a more natural context than that of Experiment 1. Children in the earlier experiment had not participated in the development of the board. They also knew they would not finish the game. Both of these factors might have led them to use different strategies than if they were involved in standard games of tic-tac-toe. To provide a more natural context, we had children in Experiment 2 play games of tic-tac-toe from beginning to end against a computer opponent.

Could games against a computer program approximate a natural situation? Our observations suggested that they could. The children acted as if the computer were an independent entity with its own personality and intelligence. The previously mentioned pilot subject provides one example of children's emotional and intellectual investment in the interaction. Another example was provided by a 5-year-old girl playing against the computer, who would only whisper as she responded to the experimenter's questions. After the session was over, the experimenter asked why: The girl said she was afraid the computer might overhear her plans. Others have made similar

observations about child-computer interactions (Lepper & Chabay, 1988; Turkle, 1984).

## Method

*Participants.* Subjects were 45 kindergartners ( $M = 72$  months) from a suburban Pittsburgh school. Two other children played a practice game but did not show any knowledge of tic-tac-toe, and therefore did not participate further.

*Materials.* A computer program that ran on a Macintosh Plus was written to play tic-tac-toe. All games began with a blank board on the computer screen. Children made moves by using the mouse to position the cursor at the desired square and then clicking the mouse button. The program made its move when the experimenter pressed a key. When going first, the program moved to one of the four corners. Its subsequent moves were determined by the following formula: win if possible, block if possible, else move to an arbitrary blank space. When either the child or the program won a game, the winning line of X's or O's flashed three times, in conjunction with three celebratory beeps from the computer.

*Procedure.* Children were assigned randomly to one of three groups: offensive focus, defensive focus, or control. One child's data had to be discarded due to experimenter error, leaving 15 children in the offensive focus group, 14 in the defensive focus group, and 15 in the control group. Children participated individually in two sessions, playing eight games in each.

All children began the first session by practicing using the mouse as an input device. After achieving sufficient skill with it and playing a practice game, children received instructions according to the group to which they had been assigned. Children in the offensive focus group were told that they should try hard to win and that for each win they would get a sticker. Children in the defensive focus group were told that they should try hard not to lose and that they would get stickers each time they did not lose. Children in the control group were told only that they should try hard and that they would get some stickers after finishing.

Children then played an initial set of eight games against the computer. They went first on even-numbered games, and second on odd-numbered ones. They played a second set of eight games within a few days. Each session lasted approximately 15 min and was videotaped.

## Results and Discussion

The main measures in Experiment 2 were percentages of games won, lost, and tied, and percentage of application of each rule. Because the emphasis

in this experiment was on making the procedure as natural as possible, we did not obtain explanations after each move; to have done so would have disrupted the flow of the game.

The kindergartners beat the computer in 1% of games, lost to it in 52%, and played it to a draw in 47%. Only 14 of the 44 children ever won, and 12 of these 14 won just once. Given the low number of wins, no analyses could be performed on the circumstances associated with winning.

In contrast to the low incidence of wins, every child lost at least twice. A 3 (Instructions: offensive focus, defensive focus, or control)  $\times$  2 (Who moved first: child or computer) repeated-measures analysis of variance (ANOVA) yielded significant main effects for instructions,  $F(2, 41) = 3.86$ ,  $p < .05$ , and for whether the child or the computer moved first,  $F(1, 41) = 15.07$ ,  $p < .001$ . Among the three experimental groups, defensive focus children lost 40% of games, control group children 51%, and offensive focus children 64%. Children in all groups lost more often when they went second than when they went first (58% vs. 45% losses).<sup>3</sup>

These results indicate that children's overall performance is affected by situational variation, but they do not provide specific information about the modifications to rule use that underlie the differences in outcomes. To learn about these modifications, we analyzed use of the rule most directly related to avoiding losses, the Block Rule.

Children in all three groups encountered similar percentages of games (85% to 88%) where they needed to apply the Block Rule at least once to avoid losing. We expressed each child's success in these games as a blocking score, calculated as the number of games where children blocked at the first opportunity, divided by the number of games where children had an opportunity to block. Performance when the first blocking opportunity arose on the ninth move of the game was excluded because blocking was unavoidable in these games.

Because the first blocking opportunity occurred on different moves in different games, the chance probability of blocking varied across games. To take account of these differing chance probabilities, we adjusted children's blocking scores as follows:

$$\text{adjusted blocking score} = \frac{\text{percentage blocks observed} - \text{percentage blocks expected by chance}}{100 - \text{percentage blocks expected by chance}}$$

where percentage blocks expected by chance for a given first opportunity to block was 100 divided by the number of open spaces on the board when that first blocking opportunity arose. An adjusted blocking score of 10 indicated

<sup>3</sup> As children rarely beat the computer, the data for tied games are almost exactly the inverse of the data for lost games. For this reason, statistical analyses on percentage of games lost and percentage of games tied yielded virtually identical results. Therefore, we only discuss the analyses for percentage of games lost.

that a child performed no better than chance; an adjusted score of 100 indicated that a child always blocked at the first opportunity.

A 3 (Instructions)  $\times$  2 (Who moved first) repeated-measures ANOVA on the adjusted blocking scores revealed significant main effects for both instructions,  $F(2, 39) = 4.63, p < .05$ , and for who moved first,  $F(1, 39) = 26.12, p < .001$ . Defensive focus children capitalized on the highest percentage of blocking opportunities, followed by control group and offensive focus children (72%, 46%, and 34%, respectively). Children in all groups blocked on a higher percentage of first blocking opportunities when they went second than when they went first (63% vs. 48% blocks).<sup>4</sup>

One aspect of the blocking data might appear at odds with the data on games lost presented earlier. Children blocked on a higher percentage of first blocking opportunities in games where they moved second. On the other hand, they also lost more often when they moved second. If children blocked on a higher percentage of opportunities in these games, why did they also lose more of them?

The answer can be found by looking beyond the initial blocking opportunity and considering the number of moves in each type of game in which blocks were needed. In the 352 games in which children moved first, they encountered 331 configurations where blocks were needed. In the 352 games where they moved second, they encountered 583 such configurations. Thus, although children blocked on a greater percentage of initial opportunities when they went second, they had many more blocks to make as the game progressed, which ultimately resulted in more losses. The difference illustrates why examining children's application of each rule, as well as their overall outcomes, is essential in evaluating strategy use in naturalistic contexts.

Thus, kindergartners adapt their strategy use in response to shifting circumstances. When situational factors supported a defensive focus, children were most likely to apply the Block Rule. When situational factors supported an offensive goal, children were least likely to apply the Block Rule, presumably because they focused on winning. Children showed their adaptations not only to situational factors inherent to tic-tac-toe (moving first or second) but also to factors unique to the particular context (the instructions and rewards).

### EXPERIMENT 3

The flexibility with which kindergartners' goals and rule use respond to changing circumstances at first appears to argue against our hypothesis that they organize rules into the type of inflexible hierarchy depicted in Table 1 and choose among the rules through the conflict resolution method of rule

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<sup>4</sup> The analysis was also conducted without adjusting blocking scores for responses made correctly by chance. The pattern of results was identical.

ordering. If children inflexibly proceed down this hierarchy, how would such flexible rule selection be possible?

One potential solution to this puzzle can be seen by considering how children might determine which rules apply to the current board configuration. Prior discussions of conflict resolution have focused on how selection occurs after the subset of applicable rules has been identified (Anderson, 1983; McDermott & Forgy, 1978; Neches, Langley, & Klahr, 1987). The implicit assumption is that the individual searches the rules and the environment sufficiently to identify all applicable rules. Although the searching of domain experts and AI programs may meet this assumption, the searching of children probably does not. Young children often conduct nonexhaustive and/or inefficient searches, even when the problem involves a single goal in relatively simple comparison, selective attention, and analogical reasoning tasks (Miller, Haynes, DeMarie-Dreblow, & Woody-Ramsey, 1986; Sternberg & Rifkin, 1979; Vurpillot, 1968).

To ensure that an applicable tic-tac-toe rule is applied, a player must search until either the rule has been applied or until all possible opportunities for application have been exhausted. However, children (and adults) do not always apply the tic-tac-toe rules they know. In Experiment 1, we saw that application of subjects' most advanced rule was often sporadic. In Experiment 2, both move order and instructions regarding what was important influenced the consistency with which children applied the Block Rule. It seems likely that the failures to block arose through not searching exhaustively, thus failing to consider all the ways in which the Block Rule could apply.

Children might use either of two distinct approaches to prevent inadequate searches from leading to failures to meet the focused-upon goal. One approach, *subgoal reorganization*, would be to change the order of search for rule-use opportunities, so that rules that could meet the focused-upon goal would be considered first. For example, children focusing on defense might try to apply rules aimed at avoiding losing (Block and, if they know it, Block Fork) before rules aimed at winning (Win and, if they know it, Fork). The flexibility of this scheme is obvious; the rules most likely to meet the current goal will always fire before other rules. But under some circumstances, this flexibility could decrease the probability of achieving even the immediate goal. For example, if children who emphasized defensive goals searched for blocks before wins, they could miss the opportunity to finish the game with an immediate win, thus opening the possibility of losing later. Similarly, if an offensively minded child's rule hierarchy favored forks over blocks, the game might end in defeat before the fork could come to fruition.

A second approach to producing flexibility, *goal-based resource allocation*, would be to maintain the rigid depth-based hierarchy, but to vary the exhaustiveness (and perhaps the carefulness) of searches of open squares for opportunities to apply each rule. Children playing defensively might devote less searching of open squares to looking for wins before going on to look



for blocks. Offensively oriented children might do the opposite. Such goal-based resource allocation would be adaptive, in that it allows some flexibility to adjust to momentary goals but also incorporates the optimal, inflexible rule hierarchy.

Solution time data could discriminate between these two means for attaining flexibility. If children try to apply rules that serve their current goal before ones serving alternative goals, solution times for applying the rules related to their current goal should be shorter. In contrast, if children try to apply rules in the order dictated by a fixed hierarchy, solution times should be shorter for rules higher in the hierarchy, regardless of how well each rule fits the current goal.

To obtain the solution time data that would allow us to discriminate among these alternatives, we used a touch-sensitive screen that enabled children to respond as soon as they decided which move to make. The method used in Experiment 3 also combined desirable elements of the Experiment 1 and 2 approaches. The one-move games in Experiment 1 allowed for a balanced presentation of problems, but at the cost of divorcing games from their familiar context. Children were asked to make a move knowing that no opponent would respond and without having played the game from the beginning. The Experiment 2 procedure overcame these advantages, but at the cost of children encountering different numbers of opportunities to use different rules and encountering the opportunities at different points in the game.

To ensure equal numbers of opportunities to use different rules at the same point in the game, subjects in Experiment 3 were presented equal numbers of partially played games where winning or blocking was possible on the next move. To increase motivation to play as well as possible, children played each game to completion. Although children still needed to enter into a partially played game, this procedure encouraged them to consider their opponent's likely responses when making a move. Children could not afford to ignore defensive goals, for instance, because defeat might soon follow.

## Method

**Participants.** Subjects were 60 first graders ( $M = 84$  months) from a suburban Pittsburgh public school. An additional 7 first graders showed no knowledge of tic-tac-toe in practice games, and were not included as subjects.

**Materials.** A computer program, running on a Macintosh SE, was used to present the experimental stimuli. The Macintosh was equipped with a touch screen, so that children could respond by simply touching the desired square. The tic-tac-toe program recorded each response and marked the time (accurate to 1/60th of a second) at which the response occurred.

All 12 games began with two X's and two O's on the board. In half of the games (the "Win" games), the best move produced victory; in the other half (the "Block" games), it avoided defeat. Within each type of game, correct moves involved corner and side locations equally often; none involved a correct move to the center. Similarly, the correct move in half of each type of game completed a horizontal winning (blocking) line; in the other half, it completed a vertical winning (blocking) line.

After the first move, the children and the program continued playing until one had won or until all spaces were occupied. The program made its moves according to the same win-block-random formula as in Experiment 2.

*Procedure.* Children were randomly assigned to one of three groups: offensive focus, defensive focus, or control. They were shown how to use the touch screen, told that they would be finishing partially completed games, and given a win and a block practice problem. After receiving the same instructions as in Experiment 2, each child played 12 games in a single session against the computer. Before each game, the experimenter asked if the child was ready. When the child was ready, the experimenter hit a key that presented the initial configuration and started a timer. Timing of the move ended when the child touched a square on the screen. The experimenter then asked the child, "Why did you make that move?" After this, the child and the program played the game to completion. Presentation order for the games was randomized separately for each individual. The session lasted approximately 15 min and was audiotaped.

## Results and Discussion

All games opened with five blank spaces on the board, creating a 20% probability that a random choice would be correct on the first move, the move of interest. As in Experiment 1, we controlled for effects of answering correctly by chance by scoring children as using a rule only if they (a) made a move consistent with the rule, and (b) gave an explanation in accord with the rule. Due to equipment failure, the verbalizations of 4 subjects were lost, leaving 19 children in the offensive focus group, 17 in the defensive focus group, and 20 in the control group.<sup>5</sup>

A 3 (Instructions)  $\times$  2 (Relevant rule) repeated-measures ANOVA on percentage of correct rule use revealed no main effects for either variable ( $F_s < 1$ ). However, the interaction between instructions and relevant rule was signifi-

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<sup>5</sup> Two sets of parallel statistical analyses were conducted. One set used the criteria of both a correct move and a correct verbalization to identify rule use and involved the 56 children for whom verbalizations were available. The other set involved all 60 children, with scores being adjusted for chance probability of being correct using the same equation as in Experiment 2. All results were similar for the two sets of analyses. For this reason, and because the use of verbalizations as converging evidence allows for more precise identification of rule use, we present only the analyses that relied on both the move and the verbalization to determine rule use.

cant,  $F(2, 53) = 3.28$ ,  $p < .05$ . Offensive focus children applied with Win Rule more often than the Block Rule: 91% versus 76%. Control children applied the two rules equally, winning on 76% of opportunities and blocking on 75%. Defensive focus children applied the Block Rule more consistently than the Win Rule, succeeding on 70% of possible wins but 85% of blocks.

Thus, in accord with the Experiment 2 results, the rule most directly related to children's current focus was applied more consistently than other rules. The Experiment 3 results also went beyond those from Experiment 2 in indicating that application of the Win Rule as well as the Block Rule was subject to such transitory influences.

The solution time data superficially appeared very different. A 3 (Instructions)  $\times$  2 (Relevant rule) repeated-measures ANOVA on median solution times for correct responses revealed only a main effect for the relevant rule,  $F(1, 48) = 12.10$ ,  $p < .01$ . Children in all groups applied the Win Rule faster than the Block Rule: 2.7 versus 5.5 s for the offensive focus children; 2.9 versus 4.4 s for the control children; and 3.4 versus 4.1 s for the defensive focus children.

These solution time data made it possible to distinguish between the two previously described approaches to attaining flexibility: changing the order in which rules are considered (subgoal reorganization), or maintaining the order but varying the amount of search for configurations matching each rule's conditions of applicability (goal-based resource allocation). If children used the first approach, we would have expected blocks to be faster than wins for the defensive focus group. Instead, children in that group, as in the others, made wins faster than blocks. This pattern suggested that subjects in all groups maintained the rigid hierarchy in which they searched first for potential wins and only then for potential blocks.

Both the accuracy and the solution time data follow from a model in which children attempt to apply rules in a fixed order, and respond to situational variation by adjusting the amount of searching of the board for opportunities to apply each rule. Children focusing on offense would be expected to locate the most wins, because they search the most exhaustively for wins; to locate the fewest blocks, because they search the least exhaustively for blocks; and to locate wins faster than blocks, because they search for wins first. Children focusing on defense would be expected to locate the fewest wins, because they search the least exhaustively for them; to locate the most blocks, because they search the most exhaustively for them; but, like the children with an offensive orientation, to have faster times for wins than for blocks, because they look for wins first. This was exactly the pattern of results that emerged.

In summary, the interpretation that children considered the rules in the fixed order by the depth-based hierarchy, but varied the amount of searching of the board for opportunities to apply each rule, allowed straightforward explanation of the superficially incongruent accuracy and solution time data of Experiment 3, as well as of the Experiment 2 accuracy data.

### A COMPUTER SIMULATION OF CHILDREN'S TIC-TAC-TOE

To test the sufficiency of this interpretation, we constructed a computer simulation of young children's tic-tac-toe. The model was tested against the Experiment 3 findings because that experiment included data on solution times as well as particular moves chosen. Because Experiment 3 only examined use of the Win Rule and the Block Rule, the model contained just three rules: Win, Block, and Random. Thus, it corresponded directly to the level of play of most kindergartners and first graders. The model could easily be extended to older age groups by including the Fork Rule and the Block Fork Rule, but with no data to test the extensions against, including them seemed premature.

The model's basic structure is quite simple. It engages in a self-terminating search for opportunities to apply the Win, Block, and Random Rules in that order. Before each game, the model chooses a search length for the Win Rule and a search length for the Block Rule. The relative search lengths of the two rules reflect the current focus. When the focus is on offense, the Win Rule's search length is longer; when the focus is on defense, the Block Rule's search length is.

These search lengths are important because they influence the likelihood of applying applicable rules. When the simulation examines a given square, it determines whether the rule currently being considered can be applied at that location. If it can, the model moves there. If not, the model must decide whether to try to apply the rule at another location or to switch to considering the next rule in the hierarchy. This decision is made by comparing the rule's current search length to the number of searches that have been made thus far in trying to apply the rule. If the number of prior searches is below the rule's search length, the model continues trying to apply the rule. If not, the model switches to searching for opportunities to apply the next rule in the hierarchy.

Formulating the simulation made clear the necessity of determining how children search open squares on the board. Although our intuition was that the search might be systematically related to the board's layout, the empirical data provided no support for this view. Because corners can be involved in horizontal, vertical, and diagonal wins, and sides only in the first two types, we thought that children might search corners before sides. Presumably, if this were the case, children would have made winning and blocking moves faster to corners than to sides. However, examination of the solution time data indicated that there were no differences between mean times for correct moves to corners and sides either for wins (3.1 vs. 3.2 s) or for blocks, (4.6 vs. 4.7 s). Separate comparisons for offensive focus, defensive focus, and control groups also failed to reveal any differences in the times. Times also were similar when wins were in a row rather than in a column

(3.2 vs. 3.1 s), as they were when blocks were in a row rather than in a column (4.5 vs. 4.7 s). Again, results were similar when offensive focus, defensive focus, and control groups were analyzed separately. These data led to the decision that the simulation would search previously unexamined open squares in a random order.

In summary, the simulation was based on the fixed Win-Block-Random rule hierarchy suggested by the solution time data. It adapted to different goals through systemically varying a single parameter, search length. The between-group differences in accuracies and solution times produced by the simulation derived entirely from variations in this single search-length parameter.

To test the usefulness of the model, we simulated performance of the offensive focus and defensive focus groups from Experiment 3. The simulation of the offensive focus group had Win Rule search lengths of four or five and Block Rule search lengths of three or four; the simulation of the defensive focus group had Win Rule search lengths of three or four and Block Rule search lengths of four or five. The models were tested by running 20 simulated children with an offensive focus and 20 with a defensive focus on the 12 partially filled boards used in Experiment 3.<sup>6</sup>

First consider the pattern of accuracy produced by the simulation. The expected interaction between instructions and relevant rule emerged. The simulation of the offensive focus group was more accurate on games where a win was possible (91% vs. 71%) but less accurate on games where a block was the best move possible (73% vs. 92%).

Now consider the simulation's solution times. The measure of solution time was the number of candidate locations considered before winning or blocking. Similar to the children, the simulations's performance varied with the relevant rule. The mean number of cycles to a correct win was less than the mean number of cycles to a correct block, regardless of whether the focus was on offense ( $M_s = 2.65$  and  $6.7$  for wins and blocks) or defense ( $M_s = 2.13$  and  $6.3$  for wins and blocks).

The model also made strong predictions about the ordering of individual children's solution times for wins and blocks. Because the simulation never began looking for blocks until it stopped trying to apply the Win Rule, it made all of its correct winning moves as fast as, or faster than, its fastest correct blocking move.

To determine whether individual children's solution times adhered to the pattern implied by the model, we returned to the empirical data and examined each child's ranked solution time data. The goal was to determine if that

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<sup>6</sup> Performance of the control group was not simulated, because in the absence of instructions encouraging any particular focus, the goals of children in that group seemed likely to have reflected an unknown mix of offensive and defensive orientations.

child's distribution of winning and blocking times was significantly different from a distribution where blocks and wins were made equally quickly. The analyses were conducted on the solution times of 31 of the 40 children in the offensive focus and defensive focus conditions, the two groups whose performance was simulated. The data of the other 9 children in these groups could not be used, in four cases because of the previously described equipment failure and in five cases because the child either never won or never blocked. For the remaining 31 children, instances of correct rule use (the sample size for each analysis) ranged from 5 to 12. Given these low numbers, we adopted a two-tailed  $\alpha$  of .10 rather than .05.<sup>7</sup>

A Kolmogorov-Smirnov test (Siegel & Castellan, 1988) revealed that 58% of children had solution time distributions significantly different from the distribution expected if latencies were the same for blocks and wins. In every case, the child's times were faster for wins; no child had significantly faster times for blocks. Thus, a majority of subjects met the strict test of the model's solution time ordering prediction, and no subject produced data inconsistent with it.

The simulation illustrates a specific mechanism by which differential allocation of resources can produce adaptive, goal-related variation within a rigid rule hierarchy. We make no claim that this is a unique solution. However, the close parallels between the data produced by the simulation and by the children do demonstrate that the simulation's mechanisms are sufficient to produce adaptive flexibility similar to that shown by kindergartners and first graders.

### GENERAL DISCUSSION

In domains with multiple, competing goals, people face a basic challenge: How to make their strategy use flexible enough to deal with shifting circumstances without losing track of overall objectives. This investigation examines how young children achieve such flexibility in one domain, tic-tac-toe. Experiment 1 indicated that children's tic-tac-toe strategies were rule based, with children acquiring the rules one-by-one and in an order predicted by the depth of search that each rule requires. Experiment 2 demonstrated that children modify their tic-tac-toe strategies in response to encouragement to focus on offense or defense, and in response to whether they go first or second in a game. Experiment 3 indicated that children achieve this constrained flexibility by varying the resources they devote to attaining each goal while pursuing the goals in the order indicated by a fixed rule hierarchy. A computer

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<sup>7</sup> An  $\alpha$  of .10 still resulted in a stringent test of whether the pattern of a child's solution times conformed to the model's predictions. Due to low sample sizes, the results for some children whose rankings were clearly consistent with the depth-based hierarchy did not prove significant in the Kolmogorov-Smirnov test. For example, one boy's solution times were as follows: wins (1.8 and 1.9 s); blocks (5.1, 6.0, 10.4, 11.5, and 16.8 s). The Kolmogorov-Smirnov on these data resulted in a maximum  $D$  of .29 ( $p > .20$ ).

simulation demonstrated exactly how such differential resource allocation could make strategies sufficiently flexible to meet changing circumstances while still pursuing longer term objectives. The simulation's performance closely paralleled that of children.

Children's reluctance to abandon their usual strategies, and the proposed mechanism by which they make strategies flexible, is not limited to tic-tac-toe. Siegler (1989) presented second graders subtraction problems under conditions in which they were rewarded solely for speed, solely for accuracy, or partially for each. The children responded by solving problems faster and less accurately when speed was rewarded, and solving them slower but more accurately when accuracy was rewarded. However, they did not change their distribution of strategy use in response to the different rewards; they simply executed the usual strategies more quickly or more carefully. Payne, Bettman, and Johnson (1988) reported a similar finding in their study of adults' decision making. When under moderate time pressure, the adults chose their usual strategies but executed each of them faster. Only when under severe time pressure did they abandon their old strategies in favor of inherently faster ones. An additional group of studies of strategy construction have indicated that even after discovering more efficient and more accurate ways of attaining goals, children often persist in using earlier discovered, less efficient, and less accurate strategies (Kuhn & Phelps, 1982; Schauble, 1990; Siegler & Jenkins, 1989).

Such findings might at first be dismissed as mere illustrations of the maladaptive consequences of habit. If the only contingency is accuracy, why not use the most accurate strategy exclusively? If a newer, more efficient strategy is discovered, why not abandon older, less efficient ones?

In considering these questions, it is useful to view goal-based resource allocation within the context of a more general space of mechanisms for generating problem-solving procedures. Each dimension within this space corresponds to a goal that an ideal adaptive mechanism would meet; each mechanism's position on a given dimension is determined by how well it meets the goal corresponding to that dimension; a mechanism's location on the set of dimensions defines its position within the space. Among the main dimensions along which such mechanisms could be evaluated are the degree to which strategies generated by each mechanism: (a) precisely fit the demands of particular circumstances; (b) result in stable pursuit of long-term goals in the face of distractions; (c) apply to novel situations; (d) avoid illegal or illegitimate activities in the domain; (e) make minimal demands on cognitive resources; and (f) allow rapid adjustments to changes in the task environment.

No one mechanism could be ideal on all of these dimensions. The very properties that make a mechanism strong on some of them necessarily make it weak on others. For example, mechanisms capable of producing strategies applicable to situations far from one's experience are inherently slow,

resource demanding, and susceptible to producing illegitimate approaches. Thus, the question is not which is the ideal strategy generation mechanism—there is no such mechanism—but rather the types of goals that each can most effectively pursue and when each is most likely to be employed.

As a first approximation, the space can be thought of in terms of a single dimension, ranging from the most resource-demanding, risky, and broadly applicable approaches at one extreme to the least resource-demanding, safest, and most narrowly applicable approaches at the other. This dimension in some ways resembles Laird and Newell's (1983) continuum of strong-to-weak methods, and in some ways Langer's (1989) continuum of mindful-to-mindless methods, but it also differs from each of them in a number of ways.

At one extreme of the dimension are *top-down mechanisms*. Among the best known of these are Miller, Galanter, and Pribram's (1960) TOTE procedure, Newell and Simon's (1972) generate and test procedure, and a variety of proposals regarding how metacognitive knowledge is used to create new strategies (e.g., Case, 1985; Flavell & Wellman, 1977; Langer, 1989; Sternberg, 1985). This class of mechanisms relies heavily on explicit, rational analysis of task demands, available resources, and the problem to be solved. For example, within Newell and Simon's model, people first create a problem space that includes critical features of the task and relevant operators for transforming the initial information. Operating within this problem space, they then construct a hierarchy of subgoals needed to solve the problem. Once they have constructed the complete hierarchy, they perform operations that satisfy each subgoal in turn until they reach the desired end state.

The strength of such mechanisms is that they can produce strategies precisely fitted to the demands of the particular task and situation. In addition, they can sometimes generate effective strategies in situations far from the problem solver's experience. However, they also have several drawbacks.

One drawback of such top-down approaches is that they are resource demanding. The subgoals created by exhaustive look ahead can quickly overwhelm an individual's cognitive resources. Even on the Tower of Hanoi, a "toy" task with a single goal, the subgoals needed to completely plan a solution often exceeds adults' working memory (Carpenter, Just, & Shell, 1990; Simon, 1975). The resource demand problem is even more severe for children trying to solve Tower of Hanoi problems (Klahr & Robinson, 1981).

Another drawback of top-down approaches is that they often are slow. This drawback is especially serious when the problem-solving situation demands rapid adaptation to changing circumstances. As Rogoff, Gauvain, and Gardner (1987) noted, such situations make it impossible to determine much in advance which subgoals must be met. This limits the analysis,



planning, and explicit consideration of situational demands and personal capacities that can be done.

A third problem is that such top-down approaches can generate illegal or illegitimate strategies. This often occurs when people do not understand the conceptual basis of legitimate procedures. For example, children often create buggy subtraction strategies because they do not understand the logic underlying borrowing, and therefore do not understand how to proceed when borrowing across a zero (Brown & Burton, 1978; VanLehn, 1983).

In sum, these explicit, analytic, high-level mechanisms can create strategies that are well fit to the details of the situation, and often are the only way to proceed effectively in novel situations. However, they also can generate illegal strategies, are resource demanding, and are often too slow to be useful when quick reactions are needed.

At the opposite extreme from top-down problem solving lies *relying on an existing strategy*. This approach is especially useful in domains in which even novel problems are highly similar to previously encountered ones. For example, first graders know and use a variety of strategies for solving arithmetic problems: counting from 1, counting from the larger addend, decomposing a single problem into two simpler ones, guessing, and so on. When presented a novel problem, they simply choose one of these existing strategies and apply it. They usually do this even when none of the existing strategies is likely to generate the right answer (Siegler & Jenkins, 1989).

This reliance on existing approaches has almost exactly opposite advantages and disadvantages as the top-down mechanisms. Its strengths are its avoidance of untested and potentially illegitimate strategies, its low resource demands, and the speed with which an existing strategy can be identified and used. Its weaknesses are its inapplicability to problems that differ substantially from those usually encountered and its frequent lack of sensitive fitting of the strategy to new situations.

A third means for generating problem-solving approaches, *opportunistic planning* (e.g., Hayes-Roth & Hayes-Roth, 1979), lies somewhere between the first two classes of mechanisms. It also combines some of their advantages and avoids some of their disadvantages. Rather than relying wholly on top-down reasoning, opportunistic planning intersperses thought and action. For example it allows attention to shift between top-down subgoal stacking and immediate responses to changing features of the environment. This reduces resource demands below those entailed by the top-down processes, where all planning is done in advance. Also, because the problem solver can act without constructing a complete subgoal hierarchy, responses can be made more quickly, which allows successful performance in situations where top-down approaches are too slow. In addition, such opportunistic approaches share an advantage with top-down mechanisms: They are sensitive to specific demands of the local environment. In situations in which

the local environment changes rapidly and unpredictably, they can be even stronger than the top-down approaches on this dimension.

The sensitivity of opportunistic mechanisms to changing circumstances is not without cost, however. Attention to the changing circumstances can lead to departures from long-term goals. This is an especially serious problem when successful performance depends on responding to the actions of external agents. For example, in many games, good players use feints, traps, and bluffs to mislead opponents. To the extent that players evaluate these plays only within the immediate context, they will be duped into moving away from their own long-term goals. Thagard (1992) eloquently described and modeled how this can occur in such adversarial domains as war, business, and poker playing.

Lying between opportunistic approaches and reliance on existing strategies is the mechanism identified by the current experiments: goal-based resource allocation. This mechanism allows more flexibility than simply applying an existing approach, avoids the risks of abandoning long-term goals that accompany opportunistic approaches, and is faster and easier to apply than top-down approaches. These qualities make it useful not only for tic-tac-toe but for a wide variety of tasks pursued by adults as well as children: everyday tasks such as commuting to work, ordering in a restaurant, getting dressed in the morning, and many others. Particular tasks within each of these domains have similar subgoal structure across situations, but also demand some degree of flexibility to accommodate situational variation.

Goal-based resource allocation may be especially important within children's problem solving. Children are frequently in the position of being taught and having to use strategies that they do not fully understand. Even when they discover a strategy on their own, they often do not understand at first why it works (Siegler & Jenkins, 1989). Not understanding why previously learned strategies are legitimate increases the likelihood that new strategies generated through top-down or opportunistic means will violate constraints of the domain. Children's limited cognitive resources also often limit their success in generating useful strategies via mindful techniques. Even when their use of such methods does result in new strategies, the demands of strategy execution often result in the otherwise useful new strategies not improving performance (DeMarie-Dreblow & Miller, 1988). For these reasons, approaches that preserve subgoal structures of known strategies and that are relatively undemanding of cognitive reasoning, but that still allow some flexibility for adapting to changing circumstances, would seem likely to occupy an especially prominent place in children's problem solving.

This analysis of mechanisms for generating problem-solving approaches also suggests when each mechanism will be most likely and when each will be used most often. Top-down approaches seem most useful in unfamiliar

situations, especially ones where all relevant information is known in advance and where existing strategies do not seem applicable. Opportunistic approaches seem most likely in unfamiliar situations in which individuals encounter some relevant information only during the course of problem solving and where they must respond rapidly to the new information. Employing a familiar strategy seems most likely in domains in which a standard algorithm can be applied to large numbers of problems, and in which the relative importance of achieving subgoals is constant across situations. Finally, the mechanism identified in this article—goal-based resource allocation—will be used especially often when cognitive resources can be allocated to different parts of the strategy in ways that confer flexible adaptation to changing situations. Such resource allocation allows rapid responses to changes in the problem environment, minimizes demands on cognitive resources, and avoids the risk of generating illegitimate strategies. When the situation is basically familiar, and when subgoals can be pursued in a consistent order, these advantages will often outweigh those of more mindful mechanisms. It is in these situations that goal-based resource allocation is likely to be of greatest importance.

## REFERENCES

- Anderson, J.H. (1983). *The architecture of cognition*. Cambridge, MA: Harvard University Press.
- Bisanz, J., & LeFevre, J. (1990). Strategic and nonstrategic processing in the development of mathematical cognition. In D.F. Bjorklund (Ed.), *Children's strategies: Contemporary views of cognitive development*. Hillsdale, NJ: Erlbaum.
- Bjorklund, D.F. (Ed.). (1990). *Children's strategies: Contemporary views of cognitive development*. Hillsdale, NJ: Erlbaum.
- Brown, J.S., & Burton, R.B. (1978). Diagnostic models for procedural bugs in basic mathematical skills. *Cognitive Science*, 2, 155-192.
- Carpenter, P.A., Just, M.A., & Shell, P. (1990). What one intelligence test measures: A theoretical account of the processing in the Raven Progressive Matrices Test. *Psychological Review*, 97, 404-431.
- Case, R. (1985). *Intellectual development: A systematic reinterpretation*. New York: Academic.
- Cooper, T.A., & Wogrin, N. (1988). *Rule-based programming with OPS5*. San Mateo, CA: Morgan Kaufmann.
- DeMarie-Dreblow, D., & Miller, P.H. (1988). The development of children's strategies for selective attention: Evidence for a transitional period. *Child Development*, 59, 1504-1513.
- Flavell, J.H., & Wellman, H.M. (1977). Metamemory. In R.V. Kail, Jr. & J.W. Hagen (Eds.), *Perspectives on the development of memory and cognition*. Hillsdale, NJ: Erlbaum.
- Geary, D.C., & Burlingham-DuBree, M. (1989). External validation of the strategy choice model for addition. *Journal of Experimental Child Psychology*, 47, 175-192.
- Goldman, S.R., Pellegrino, J.W., & Mertz, D.L. (1988). Extended practice of basic addition facts: Strategy in learning disabled students. *Cognition and Instruction*, 5, 223-265.
- Hayes-Roth, B., & Hayes-Roth, F. (1979). A cognitive model of planning. *Cognitive Science*, 3, 275-310.

- Klahr, D., & Robinson, M. (1981). Formal assessment of problem-solving and planning processes in preschool children. *Cognitive Psychology*, 13, 113-148.
- Kuczaj, S. (1977). The acquisition of regular and irregular past tense forms. *Journal of Verbal Learning and Verbal Behaviour*, 16, 589-600.
- Kuhn, D., & Phelps, E. (1982). The development of problem-solving strategies. In H. Reese & L. Lipsett (Eds.), *Advances in child development and behavior*. San Diego, CA: Academic.
- Laird, J.E., & Newell, A. (1983). *A universal weak method* (Tech. Rep. No. CMU-CS-83-114). Pittsburgh, PA: Carnegie Mellon University.
- Langer, E.J. (1989). *Mindfulness*. Reading, MA: Addison-Wesley.
- Lepper, M.R., & Chabay, R.W. (1988). Socializing the intelligent tutor: Bringing empathy to computer tutors. In H. Mandl & A.M. Lesgold (Eds.), *Learning issues for intelligent tutoring systems*. New York: Springer-Verlag.
- Maratsos, M.P. (1983). Some current issues in the study of the acquisition of grammar. In J.H. Flavell & E. Markman (Eds.), *Handbook of child psychology*. New York: Wiley.
- McDermott, J., & Forgy, C. (1978). Production system conflict resolution strategies. In D.A. Waterman & F. Hayes-Roth (Eds.), *Pattern directed inference systems*. Orlando, FL: Academic.
- McGilly, K., & Siegler, R.S. (1989). How children choose among serial recall strategies. *Child Development*, 60, 172-182.
- Miller, G.A., Galanter, E., & Pribram, K.H. (1960). *Plans and the structure of behavior*. New York: Holt.
- Miller, P.H., Haynes, V.F., DeMarie-Dreblow, D., & Woody-Ramsey, J. (1986). Children's strategies for gathering information in three tasks. *Child Development*, 57, 1429-1439.
- Neches, R., Langley, P., & Klahr, D. (1987). Learning, development, and production systems. In D. Klahr, P. Langley, & R. Neches (Eds.), *Production system models of learning and development*. Cambridge, MA: MIT Press.
- Newell, A., & Simon, H.A. (1972). *Human problem solving*. Englewood Cliffs, NJ: Prentice-Hall.
- Payne, J.W., Bettman, J.R., & Johnson, E.J. (1988). Adaptive strategy selection in decision making. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 14, 534-552.
- Rogoff, B., Gauvain, M., & Gardner, W. (1987). Children's adjustment of plans to circumstances. In E.K. Scholnick, S.L. Friedman, & R.R. Cocking (Eds.), *Blueprints for thinking: The role of planning in cognitive development*. Cambridge: Cambridge University Press.
- Schauble, L. (1990). Belief revision in children: The role of prior knowledge and strategies for generating evidence. *Journal of Experimental Child Psychology*, 49, 31-57.
- Shultz, T.R., Fisher, G.W., Pratt, C.C., & Rulf, S. (1986). Selection of causal rules. *Child Development*, 57, 143-152.
- Siegel, S., & Castellan, J.N. (1988). *Nonparametric statistics*. New York: McGraw-Hill.
- Siegler, R.S. (1989). How domain-general and domain-specific knowledge interact to produce strategy choices. *Merrill-Palmer Quarterly*, 35, 1-26.
- Siegler, R.S., & Jenkins, E. (1989). *How children discover new strategies*. Hillsdale, NJ: Erlbaum.
- Siegler, R.S., & Shrager, J. (1984). Strategy choices in addition and subtraction: How do children know what to do? In C. Sophian (Ed.), *Origins of cognitive skills*. Hillsdale, NJ: Erlbaum.
- Simpson, J.A., & Weiner, E.S.C. (1989). *The Oxford English Dictionary*. New York: Oxford University Press.
- Simon, H.A. (1975). The functional equivalence of problem-solving skills. *Cognitive Psychology*, 8, 268-288.
- Spitz, H.H., & Winters, E.A. (1977). Tic-tac-toe performance as a function of maturational level of retarded adolescents and nonretarded children. *Intelligence*, 1, 108-117.

- Sternberg, R.J. (1985). *Beyond IQ: A triarchic theory of human intelligence*. New York: Cambridge University Press.
- Sternberg, R.J., & Rifkin, B. (1979). The development of analogical reasoning processes. *Journal of Experimental Child Psychology*, 27, 195-232.
- Sutton-Smith, B., & Roberts, J.M. (1967). Studies of an elementary game of strategy. *Genetic Psychology Monographs*, 75, 3-41.
- Thagard, P. (1992). Adversarial problem solving: Modeling an opponent using explanatory coherence. *Cognitive Science*, 16, 123-149.
- Turkle, S. (1984). *The second self: Computers and the human spirit*. New York: Simon & Schuster.
- VanLehn, K. (1983). On the representation of procedures in repair theory. In H.P. Ginsburg (Ed.), *The development of mathematical thinking*. New York: Academic.
- Vurpillot, E. (1968). The development of scanning strategies and their relation to visual differentiation. *Journal of Experimental Child Psychology*, 6, 632-650.
- Zaslavsky, C. (1982). *Tic-tac-toe and other three in a row games from ancient Egypt to the modern computer*. New York: Thomas Y. Crowell.

## APPENDIX

We examined the usefulness of six methods for resolving conflicts among the eight tic-tac-toe rules in Table 1. Four of these conflict resolution methods were described by Neches et al. (1987): rule ordering; recency, specificity, and rule recency. A fifth was a composite method used by the OPS5 production system language. The sixth was arbitrary selection among satisfied rules, and was included to provide a point of comparison for the systematic approaches. In the following we describe each of these conflict resolution methods and then compare their effectiveness in the tic-tac-toe context.

1. *Rule ordering* resolves conflicts by arranging satisfied rules into a hierarchy and selecting the highest applicable rule in the hierarchy for firing. The simulation embodying this method used the hierarchical ordering suggested by Newell and Simon (1972) and shown in Table 1.
2. *Recency* guides the system to prefer production rules that match the data most recently entered into working memory. In a turn-taking game like tic-tac-toe, this at first glance appears to be a powerful guide for rule selection. The most recent entry into working memory is likely to be the opponent's latest move, so a preference for recency would lead to firing the rule most likely to thwart the opponent's advances. This same preference, however, can cripple offensive play. Consider a situation in which a player has put two X's in a row. The opponent fails to block, instead creating a potential win by placing two O's in a row. Two of the player's rules are now satisfied: Win and Block. If the player's move follows the recency principle, the Block Rule will be chosen, because it matches the most recent development in the game, the opponent's last move. However, this could lead the player to miss an opportunity to win.

In the simulation that used recency as a conflict resolution principle, every square was assigned a recency value reflecting the order in which the squares were filled. The most recently filled square received the highest value; squares that had not yet been filled received the lowest. On each cycle, the simulation examined the squares in order of recency and determined which rule(s) could be fired. If a single rule could fire at the most recently filled location, that rule was chosen. If no rule could fire, the simulation examined if any rules could fire at the square with the next highest recency value. If two or more rules fired at the initial location, the simulation would examine if any of those rules would fire at the location with the next highest recency value. The process continued until a rule was chosen or the last square had been examined, in which case a rule was chosen arbitrarily among the remaining contenders.

3. *Specificity* directs the system to favor satisfied rules with the greatest number of tests on their condition side. For example, the Fork Rule and the Block Fork Rule would be favored over other rules because they require the most conditions to be met. This is an adaptive solution only when the rules with the most conditions provide the best move. Such cases sometimes arise in tic-tac-toe. For example, if Fork and Play Center were both matched, the specificity principle would lead to the choice of the more powerful Fork Rule. In other contexts, however, the best applicable rule does not include the most conditions. For example, if the conditions of both Fork and Block were matched, the specificity criterion would lead to the Fork Rule being chosen, resulting in a missed blocking opportunity, and thus, probably, to a loss before the fork could be executed.

For purposes of the simulation, we assigned each rule a specificity value equal to the number of elements on the rule's condition side. On each cycle, the simulation fired the satisfied rule with the highest specificity value. If there were a tie, one of the tied rules was fired arbitrarily.

4. *Rule recency* specifies that recently learned rules will be selected over earlier learned ones. According to this principle, a child who has just learned to set up a fork would pass up applying other rules if the Fork Rule could be fired. The assumption is that newer rules will, in general, represent better responses, and so should be favored. Although this assumption sometimes is justified, for instance, if a newer Fork Rule competes against an earlier learned Play Center Rule, at other times, the earlier learned rule generates the better move. For instance, passing up the older Block Rule to fire the newer Fork Rule would usually lead to a worse outcome (losing).

The simulation of rule recency was based on the developmental sequence suggested by the Experiment 1 results: (in order of recency) Block Fork, Fork, Block, and Win. When multiple rules were satisfied,

the simulation fired the most recently acquired one. If none were satisfied, the simulation arbitrarily chose among the remaining possibilities (Play Center, Play Opposite Corner, Play Empty Corner, Play Side). The latter choice was arbitrary because we had no empirical evidence as to the order in which children acquire these rules.

5. The *OPSS conflict resolution scheme* was simulated by using a hierarchical combination of two simpler principles and arbitrary choice (Cooper & Wogrin, 1988).<sup>8</sup> It began by using recency to eliminate satisfied rules that matched less recently entered data elements. If this resulted in only one rule remaining, the rule fired. If multiple rules remained, specificity was used to eliminate rules with fewer tests on their condition sides. If a single rule remained, it fired. Otherwise, the simulation chose arbitrarily among the remaining rules.
6. *Arbitrary choice* resolved conflicts by choosing arbitrarily within the set of applicable rules.

Each simulation played 1,000 games of tic-tac-toe against itself and each of the five other simulations. Results of these games indicated that the simulation using rule ordering as the conflict resolution principle was the only one to achieve an expert level of play; it had 42% wins and 0% losses. The simulations embodying the other principles did considerably less well, ranging in effectiveness from specificity, which had 32% wins and 28% losses, to arbitrary choice, which had 20% wins and 47% losses. The success of rule ordering should not be surprising because it is the only conflict resolution principle that relies on knowledge of what each rule accomplishes rather than on general information about the current state of working memory or about the history of rule acquisition in a domain. As discussed earlier, attending to the subgoals that each rule satisfies and the depth into the game at which they are satisfied is critical to integrating offensive and defensive goals in tic-tac-toe.

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<sup>8</sup> OPS5 actually uses a combination of three conflict resolution principles and arbitrary choice. However, one of the conflict resolution principles, refraction, is not applicable in the current context. Refraction specifies that once a rule fires, it cannot be fired again unless it matches different information in working memory. Because every firing of a tic-tac-toe rule changes the game board, it is impossible for a rule to fire twice with the same data; thus, refraction would never apply.