Submitted to *Manufacturing & Service Operations Management*
manuscript (Please, provide the manuscript number!)

The pickup and delivery problem with time windows and scheduling on edges

Rafael A. Melo
Institute of Computing, Universidade Federal da Bahia, Salvador, Brazil, rafael.melo@ufba.br

Sunil Tiwari
ESSCA School of Management, 4 Pont Pasteur, 69007 Lyon, France, sunil.tiwari@essca.fr

We introduce and consider a generalization of the well-known pickup and delivery problem with time windows, namely, the PDPTW and scheduling on edges (PDPTW-SE). The problem consists in determining routes for transportation requests, each of which composed of pickup and delivery locations, using a heterogeneous fleet of vehicles. The goal consists in minimizing the total completion time while capacity, time windows, and precedence constraints have to be met. Besides, there are machines that must be employed when traversing certain edges, and the routing of the vehicles must also define the schedule of these machines. We propose a compact mixed integer programming (MIP) formulation and a biased random-key genetic algorithm (BRKGA)* for the problem. The performed computational experiments indicate that ... Furthermore, we tackle a special case of PDPTW-SE arising in an innovative practical context of automated guided vehicles (AGVs) routing and scheduling in a multi-floor hospital.

Key words: Routing; Pickup and delivery problem with time windows; Machine scheduling; Integrated routing and scheduling; Healthcare supply chain; Automated guided vehicles

---

1. Introduction

https://www.sciencedirect.com/science/article/pii/S0377221722005252?via

the Pickup and delivery problem with time windows and scheduling on edges (PDPTW-SE) PDPTW-SE is NP-hard as it generalizes PDPTW, which is known to be NP-hard (Dumas et al.

* RAFAEL: confirm if we are really going to propose this

1991, Lenstra and Kan 1981). A constrained special case of PDPTW-SE finds an innovative application in the routing and scheduling of automated guided vehicles (AGV) in a multi-floor hospital with the synchronization of elevators.

Pickup and delivery problems (PDP) Savelsbergh and Sol (1995) Berbeglia et al. (2007) Parragh et al. (2008) ... also named dial a ride problem (DARP) when the transportation of people is considered Bodin and Berman (1979) Cullen et al. (1981)

PDP with time windows (PDPTW) Dumas et al. (1991) PDPTW with scheduling Ghilas et al. (2016) Ghilas et al. (2018) other VRP with scheduling

AGV x autonomous vehicles (AV) or autonomous mobile robots (AMR)

AGV Vis (2006) AGV hospital logistics Bačík et al. (2017) Chikul et al. (2017) Aziez et al. (2022)

1.1. Contributions and organization

The contributions of our work can be summarized as follows. ... mixed integer programming (MIP) formulation...

The remainder of this paper is organized as follows. Section 2 formally defines PDPTW-SE. Section 3 describes the proposed compact MIP formulation. Section 4 presents the innovative application of a constrained special case of PDPTW-SE in the routing and scheduling of AGVs in a multi-floor healthcare supply chains with the synchronization of elevators. The performed computational experiments are summarized in Section 5. Section 6 discusses some concluding remarks.

2. Problem formalization

PDPTW-SE can be formally defined using an undirected graph $G = (V, E)$ as follows. Let $V = V_p \cup V_d \cup \{0\}$, where $V_p = \{1, \ldots, n\}$ is a set of pickup nodes, $V_d = \{n+1, \ldots, 2n\}$ is a set of delivery nodes, and node 0 represents the depot. In addition, let $E = E^s \cup E^m$ be the set of edges, where E^s is the set of edges that can be directly traversed by a vehicle and E^m is the set of edges whose traversals involve using a machine that has to be scheduled. Let $R = \{r_1, r_2, \ldots, r_n\}$ be a set of unsplittable requests, each of which denoted by a tuple $r_i = \langle v_i, v_{n+i}, q_i \rangle$, where $v_i \in V_p$ denotes its origin (pickup or loading node), $v_{n+i} \in V_d$ its destination (delivery or unloading node), and q_i its weight. It is assumed that $q_i \in \mathbb{Z}_{>0}$ for $i \in V_p$ and $q_i \in \mathbb{Z}_{<0}$ for $i \in V_d$, with $q_i = -q_{n+i}$ for $i \in V_p$. Each node $i \in V^p \cup V^d$ has a strict time window $[e_i, l_i]$ in which the vehicles should start the corresponding loading/unloading operation. The time for loading/unloading a vehicle at node $i \in V^p \cup V^d$ is denoted by s_i.

There is a set of heterogeneous vehicles K, with each $k \in K$ having a capacity Q_k. Besides, traversing the edge $ij \in E^s$ with vehicle $k \in K$ takes time $\hat{d}^k_{ij}$. The time $\hat{d}^k_{ij}$ for an edge $ij \in E^m$ defines a lower bound on the real time to traverse the edge, as it does not take the schedule of the machine into consideration. Consider a set of machines H used to carry the vehicles between intermediate locations to transpose certain obstacles. Besides, let $H_e \subseteq H$ be the set of machines that can be used in the traversal of edge $e \in E^m$. It is assumed that a machine can only carry a vehicle at a time, no preemption is allowed, and a single machine is necessary to cross an edge $ij \in E^m$. Denote by $\bar{d}^k_{if^h_i}$ the time to travel between node $i \in V$ and the location of machine $h \in H$ corresponding to node i, f^h_i, using vehicle $k \in K$. The time for the machine $h \in H$ to traverse the obstacle corresponding to edge $ij \in E^m$ is denoted by $O^h_{f^h_i f^h_j}$. It is assumed that $O^h_{f^h_i f^h_j}$ also includes an estimate of the time to wait for the machine to arrive at f^h_i.

Denote by C_k the completion time of vehicle k and let $C_{total} = \sum_{k \in K} C_k$ denote the total completion time. The problem thus consists in defining a feasible routing and scheduling of the vehicles and machines that minimizes the total completion time.

3. MIP formulation

In this section, we present an arc-based formulation for PDPTW-SE. Consider the digraph $G' = (V', A)$, where $V' = V \cup \{2n+1\}$ and node $2n+1$ is a copy of the depot. Besides, define $V'_p = V_p \cup \{0\}$ and $V'_d = V_d \cup \{2n+1\}$. The set of arcs is defined as $A = \{(i,j) \mid i,j \in V_p \cup V_d\} \cup \{(0,j) \mid j \in V_p \cup \{2n+1\}\} \cup \{(i, 2n+1) \mid i \in V_d\}$. For every $k \in K$, set: $d^k_{ij} = d^k_{ji} = \hat{d}^k_{ij}$ for $ij \in E$ with $i,j \in V_p \cup V_d$; $d^k_{i,2n+1} = \hat{d}^k_{0i}$ for $i \in V_d$; $d^k_{0j} = \hat{d}^k_{0j}$ for $j \in V_p$; and $d^k_{0,2n+1} = 0$. Additionally, let A^s and A^m be the arcs corresponding to the edges in E^s and E^m, respectively.

In what follows, we divide the presentation of the formulation into its routing and scheduling parts. The routing part of the formulation is described in Section 3.1 while its scheduling part is detailed in Section 3.2. The complete formulation is presented in Section 3.3.

3.1. Routing constraints

We define the following routing related variables. Binary variable x_{ij}^k is equal to one if vehicle k goes directly from node $i \in V^+$ to node $j \in V \cup \{2n+1\}$ (i.e., traverses arc $(i,j) \in A$), zero otherwise. Variable z_i^k denotes the weight of vehicle $k \in K$ just after leaving node $i \in V'$. The routing part of the solution can thus be described by:

$$\sum_{j \in V_p} x_{0j}^k = 1, \qquad \forall\, k \in K, \tag{1}$$

$$\sum_{j:(j,i) \in A} x_{ji}^k - \sum_{j:(i,j) \in A} x_{ij}^k = 0, \qquad \forall\, k \in K,\ i \in V_p \cup V_d, \tag{2}$$

$$\sum_{j \in V_d} x_{j,2n+1}^k = 1, \qquad \forall\, k \in K, \tag{3}$$

$$\sum_{k \in K} \sum_{j:(j,i) \in A} x_{ji}^k = 1, \qquad \forall\, i \in V_p \cup V_d, \tag{4}$$

$$\sum_{j:(j,i) \in A} x_{ji}^k = \sum_{j:(j,n+i) \in A} x_{j,n+i}^k, \qquad \forall\, k \in K,\ i \in V_p, \tag{5}$$

$$z_0^k = 0, \qquad \forall\, k \in K, \tag{6}$$

$$z_j^k \geq z_i^k + d_i - M_1(1 - x_{ij}^k), \qquad \forall\, k \in K,\ (i,j) \in A, \tag{7}$$

$$z_i^k \leq \min\{Q^k, \max\{0, Q^k + d_i\}\} \sum_{j:(j,i) \in A} x_{ji}^k, \qquad \forall\, k \in K,\ i \in V_p \cup V_d, \tag{8}$$

$$z_i^k \geq d_i \sum_{j:(j,i) \in A} x_{ji}^k, \qquad \forall\, k \in K,\ i \in V_p, \tag{9}$$

$$0 \leq x_{ij}^k \leq 1, \qquad \forall\, k \in K,\ (i,j) \in A, \tag{10}$$

$$z_i^k \geq 0, \qquad \forall\, k \in K,\ i \in V'. \tag{11}$$

Constraints (1)-(3) are balance constraints for the vehicles. Constraints (4) ensure that every pickup and delivery node is visited. Constraints (5) guarantee that the pickup and delivery nodes corresponding to a given request are visited by the same vehicle. Constraints (6)-(9) enforce that the capacities of the vehicles are respected. Constraints (10)-(11) restrain the domains of the $\boldsymbol{x}$ and $\boldsymbol{z}$ variables.

3.2. Scheduling constraints

We introduce the following vehicle scheduling related variables. Variable t_i provides the starting time to serve node $i \in V$. Variable C_k gives the completion time of vehicle $k \in K$. Furthermore, consider the following machine scheduling related variables. Binary variable ϕ^h_{ij} is equal to one if machine $h \in H$ is used to cross arc $(i,j) \in A^m$, zero otherwise. Binary variable $\gamma^h_{iji'j'}$ is equal to one if the use of machine $h \in H$ to cross arc $(i,j) \in A^m$ precedes its use to cross arc $(i',j') \in A^m$, zero otherwise. Variable α^h_{ij} denotes the start time to use machine $h \in H$ to cross arc $(i,j) \in A^m$. In this way, the scheduling part of the solution can be cast as:

$$t_j \geq t_i + s_i + d^k_{ij} - M_2(1 - x^k_{ij}), \qquad \forall\, k \in K,\ (i,j) \in A,\ \text{with } j \in V, \tag{12}$$

$$t_i + s_i \leq t_{n+i}, \qquad \forall\, i \in V_p, \tag{13}$$

$$e_i \leq t_i \leq l_i, \qquad \forall\, i \in V_p \cup V_d, \tag{14}$$

$$t_0 = 0, \tag{15}$$

$$\sum_{h \in H} \phi^h_{ij} = \sum_{k \in K} x^k_{ij}, \qquad \forall\, (i,j) \in A^m, \tag{16}$$

$$\alpha^h_{ij} \geq t_i + s_i + \bar{d}^k_{if^h_i} - M_3(1 - \phi^h_{ij}), \qquad \forall\, h \in H,\ k \in K,\ (i,j) \in A^m, \tag{17}$$

$$t_j \geq \alpha^h_{ij} + O^h_{f^h_i f^h_j} + \bar{d}^k_{f^h_j j} - M_4(2 - \phi^h_{ij} - x^k_{ij}), \qquad \forall\, h \in H,\ k \in K,\ (i,j) \in A^m,\ \text{with } j \in V, \tag{18}$$

$$\gamma^h_{iji'j'} + \gamma^h_{i'j'ij} \geq \phi^h_{ij} + \phi^h_{i'j'} - 1, \qquad \forall\, h \in H,\ (i,j),(i',j') \in A^m, \tag{19}$$

$$\gamma^h_{iji'j'} \leq \phi^h_{ij}, \qquad \forall\, h \in H,\ (i,j),(i',j') \in A^m, \tag{20}$$

$$\gamma^h_{i'j'ij} \leq \phi^h_{ij}, \qquad \forall\, h \in H,\ (i,j),(i',j') \in A^m, \tag{21}$$

$$\alpha^h_{i'j'} \geq \alpha^h_{ij} + O^h_{f^h_i f^h_j} - M(1 - \gamma^h_{iji'j'}), \qquad \forall\, h \in H,\ (i,j),(i',j') \in A^m, \tag{22}$$

$$C_k \geq t_i + s_i + d^k_{i,2n+1} - M_2(1 - x^k_{i,2n+1}), \qquad \forall\, k \in K,\ (i,2+1) \in A, \tag{23}$$

$$\begin{gathered} C_k \geq \alpha^h_{i,2n+1} + O^h_{f^h_i f^h_{2n+1}} + \bar{d}^k_{f^h_{2n+1} 2n+1} - M_4(2 - \phi^h_{i,2n+1} - x^k_{i,2n+1}), \\ \forall\, h \in H,\ k \in K,\ (i,2n+1) \in A^m, \end{gathered} \tag{24}$$

$$C_k \geq 0, \qquad \forall\ k \in K, \tag{25}$$

$$t_i \geq 0, \qquad \forall\ i \in V, \tag{26}$$

$$0 \leq \phi^h_{ij} \leq 1, \qquad \forall\ h \in H,\ (i,j) \in A^m, \tag{27}$$

$$\alpha^h_{ij} \geq 0, \qquad \forall\ h \in H,\ (i,j) \in A^m, \tag{28}$$

$$0 \leq \gamma^h_{iji'j'} \leq 1, \qquad \forall\ h \in H,\ (i,j),(i',j') \in A^m\}. \tag{29}$$

Constraints (12) ensure that whenever an arc is traversed by a vehicle, a lower bound on the time to serve its destination node is determined by the times of its origin node. Constraints (13) guarantee that a pickup node of a request is visited before its delivery node. Constraints (14) determine that service of each node starts within its time window. Constraints (15) set the start time of the depot. Constraints (16) imply that a machine is used for an arc if and only if it is traversed by a vehicle. Constraints (17) define that whenever a machine is used to traverse an arc, a lower bound on its starting time on the machine is determined by the times corresponding to its origin node. Constraints (18) determine that whenever a machine is used to traverse an arc, the machine times define a lower bound on the starting time of its destination node. Constraints (19)-(21) define that there is an order between the traversal of two distinct arcs if and only if they are both scheduled on the same machine. Constraints (22) define a lower bound on the time to traverse an arc whenever it is preceded by another arc. Constraints (23)-(24) determine lower bounds on the completion times for the vehicles. Constraints (25)-(29) define the domains of the $\boldsymbol{t}$, $\boldsymbol{C}$, $\boldsymbol{\phi}$, $\boldsymbol{\alpha}$, and $\boldsymbol{\gamma}$ variables.

3.3. The complete formulation

A formulation for PDPTW-SE can thus be cast as:

$$\min \qquad \sum_{k \in K} C_k \tag{30}$$

$$(1) - (29).$$

The objective function (30) minimizes the total completion time.

4. A constrained special case: AGV routing and scheduling in a multi-floor hospital supply chain

In this section we present a practical application of PDPTW-SE in healthcare supply chain. More specifically, a constrained special case of the problem arises in the context of a multi-floor hospital supply chain with synchronization of the elevators.

In the considered problem, the requests correspond to ... The pickup and delivery locations represent the ... The vehicles are related to the automated guided vehicles (AGVs). Besides, the machines correspond to the elevators that are used to move between the different floors. This constrained special case has a particularity that, after collecting a request at pickup node $i \in V_p$, the vehicle has to go directly to its delivery node $n+i \in V_d$.

4.1. The adapted formulation

Based on the formulation for PDPTW-SE described in Section 3. The adapted formulation uses the already defined variables $\boldsymbol{x}$, $\boldsymbol{t}$, $\boldsymbol{C}$, $\boldsymbol{\phi}$, $\boldsymbol{\alpha}$, and $\boldsymbol{\gamma}$. Notice that variables $\boldsymbol{z}$ used to formulate PDPTW-SE are not needed in this setting. We set $x_{ij}^k = 0$ for:

- $(i,j) \in A$ when $i \in V_p$ and $j \neq n+i$,
- $(i,j) \in A$ when $i \in V_d$ and $j = n-i$,
- $(i,j) \in A$ s.t. $i \in V_p$ and $d_i > Q_k$ or $i \in V_d$ and $d_i < -Q_k$.

Besides, we set $\alpha_{ij}^h = \phi_{ij}^h = \gamma_{iji'j'}^h = \gamma_{i'j'ij}^h = 0$ whenever $(i,j) \in A^m$ and any of the conditions to set the $\boldsymbol{x}$ variables to zero hold.

Thus, the special case can be formulated as:

$$\min \quad \sum_{k \in K} C_k \tag{30 revisited}$$

$$(1)-(5),(10),(12)-(29).$$

5. Computational experiments

All the computational experiments were carried out on a machine running under Ubuntu x86-64 GNU/Linux, with an Intel Core i7-8700 Hexa-Core 3.20GHz processor and 16Gb of RAM. The metaheuristic was coded in C++ and the formulation solved using Gurobi XXX under standard configurations, except for .... Each execution of the solver was limited to one hour (3,600s).

6. Concluding remarks

...

Acknowledgments

Work of Rafael A. Melo was supported by the Brazilian National Council for Scientific and Technological Development (CNPq).

References

Aziez I, Côté JF, Coelho LC (2022) Fleet sizing and routing of healthcare automated guided vehicles. *Transportation Research Part E: Logistics and Transportation Review* 161:102679, URL `http://dx.doi.org/https://doi.org/10.1016/j.tre.2022.102679`.

Bačík J, Ďurovský F, Biroš M, Kyslan K, Perduková D, Padmanaban S (2017) Pathfinder–development of automated guided vehicle for hospital logistics. *IEEE Access* 5:26892–26900, URL `http://dx.doi.org/10.1109/ACCESS.2017.2767899`.

Berbeglia G, Cordeau JF, Gribkovskaia I, Laporte G (2007) Static pickup and delivery problems: a classification scheme and survey. *TOP* 15(1):1–31, URL `http://dx.doi.org/10.1007/s11750-007-0009-0`.

Bodin LD, Berman L (1979) Routing and scheduling of school buses by computer. *Transportation Science* 13(2):113–129, URL `http://dx.doi.org/10.1287/trsc.13.2.113`.

Chikul M, Maw HY, Soong YK (2017) Technology in healthcare: A case study of healthcare supply chain management models in a general hospital in singapore. *Journal of Hospital Administration* 6(6):63–70.

Cullen FH, Jarvis JJ, Ratliff HD (1981) Set partitioning based heuristics for interactive routing. *Networks* 11(2):125–143, URL `http://dx.doi.org/10.1002/net.3230110206`.

Dumas Y, Desrosiers J, Soumis F (1991) The pickup and delivery problem with time windows. *European Journal of Operational Research* 54(1):7–22, URL `http://dx.doi.org/10.1016/0377-2217(91)90319-Q`.

Ghilas V, Cordeau JF, Demir E, Van Woensel T (2018) Branch-and-price for the pickup and delivery problem with time windows and scheduled lines. *Transportation Science* 52(5):1191–1210, URL `http://dx.doi.org/10.1287/trsc.2017.0798`.

Ghilas V, Demir E, Van Woensel T (2016) The pickup and delivery problem with time windows and scheduled lines. *INFOR: Information Systems and Operational Research* 54(2):147–167, URL `http://dx.doi.org/10.1080/03155986.2016.1166793`.

Lenstra JK, Kan AR (1981) Complexity of vehicle routing and scheduling problems. *Networks* 11(2):221–227.

Parragh SN, Doerner KF, Hartl RF (2008) A survey on pickup and delivery problems. *Journal für Betriebswirtschaft* 58(1):21–51, URL `http://dx.doi.org/10.1007/s11301-008-0033-7`.

Savelsbergh MW, Sol M (1995) The general pickup and delivery problem. *Transportation Science* 29(1):17–29, URL `http://dx.doi.org/10.1287/trsc.29.1.17`.

Vis IF (2006) Survey of research in the design and control of automated guided vehicle systems. *European Journal of Operational Research* 170(3):677–709, URL `http://dx.doi.org/https://doi.org/10.1016/j.ejor.2004.09.020`.