

Unit 8

Introduction to Linear Programming

8.1 Industry and life is optimization

Optimization is key to any industrial process. This is because optimization pursues the best way to achieve a particular **objective** (or multiple objectives) related to that process under a number of resource or other **constraints**. Optimization is key to life as well, as people (typically) strive to achieve the best for their lives under a number of personal and environmental constraints. Most optimization problems involve three main elements, which are listed below.

- The **decision variables**
- The **objective function**
- The **constraints**
- A **feasible solution**
- An **infeasible solution**
- An **optimal solution**

8.2 Linear Optimization Problems

8.3 Motivating Examples

8.4 The Standard Form a LPP

8.5 Basic Feasible Solutions and Optimality

8.6 A Clever Partition

8.7 The Simplex Method

8.8 Exercises

Exercise 8.1. Create a function in Python that receives a ‘tableau’ with $\tilde{b} \geq 0$, and returns the resulting ‘tableau’ of conducting the iteration process of the Simplex method.

Exercise 8.2. Consider the tableau shown in Table 8.1. Conduct Simplex iterations to solve the associated linear optimization problem. What is the optimal solution found?

	1	x_5	x_2	x_4
z	-3	21	-1	-1/2
x_3	6	3	-1	-2
x_1	3	-1	0	1

Table 8.1: Tableau for Exercise 8.2

Exercise 8.3. Consider the Table 8.2. Conduct the Simplex method.

	1	x_1	x_2
z	0	-1	-1
x_3	12	-2	-1
x_4	-3	1	0

Table 8.2: Tableau for Exercise 8.3

Exercise 8.4. Conduct Simplex method to solve the Gasoline-Mixture Problem stated in subsection 2.1.3 of [1].

Exercise 8.5. Considering the following linear optimization problem:

$$\begin{aligned} \max_{x_1, x_2} \quad & z = x_1 + 2x_2 \\ \text{s.t.} \quad & 2x_1 + x_2 \leq 12 \\ & x_1 - x_2 \geq 2 \\ & x_1, x_2 \geq 0, \end{aligned}$$

1. Conduct a geometric analysis: find the feasible region, find the level sets of the objective function and find the optimal solution.
2. Conduct the Simplex method.

Bibliography

- [1] SIOSHANSI, R., AND CONEJO, A. J. *Optimization in Engineering: Models and Algorithms*. Springer Optimization and Its Applications. Springer, 2017.