School of Computer Science
Faculty of Science
National University of Engineering

Test 4

Topics: convex sets; convex functions; optimality conditions; Iterative Solution Algorithm for Unconstrained NLPPs
Subject: Computational Mathematics **Period: 2021-1**

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1. Let $A, B \subseteq \mathbb{R}^p$ and $C \subseteq \mathbb{R}^q$ be convex sets and let $\alpha \in \mathbb{R}$. Then show that
   (a) (2 pts.) $A + B$ is convex and
   (b) (2 pts.) αA is convex.
2. Let $f : [1, \infty) \to \mathbb{R}$, given by $f(x) = -\sqrt{x-1}$.
   (a) (2 pts.) Determine whether f is convex. The assertion must be proven.
   (b) (2 pts.) Determine whether f has a global minimum. The assertion must be proven.
   (c) (1 pt.) Sketch the graph of f.
3. Consider the following unconstrained NLOP:

$$\min_{(x_1, x_2) \in \mathbb{R}^2} \quad f(x_1, x_2),$$

where $f(x_1, x_2)$ is defined by

$$x_1{}^2 + 3 \cdot x_1 \cdot x_2 + x_2{}^2.$$

In a .ipynb file using only python code:

(a) (1 pt.) Plot the graph of f over an appropriate domain.

(b) (2 pts.) Express $f(x)$ in the form

$$\frac{1}{2} \cdot x^T \cdot Q \cdot x,$$

where Q is a 2×2 symmetric matrix, and determine the eigen values λ_{min} and λ_{max} (with $\lambda_{min} \leq \lambda_{max}$) of Q.

(c) (3 pts.) Let u_{min} be a nonzero eigenvector associated with λ_{min}, and taking as initial point $x^0 := u_{min}$, conduct iterations of the Generic Algorithm for Unconstrained Nonlinear Optimization Problems using the steepest descent search direction and an exact line search to solve this problem.

(d) (2 pts.) Is the point at which the algorithm terminates guaranteed to be a local or global optimum? Why?

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