



**School of Computer Science**  
**Faculty of Science**  
**National University of Engineering**

## **Final Exam**

**Topics: Simplex method; convexity; constrained NLPP**

**Subject: Computational Mathematics**

**Period: 2021-1**

---

1. Gulf Coast Electronics is ready to award contracts for printing its annual report. For the past several years, the four-color annual report has been printed by Johnson Printing and Lakeside Litho. A new firm, Benson Printing, inquired into the possibility of doing a portion of the printing. The quality and service level provided by Lakeside Litho has been extremely high; in fact, only 0.5% of Gulf Coast's annual reports have had to be discarded because of quality problems. Johnson Printing has also had a high quality level historically, producing an average of only 1% unacceptable reports. Because Gulf Coast Electronics has had no experience with Benson Printing, it estimated Benson's defective rate to be 10%. Gulf Coast would like to determine how many reports should be printed by each firm to obtain 75,000 acceptable-quality reports. To ensure that Benson Printing will receive some of the contract, management specified that the number of reports awarded to Benson Printing must be at least 10% of the volume given to Johnson Printing. In addition, the total volume assigned to Benson Printing, Johnson Printing, and Lakeside Litho should not exceed 30,000, 50,000, and 50,000 copies, respectively. Because of the long-term relationship with Lakeside Litho, management also specified that at least 30,000 reports should be awarded to Lakeside Litho. The cost per copy is

\$2.45 for Benson Printing, \$2.50 for Johnson Printing, and \$2.75 for Lakeside Litho.

- (a) (4 pts.) Formulate and solve a linear program for determining how many copies should be assigned to each printing firm to minimize the total cost of obtaining 75,000 acceptable-quality reports. Program the algorithm studied in class.
- (b) (2 pts.) Suppose that the quality level for Benson Printing is much better than estimated. What effect, if any, would this quality level have?
- (c) (2 pts.) Suppose that management is willing to reconsider its requirement that Lakeside Litho be awarded at least 30,000 reports. What effect, if any, would this consideration have?

2. Let

$$C := \{(x, y) \in [0, +\infty) \times [0, +\infty)\}$$

and let  $f : C \rightarrow \mathbb{R}$ , given by

$$f(x, y) := -\frac{x \cdot y}{x + y}$$

if  $x > 0$  and  $y > 0$ , and  $f(x, y) = 0$  otherwise.

- (a) (1 pt.) Show that  $C$  is convex.
- (b) (3 pts.) Determine whether the function  $f$  is convex (on  $C$ .) Why?

3. The **Himmelblau function**  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  is defined by

$$f(x_1, x_2) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2.$$

- (a) (2 pts.) Plot the graphic of the Himmelblau function on  $D := [-6, 6] \times [-6, 6]$ .
- (b) (1 pt.) Give an argument based on the findings of the previous item in order to assert that the function is not convex on  $D$ .

- (c) (3 pts.) This function has four global minima where the function is equal to 0. In order to find them implement a function `solve` in Python that conducts iterations of the Generic Algorithm for Unconstrained Nonlinear Optimization Problems using:
- i. the steepest descent search direction and the Armijo rule with  $\sigma = 0.1$ ,  $\beta = 0.1$  and  $s = 1$ ;
  - ii. a tolerance of  $10^{-13}$ ;
  - iii. this function must have a parameter in order to receive the starting point of the iteration process to be conducted.
- (d) (2 pts.) Call the `solve` function above giving the starting points:  $(-4, -4)$ ,  $(-4, 4)$ ,  $(0, 0)$  and  $(4, 0)$ . All the iterations of each process must be printed.

August 4, 2021