School of Computer Science
Faculty of Science
National University of Engineering

Test 4

Topics: convex sets; convex functions; optimality conditions; Iterative Solution Algorithm for Unconstrained NLPPs
Subject: Computational Mathematics **Period: 2020-2**

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1. Let a in $\mathbb{R}^{\mathrm{n}}$, d in $\mathbb{R}^{\mathrm{n}} \setminus \{0\}$, c in $\mathbb{R}$ and r be a positive number. Show whether the following sets are convex or not:
   (a) (1 pt.) $\mathcal{H}(d, c) := \{v \in \mathbb{R}^n \ ; \ \langle v, d \rangle = c\}$;
   (b) (2 pts.) $Hole := \{v \in \mathbb{R}^n \ ; \ |v - a| \geq r\}$;
   (c) (2 pts.) $X := \{v \in \mathbb{R}^n \ ; \ A \cdot v \leq b\}$, where A is an $m \times n$ matrix and b is in $\mathbb{R}^m$.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be such that $f(x) = \ln(1 + e^x)$.
   (a) (2 pts.) Determine whether f is convex. The assertion must be proven.
   (b) (2 pts.) Determine whether f has a global minimum. The assertion must be proven.
   (c) (1 pt.) Sketch the graph of f.

3. Consider the following unconstrained NLOP:

$$\min_{(x_1, x_2) \in \mathbb{R}^2} \quad f(x_1, x_2),$$

where $f(x_1, x_2)$ is defined by

$$14\cdot\sqrt{(x_1-7)^2+(x_2-2)^2}+20\cdot\sqrt{(x_1-5)^2+(x_2+3)^2}+30\cdot\sqrt{(x_1+6)^2+(x_2-4)^2}.$$

In a .ipynb file using only python code:

(a) (3 pts.) Plot the graph of f over an appropriate domain.

(b) (5 pts.) Starting from the point $x^0 := (0,0)$ conduct iterations of the Generic Algorithm for Unconstrained Nonlinear Optimization Problems using the steepest descent search direction and an exact line search to solve this problem.

(c) (2 pts.) Is the point at which the algorithm terminates guaranteed to be a local or global optimum? Why? (only for this it is not necessary to use python code.)

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