

Diketahui $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$

Jika $f(x) = x^3 - 3x^2 - 2x + 5I_2$ maka tentukanlah $f(A)$

$$= 3 \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} - 2 \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 3 \begin{bmatrix} -1 \cdot -1 + (-2) \cdot 3 & (-1) \cdot 2 + (-1) \cdot 2 \\ 3 \cdot 1 + 1 \cdot 3 & 3 \cdot (-1) + 1 \cdot 2 \end{bmatrix} - \begin{bmatrix} 2 \cdot -1 & 2 \cdot 2 \\ 2 \cdot 3 & 2 \cdot 1 \end{bmatrix} + \begin{bmatrix} 5 \cdot 1 & 5 \cdot 0 \\ 5 \cdot 0 & 5 \cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 4 \\ 6 & 5 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -5 - (-2) + 5 & 4 - 4 + 0 \\ 6 - 6 + 0 & 5 - 2 + 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$$

2, Hitunglah Determinan dari matriks

$$\begin{bmatrix} 2 & 1 & 0 & -1 \\ -1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 1 \\ 3 & -2 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 0 & -1 \\ -1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 1 \\ 3 & -2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 & 1 \\ -1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 1 \\ 3 & -2 & 2 & 0 \end{bmatrix}$$

$$= (2 \times 0 \times -1 \times 0) - (1 \times 1 \times 1 \times 3) + (0 \times 2 \times 0 \times -2) - (-1 \times -1 \times 1 \times 2) -$$

$$(2 \times 2 \times -1 \times 2) + (1 \times -1 \times 1 \times 2) - (0 \times 0 \times 0 \times 0) + (1 \times 1 \times 1 \times 3)$$

$$= 0 - 3 + 0 - 2 - 8 + (-2) - 0 + 3$$

$$= -12$$

$$A = \begin{bmatrix} 2 & -1 & -2 \\ 1 & -3 & 1 \\ -2 & 2 & 3 \end{bmatrix}$$

$$2. |A| = -1 \cdot (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} + (-3) \cdot (-1)^{2+2} \begin{vmatrix} 2 & -2 \\ -2 & 3 \end{vmatrix} + 2 \cdot (-1)^{3+2} \begin{vmatrix} 2 & -2 \\ 1 & 1 \end{vmatrix}$$

$$= (3 - (-2)) + (-3) \cdot (6 - 4) + (-2) \cdot (2 + 2)$$

$$= 5 + (-6) + (-8)$$

$$= -9$$

3, Diketahui Matriks $A = \begin{bmatrix} 2 & -1 & -2 \\ 1 & -3 & 1 \\ -2 & 2 & 3 \end{bmatrix}$

$\text{adj } A = (K_A)^t$

Koefaktor dari $2 = A_{11} = \begin{vmatrix} -3 & 1 \\ 2 & 3 \end{vmatrix} = -11$

$-1 = A_{12} = \begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} = 5$

~~$-2 = A_{13}$~~

$-2 = A_{13} = \begin{vmatrix} 1 & -3 \\ -2 & 2 \end{vmatrix} = -4$

$-2 = A_{31} = \begin{vmatrix} 1 & -3 \\ -2 & 2 \end{vmatrix} = -4$

$2 = A_{32} = \begin{vmatrix} 2 & -1 \\ -2 & 2 \end{vmatrix} = 2$

$1 = A_{21} = \begin{vmatrix} -1 & -2 \\ 2 & 3 \end{vmatrix} = 1$

$3 = A_{33} = \begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix} = -5$

$-3 = A_{22} = \begin{vmatrix} 2 & -2 \\ -2 & 3 \end{vmatrix} = 2$

$1 = A_{23} = \begin{vmatrix} 2 & -1 \\ -2 & 2 \end{vmatrix} = 2$

~~$\text{adj } A = \begin{bmatrix} 5 & -4 & -1 \end{bmatrix}^t$~~

$\text{adj } A = \begin{bmatrix} -11 & 5 & -4 \\ 1 & 2 & 2 \\ -4 & 2 & -5 \end{bmatrix} = \begin{bmatrix} -11 & -5 & -4 \\ -1 & 2 & -2 \\ -4 & -2 & -5 \end{bmatrix}$

1/ Tentukan matrik invers dengan metode penjumlahan dari matriks

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & -2 & 2 \\ 3 & 2 & -1 \end{bmatrix}$$

$$\therefore \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ -1 & -2 & 2 & 0 & 1 & 0 \\ 3 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} \\ 2R_2 + R_1 \\ R_3 - 3R_1 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & -3 & 1 & 1 & 1 & 0 \\ 0 & 5 & 2 & 3 & 0 & 1 \end{array} \right] \begin{array}{l} 3R_1 - R_2 \\ \\ -3R_3 - 5R_2 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 3 & 0 & -2 & 2 & -1 & 0 \\ 0 & -3 & 1 & 1 & 1 & 0 \\ 0 & 0 & -11 & -14 & -5 & -1 \end{array} \right] \begin{array}{l} 11R_3 + -2R_3 \\ -11R_2 - R_3 \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 33 & 0 & 0 & 50 & -1 & 2 \\ 0 & 33 & 0 & 3 & 6 & 1 \\ 0 & 0 & -11 & -14 & -5 & -1 \end{array} \right] \begin{array}{l} \frac{1}{33} R_1 \\ \frac{1}{33} R_2 \\ -\frac{1}{11} R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 50/33 & -1/33 & 2/33 \\ 0 & 1 & 0 & 1/11 & 2/11 & 1/11 \\ 0 & 0 & 1 & -14/11 & -5/11 & -1/11 \end{array} \right]$$

$$\begin{aligned} \text{Dröbetanvi} &= x_1 - 2x_2 + 2x_3 = -1 \\ &-2x_1 - x_2 + 2x_3 = 3 \\ &x_1 + 2x_2 - 3x_3 = -2 \end{aligned}$$

$$= A_1 = \begin{bmatrix} -1 & -2 & 2 \\ 3 & -1 & 2 \\ -2 & 2 & 3 \end{bmatrix} \rightarrow \det A_1 = 25$$

$$= A_2 = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 2 \\ 1 & -2 & -3 \end{bmatrix} \rightarrow \det A_2 = 1$$

$$= A_3 = \begin{bmatrix} 1 & -2 & -1 \\ -2 & -1 & 3 \\ 1 & 2 & -2 \end{bmatrix} \rightarrow \det A_3 = 1$$

$$= A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & -1 & 2 \\ 1 & 2 & -3 \end{bmatrix} \rightarrow \det A = 1$$

$$x_1 = \frac{\det A_1}{\det A} = \frac{25}{1} = 25 //$$

$$x_2 = \frac{\det A_2}{\det A} = \frac{1}{1} = 1$$

$$x_3 = \frac{\det A_3}{\det A} = \frac{1}{1} = 1$$

