Financial Analytics

Simple Rate of Return

- Simple Rate of Return is defined as the net gain or loss on an investment over a specified time period, expressed as a percentage of the investment's initial cost.

$$\frac{(CurrentValue - InitialValue) * 100}{(InitialValue)}$$

$$\frac{P_1 - P_0}{P_0} = \frac{P_1}{P_0} - 1$$

Log Returns

- As time progresses, the investments that we take part in increase and earn a compounded return. For bonds, the compound interest is fixed but for stocks, it varies. To calculate this continuously compounded interest, natural logarithm function is used by investors.
- For bonds, the formula of log return is given by ln(1 + stated rate of interest)
- For stocks, this becomes

$$ln(\frac{Current Price}{Original Price})$$

$$ln(\frac{P_t}{P_{t_0}})$$

- **Calculating Daily Average Returns**
- Daily Average Returns are given by computing the mean of the log rate of return series.
- **Calculating Annual Average Returns**
- Annual Average Returns are given by computing the mean of the log rate of return series and then multiplying the value by 250 since 250 days exist in a business day system.

Portfolios

- Portfolios are groups of assets, such as stocks and bonds, that are held by an investor.
- One convenient way to describe a portfolio is by listing the proportion of the total value of the portfolio that is invested into each asset.
- These proportions are called portfolio weights.
  - Portfolio weights are sometimes expressed in percentages.
  - However, in calculations, make sure you use proportions (i.e., decimals).

Portfolio Returns

- Suppose, we have three assets A_1, A_2 and A_3. The weights associated with these assets are w_1, w_2 and w_3. Also, the returns associated with these assets are r_1, r_2 and r_3. Let the cumulative return be represented as R. Thus, R is given by
- $R = w_1 r_1 + w_2 r_2 + w_3 r_3$
- If there are N assets, the same R is given by
- $R = \Sigma w_n r_n$
- where n varies from 1 to N.

Markowitz Model

- Portfolio theory published by Harry Markowitz in 1952 helps us in the formulation of portfolio theory. Following are the highlights of this theory:
- Rather than putting everything on a single place as one investment, he emphasized on creating a diverse portfolio from different industries and then optimizing the risk involved in it.
- These industries shouldn't be having correlations between them or the correlations between them should be minimum in order to prevent additional risks from coming up. This optimized portfolio leads to higher returns also.

Why Diversification Works, I.

- **Correlation:** The tendency of the returns on two assets to move together. Imperfect correlation is the key reason why diversification reduces portfolio risk as measured by the portfolio standard deviation.

- ***Positively*** correlated assets tend to move up and down together.
- ***Negatively*** correlated assets tend to move in opposite directions.

- Imperfect correlation, positive or negative, is why diversification reduces portfolio risk.

- Combining stocks into portfolios can reduce standard deviation below the level obtained from a simple weighted average calculation.
- Correlation coefficients make this possible.
- The various weighted combinations of stocks that create this standard deviations constitute the set of *efficient portfolios*.

- Let’s assume there are two shares:
- Share X – Expected return = 8%; Standard deviation = 4%
- Share Y – Expected return = 10%; Standard deviation = 3%
- The correlation between share X and share Y = 0.2 or 20%. We know that 0.2(20%) is very far away from 1(100%). The positive correlation denotes that as share X increases, share Y also increases and as share X decreases, share Y decreases but this relation is very small.

Portfolio	Weight of Share X	Weight of Share Y	Expected return	Standard deviation (Risk involved)
Portfolio 1	100%	0%	?	?
Portfolio 2	75%	25%	?	?
Portfolio 3	50%	50%	?	?
Portfolio 4	25%	75%	?	?
Portfolio 5	0%	100%	?	?

Expected return

$r = w_X r_X + w_Y r_Y$

Portfolio	Weight of Share X	Weight of Share Y	Expected return	Standard deviation (Risk involved)
Portfolio 1	100%	0%	1 x 0.08 + 0 x 0.1	?
Portfolio 2	75%	25%	0.75 x 0.08 + 0.25 x 0.1	?
Portfolio 3	50%	50%	0.5 x 0.08 + 0.5 x 0.1	?
Portfolio 4	25%	75%	0.25 x 0.08 + 0.75 x 0.1	?
Portfolio 5	0%	100%	0 x 0.08 + 1 x 0.1	?

Portfolio	Weight of Share X	Weight of Share Y	Expected return	Standard deviation (Risk involved)
Portfolio 1	100%	0%	8%	?
Portfolio 2	75%	25%	8.5%	?
Portfolio 3	50%	50%	9%	?
Portfolio 4	25%	75%	9.5%	?
Portfolio 5	0%	100%	10%	?

Standard deviation

$$s_C^2 = (w_X s_X + w_Y s_Y)^2 = w_X^2 s_X^2 + w_Y^2 s_Y^2 + 2 w_X s_X w_Y s_Y p$$

where w_X is the weight of share X, w_Y is the weight of share Y, s_X is the standard deviation of share X, s_Y is the standard deviation of share Y, p is correlation between share X and share Y.

We have

- Share X – Expected return = 8%; Standard deviation = 4%
- Share Y – Expected return = 10%; Standard deviation = 3%

  The correlation between share X and share Y = 0.2 or 20%

Standard deviation

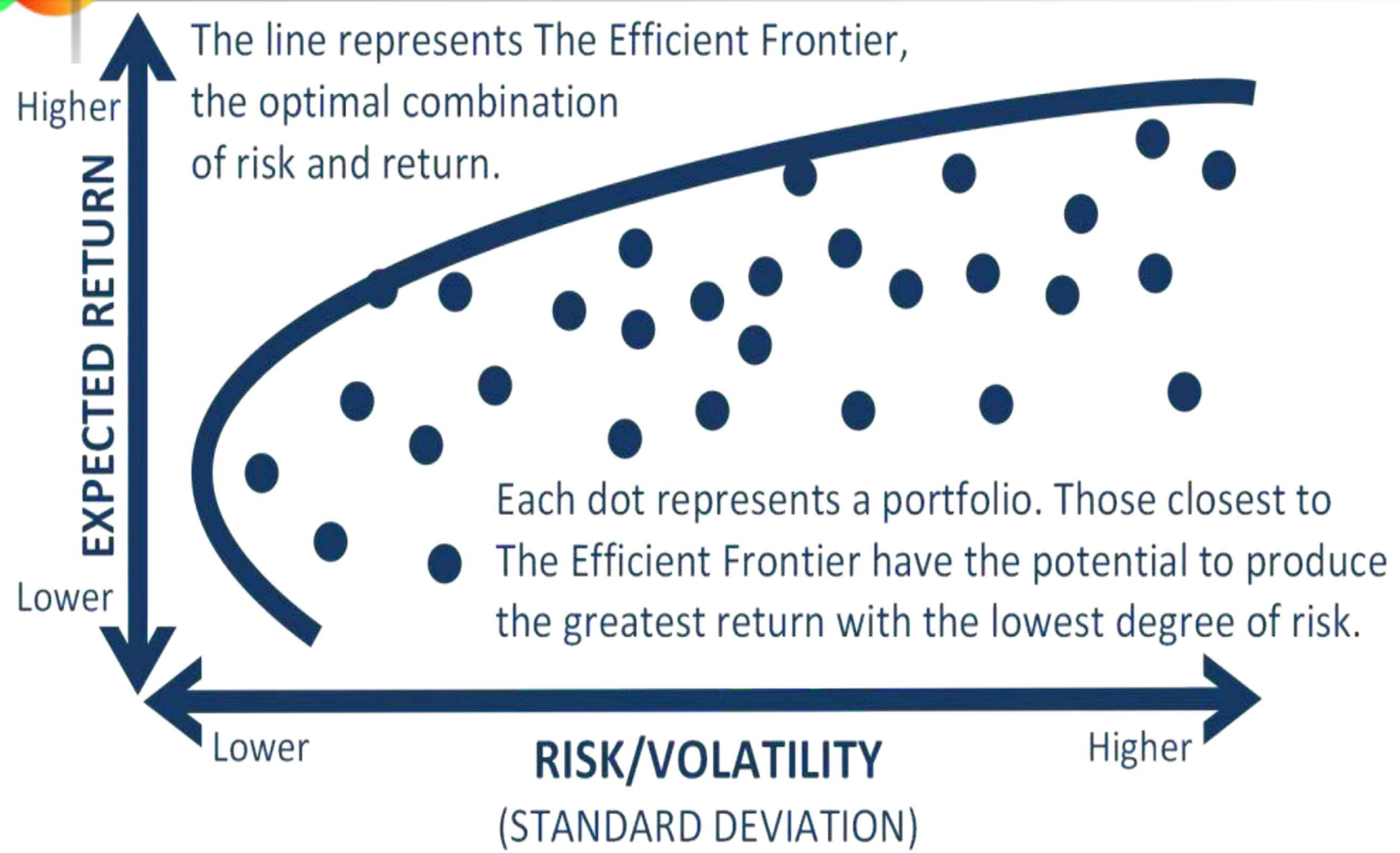
Portfolio	Weight of Share X	Weight of Share Y	Expected return	Standard deviation (Risk involved)
Portfolio 1	100%	0%	8%	4%
Portfolio 2	75%	25%	8.5%	3.16%
Portfolio 3	50%	50%	9%	2.7%
Portfolio 4	25%	75%	9.5%	2.6%
Portfolio 5	0%	100%	10%	3%

Markowitz Efficient frontier

- The Markowitz Efficient frontier is the set of portfolios with the maximum return for a given risk AND the minimum risk given a return.

- For the plot, the upper left-hand boundary is the Markowitz efficient frontier.

- All the other possible combinations are inefficient. That is, investors would not hold these portfolios because they could get either
  - more return for a given level of risk, or
  - less risk for a given level of return.



The line represents The Efficient Frontier,
the optimal combination
of risk and return.
Each dot represents a portfolio. Those closest to
The Efficient Frontier have the potential to produce
the greatest return with the lowest degree of risk.
Higher
EXPECTED RETURN
Lower
Lower
RISK/VOLATILITY
(STANDARD DEVIATION)
Higher

- The chart above shows a **hyperbola showing all the outcomes for various portfolio** combinations of risky assets, where Standard Deviation is plotted on the X-axis and Return is plotted on the Y-axis.
- **Tangency Portfolio** is the point where the **portfolio of only risky assets meets the combination of risky and risk-free assets.** This portfolio maximizes return for the given level of risk.
- Portfolio along the lower part of the hyperbola will have lower return and eventually higher risk. Portfolios to the right will have higher returns but also higher risk.

Basic Parameters

- **Portfolio Expected Return**

  - Expected return:

$$\mathrm{E}(R_p) = \sum_i w_i \, \mathrm{E}(R_i)$$

- **Portfolio Variance**

$$\sigma_{Portfolio} = \sqrt{w_T \cdot \Sigma \cdot w}$$

- $\sigma_{Portfolio}$: Portfolio volatility
- Σ: Covariance matrix of returns
- w: Portfolio weights (w_T is transposed portfolio weights)
- · The dot-multiplication operator

Basic Parameters

- **Sharpe Ratio**
- The Sharpe ratio measures the return of an investment in relation to the risk-free rate (Treasury rate) and its risk profile. In general, a higher value for the Sharpe ratio indicates a better and more lucrative investment. Thus if comparing two portfolio’s with similar risk profiles, given all else equal it would be better to invest in the portfolio with a higher Sharpe Ratio.

Sharpe Ratio = K * (average return – risk free rate) / standard deviation of return

An annual value:

K = SQRT(250) if we sample the portfolio on every trading day.