CSC343 Assignment 3

Quan Xu, Zhiyang Yu, Zijin Zhang

Due: Friday, April 5th, 11:59 p.m.

Database Design

1.

(a)

$B^+ = BD$ # By $B \to D$

$BC^+ = ABCDEF$ # By $BC \to A, B \to D, AD \to E, E \to F$

$E^+ = EF$ # By $E \to F$

$AB^+ = ABCDEF$ # By $AB \to C, B \to D, AD \to E, E \to F$

$AC^+ = ACBDEF$ # By $AC \to, B \to D, AD \to E, E \to F$

$AD^+ = ADEF$ # By $AD \to E, E \to F$

Since there is no functional dependencies involve G, G is trivial and we should add G to BC, AB, AC in order to get $R(A, B, C, D, E, F, G)$.
Therefore, the candidate keys is BCG, ABG, ACG.

(b)

(1) Check if we can reduce any FD
- $BC \to A$ cannot be reduced since $B^+ = BD, C^+ = C$ and neither of them have attribute A.
- $AB \to C$ cannot be reduced since $A^+ = A, B^+ = BD$ and neither of them have attribute C.
- $AC \to B$ cannot be reduced since $A^+ = A, C^+ = C$ and neither of them have attribute B.
- $AD \to E$ cannot be reduced since $A^+ = A, D^+ = D$ and neither of them have attribute E.
So, none of the FD can be reduced.

(2) Check if we can remove any FD

- $B \to D$ cannot be removed since $B^+ = B$ and it does not have attribute D if $B \to D$ is removed.
- $BC \to A$ cannot be removed since $BC^+ = BCD$ and it does not have attribute A if $BC \to A$ is removed.
- $E \to F$ cannot be removed since $E^+ = E$ and it does not have attribute F if $E \to F$ is removed.
- $AB \to C$ cannot be removed since $AB^+ = ABDEF$ and it does not have attribute C if $AB \to C$ is removed.
- $AC \to B$ cannot be removed since $AC^+ = AC$ and it does not have attribute B if $AC \to B$ is removed.
- $AD \to E$ cannot be removed since $AD^+ = AD$ and it does not have attribute E if $AD \to E$ is removed.

So, none of the FD can be removed.

Therefore, the original set FD of functional dependencies $\{B \to D, BC \to A, E \to F, AB \to C, AC \to B, AD \to E\}$ is a minimal cover for FD.

(c)

No, the relation $R(A, B, C, D, E, F, G)$ is not in $BCNF$ by the given set FD of functional dependencies $\{B \to D, BC \to A, E \to F, AB \to C, AC \to B, AD \to E\}$. $BCNF$ require the LHS of an FD to be a superkey; however, $B^+ = BD$ which implies that B is not a superkey and $B \to D$ violates $BCNF$.

We provide a $BCNF$ decomposition for R
- Decompose R using $AD \to E, AD^+ = ADEF$. So this yields two relationships: $R_1 = ADEF, R_2 = ABCDG$

- Project the FDs onto $R_1 = ADEF$

A	D	E	F	Closure	FDs
✓				$A^+ = A$	Nothing
	✓			$D^+ = D$	Nothing
		✓		$E^+ = EF$	$E \to F$, violates $BCNF$, abort the projection.

We need to decompose R_1 further.
- Decompose R_1 using FD: $E \to F$. This yields two relations $R_{11} = EF$ and $R_{12} = ADE$
- Project the FDs onto $R_{11} = EF$

E	F	Closure	FDs
✓		$E^+ = EF$	$E \to F$, E is the superkey of R_{11}
	✓	$F^+ = F$	Nothing

This relation R_{11} satisfies $BCNF$.

- Project the FDs onto $R_{12} = ADE$

A	D	E	Closure	FDs
✓			$A^+ = A$	Nothing
	✓		$D^+ = D$	Nothing
		✓	$E^+ = EF$	Nothing
✓	✓		$AD^+ = ADEF$	$AD \to E$, AD is superkey of R_{12}
✓		✓	$AE^+ = AEF$	Nothing
	✓	✓	$DE^+ = DE$	Nothing

This relation R_{12} satisfies $BCNF$.

- Project the FDs onto $R_2 = ABCDG$

A	B	C	D	G	Closure	FDs
✓					$A^+ = A$	Nothing
	✓				$B^+ = BD$	$B \to D$, violates $BCNF$, abort the projection.

We need to decompose R_2 further.
- Decompose R_2 by using FD: $B \to D$.
- This yields two relations $R_{21} = BD$ and $R_{22} = ABCG$

- Project the FDs onto $R_{21} = BD$

B	D	Closure	FDs
✓		$B^+ = BD$	$B \to D$, B is the superkey of R_{21}
	✓	$D^+ = D$	Nothing

This relation R_{21} satisfies $BCNF$.

- Project the FDs onto $R_{22} = ABCG$

A	B	C	G	Closure	FDs
✓				$A^+ = A$	Nothing
	✓			$B^+ = BD$	Nothing
		✓		$C^+ = C$	Nothing
			✓	$G^+ = G$	Nothing
✓	✓			$AB^+ = ABCDEF$	$AB \to C$, violates $BCNF$, abort the projection

We need to decompose R_{22} further.
- Decompose R_{22} using DF: $AB \to C$.
- This yield two relations: $R_{221} = ABC$ and $R_{222} = ABG$

- Project FDs onto $R_{221} = ABC$

A	B	C	Closure	FDs
✓			$A^+ = A$	Nothing
	✓		$B^+ = BD$	Nothing
		✓	$C^+ = C$	Nothing
✓	✓		$AB^+ = ABCDEF$	$AB \to C$, AB as superkey of R_{221}
✓		✓	$AC^+ = ABCDEF$	$AC \to B$, AC as superkey of R_{221}
	✓	✓	$BC^+ = ABCDEF$	$BC \to A$, BC as superkey of R_{221}

- The relation R_{221} satisfies $BCNF$

- Project FDs onto $R_{222} = ABG$

A	B	G	Closure	FDs
✓			$A^+ = A$	Nothing
	✓		$B^+ = BD$	Nothing
		✓	$G^+ = G$	Nothing
✓	✓		$AB^+ = ABCDEF$	Nothing
✓		✓	$AG^+ = AG$	Nothing
	✓	✓	$BG^+ = BDG$	Nothing

- The relation R_{222} satisfies $BCNF$

- Find Decomposition:
  (1)R_{11} with FD: $\{E \rightarrow F\}$
  (2)R_{12} with FD: $\{AD \rightarrow E\}$
  (3)R_{21} with FD: $\{B \rightarrow D\}$
  (4)R_{221} with FD: $\{AB \rightarrow C, AC \rightarrow B, BC \rightarrow A\}$
  (5)R_{222} with FD: $\{\}$

(d)

No, the relation $R(A, B, C, D, E, F, G)$ is not in $3NF$ by the given set FD of functional dependencies $\{B \rightarrow D, BC \rightarrow A, E \rightarrow F, AB \rightarrow C, AC \rightarrow B, AD \rightarrow E\}$. By the definition of $3NF$, $X \rightarrow A$ violates $3NF$ if X is not a superkey or A is not a prime(a memeber of any key). So the FD: $B \rightarrow D$ violates $3NF$ since B is not a superkey and D is not a prime. (candidate keys: $\{BCG, ABG, ACG\}$).

We provide a $3NF$ decomposition for R
- By part (b), the minimal cover is:
  - $B \rightarrow D$
  - $BC \rightarrow A$
  - $E \rightarrow F$
  - $AB \rightarrow C$
  - $AC \rightarrow B$
  - $AD \rightarrow E$
- The set of relations that would result would have these attributes correspondingly:
  - $R_1(B, D)$
  - $R_2(A, B, C)$
  - $R_3(E, F)$
  - $R_4(A, B, C)$
  - $R_5(A, B, C)$
  - $R_6(A, D, E)$
- Since the attributes A, B, C occurs within relation R_2, we do not need to keep the relations R_4 and R_5. So,
  - $R_1(B, D)$

- $R_2(A, B, C)$
- $R_3(E, F)$
- $R_6(A, D, E)$

- Since R_1, R_2, R_3, R_6 do not contain a key of R, so we need to add another relation that contain a key:
  - $R_7(A, B, G)$ with no FD
- So the final set of relation is :
  - $R_1(B, D)$
  - $R_2(A, B, C)$
  - $R_3(E, F)$
  - $R_6(A, D, E)$
  - $R_7(A, B, G)$

2.

WTS :
1) S is in $BCNF \implies S$ is in $3NF$.
2) S is in $3NF \implies S$ is in $BCNF$.

Proof. (1)
Assume that relation S is in $BCNF$.
Since it is given that relation S has only one-attribute keys, we know that there is only one key in relation S and denotes it as K. By our assumption and according to the definition of $BCNF$, we know that every attributes of S should only depend on the whole key, in our case, K. So for any attribute A is relation S other than K, we have $K \rightarrow A$. Since K is the key of S, K must also be a superkey of S and A cannot be a prime since A is the attribute other than K. Thus, we have $K \rightarrow A$ such that K is a superkey of S and A is not a prime. Therefore, the relation S must be in $3NF$.

□

Proof. (2)
Assume that relation S is in $3NF$.
According to the definition of $3NF$, we know that the dependency function $X \rightarrow A$ could have following two situation:
1) X is a superkey and A is any other attribute in the relation
2) X is not the superkey but A is a prime(a member of any key)

Since it is given that relation S has only one-attribute keys, we know that there is only one key in relation S and denotes it as K. So in situation (1), X must be the only key K. For situation (2), A must be the only key K; however, this situation is not realistic since a key should not depend on any non-key attribute. Thus, we have dependency function $K \rightarrow A$ which K is the only key in relation S and A is any other attribute. This means that every attributes of S is dependent on the only key K for this relation. Therefore, the realtion S is also in $BCNF$.

□

Entity-Relationship Model