Assignment 2

Autonomous Systems

(Report)

Submitted By:

Anirban Maitra (M23CSA005)

Submitted To:

Dr. Gaurav Harit
Assistant Professor
Department of Computer Science
and Engineering

INDEX

1. Objective:

This report presents the implementation and analysis of a combinatorial auction scenario involving advertising space on lecture slides, utilising the Vickrey-Clarke-Groves (VCG) auction mechanism. The problem includes defining the revenue generated under VCG based on the bidders' types, examining the strategic implications of identity manipulation, and evaluating an alternative auction mechanism to compare its revenue and truthfulness properties.

We aim to:

1. Implement the VCG auction mechanism and compute revenue based on different bidding scenarios.
2. Demonstrate that VCG is not dominant strategy truthful when agents can submit bids using multiple identities.
3. Analyse an alternative auction mechanism and verify its properties of truthfulness and revenue generation.

2. Introduction to VCG:

Vickrey–Clarke–Groves (**VCG**) **mechanism** is a generic truthful mechanism for achieving a socially optimal solution. A VCG auction performs a specific task: dividing items among people. A VCG *mechanism* is more general: it can be used to select any outcome out of a set of possible outcomes.

Scenario Overview

The professor auctions off two advertising spaces:

1. Top Banner Space (t)
2. Sidebar Space (s)

Each agent's utility is defined by two parts:

- **Utility value** ($\mathbf{V}_i$) : The utility derived from winning the allocation.
- **Slot preference** ($fi = \{t, s, b\}$) : Indicates the preferred slot, where:
  - **t** : Prefers the top banner space.
  - **s** : Prefers the sidebar space.
  - **b** : Prefers both slots and does not want any other ad alongside its own.

VCG Mechanism

The VCG mechanism allocates goods to bidders in a way that maximises total value and charges each winning bidder based on the harm they cause to others (i.e., second-highest bids). In this context, we define:

This report addresses the revenue generated through the VCG mechanism, investigates the potential for identity manipulation among bidders, and evaluates an alternative auction mechanism.

Let:

- $\mathbf{v}_{1s}$: First-highest bid for sidebar space.
- $\mathbf{v}_{1t}$: First-highest bid for top banner space.
- $\mathbf{v}_{1b}$: First-highest bid for both spaces.
- $\mathbf{v}_{2s}$: Second-highest bid for sidebar space.
- $\mathbf{v}_{2t}$: Second-highest bid for top banner space.
- $\mathbf{v}_{2b}$: Second-highest bid for both spaces.

The total revenue generated by the auction depends on the configuration of bids. If a bidder wins both slots, they pay the second-highest combined bid. Otherwise, separate payments are made for the top banner and the sidebar.

Revenue Calculation

The revenue from the VCG auction is determined as:

- If a bidder wins both slots, the revenue is **v2b**.
- If separate bidders win the top and sidebar spaces, the revenue is **v2t + v2s**.

3. Implementation

Code Structure

We implemented the following scripts:

1. *vcg_auction.py* : Implements the VCG mechanism and calculates revenue.
2. *collusion_experiment.py* : Simulates agents colluding by creating multiple identities.

3. *alternative_mechanism.py* : Implements an alternative auction mechanism that allocates both goods to the highest bidder and charges the second-highest bid.

Input Data Format

The input data is provided in JSON format, where each bid is specified by the slot the agent is bidding for and the bid amount. Example structure:

```
[
    {"slot": "t", "value": 120},
    {"slot": "s", "value": 110},
    {"slot": "b", "value": 150},
    {"slot": "t", "value": 100},
    {"slot": "s", "value": 90}
     .
     .
     .
]
```

4. Results and Analysis:

Scenario 1 :- VGC Auction Mechanism

Assume we have 5 bidders in which 2 bidders are bidding for top banner space, 2 bidders bidding for side bar space and only 1 bidder bidding for both banner space, which can be seen below :

```
[
  {"slot": "t", "value": 100},      Bidder 1
  {"slot": "s", "value": 90},       Bidder 2
  {"slot": "b", "value": 150},      Bidder 3
  {"slot": "t", "value": 80},       Bidder 4
  {"slot": "s", "value": 85}        Bidder 5
]
```

The results that we achieved after performing VGC auction mechanism is :

Separate bidders win top banner and sidebar

Winning bid for top banner: 100

Winning bid for sidebar: 90

Total VCG Revenue: 165

Winning Bids Information:

Top banner won with bid: 100

Sidebar won with bid: 90

Analysis :-

Bidder 3 who bids for both the spaces (top banner space and side banner space) with bidding value of 150

Check [max(top banner bids) + max (side banner bids)] (< or >) Both banner space bid

Bidder 1 Bid + Bidder 2 Bid = (100 + 90) = 190

190 > 150 so Bidder 3 looses.

For top banners space the bids are [Bidder 1, Bidder 4] = [100, 80], so Bidder 1 wins the top banner space with expenditure of 80.

For side banner space the bids are [Bidder 2, Bidder 5] = [90, 85], so Bidder 2 wins the side banner space with expenditure of 85.

Now the VGC Revenue is [Bidder 1 Bid + Bidder 2 Bid] = 80 + 85 = 165.

Scenario 2 :- Collusion Experiment

The collusion experiment allows an agent to submit bids using two identities to influence the outcome of the auction. By slightly lowering the second bid through a fake identity, the agent may lower the price they pay for the winning slot.

Assume this time there are only 3 bidders among which 1 bidder is bidding for top banner space with 2 different identities and 2 different Bidding prices [100 and 80]. 1 Bidder is bidding for side banner space with 2 different identities and 2 different Bidding prices [120 and 90]. Only 1 bidding is placed for both the banner spaces by Bidder 3 at bidding price of [150].

[

{"slot": "t", "value": 100},	Bidder 1
{"slot": "t", "value": 80},	Bidder 1
{"slot": "s", "value": 120},	Bidder 2
{"slot": "s", "value": 90},	Bidder 2
{"slot": "b", "value": 150}	Bidder 3

]

The results that we achieved after performing VGC auction mechanism where bids are given by same Bidder with different identities is :

Separate bidders win top banner and sidebar

Winning bid for top banner: 100

Winning bid for sidebar: 120

Total VCG Revenue: 170.1

Winning Bids Information:

Separate bidders win top banner and sidebar

Winning bid for top banner: 100

Winning bid for sidebar: 120

Analysis :-

Bidder 3 who bids for both the spaces (top banner space and side banner space) with bidding value of 150

Check [max(top banner bids) + max (side banner bids)] (< or >) Both banner space bid

Bidder 1 Bid + Bidder 2 Bid = (100 + 120) = 220

220 > 150 so Bidder 3 looses.

For top banners space the bids are [Bidder 1, Bidder 1] = [100, 80], so Bidder 1 wins the top banner space with expenditure of 80.

For side banner space the bids are [Bidder 2, Bidder 2] = [120, 90], so Bidder 2 wins the side banner space with expenditure of 90.

Now the VGC Revenue is [Bidder 1 Bid + Bidder 2 Bid] = 80 + 90 = 170.

This shows that VCG is not dominant-strategy truthful in settings where agents can collude or create multiple identities. By splitting bids across multiple identities, the agent can reduce its payments and gain an unfair advantage, violating the truthfulness property of the VCG mechanism.

Scenario 3 :- Alternative Auction Mechanism

In alternative auction mechanism where multiple bidders compete for two advertising slots: a top banner and a sidebar. The auction allocates both slots to the highest bidder, and the winning bidder pays an amount equivalent to the second-highest bid, regardless of individual preferences for the slots.

Assume we have 5 bidders in which 2 bidders are bidding for top banner space, 2 bidders bidding for side bar space and only 1 bidder bidding for both banner space, which can be seen below :

```
[
   {"slot": "t", "value": 100},
   {"slot": "s", "value": 90},
   {"slot": "b", "value": 150},
   {"slot": "t", "value": 80},
   {"slot": "s", "value": 85}
]
```

{"slot": "t", "value": 100},	Bidder 1
{"slot": "s", "value": 90},	Bidder 2
{"slot": "b", "value": 150},	Bidder 3
{"slot": "t", "value": 80},	Bidder 4
{"slot": "s", "value": 85}	Bidder 5

The results that we achieved after performing Alternative Auction Mechanism is :

Highest bidder wins with bid: 150

Amount paid (second-highest bid): 100

Total Revenue under Alternative Mechanism: 100

Analysis :-

Highest Bidder is Bidder 3 with bidding value of 150

Second Highest Bidder is Bidder 1 with bidding value of 100

So Bidder 3 wins the Auction at the expenditure of 100, and Revenue generate is only 100 but in case of VGC auction mechanism the revenue was 165.

The alternative mechanism incentivises truthful bidding because:

- ❖ If a bidder reports a value higher than their true value, they risk overpaying if they win.
- ❖ If a bidder reports a lower value, they may loose the auction even if they could have won at their true value.

5. Revenue Comparison

❖ The alternative mechanism guarantees that the revenue generated will be at least half of what the VCG mechanism generates. This happens because:

❖ In both mechanisms, the winning bidder pays the second-highest bid.

❖ The alternative mechanism simply disregards any bidder preferences but still charges the second-highest bid, which is also true for the VCG mechanism in some cases.

6. Conclusion

This experiment demonstrated that the VCG mechanism, while designed to encourage truthful bidding, can be manipulated when agents create fake identities. The alternative mechanism, where the highest bidder wins and pays the second-highest bid, ensures truthfulness but results in lower revenue than the VCG auction. However, it still generates at least half the revenue of the truthful VCG auction, as expected.

This analysis highlights the importance of designing auction mechanisms that are robust to manipulation while balancing fairness and revenue generation.

7. References

- *Weiss, G. (2013). Multiagent Systems: A Modern Approach to Distributed Artificial Intelligence. MIT Press.*
- *Vickrey, W. (1961). Counterspeculation, Auctions, and Competitive Sealed Tenders. Journal of Finance.*