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Poster

Fast simulation of viscous lava flow using Green's functions as a smoothing kernel

Y. Kedadry[1] and G. Cordonnier[2]

[1]École Normale Supérieure, France
[2]Inria, Université Côte d'Azur, France

Abstract

We present a novel approach to simulate large-scale lava flow in real-time. We use a depth-averaged model from numerical vulcanology to simplify the problem to 2.5D using a single layer of particle with thickness. Yet, lava flow simulation is challenging due to its strong viscosity which introduces computational instabilities. We solve the associated partial differential equations with approximated Green's functions and observe that this solution acts as a smoothing kernel. We use this kernel to diffuse the velocity based on Smoothed Particle Hydrodynamics. This yields a representation of the velocity that implicitly accounts for horizontal viscosity which is otherwise neglected in standard depth-average models. We demonstrate that our method efficiently simulates large-scale lava flows in real-time.

CCS Concepts

• ***Computing methodologies*** → ***Modeling and simulation;*** *Real-time simulation;*

1. Introduction

Lava is a dangerous yet fascinating phenomenon, which has been staged in many digital productions. Lava is simulated in computer graphics as a viscous fluid [PICT15, JST*16], but requires either a prohibitively low time step or the solution of a linear system, which hinders their application to the real-time simulation of large volcanic landscapes. Faster models have been proposed at the cost of a simpler viscosity representation [SAC*99].

Some approximations are used in the specialized literature [KV15, HDP22], mainly that lava is shallow enough to be treated as a vertically-averaged single-layer of fluid, and that the viscous forces are so strong that the inertial terms are neglectable in momentum conservation:

$$\vec{0} = \nabla \cdot \tau - \rho g \nabla S \tag{1}$$

where S is the lava surface elevation, ρ the lava density and g the gravity. The stress tensor τ is related to the strain rate tensor $\epsilon = \frac{1}{2}\left(\nabla u + \nabla u^T\right)$ with $\tau = \mu(\theta)\epsilon$. Several non-linear rheologies have been proposed for the viscosity μ, from the Bingham model used by [HBV*11] to viscoplastic Herschel-Bulkley in [BSS16], and they all agree on the importance of the temperature θ. Here, we simplify our derivation by assuming that the viscosity depends on the temperature only, but we assume that our model could adapt to more complex rheologies. The main weakness of vertically averaged models is that they neglect the horizontal components of the viscosity. While this approximation leads to a closed-form formulation for the velocity, it is only valid for flat and shallow flows and yields a non-linear diffusion equation whose solution requires extremely small time steps.

We propose a solution to integrate the horizontal components of the viscosity in a vertically averaged model, using Green's functions. We observe that these functions behave as a smoothing kernel. Therefore, we devise a new Smoothed Particle Hydrodynamics (SPH) approach, where a kernel for velocity is chosen in a way that implicitly models viscosity.

We implemented our approach on the GPU and showed that our model provides a good balance between physical realism and fast computational time to efficiently simulate and render lava flow in real-time.

2. Method

On large landscapes, a full 3D simulation is costly and not necessary as lava flows are mostly shallow. Therefore, we use a depth-averaged simulation where each particle embeds the lava flow thickness. The motions of the particles are driven vertically by the altitude of the terrain and horizontally by a 2D depth-averaged velocity $\bar{u}$. The thickness of the lava flow h has to accommodate for the divergence or convergence of the 2D velocity vectors [SBC*11], and evolve with:

$$\frac{\mathrm{D}h}{\mathrm{D}t} = -h \nabla \cdot \bar{u}, \tag{2}$$

where $\mathrm{D}/\mathrm{D}t$ is the notation for material derivative, *i.e.,* the variation of a quantity attached to the particle.

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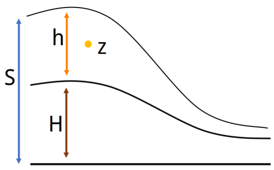
Let us consider a volume of lava of local thickness h, flowing on a bedrock of height H, and a surface $S = h + H$ (see inset Figure).



We assume a linear stress-strain relationship and low temperature variations. From the incompressibility of the lava $\nabla \cdot u = 0$ we can rewrite the momentum equation (Eq. 1) as:

$$\vec{0} = \Delta u + \frac{\partial^2 u}{\partial z^2} - \frac{\rho g}{\mu(\theta)} \nabla S. \tag{3}$$

Note that we separated the 2D Laplacian of the velocity Δu from the vertical term $\frac{\partial^2 u}{\partial z^2}$.

Shallow models are derived by neglecting the horizontal Laplacian [BSS16] and integrating twice, with the boundary conditions $u(H) = 0$ and $\frac{\partial u}{\partial z}(S) = 0$. This yields an expression for the velocity inside the lava column that we note $u_0(z)$ as it is often called a 0-th order approximation [BSS16]. The associated depth-averaged velocity is $\bar{u}_0(z) = \frac{\rho g h^2}{3\mu(\theta)} \nabla S$. This formulation is convenient as it gives a closed-form solution for the velocities. However, neglecting the horizontal viscosity is inaccurate for sharp terrain gradients, and, coupled with the particle advection, leads to a simulation that requires very small time steps to be stable.

Instead of neglecting the horizontal viscosity, we approximate it. We integrate vertically Eq. 3 and set $U = h^3 \bar{u}$; $\bar{u}$ being the vertically averaged velocity, which gives:

$$\Delta U - \alpha \frac{U}{h^2} = \alpha \frac{h^3 \rho g}{3\mu(\theta)} \nabla S, \tag{4}$$

where α is a shape factor that depends on the vertical profile velocity, for instance, $\alpha = 5/2$ for $u \approx u_0$. For a detailed derivation of Eq. 4 you can refer to the supplementary material.

Assuming that h does not vary significantly, Eq. 4 is of the form $\Delta u - k^2 u = b$, which we solve by convolving the right-hand term b with the associated Green's function $G_k(r) = -\frac{1}{2\pi} K_0(kr)$, where r is the convolution distance and K_0 the modified Bessel function of the second kind. We approximate K_0 with a series expansion at $r = 0$: $K_0(r) \approx (-\log(r) - \gamma + \log(2)) + \frac{1}{4} r^2 (-\log(x) - \gamma + 1 + \log(2))$ where γ is the Euler's constant. Finally, we obtain U with:

$$U(x) = \iint G_k(\|s - x\|) b(s) \mathrm{d}s, \tag{5}$$

Note that this equation is similar to the smoothing operation central to the SPH method. Therefore, we assign to each particle a pseudo velocity $b = \alpha \frac{h^3 \rho g}{3\mu(\theta)} \nabla S$, and use G as the smoothing operator. We can then evaluate $U(x)$ and its derivatives for any $x \in \mathbb{R}^2$, and eventually recover the velocity $\bar{u} = U/h^3$. Finally, we use the $\bar{u}$ to advect the particles and update the ice thickness with Eq. 2.

Figure 1 shows a volcano (real topography from Mount St. Helens, US), and a lava flow simulated with our method. Using an NVIDIA RTX A6000 as the GPU we were able to simulate around 10000 particles at more than 100 FPS consistently. Compared to a version that does not take into account the lateral components of the viscosity, our approach yields smoother velocity fields and is no longer constrained by a diffusive stability condition (with very small time steps, which also depend locally on the lava thickness). In contrast, we constrain our time step solely by a CFL-like criterion that forces the particles to advance less than the distance between neighboring particles. Please see our video and poster for additional results.

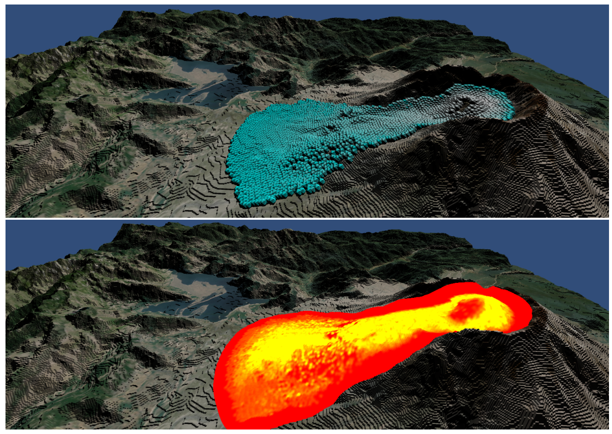


Figure 1: *Simulation of a viscous lava flow over Mount St Helens. The top view shows the particles, and the bottom view the reconstructed lava surface, color-coded with the height of each column.*

Future work includes the improvement of our simulation by exploring non-linear viscosity laws [HBV*11] and vertical temperature variations [BSS16], which model the spatially changing behavior of lava flow from fluid to solid. Further experimentation is needed to assess the impact of our different approximations.

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