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Fast simulation of viscous lava flow using Green's functions as a smoothing kernel

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Abstract

We present a novel approach to model large-scale lava flow in real-time. We use a depth-averaged model from numerical vulcanology to simplify the problem to 2D. Yet, lava flow simulation is challenging due to its strong viscosity. We solve the associated partial differential equations with approximated Green's functions and observe that this solution acts as a smoothing kernel. We use this kernel in a discretization of the velocity based on Smoothed Particle Hydrodynamics, yielding a representation of the velocity that implicitly accounts for viscosity. We demonstrate that our method efficiently simulates large-scale lava flows in real-time.

CCS Concepts
• ***Computing methodologies*** $\rightarrow$ ***Modeling and simulation;*** *Real-time simulation;*

1. Introduction

Lava is a dangerous yet fascinating phenomenon, which has been staged in many digital productions. Lava is simulated in computer graphics as a viscous fluid [SAC*99, PICT15, JST*16], but requires either a prohibitively low time step or the solution of a linear system, which hinders their application to the real-time simulation of large volcanic landscapes. Some approximations are common in the specialized literature [KV15, HDP22], mainly that lava is shallow enough to be trained as a vertically-averaged single-layer of fluid, and that the viscous forces are so strong that the internal terms are neglectable in momentum conservation:

$$\vec{0} = \nabla \cdot \tau - \rho g \nabla S \tag{1}$$

where S is the lava surface elevation, ρ the lava density and g the gravity. The stress tensor τ is related to the strain rate tensor $\epsilon = \frac{1}{2}\left(\nabla u + \nabla u^T\right)$ with $\tau = \mu(\theta)\epsilon$. Several non-linear rheologies have been proposed for the viscosity μ, from the Bingham model used by [HBV*11] to viscoplastic Herschel-Bulkley in [BSS16], and they all agree on the importance of temperature. Here, we simplify our derivation by assuming that the viscosity depends on the temperature only, but we assume that our model could adapt more complex rheologies. The main weakness of vertically averaged models is that they neglect the horizontal components of the viscosity. While this approximation leads to closed-form solutions for the velocity, it is only valid for flat and shallow flows and yields a non-linear diffusion equation whose solution requires extremely small time steps.

We propose a solution to integrate the horizontal components of the viscosity in a vertically averaged model, using Green's functions. We observe that these functions behave as a smoothing kernel. Therefore, we devise a new Smoothed Particle Hydrodynamics (SPH) approach, where a kernel for velocity is chosen in a way that implicitly models viscosity.

We implemented our approach on the GPU and showed that our model provides a good balance between physical realism and fast computational time to efficiently simulate and render lava flow in real-time.

2. Method

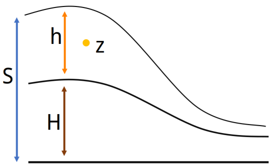
We simulate the lava flow with Smoothed Particle Hydrodynamics (SPH) on the GPU. We will present how we use Green's functions as a smoothing kernel to implicitly account for the viscous forces.

On large landscapes, a full 3D simulation is costly and not necessary as lava flows are mostly shallow. Therefore, we use a depth-averaged simulation where each particle embeds the lava flow thickness. The motions of the particles is driven vertically by the altitude of the terrain and horizontally by a 2D depth-averaged velocity $\bar{u}$. The thickness of the lava flow h has to accommodate for the divergence or convergence of the 2D velocity vectors [SBC*11], and evolve with:

$$\frac{\mathrm{D}h}{\mathrm{D}t} = -h\nabla \cdot \bar{v}, \tag{2}$$

where $\mathrm{D}/\mathrm{D}t$ is the notation for material derivative, *i.e.,* the variation of a quantity attached to the particle.

Let us consider a volume of lava of local thickness h, flowing on a bedrock of height H, and a surface $S = h + H$ (see inset Figure).



We assume a linear stress-strain relationship and low temperature variations, and from the compressibility of the ice $\nabla \cdot u = 0$ we can rewrite the momentum equation (Eq.2) as:

$$\vec{0} = \Delta u + \frac{\partial^2 u}{\partial z^2} - \frac{\rho g}{\mu(\theta)} \nabla S. \tag{3}$$

Note that we separated the 2D Laplacian of the velocity Δu from the vertical term $\frac{\partial^2 u}{\partial z^2}$.

Shallow models are derived by neglecting the horizontal Laplacian [BSS16] and integrating twice, with the boundary conditions $u(H) = 0$ and $\frac{\partial u}{\partial z}(S) = 0$. This yields an expression for the velocity inside the lava column that we note $u_0(z)$ as it is often called a 0-th order approximation [BSS16]. The associated depth-averaged velocity is $\bar{u}_0(z) = \frac{\rho g h^2}{3\mu(\theta)} \nabla S$. This formulation is convenient as it gives a closed-form solution for the velocities. However, neglecting the horizontal viscosity is inaccuate for sharp terrain gradients, and, coupled with the particle advection, leads to a simulation that requires very small time steps to be stable.

Instead of neglecting the horizontal viscosity, we approximate it. We integrate vertically Eq 3 and set $U = h^3 \bar{u}$, which gives:

$$\Delta U - \alpha \frac{U}{h^2} = \alpha \frac{h^3 \rho g}{3\mu(\theta)} \nabla S, \tag{4}$$

where α is a shape factor that depends of the vertical profile velocity, for instance $\alpha = 72/33$ for $u \approx u_0$.

Assuming that h does not vary significantly, Eq. 4 is of the form $\Delta u - k^2 u = b$, which can be solved by convolving the right-hand term b with the associated Green's function $G_k(r) = -\frac{1}{2\pi} K_0(kr)$, where r is the convolution distance and K_0 the modified Bessel function of the second kind. We approximate K_0 with a series expension at $r = 0$: $K_0(r) \approx (-\log(r) - \gamma + \log(2)) + \frac{1}{4} r^2 (-\log(x) - \gamma + 1 + \log(2))$ where γ is the Euler's constant. Finally, we obtain U with:

$$U(x) = \iint G_k(\|s - x\|) b(s) \mathrm{d}s, \tag{5}$$

Note that this equation is similar to the smoothing operation central to the SPH method. Therefore, we assigne to each particle a pseudo velocity $b = \alpha \frac{h^3 \rho g}{3\mu(\theta)} \nabla S$, and use G as the smoothing operator. We can then evaluate $U(x)$ and its derivatives for any $x \in \mathbb{R}^2$, and eventually recover the velocity $\bar{u} = U/h^3$.

3. Results and Discussions

Figure 1 shows a volcano (real topography from Mount St. Helens, US), and a laval flow with simulated with our method. Using an Intel(R) Xeon(R) Gold 5218R CPU at 2.10GHz and an NVIDIA RTX A6000 as the GPU we were able to simulate around 10000 particles at more than 100 FPS consistently. Please see our video and poster for additional results.

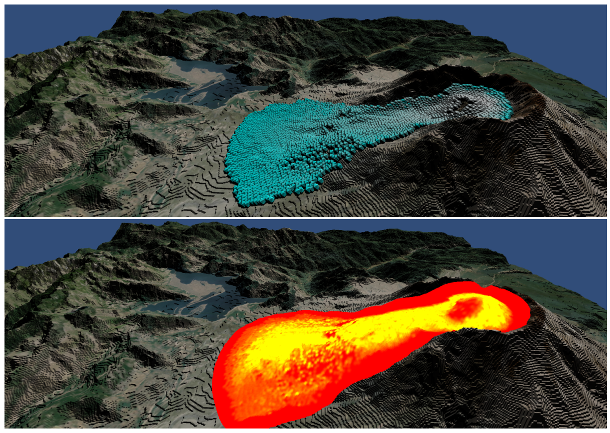


Figure 1: Simulation of a viscous lava flow over Mount St Helens. The top view shows the paricles, and the bottom view the reconstructed lava surface, color-coded with the height of each column.

Future work include the improvement of our simulation by exploring non-linear viscosity laws [HBV*11] and vertical temperature variations [BSS16], which are fundamental in lava flows. Further experimentation is needed to assess the impact of our different approximations.

References

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