Lava Flow Simulation

ENS ÉCOLE NORMALE SUPÉRIEURE | PSL RESEARCH UNIVERSITY PARIS

Inria

Why lava flows ?

Why lava flows ?

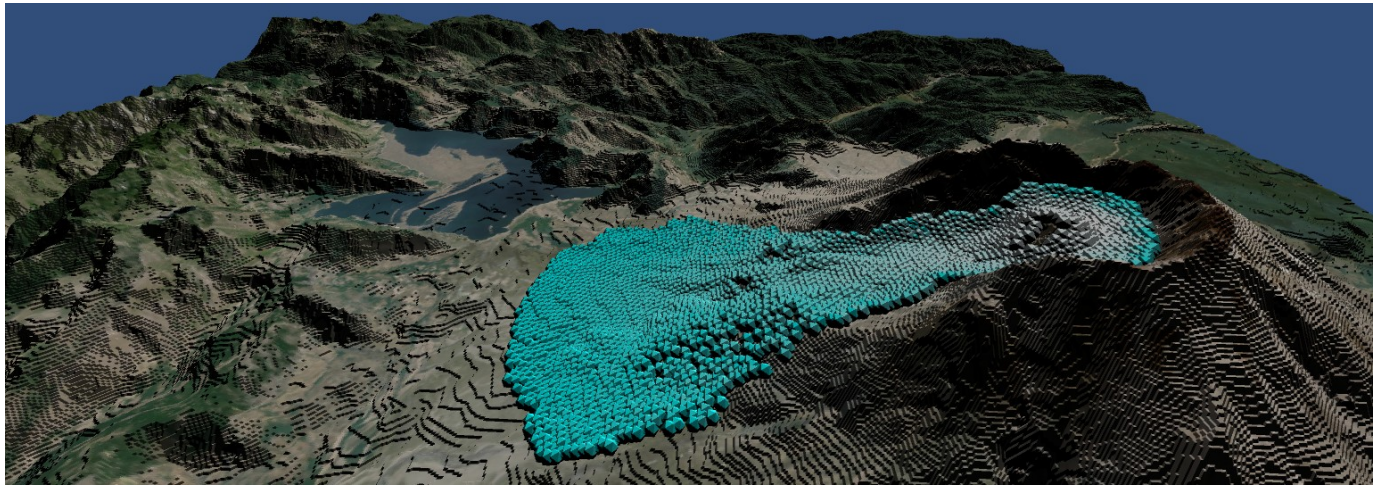
- Entertainment

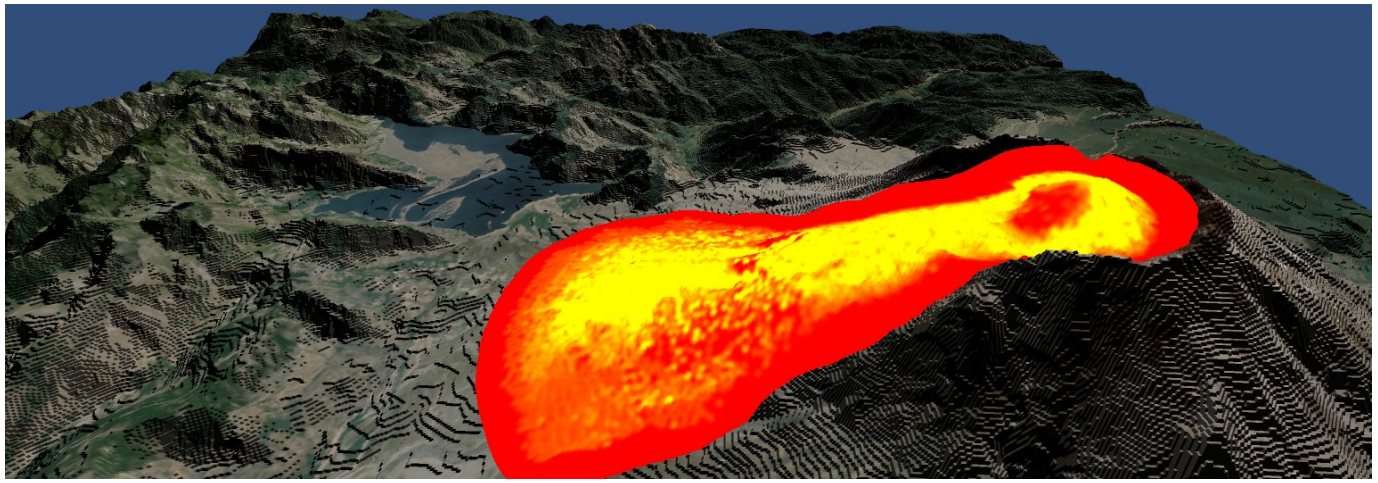
Why lava flows ?

- Entertainment

- Prevention

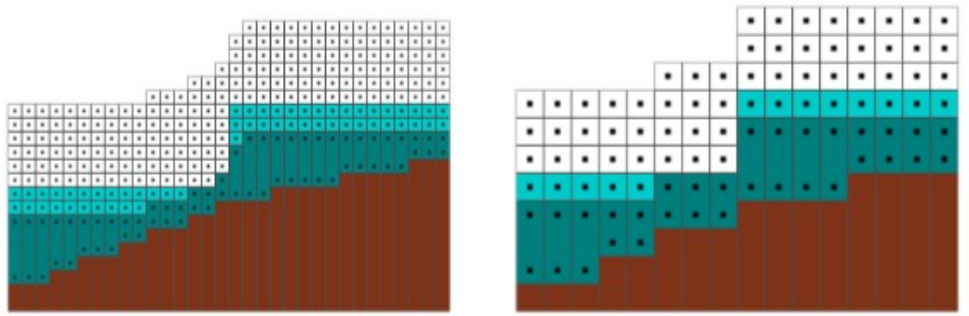
Our objectives





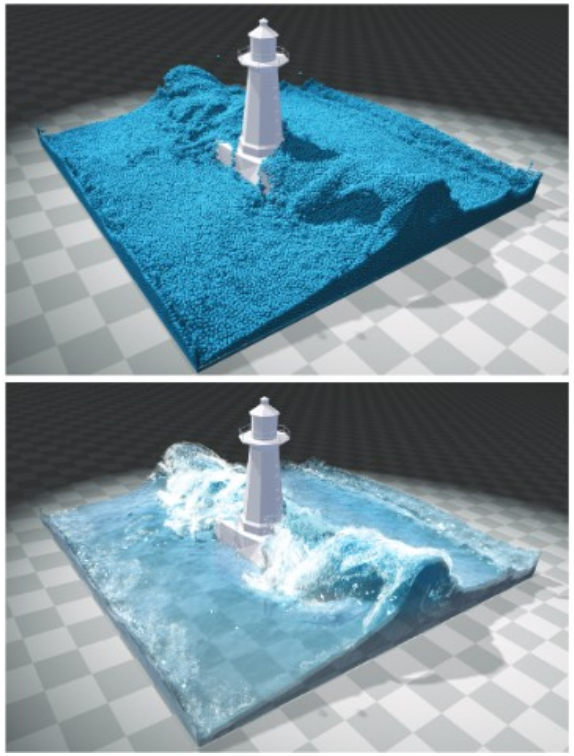
Existing methods: Fluid simulation

Eulerian models



from *Real-Time Eulerian Water Simulation Using a Restricted Tall Cell Grid*, Müller et al.

Lagrangian models



from *SPH Fluids in Computer Graphics*, Ihmsen et al.

Existing methods: From 3D to 2D

3D Navier-Stokes Equations

$$\rho(\frac{d\vec{u}}{dt} + (\vec{u} \cdot \nabla)\vec{u}) = \rho\vec{g} - \nabla \cdot \mathrm{pI} + \rho\vec{F}_{ext}$$

ρ	Density
$\vec{u}$	Velocity
$\vec{g}$	Gravity
p	Pressure
I	Identity tensor
$\vec{F}_{ext}$	External forces

2D Shallow Water Equations

$$\frac{Du}{Dt} = -\vec{g}\nabla S + \vec{F}_{ext}$$

$\vec{u}$	Velocity
$\vec{g}$	Gravity
$\vec{F}_{ext}$	External forces
S	Surface

Existing methods: Viscosity

Explicit schemes

x(t+dt) = f(x(t))

\+ Fast per time step

\- Needs small steps to be stable

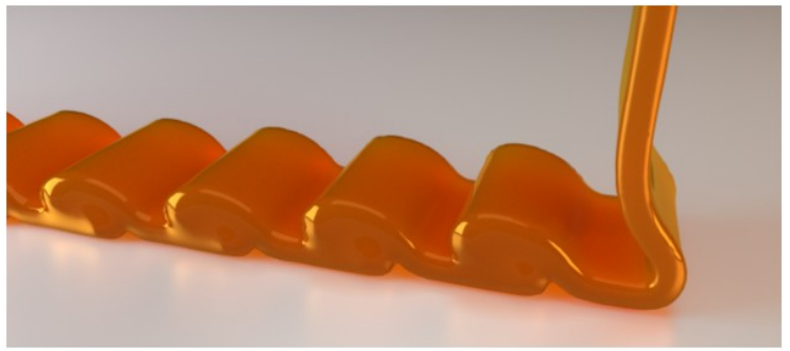
\- Steps sizes depends on 1/viscosity

Implicit schemes

x(t+dt) = f(x(t+dt), x(t))

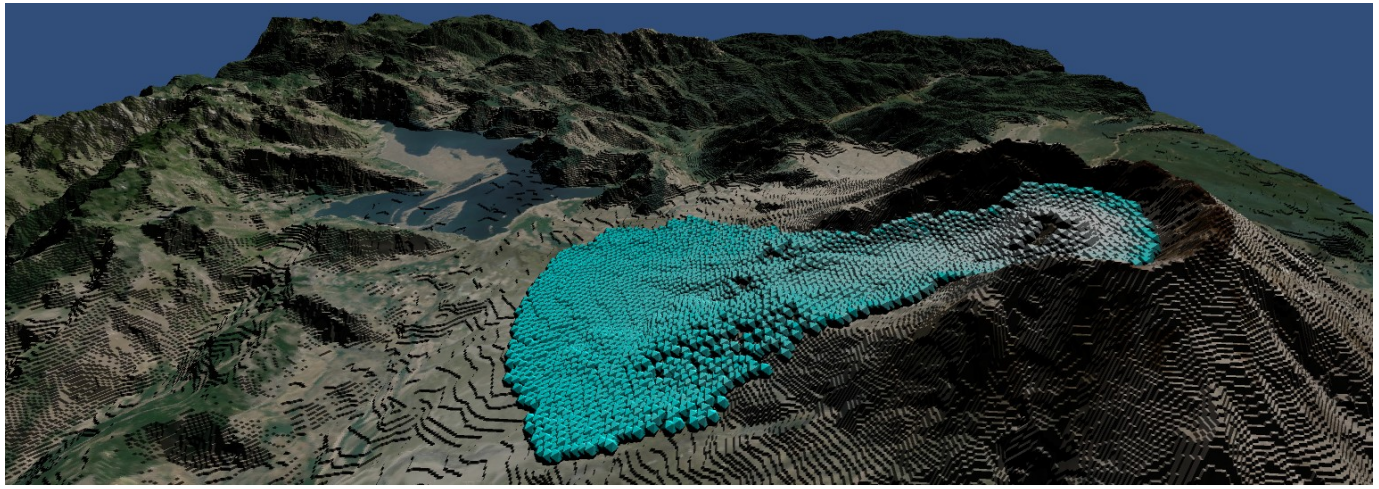
\+ Time step size independent

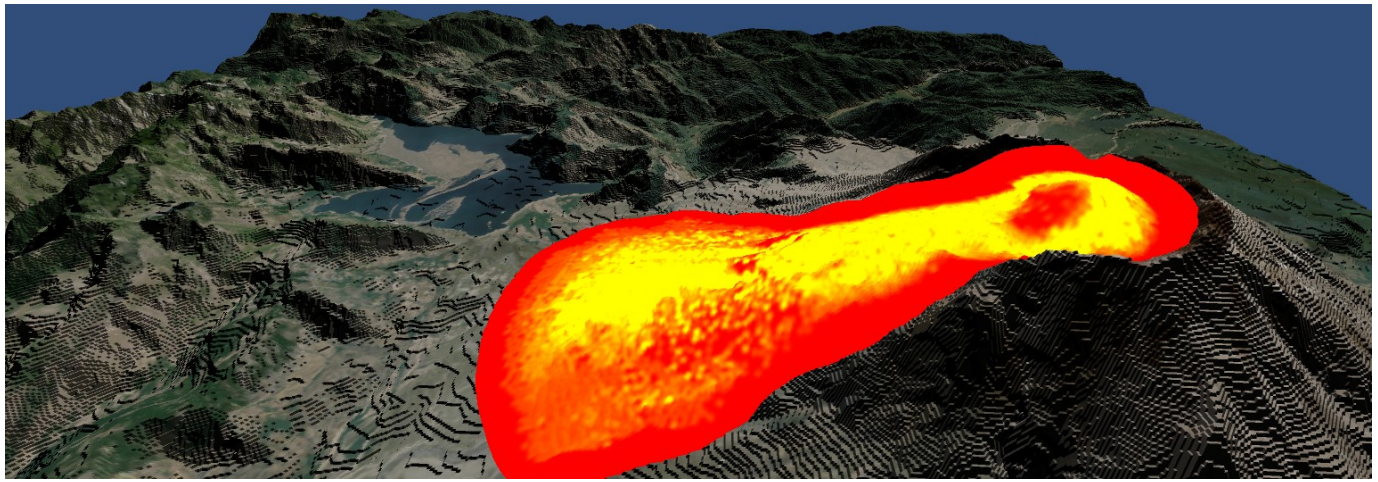
\- Linear system to solve



from *A Physically Consistent Implicit Viscosity Solver for SPH Fluids*, Weiler et al.

Our method





The algorithm

Algorithm The main loop of the simulation

```
Require: Grids Initialization
  while True do
    if elapsedTime > dt then
      if Current number of particles < Maximum number of particles then
        Generate (elapsedTime/dt) particles
      end if
      Update neighbors
      Update heights
      Update temperatures
      Update velocities
      Update positions
      Update terrain and render
      Reset elapsedTime
    end if
  end while
```

The algorithm

Algorithm The main loop of the simulation

Require: Grids Initialization
  while True **do**
    if *elapsedTime* $>$ *dt* **then**
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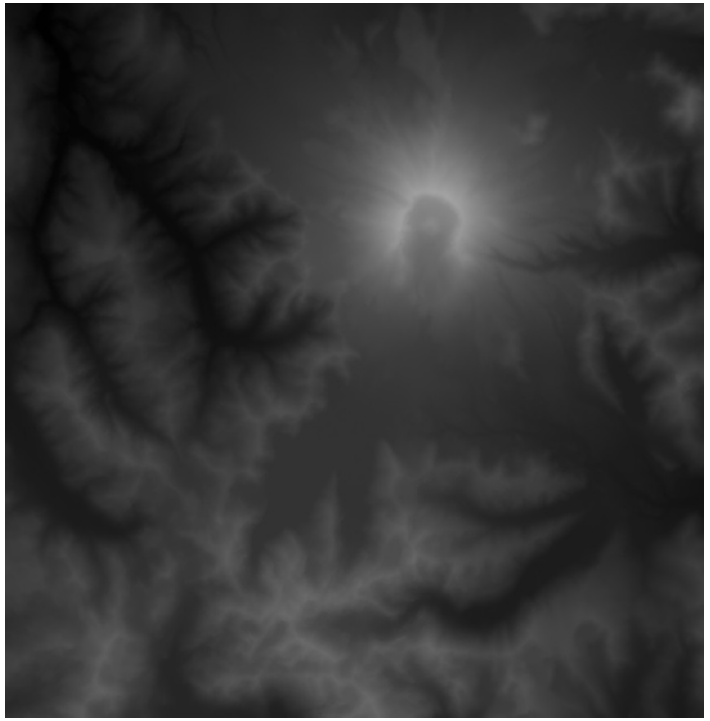
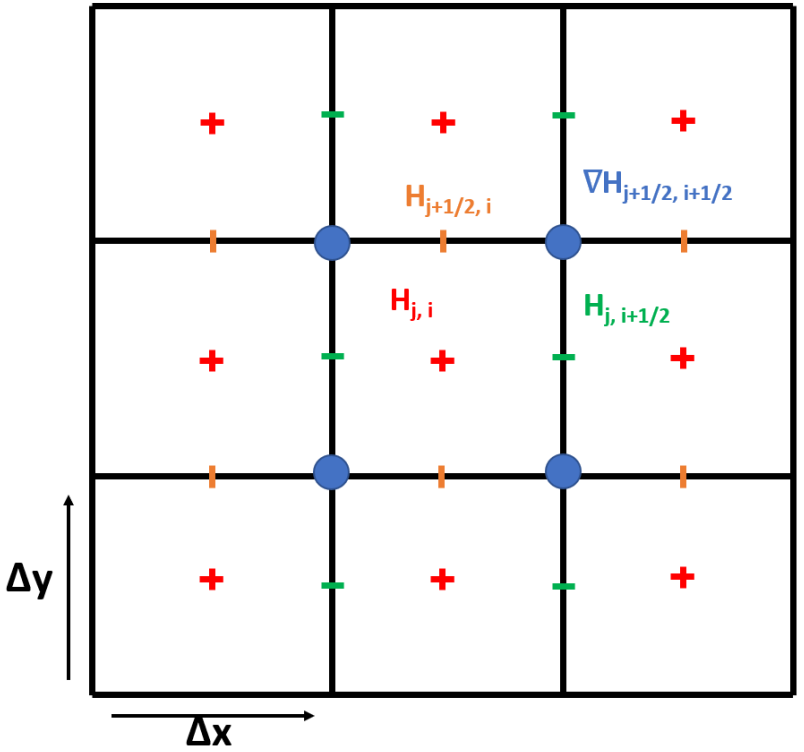
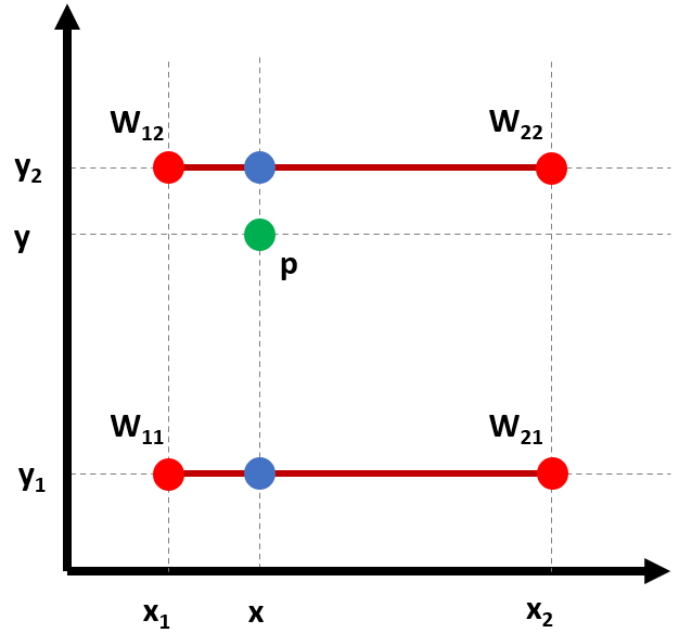
      Reset *elapsedTime*
    end if
  end while

The Staggered grid







Heightmap → Staggered grid → Interpolation

The algorithm

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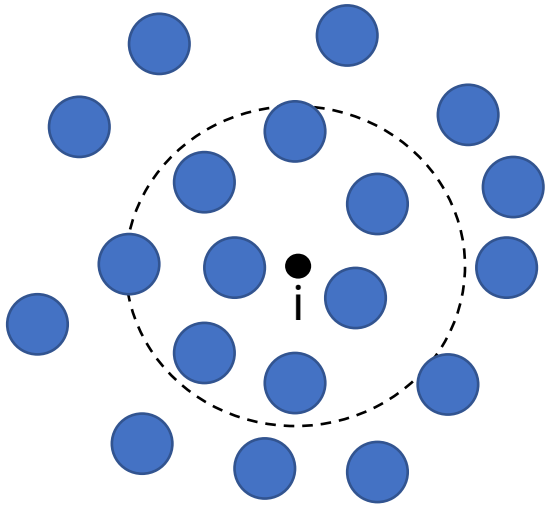
    end if
  end while

Smoothed Particle Hydrodynamics

3D Version



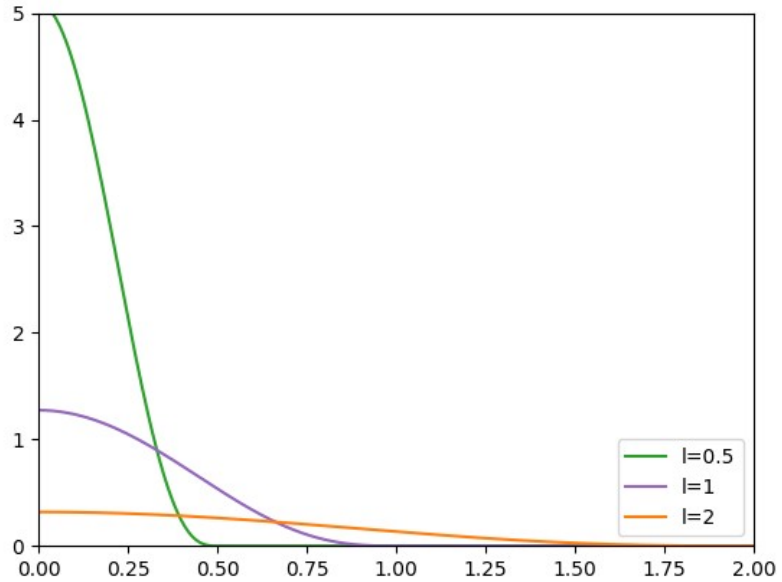
$$q_i = \sum_j \frac{m_j}{\rho_j} q_j W_{ij}$$

$$\nabla q_i = \sum_j \frac{m_j}{\rho_j} q_j \nabla W_{ij}$$

$$\nabla \cdot q_i = \sum_j \frac{m_j}{\rho_j} q_j \cdot \nabla W_{ij}$$

$$\nabla^2 q_i = \sum_j \frac{m_j}{\rho_j} q_j \nabla^2 W_{ij}$$

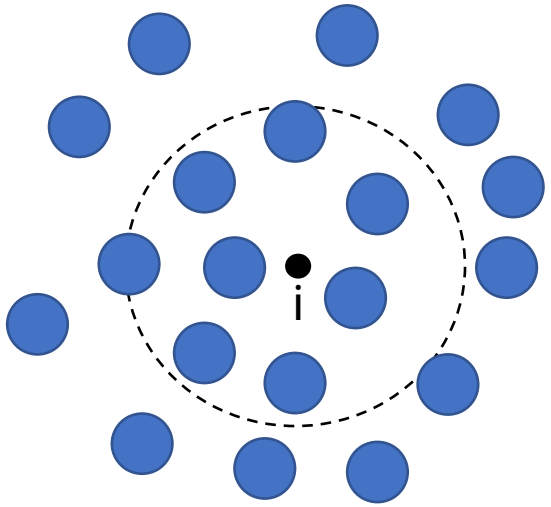
$$W_{ij} = W(x_i - x_j, l)$$



Usual Smoothing kernel : Wpoly6

Smoothed Particle Hydrodynamics

2D Version



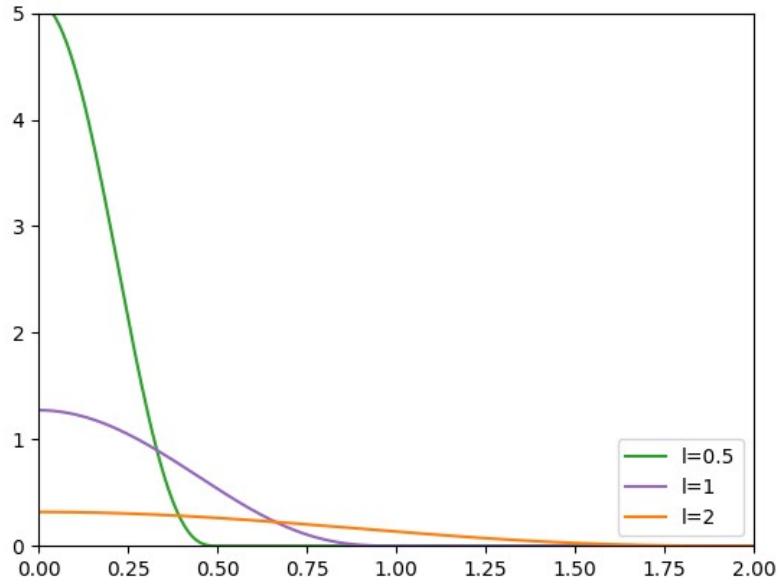
$$q_i = \sum_j \frac{V_j}{h_j} q_j W_{ij}$$

$$\nabla q_i = \sum_j \frac{V_j}{h_j} q_j \nabla W_{ij}$$

$$\nabla \cdot q_i = \sum_j \frac{V_j}{h_j} q_j \cdot \nabla W_{ij}$$

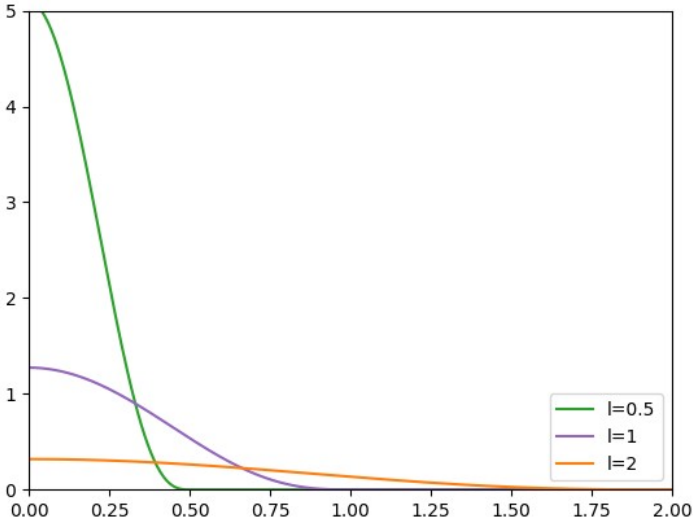
$$\nabla^2 q_i = \sum_j \frac{V_j}{h_j} q_j \nabla^2 W_{ij}$$

$$W_{ij} = W(x_i - x_j, l)$$

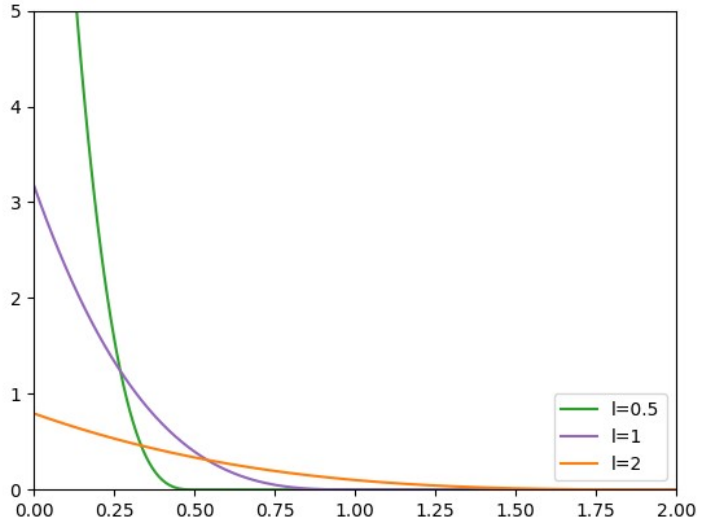


Usual Smoothing kernel : Wpoly6

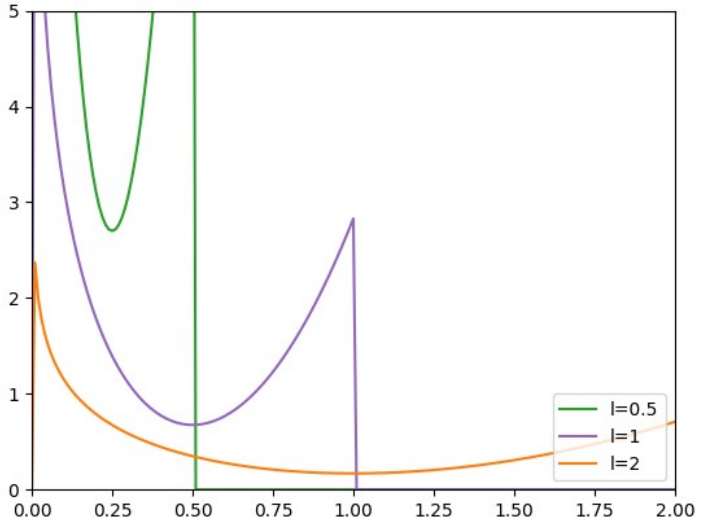
Smoothed Particle Hydrodynamics



Wpoly6



Wspiky



Wvisc

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    end if
  end while
```

The viscosity

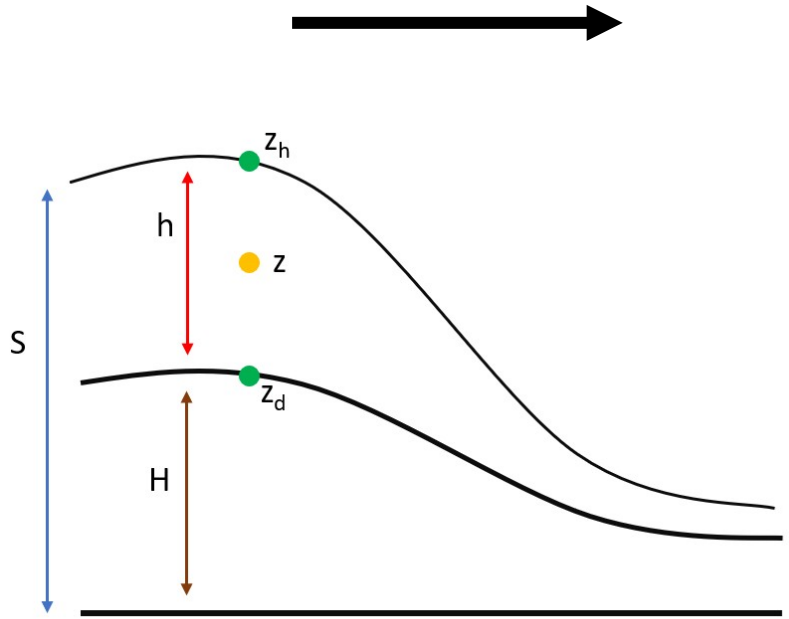
$$\frac{d\vec{u}}{dt} + (\vec{u}\cdot\nabla)\vec{u} = -\frac{\nabla\cdot \mathrm{pI}}{\rho} + \frac{\nabla\cdot\tau}{\rho} + \vec{g}$$

Stokes

Blatter

$$\vec{0} = \nabla\cdot\tau + \rho\vec{g}\nabla S$$

$\rho \rightarrow$ Density
$\vec{u} \rightarrow$ Velocity
$\vec{g} \rightarrow$ Gravity
$p \rightarrow$ Pressure
$I \rightarrow$ Identity tensor
$\tau \rightarrow$ Stress tensor



$\rho \rightarrow$ Density
$\vec{g} \rightarrow$ Gravity
$\tau \rightarrow$ Stress tensor
$S \rightarrow$ Surface

The viscosity

$$\vec{0} = \nabla \cdot \tau + \rho \vec{g} \nabla S$$

$$\tau = f(\theta)\mu\epsilon$$

Volcanology

$$f(\theta)\mu \begin{pmatrix} \frac{\partial^2 u_x}{\partial x^2} + \frac{1}{2}\frac{\partial^2 u_x}{\partial y^2} + \frac{1}{2}\frac{\partial^2 u_x}{\partial z^2} \\ \frac{\partial^2 u_y}{\partial y^2} + \frac{1}{2}\frac{\partial^2 u_y}{\partial x^2} + \frac{1}{2}\frac{\partial^2 u_y}{\partial z^2} \end{pmatrix} = -\rho \vec{g} \nabla S$$

$\rho \rightarrow$ Density

$\vec{g} \rightarrow$ Gravity

$\tau \rightarrow$ Stress tensor

$S \rightarrow$ Surface

$\rho \rightarrow$ Density

$\vec{g} \rightarrow$ Gravity

$\tau \rightarrow$ Stress tensor

$S \rightarrow$ Surface

$\theta \rightarrow$ Temperature

$\mu \rightarrow$ Viscosity

$\epsilon \rightarrow$ Strain rate tensor

$u_x, u_y \rightarrow$ 2D Velocity components

The viscosity

$$f(\theta)\mu\begin{pmatrix}\frac{\partial^2 u_x}{\partial x^2}+\frac{1}{2}\frac{\partial^2 u_x}{\partial y^2}+\frac{1}{2}\frac{\partial^2 u_x}{\partial z^2}\\ \frac{\partial^2 u_y}{\partial y^2}+\frac{1}{2}\frac{\partial^2 u_x}{\partial x\partial y}+\frac{1}{2}\frac{\partial^2 u_y}{\partial z^2}\end{pmatrix}=-\rho\vec{g}\nabla S$$

Integration

$$\frac{\partial}{\partial x^2}u'+\frac{1}{2}\frac{\partial}{\partial y^2}u'-\frac{3}{2}u'h^4=-(h^3\frac{\rho\vec{g}\nabla S}{f(\theta)\mu})$$

$\rho \rightarrow$ Density

$\vec{g} \rightarrow$ Gravity

$\tau \rightarrow$ Stress tensor

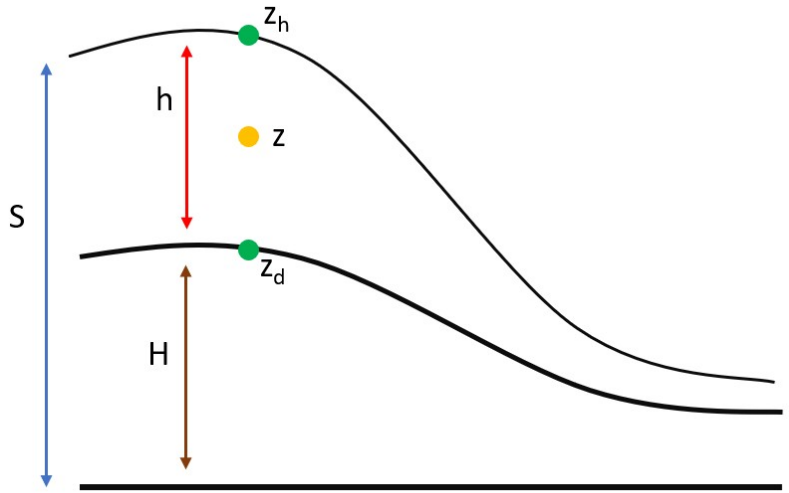
$S \rightarrow$ Surface

$\theta \rightarrow$ Temperature

$\mu \rightarrow$ Viscosity

$\epsilon \rightarrow$ Strain rate tensor

$u_x, u_y \rightarrow$ 2D Velocity components



$$u' = h^3\bar{u}$$

The viscosity

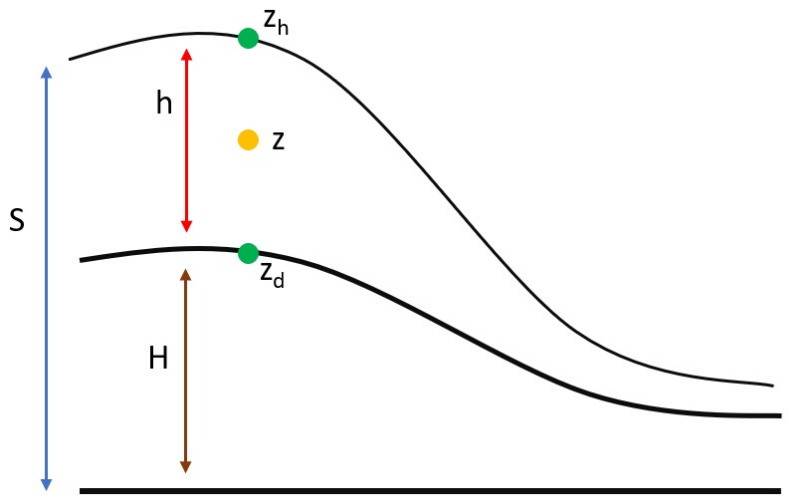
$$\frac{\partial}{\partial x^2}u' + \frac{1}{2}\frac{\partial}{\partial y^2}u' - \frac{3}{2}u'h^4 = -(h^3\frac{\rho\vec{g}\nabla S}{f(\theta)\mu})$$

Known Equation

$$\nabla^2 u' - k^2 u' = b_j$$

$$G(u) = -\frac{1}{2\pi}K_0(kr)$$

$$u' = h^3\bar{u}$$



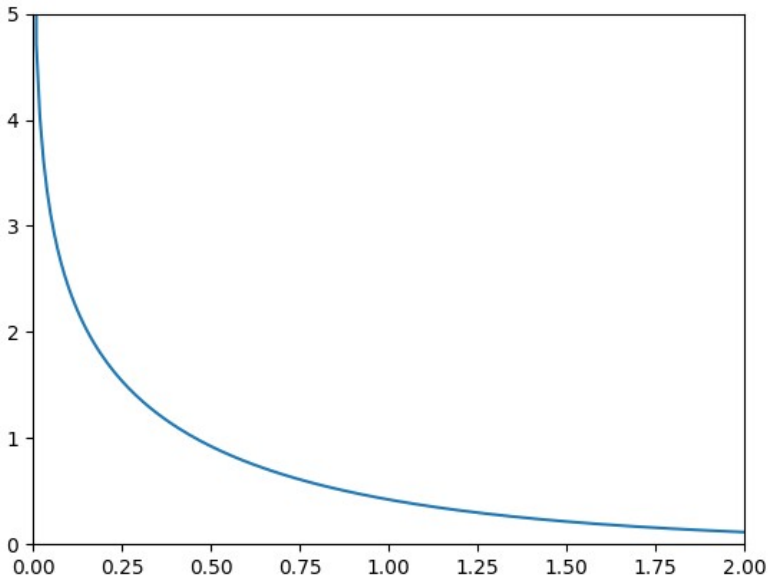
$$\bar{u_i} = h_i^3\frac{\sum_j G(u_j) * b_j}{\sum_j G(u_j)}$$

$$\bar{u_i} = h_i^3\frac{\sum_j K_0(kr) * b_j}{\sum_j K_0(kr)}$$

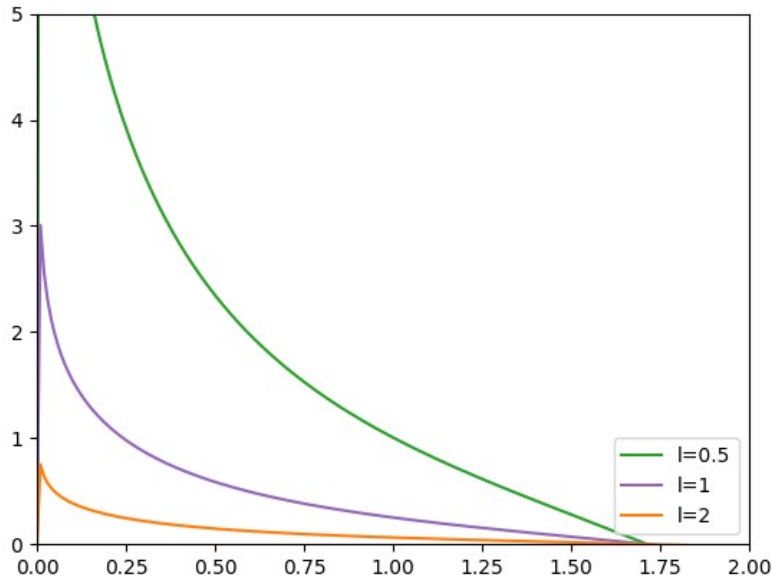
The viscosity

$$W_{\text{new}}(r,l) = \frac{2}{\pi l^2}\begin{cases}((-\log(r) - \gamma + \log(2)) + \frac{1}{4}r^2(-\log(r) - \gamma + 1 + \log(2))) & \text{if } 0 < r \leq l \\ 0 & \text{otherwise}\end{cases}$$

$$K_0 = \int_0^{+\infty} \frac{\cos(xt)}{\sqrt{t^2+1}}\, dt$$

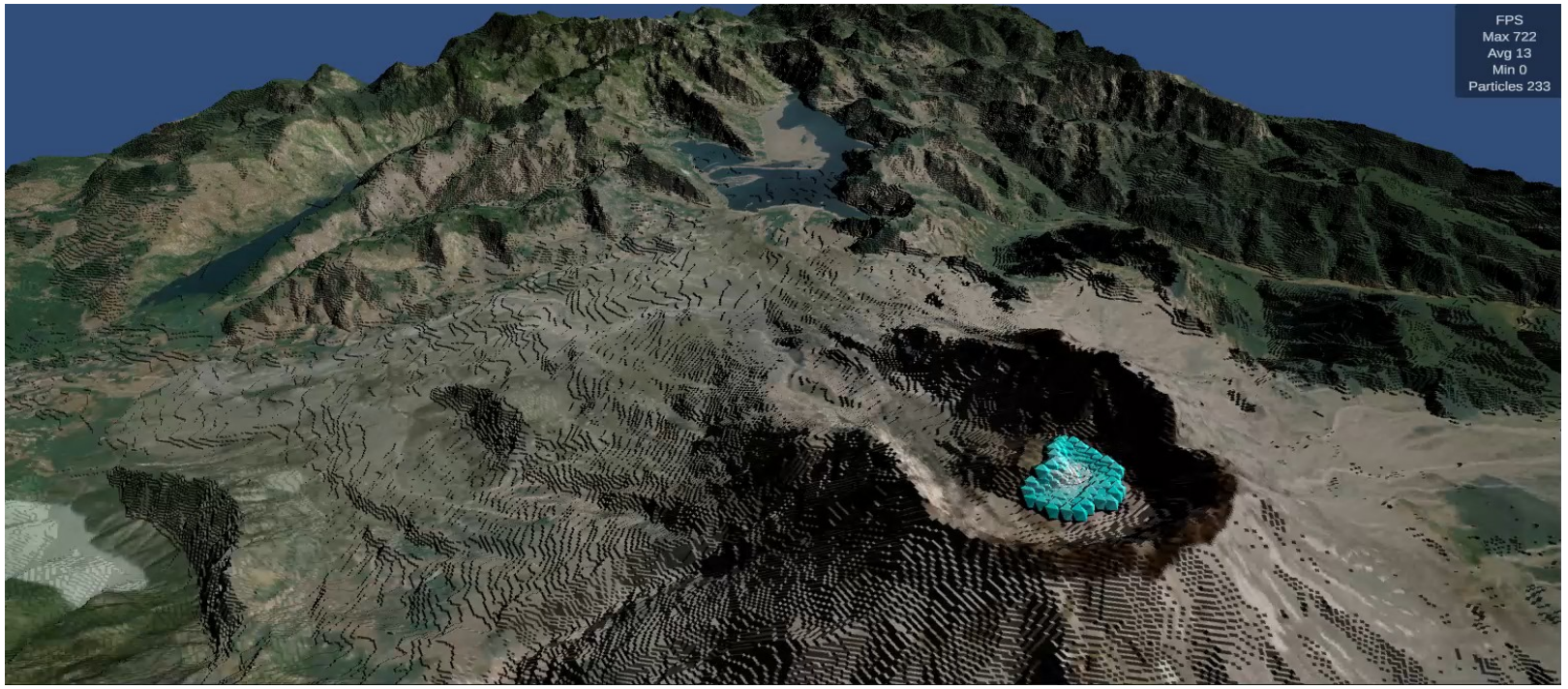


Modified Bessel function K_0



New smoothing kernel W_{new}

Final results



Simulation running in a NVIDIA RTX A6000, using Mount St.Helens topography

What’s next

- Implement a validation model
- Change the model for μ
- Change the model for θ

Thanks for your attention