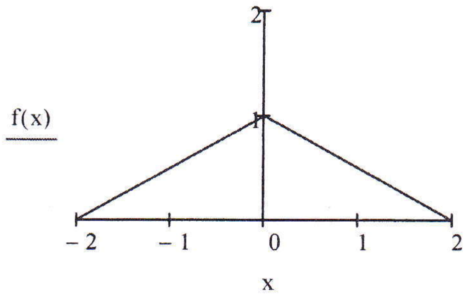
3. Schularbeit 4AHELT 8. 4. 2013

Gruppe: B

1) Ermittle die Fourier-Reihe folgender Funktion: 8



$A_0 = 0{,}5$

2) Ermittle die allgemeine Lösung: 7

$$y' \cdot \sqrt{2x} - 2y = 1$$

3) Sei $y'' + p\,y' + y = 2x + 1$ die DG einer ged. Schwingung! 9

Welchen Wert muss p haben, dass der aperiodische Grenzfall eintritt? Bestimme die spezielle Lösung für $y(0)=0$ und $y'(0)=2$!

4) Löse folgende Anfangswertaufgabe:

$$y' + \frac{2y}{x} = \cos(x) \qquad y(\pi) = 1$$ 8

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Schuljahr: 2012/13 Vor- und Zuname: [illegible]

Klasse: 4AHEL T

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3. **Schularbeit**

aus Mathematik am 8.4.13

Aufgabe:

1) $b_n = 0$ $\omega = \frac{2\pi}{T} = \frac{\pi}{2}$

$a_n = \frac{2}{T} \cdot 2 \cdot \int_0^2 \left(-\frac{1}{2}x + 1\right) \cdot \cos(n\omega x)\, dx$

~~$= \frac{4}{4} \cdot \int_0^2 \left(\frac{1}{2}x \cdot \cos(n\omega x) + \cos(n\omega x)\right) dx$~~

$u = -\frac{1}{2}x + 1$

$\int u \cdot v' = u \cdot v - \int u' \cdot v$

$u' = -\frac{1}{2}$

$v' = \cos(n\omega x)$

$v = \frac{\sin(n\omega x)}{n\omega}$

$\rightarrow \left(-\frac{1}{2}x + 1\right) \cdot \frac{\sin(n\omega x)}{n\omega}$

$- \int -\frac{\sin(n\omega x)}{2n\omega}\, dx =$

$= \left(\frac{1}{2}x + 1\right) \frac{\sin(n\omega x)}{n\omega} - \frac{\cos(n\omega x)}{2(n\omega)^2}$

$$a_n = \left(\left(\frac{1}{2}x+1\right)\cdot\frac{\sin(n\omega x)}{n\omega} - \frac{\cos(n\omega x)}{2(n\omega)^2}\right)\Bigg|_0^2$$

$$= \underbrace{\left(\frac{1}{2}\cdot 2+1\right)\cdot\frac{\sin(n\frac{\pi}{2}\cdot 2)}{n\omega}}_{=0} - \frac{\cos(n\frac{\pi}{2}\cdot 2)}{2n^2\omega^2} - \frac{\sin(n\frac{\pi}{2}\cdot 0)}{n\omega} + \frac{\cos(0)}{2n^2\omega^2} =$$

$$= \frac{\cos(n\pi)}{2n^2\omega^2} + \frac{1}{2n^2\omega^2} = \frac{1}{2n^2\omega^2}(-\cos(n\pi)+1)$$

$$\frac{1}{2n^2\omega^2} = \frac{2}{n^2\pi^2}$$

$a_1 = \cancel{\frac{2}{\pi^2}}\ \cancel{-\frac{4}{\pi^2}}$

$\cancel{a_2 = 0}$

$\cancel{a_3 = -\frac{4}{9\pi^2}}$

$\cancel{a_4 = 0}$

$\cancel{a_5 = -\frac{4}{25\pi^2}}$

$\cancel{a_6 = 0}$

$a_1 = 0 \quad a_2 = \frac{1}{\pi^2} \quad a_3 = 0 \quad a_4 = \frac{1}{4\pi^2}$

$a_5 = 0$

$\cancel{f(x) = 0{,}5 - \frac{4}{\pi^2}\cos(\omega x) - \frac{4}{9\pi^2}\cdot\frac{\cos(3\omega x)}{3} - \frac{4}{25\pi^2}\cdot\frac{\cos(5\omega x)}{5} + \ldots}$

$$f(x) = 0{,}5 + \frac{1}{\pi^2}\cdot\frac{\cos(2\omega x)}{2} + \frac{1}{4\pi^2}\cdot\frac{\cos(4\omega x)}{4}$$

7

2) $y'\sqrt{2x} - 2y = 1$

→ DF?

$d(x)=0 \Rightarrow \quad \frac{dy}{dx}\sqrt{2x} = 2y$

$$\frac{dy}{y} = \frac{2}{\sqrt{2x}}\,dx$$

$$\int \frac{dy}{y} = \int \frac{2}{\sqrt{2x}}\,dx$$

$$\int (2x)^{-\frac{1}{2}}\,dx = 2\cdot(2x)^{\frac{1}{2}}\cdot\frac{1}{2} = \sqrt{2x}$$

$$\ln y = 2\sqrt{2x} + c_1$$

$$y = e^{2\sqrt{2x}+c_1} = e^{2\sqrt{2x}}\cdot e^{c_1} = c\cdot e^{2\sqrt{2x}} = y_h$$

$$\left(2\cdot\sqrt{2}\cdot\sqrt{x}\right)' = \sqrt{2}\cdot\frac{1}{\sqrt{x}} = \sqrt{\frac{2}{x}\cdot\frac{2}{2}} = \frac{2}{\sqrt{2x}}$$

$$\int \frac{2}{\sqrt{2}}\cdot\frac{1}{\sqrt{x}}\,dx = \frac{2\cdot 2}{\sqrt{2}}\sqrt{x} = 4\cdot\sqrt{\frac{x}{2}} = \cancel{\sqrt{\frac{16x}{2}}}$$

y_p:
$$y = c(x)\cdot e^{2\sqrt{2x}}$$

$$y' = c'(x)\cdot e^{2\sqrt{2x}} + c(x)\cdot\frac{2}{\sqrt{2x}}\cdot e^{2\sqrt{2x}}$$

$$\Rightarrow \left(c'(x)\cdot e^{2\sqrt{2x}} + c(x)\cdot\frac{2}{\sqrt{2x}}\cdot e^{2\sqrt{2x}}\right)\cdot\sqrt{2x} - 2\cdot c(x)\cdot e^{2\sqrt{2x}} = 1$$

$$\Rightarrow \sqrt{2x}\cdot c'(x)\cdot e^{2\sqrt{2x}} = 1$$

$$c'(x) = \frac{1}{\sqrt{2x}\cdot e^{2\sqrt{2x}}} = (2x)^{-\frac{1}{2}}\cdot e^{-2\sqrt{2x}}$$

$\int$!

$$c(x) = -\frac{1}{2}\cdot(2x)^{-\frac{3}{2}}\cdot 2\cdot e^{-2\sqrt{2x}} + (2x)^{\frac{1}{2}}\cdot\left(-\frac{2}{\sqrt{2x}}\right)\cdot e^{-2\sqrt{2x}}$$

$$c(x) = -\sqrt{(2x)^3}\cdot e^{-2\sqrt{2x}} + \frac{1}{\sqrt{2x}}\cdot\left(-\frac{2}{\sqrt{2x}}\right)\cdot e^{-2\sqrt{2x}}$$

$$c(x) = -\sqrt{(2x)^3}\cdot e^{-2\sqrt{2x}} - \frac{1}{x}\cdot e^{-2\sqrt{2x}}$$

$$y_p = -\sqrt{(2x)^3} - \frac{1}{x}$$

$$\Rightarrow \quad y = c\cdot e^{2\sqrt{2x}} - \sqrt{8x^3} - \frac{1}{x}$$

4)

$y' + \frac{2y}{x} = \cos x \qquad y(\pi) = 1$

$s(x) = 0 \Rightarrow \frac{dy}{dx} = -\frac{2y}{x}$

$\frac{dy}{y} = -\frac{2}{x} \cdot dx$

$\ln c$

$\ln y = -2 \cdot \ln x + c_1$

$y_h = x^{-2} \cdot c = \frac{c}{x^2}$

$y_p: \quad y = a \cdot \sin(x) + b \cdot \cos x$ // DF!

$y' = a \cdot \cos x - b \sin x$

$a \cos x - b \sin x + \frac{2a \sin x}{x} + \frac{2b \cos x}{x} = \cos x$

$a + \frac{2b}{x} = 1$

$\cancel{a\left(1+\frac{2}{x}\right) = 1 \Rightarrow a = \frac{1}{1+\frac{2}{x}}}$

$-b - \frac{2a}{x} = 0 \qquad \cancel{b\left(-1-\frac{2}{x}\right) = 0 \rightarrow b_1 = 0}$

$b = -\frac{2a}{x}$

$\Rightarrow a + \frac{2}{x}\left(-\frac{2a}{x}\right) = 1$

$a\left(1 - \frac{4}{x^2}\right) =$

$a = \frac{1}{1-\frac{4}{x^2}} = \frac{x^2}{x^2-4}$

$\cancel{y_p = \frac{1}{1+\frac{2}{x}} \sin x}$

$\cancel{y = \frac{c}{x^2} + \frac{x}{x+2} \sin x}$

$\cancel{1 = \frac{c}{\pi^2} \Rightarrow c = \pi^2}$

$\cancel{y = \frac{\pi^2}{x^2} + \frac{x}{x+2} \sin x}$

Fortsetzung auf Zettel

26/32

Gut

3) $\ddot{y} + p\dot{y} + y = 2t + 1$ Ulrich Clemens

$$\sqrt{+\frac{p^2}{4} - 1} = 0$$

$$+\frac{p^2}{4} = 1$$

$$+p^2 = 4 \Rightarrow p = 2$$ (nur positiver Dämpfungsgrad)

$$\ddot{y} + 2\dot{y} + y = 2t + 1$$

$$\cancel{\lambda^2 + 2\lambda + 1 = 0}$$

$$y_h = e^{-t}(c_1 + c_2 t)$$

$$\cancel{\dot{y} = -e^{-t}(c_1 + c_2 t) + e^{-t} \cdot c_2}$$

$$\cancel{0 = c_1}$$

$$\cancel{2 = -c_1 + c_2 \Rightarrow c_2 = 2}$$

y_p: $y = a \cdot t + b$

$\dot{y} = a$

$\ddot{y} = 0$

$$2a + a \cdot t + b = 2t + 1$$

$$a = 2 \qquad 2a + b = 1$$

$$b = -3$$

$$y = y_h + y_p = e^{-t}(c_1 + c_2 t) + 2t - 3$$

$$\dot{y} = -e^{-t}(c_1 + c_2 t) + e^{-t} \cdot c_2 + 2$$

3) $y(0)=0 \qquad \dot{y}(0)=2$

$$0 = c_1 - 3 \quad \Rightarrow \quad c_1 = 3$$

$$2 = -c_1 + c_2 + 2 \quad \Rightarrow \quad c_2 = 3$$

$$\Rightarrow y = e^{-t}(3+3t) + 2t - 3$$

9

4) $\cancel{y_p = \frac{x^2}{x^2-4} \cos x + \frac{2\sin}{x}}$ $\qquad b = -\frac{2}{x}\left(\frac{x^2}{x^2-4}\right) = -\frac{2x^2}{x^3-4x} = -\frac{2x}{x^2-4}$

$$y_p = \frac{x^2}{x^2-4} \cdot \sin x - \frac{2x}{x^2-4} \cdot \cos x$$

$$y = \frac{c}{x^2} + \frac{x^2}{x^2-4} \cdot \sin x - \frac{2x}{x^2-4} \cdot \cos x$$

$$1 = \frac{c}{\pi^2} + \frac{\pi^2}{\pi^2-4} \cdot \sin \pi - \frac{2\pi}{\pi^2-4} \cdot \cos \pi$$

$$1 = \frac{c}{\pi^2} + \frac{2\pi}{\pi^2-4}$$

$$\cancel{c = \pi^2 - \frac{2\pi^3}{\pi^4 - 4\pi^2}}$$

$$c = \pi^2 - \frac{2\pi^3}{\pi^2-4}$$

$$\Rightarrow y = \left(\pi^2 - \frac{2\pi^3}{\pi^2-4}\right) \cdot \frac{1}{x^2} + \frac{x^2}{x^2-4} \cdot \sin x - \frac{2x}{x^2-4} \cdot \cos x$$

5