Uhl

1. Schularbeit 4AHELT 31. 10. 2012

Gruppe: B

1) Geg.: k: $x^2 + y^2 = 36$ g: $y = \sqrt{3} \cdot x = x\sqrt{3}$ 8

Ges.: V_y von $A_{k,g}$ und der y-Achse im 1. Quadranten

2) Berechne das Integral: 6

$$\int \frac{3x - 5}{x^2 + 2x - 8}\,dx$$

3) Berechne den Konvergenzradius folgender Potenzreihe ! 6

$$\sum_{n=1}^{\infty} \left(\frac{n!}{n^3} \cdot x^n \right)$$

4) Die Funktion f ist an der Stelle $x = 0$ in eine Taylorreihe zu entwickeln: 7

f: $y = \ln(2 + 2x)$ 5 Glieder

5) Berechne das Integral : 5

$$\int \frac{\ln(x)}{x^2}\,dx$$

HTBL u. VA. ST. PÖLTEN

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Klasse: 4AHELT

Kat.-Nr. 18

1. Schularbeit

aus Mathematik am 31.10.12

Aufgabe:

5) $\int \frac{\ln(x)}{x^2}\,dx$

$u = \ln x$

$u' = \frac{1}{x}$

$v' = \frac{1}{x^2} = x^{-2}$

$v = -\frac{1}{x}$

$u \cdot v - \int u' \cdot v$

$\ln x \cdot \left(-\frac{1}{x}\right) - \int \frac{1}{x}\left(-\frac{1}{x}\right)dx$

$-\frac{\ln|x|}{x} - \frac{1}{x} = \frac{-\ln|x| - 1}{x} + C$

5

2) $x^2 + 2x - 8 = 0 \Rightarrow x_1 = +2$

$x_2 = -4$

$3x - 5 = A(x+4) + B(x-2)$

$x = 2 \Rightarrow 1 = 6A \Rightarrow A = \frac{1}{6}$

$x = -4 \Rightarrow -17 = -6B \Rightarrow B = \frac{17}{6}$

$\int \frac{1}{6(x-2)}\,dx + \int \frac{17}{6(x+4)}\,dx = \frac{\ln|x-2|}{6} + \frac{17\ln|x+4|}{6} + C$ 5

4) $f(x) = \ln(2x+2)$ $f(0) = \ln 2$

$f'(x) = \frac{2}{2x+2} = \left(\frac{1}{x+1}\right)$ $f'(0) = 1$

$f''(x) = -\frac{4}{(2x+2)^2}$ $f''(0) = -1$

$f'''(x) = \frac{16}{(2x+2)^3}$ $f'''(0) = 2$

$f^{IV}(x) = -\frac{96}{(2x+2)^4}$ $f^{IV}(0) = -6$

$f^{V}(x) = \frac{768}{(2x+2)^5}$

$f^{V}(0) = 24$

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4) $\ln(2+2x) = \ln 2 + x - \frac{x^2}{2} + \frac{2x^3}{3!} - \frac{6x^4}{4!} + \frac{24x^5}{5!} = \ldots$ 7

$~~= \ln 2 + \sum_{n=0}^{\infty} n!~~$

$= \ln 2 + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(n-1)!}{n!} x^n = \ln 2 + \sum_{n=1}^{\infty} \frac{x^n}{n} (-1)^{n+1}$

3) $r = \lim_{n\to\infty} \frac{a_n}{a_{n+1}} = \lim_{n\to\infty} \frac{\frac{n!}{n^3}}{\frac{(n+1)!}{(n+1)^3}} = \lim_{n\to\infty} \frac{(n+1)^3}{\cancel{(n+1)}n^3} =$ 2

$= \lim_{n\to\infty} \frac{n^3+3n^2+3n+1}{n^4+n^3} = 0 \Rightarrow$ ~~divergent~~

$\Rightarrow$ nirgends konvergent 6

$(n^2+2n+1)(n+1)$
$= n^3+n^2+2n^2+2n+n+1$
$= n^3+3n^2+3n+1$ 2

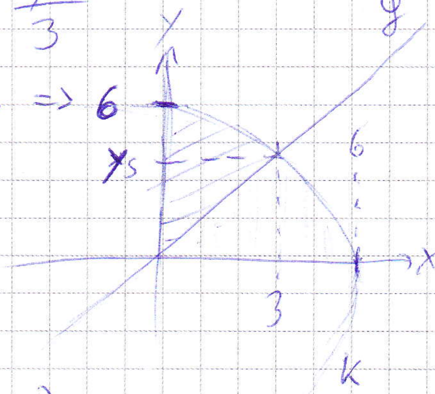
1) $k: y^2 = 36 - x^2$ $\qquad g: y = x\sqrt{3}$

$x^2 = 36 - y^2$ $\qquad y^2 = 3x^2$

$x^2 = \frac{y^2}{3}$

~~$V_y = \pi(\int$~~

$y^2 = 36 \Rightarrow 6$



$36 - x^2 = 3x^2$

$4x^2 = 36 \Rightarrow x_s = 3$

$y_s = g(3) = 3\sqrt{3}$

$V_y = \pi\left(\int_0^{3\sqrt3} 3x^2\,dx + \int_{3\sqrt3}^{6} (36-x^2)\,dx\right)$

$\pi\left(x^3\Big|_0^{3\sqrt3} + \left(36x - \frac{x^3}{3}\right)\Big|_{3\sqrt3}^{6} \right)$

$\pi\left((3\sqrt3)^3 + ~~216 - \frac{(3\sqrt3)^3}{3}~~ + 216 - \frac{6^3}{3} - (36\cdot 3\sqrt3 - \ldots\right.$

$\pi\left(27\sqrt{27} + 144 - 108\sqrt3 + 9\sqrt{27}\right)$

$81\sqrt3 + 144 - 108\sqrt3 + 27\sqrt3$

$\pi(144 +$

4

$$V_y = \pi \int x^2 \, dy$$

$$= \pi \left(\int_0^{3\sqrt{3}} \frac{y^2}{3} \, dy + \int_{3\sqrt{3}}^{6} (36 - y^2) \, dy \right)$$

$$= \pi \left(\frac{y^3}{9} \Big|_0^{3\sqrt{3}} + \left(36y - \frac{y^3}{3}\right) \Big|_{3\sqrt{3}}^{6} \right)$$

$\sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$

$$= \pi \left(3\sqrt{27} + 216 - 72 - \left(108\sqrt{3} - 9\sqrt{27}\right) \right)$$

$$\pi \left(\cancel{3\sqrt{27}}^{\,9\sqrt{3}} + 144 - 108\sqrt{3} + 27\sqrt{3} \right)$$

$$\pi \left(144 - 72\sqrt{3} \right) = 19{,}3\,\pi$$

8

$$V_x = \pi \int y^2 \, dx$$

~~$V_{x_g} = 3 \cdot 3\sqrt{3} \cdot \frac{1}{2} = 3\sqrt{3}$~~

$$\frac{r^2 \pi \cdot h}{3}$$

$$= \frac{27\pi \cdot 3}{3}$$

$$V_{x_g} = 27\pi$$

$$V_x = \pi \left(\cancel{3\sqrt{3}}^{\,27} + \int_3^6 (36 - x^2) \, dx \right)$$

$$\pi \left(\cancel{3\sqrt{3}}_{\,27} + \left(36x - \frac{x^3}{3}\right) \Big|_3^6 \right)$$

$$\pi \left(\cancel{3\sqrt{3}}_{\,27} + 36 \cdot 6 - \frac{6^3}{3} - 36 \cdot 3 + \frac{3^{3 \cdot 2}}{3} \right.$$

$$V_x = 72\pi$$

3) $x + \frac{2}{8}x^2 + \frac{6}{27}x^3 + \frac{24}{64} \cdot x^4 + \frac{120}{125}x^5$

~~$\lim_{n \to \infty} a_n = n!$~~

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \cancel{\lim_{n \to \infty} \frac{(n+1)!}{(n+1)^3}} = \frac{1}{1} = \infty$$

$\Rightarrow$ divergent

$\left.\frac{(\sqrt{3})^3}{3}\right)$

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