A Bandit Approach to Indirect Inference

Felix Steinberger Eriksson

17 May 2023

How to Successfully Gamble Your Way to a Parameter Estimate

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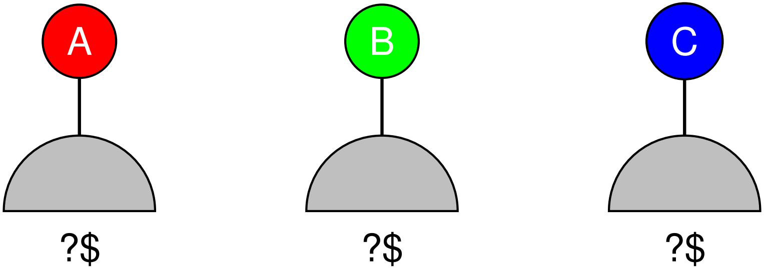
Reward Levers



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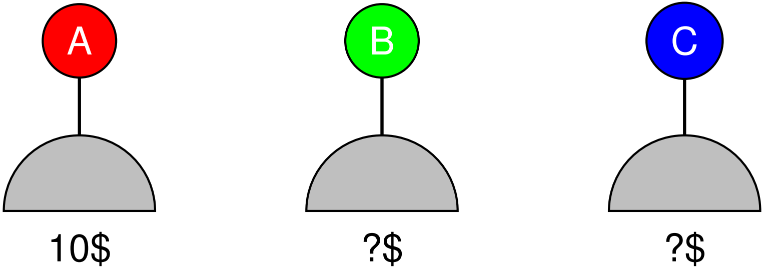

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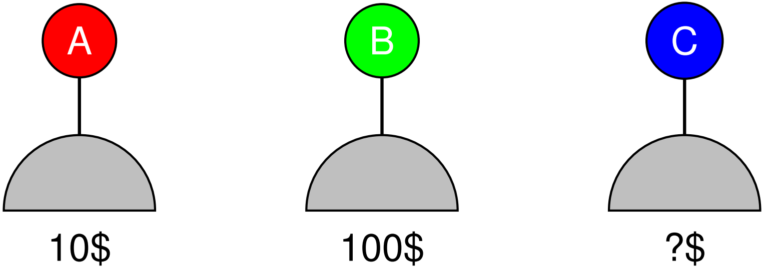

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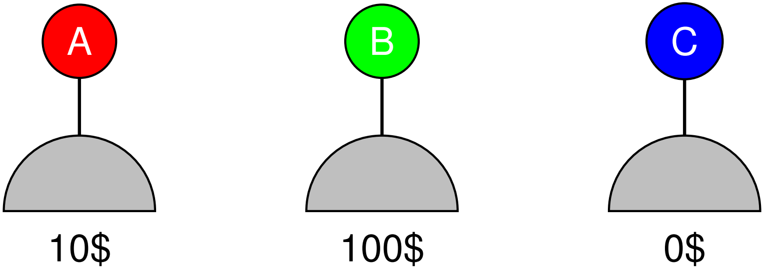

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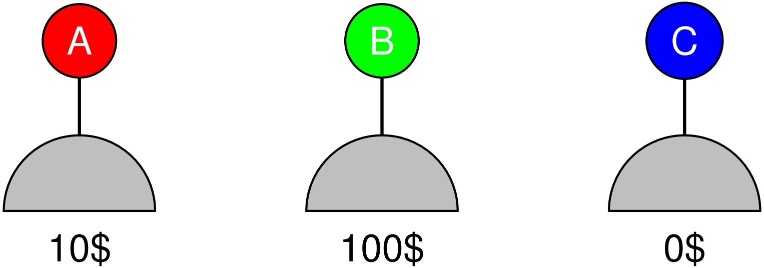
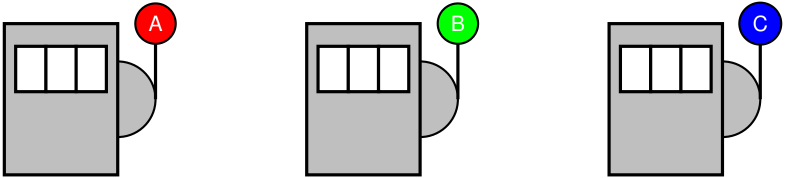


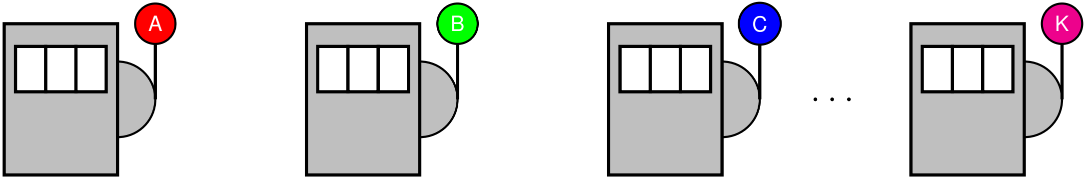
Pull green lever lots, get rich!

Stochastic Bandits



$r_A \sim F_A$ $\quad$ $r_B \sim F_B$ $\quad$ $r_C \sim F_C$

Stochastic Bandits



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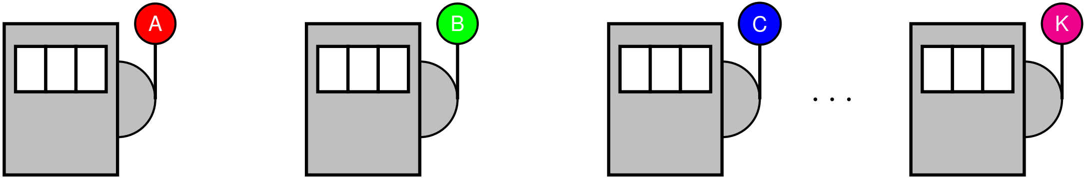
Stochastic Bandits



$r_A \sim F_A$ $r_B \sim F_B$ $r_C \sim F_C$ $r_K \sim F_K$

Random rewards, need some strategy to select arms.

Bandit algorithms

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ϵ-greedy

Keep estimates of rewards of arms. With some small probability ϵ, select an arm uniformly at random. Otherwise, select the arm with best estimated reward.

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Upper Confidence Bound (UCB)

Keep estimates of rewards that incorporate growing uncertainty for arms not played in a long time.

Bandit algorithms

Takeaway

There is theory on **smart** ways to select which arm to play.

Parameter estimation

Collection $\mathcal{M} = \{M(\theta) \big| \theta \in \Theta\}$ of parametric models.

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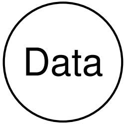
Given data sampled from some $M(\theta^\star)$, want to identify $\theta^\star$.

Parameter estimation



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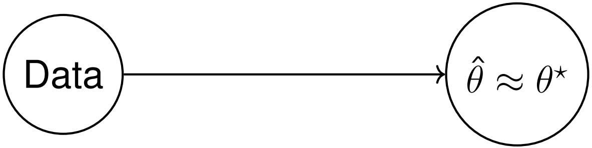
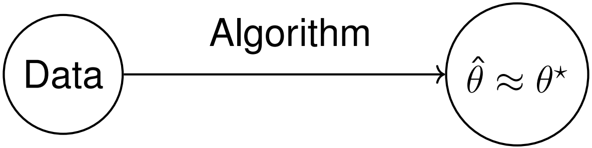


Parameter estimation

$$\text{Data} \xrightarrow{\mapsto} \hat{\theta} \approx \theta^{\star}$$

Parameter estimation



Parameter estimation

$$\text{Data} \xrightarrow{\text{Divine intuition}} \hat{\theta} \approx \theta^{\star}$$

Indirect inference

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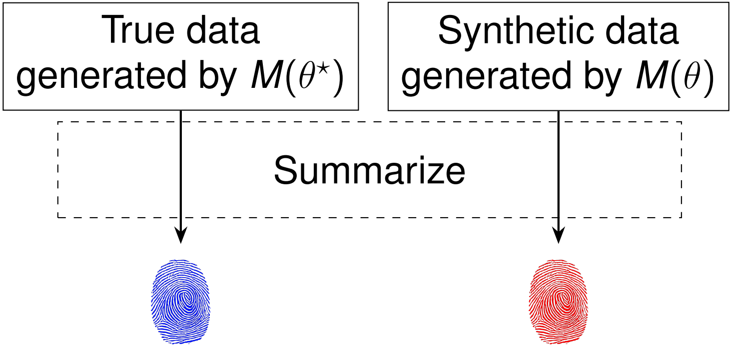
$$\boxed{\begin{array}{c}\text{True data}\\ \text{generated by } M(\theta^\star)\end{array}}$$

Indirect inference

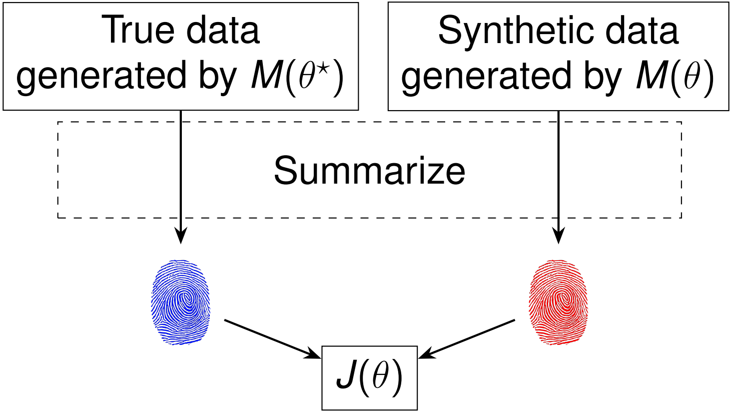
True data generated by $M(\theta^\star)$

Synthetic data generated by $M(\theta)$

Indirect inference



Indirect inference



Indirect inference

True data generated by $M(\theta^\star)$

Synthetic data generated by $M(\theta)$

Summarize

$J(\theta)$

Pick $\hat{\theta} \in \arg\min_{\theta \in \Theta} J(\theta)$!

Easy, right?

$J(\theta)$ has no closed form, but we can evaluate it, right?

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Generate some data, calculate thumbprint, compare similarity.

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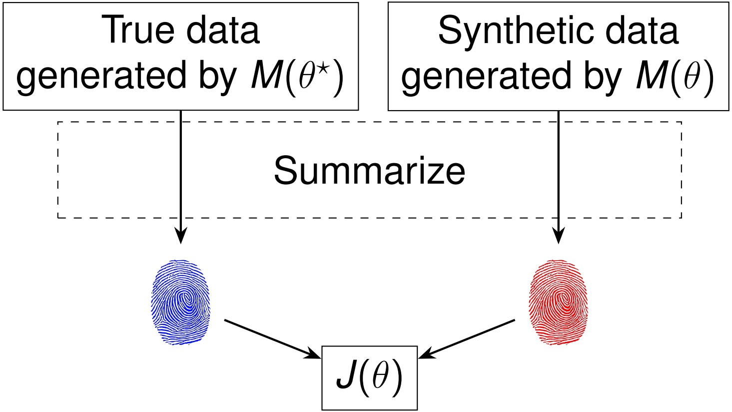
Generate some data, calculate thumbprint, compare similarity.

Use gradient descent or any other standard optimization tool (and hope the function is convex...)

Problem!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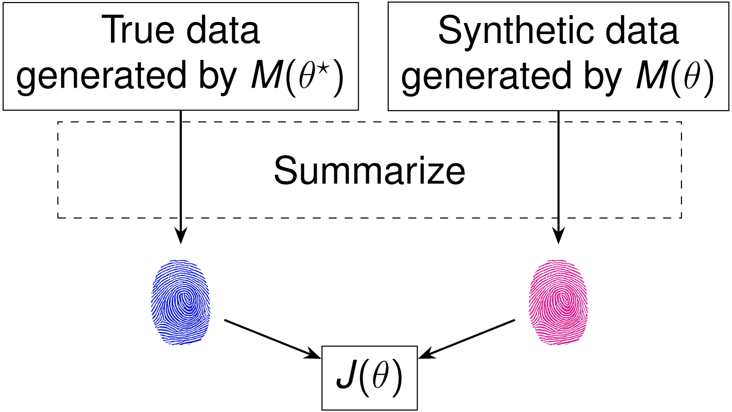
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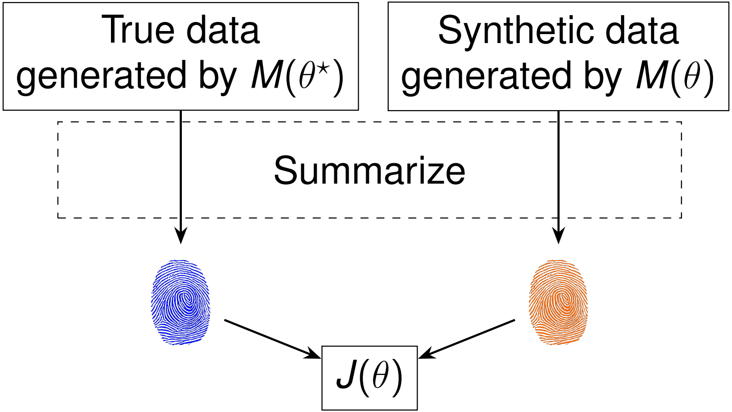
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Problem!

Each "evaluation" of $J(\theta)\dots$

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Each "evaluation" of $J(\theta)\ldots$

is a sample from $F_{J(\theta)}$!

Treat Indirect Inference as a Bandit Problem

Each parameter θ_i in $\Theta = \{\theta_1, \ldots, \theta_K\}$ is an arm with reward distribution $F_{J(\theta_i)}$.

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Each parameter θ_i in $\Theta = \{\theta_1, \ldots, \theta_K\}$ is an arm with reward distribution $F_{J(\theta_i)}$.

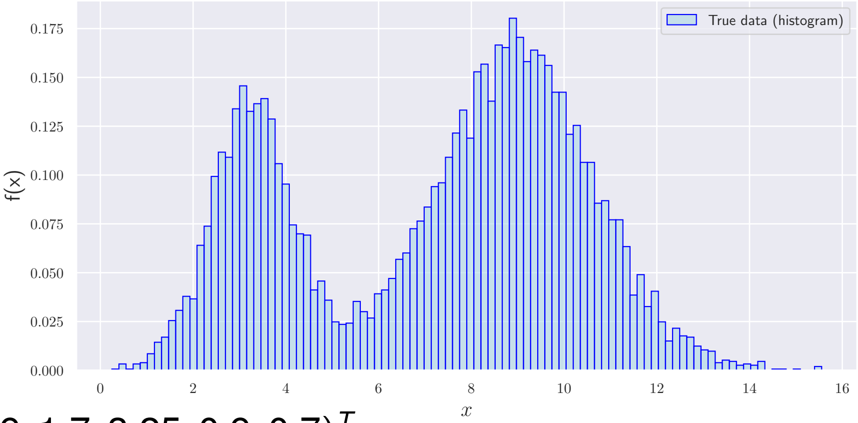
Use bandit optimization methods to find best arm (i.e. best $\theta \in \Theta$).

Bandit Optimization for Indirect Inference works!

Mixture normal model:

$$f_\theta(x) = pN(x; \mu_1, \sigma_1^2) + (1-p)N(x; \mu_2, \sigma_2^2)$$

Bandit Optimization for Indirect Inference works!



$$\theta^{\star} = (9, 1.7, 3.25, 0.9, 0.7)^T$$

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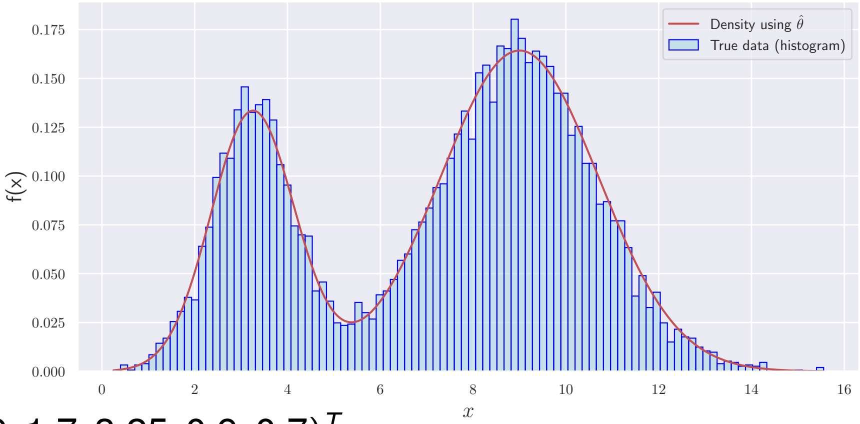

$$\hat{\theta} = (9, 1.7, 3.25, 0.9, 0.7)^T$$

First steps...

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- Discretization

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- Discretization
- More difficult for finer grids due to similar rewards.

Next steps?

Continuous action spaces!

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- Extant theory

Next steps?

Continuous action spaces!

- Extant theory
- Function approximation

Thank You!

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