Monte Carlo Tree Search on a Graphics Processing Unit in the Context of Online Robotic Decision Making

Felix Steinberger Eriksson

Mentors: Soon-Jo Chung and Ben Riviere

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Caltech

Introduction

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- In real scenarios: sequences of such problems, require fast solutions
- Critically, we want anytime algorithms that can provide reasonable solutions if terminated early.

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- For simplicity, consider only finite $\mathcal{X}$ ($|\mathcal{X}| = X < \infty$) and $\mathcal{U}$ ($|\mathcal{U}| = U < \infty$).

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At time t, in state $x_t \in \mathcal{X}$, an agent

- selects an action $a_t \in \mathcal{U}$,
- receives reward $R(x_t, a_t)$, and
- transitions to state $x_{t+1} = F(x_t, a_t)$.

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- In the finite horizon case, a policy is a sequence $(a_h)_h \in \mathcal{U}^H$.
- Objective: find optimal policy/action sequence $(a_h^*)_h \in \mathcal{U}^H$.
- Equivalently: determine $(a_h^*)_h \in \arg\max_{(a_h)_h} \sum_{h=0}^{H-1} R(x_h, a_h)$ where $x_{h+1} = F(x_h, a_h)\ \forall h = 0, \ldots, H-1$.

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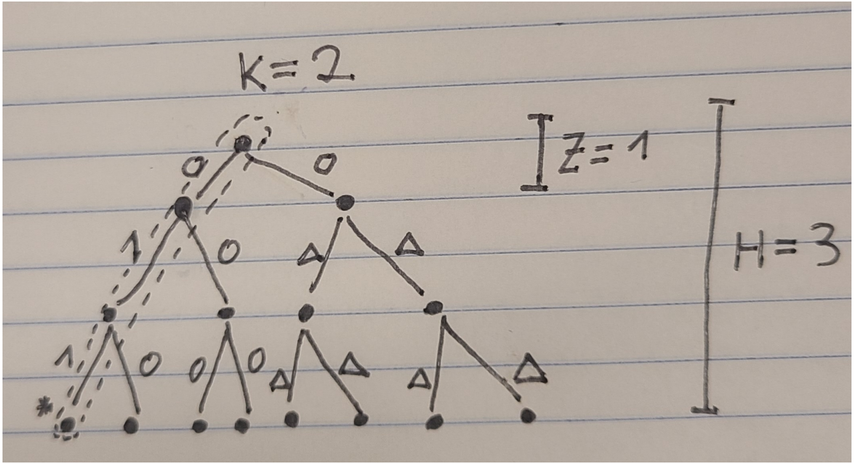
- Canonical MDP: Delayed Dense Needle with Gap

Delayed Dense Needle with Gap



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4) Backpropagate collected statistics about states on trajectory.

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- Similar to Model Predictive Control. To compute the next optimal action, explore exhaustively for the next κ steps, then sample m rollouts until termination.
- With exploration horizon κ and per-node sampling budget m selected appropriately, can guarantee an optimal policy with sample complexity $\mathcal{O}(H^2|U|^\kappa m)$, where H is the horizon of the MDP and $|U|$ is the number of actions in the fixed, finite action set U.

Problems with GORP

- Not adaptive: κ and m need to be large enough to find the correct action at each iteration. In many cases, this leads to over-computation for easier subproblems. Future research direction.

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- Often, computing these is at least as expensive as solving the MDP in question.

Doubling trick

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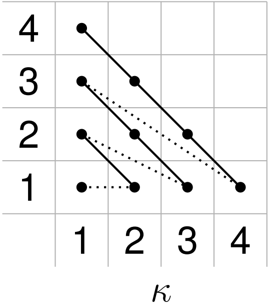
$$p = \log_{|U|} m$$



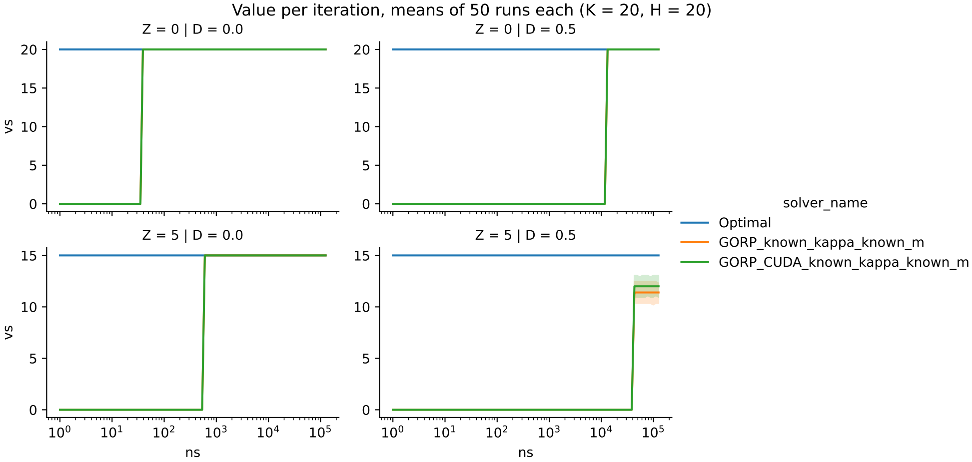
Implementation

- Employ GPGPU programming in CUDA to achieve orders of magnitude better real-time performance than possible on a CPU.

Results

- Speedups of factor 70 to 140 on canonical evaluation MDP.

Results


Value per iteration, means of 50 runs each (K = 20, H = 20)
Z = 0 | D = 0.0
Z = 0 | D = 0.5
Z = 5 | D = 0.0
Z = 5 | D = 0.5
vs
ns
solver_name
Optimal
GORP_known_kappa_known_m
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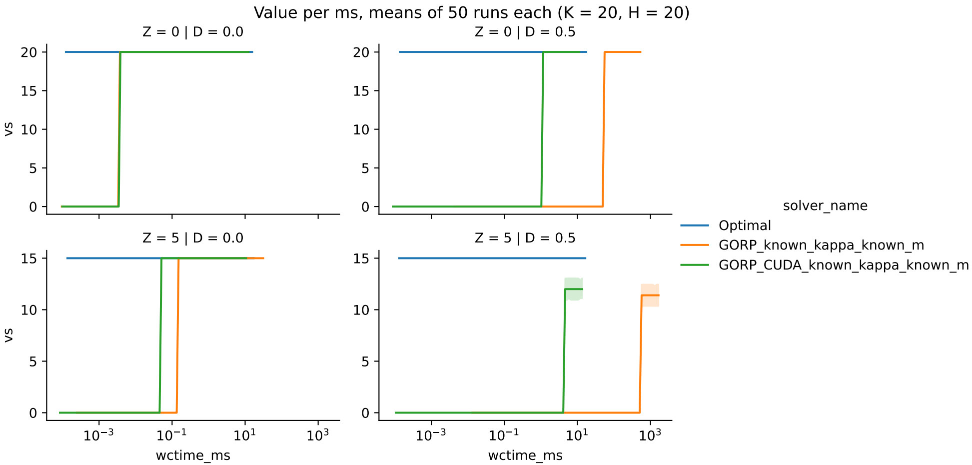
Value per ms, means of 50 runs each (K = 20, H = 20)
Z = 0 | D = 0.0
Z = 0 | D = 0.5
Z = 5 | D = 0.0
Z = 5 | D = 0.5
vs
wctime_ms
solver_name
Optimal
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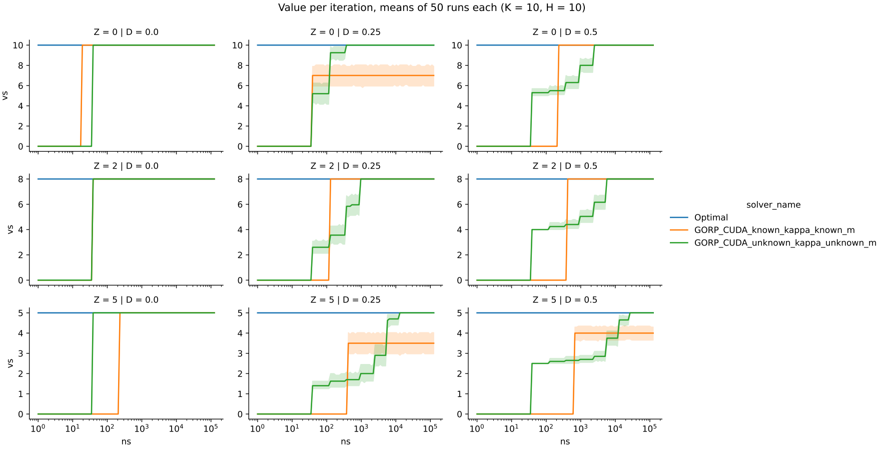

Value per iteration, means of 50 runs each (K = 10, H = 10)
Z = 0 | D = 0.0
Z = 0 | D = 0.25
Z = 0 | D = 0.5
Z = 2 | D = 0.0
Z = 2 | D = 0.25
Z = 2 | D = 0.5
Z = 5 | D = 0.0
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$\Longrightarrow$ A strong case for feasibility of MCTS in real-time applications like robotic planning.

Acknowledgements

Thank you!
Reach out at felixse@kth.se.

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