7.18 Theorem *There exists a real continuous function on the real line which is nowhere differentiable.*

Proof Define

(34) $$\varphi(x) = |x| \qquad (-1 \le x \le 1)$$

and extend the definition of $\varphi(x)$ to all real x by requiring that

(35) $$\varphi(x+2) = \varphi(x).$$

Then, for all s and t,

(36) $$|\varphi(s) - \varphi(t)| \le |s - t|.$$

In particular, φ is continuous on R^1. Define

(37) $$f(x) = \sum_{n=0}^{\infty} \left(\tfrac{3}{4}\right)^n \varphi(4^n x).$$

Since $0 \le \varphi \le 1$, Theorem 7.10 shows that the series (37) converges uniformly on R^1. By Theorem 7.12, f is continuous on R^1.

Now fix a real number x and a positive integer m. Put

(38) $$\delta_m = \pm \tfrac{1}{2} \cdot 4^{-m}$$

where the sign is so chosen that no integer lies between $4^m x$ and $4^m(x + \delta_m)$. This can be done, since $4^m |\delta_m| = \frac{1}{2}$. Define

(39) $$\gamma_n = \frac{\varphi(4^n(x+\delta_m)) - \varphi(4^n x)}{\delta_m}.$$

When $n > m$, then $4^n \delta_m$ is an even integer, so that $\gamma_n = 0$. When $0 \le n \le m$, (36) implies that $|\gamma_n| \le 4^n$.

Since $|\gamma_m| = 4^m$, we conclude that

$$\begin{aligned} \left| \frac{f(x+\delta_m) - f(x)}{\delta_m} \right| &= \left| \sum_{n=0}^{m} \left(\frac{3}{4}\right)^n \gamma_n \right| \\ &\ge 3^m - \sum_{n=0}^{m-1} 3^n \\ &= \tfrac{1}{2}(3^m + 1). \end{aligned}$$

As $m \to \infty$, $\delta_m \to 0$. It follows that f is not differentiable at x.

Rudin, Walter (1976). Principles of mathematical analysis (Third ed.). New York

A Bandit Approach to Indirect Inference

Erik Ildring

2

A Bandit Approach to Indirect Inference

1

Parameter estimation

Family of parametric models:

$$\mathscr{M} = \{M(\theta) \ : \theta \in \Theta\}$$

Parameter estimation method:

$$D \mapsto \theta \in \Theta$$

Indirect Inference

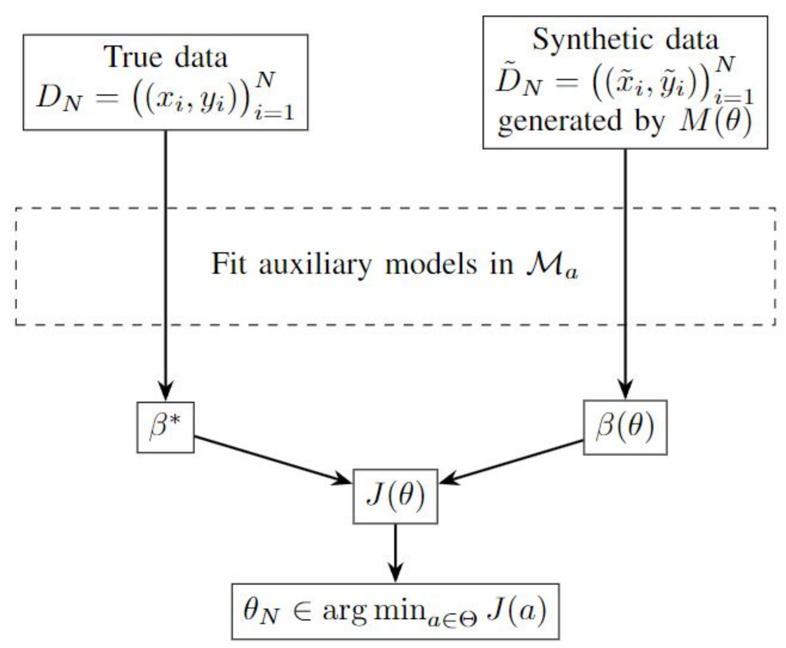
Data points: D_N (By physical process)

Models: $\mathscr{M} = \{M(\theta) : \theta \in \Theta\}$ (Intractable)

Intractable: Mapping from D_N to θ difficult to describe or computationally expensive to calculate.

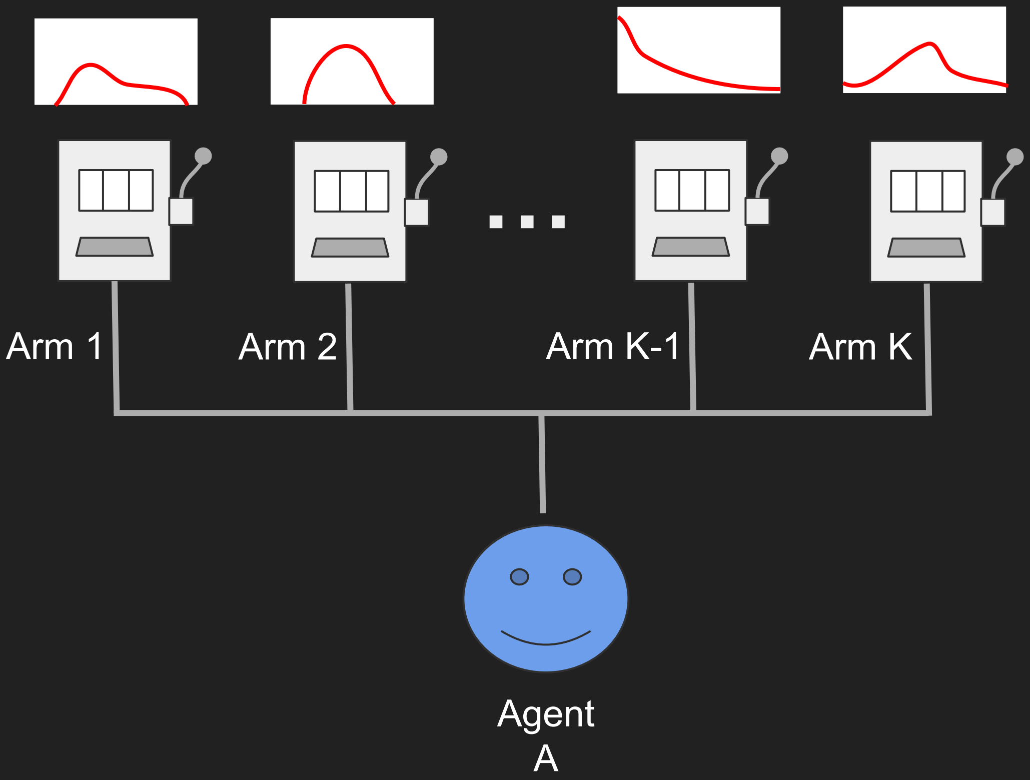
Auxiliary models: $\mathscr{M}_a = \left\{M_a(\beta) : \beta \in B \subseteq \mathbb{R}^k\right\}$

Indirect Inference (cont.)



$$J(\theta) = (\beta(\theta) - \beta^*)^T W (\beta(\theta) - \beta^*)$$

Stochastic bandit



At time t…

Goal: In T rounds, find the arm with the best expected reward.

Regret

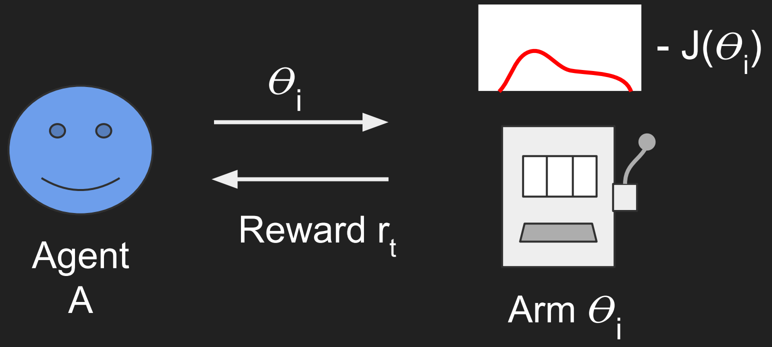
Expected reward of a_t	Best expected reward
$\mu(a_t)$	μ^*

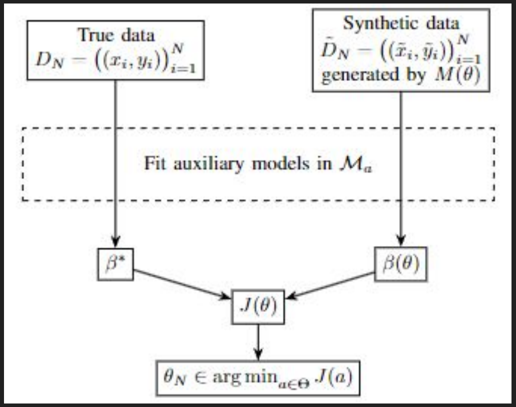
$$R^A(T) = T\mu^* - \sum_{t=1}^{T} \mu(a_t)$$

Cost of learning

What we have done…

- Treat each parameter as an arm
- Let reward distribution for each arm θ_i be - $J(\theta_i)$
- Agent explores set of arms to determine the arm with largest expected reward, in other words finds the parameter which minimizes J.





Example: Gaussian random variables

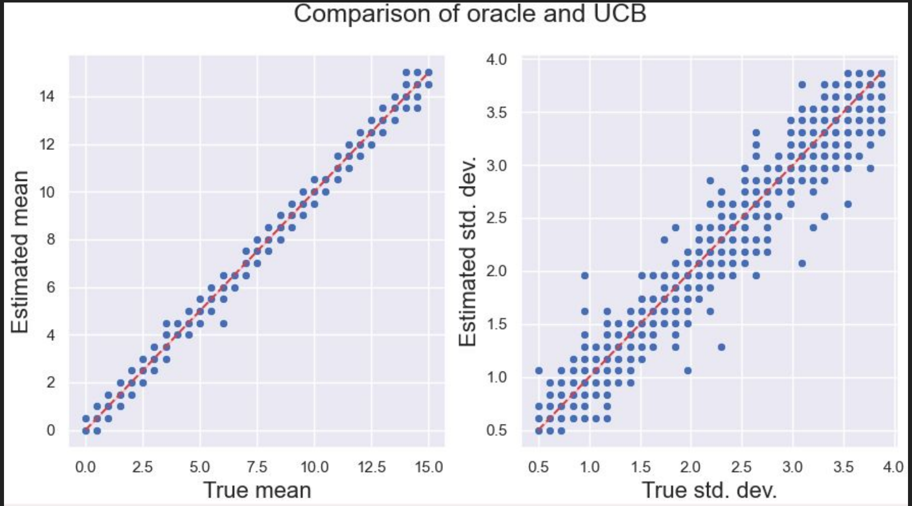
Data: $D \sim N\left(\mu, \sigma^2\right)$

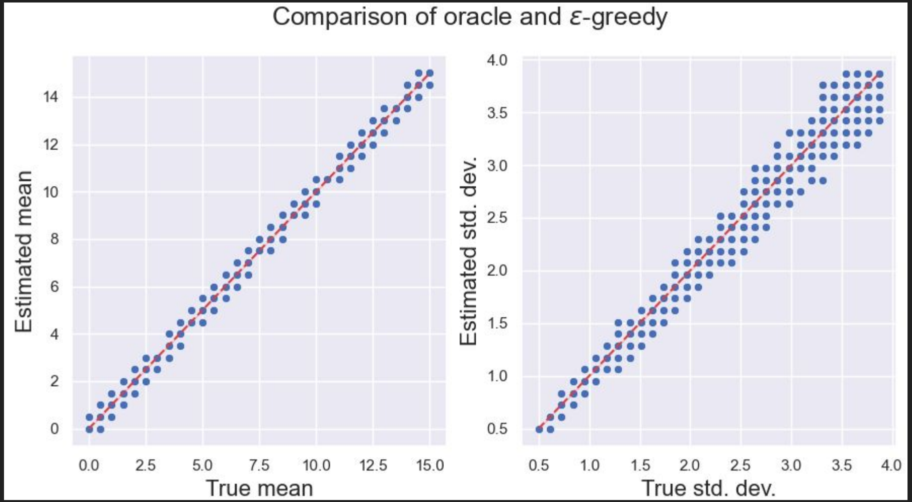
Models: $M(\theta) = N\left(\theta_1, {\theta_2}^2\right)$

Auxiliary parameters: $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ $\quad S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}$

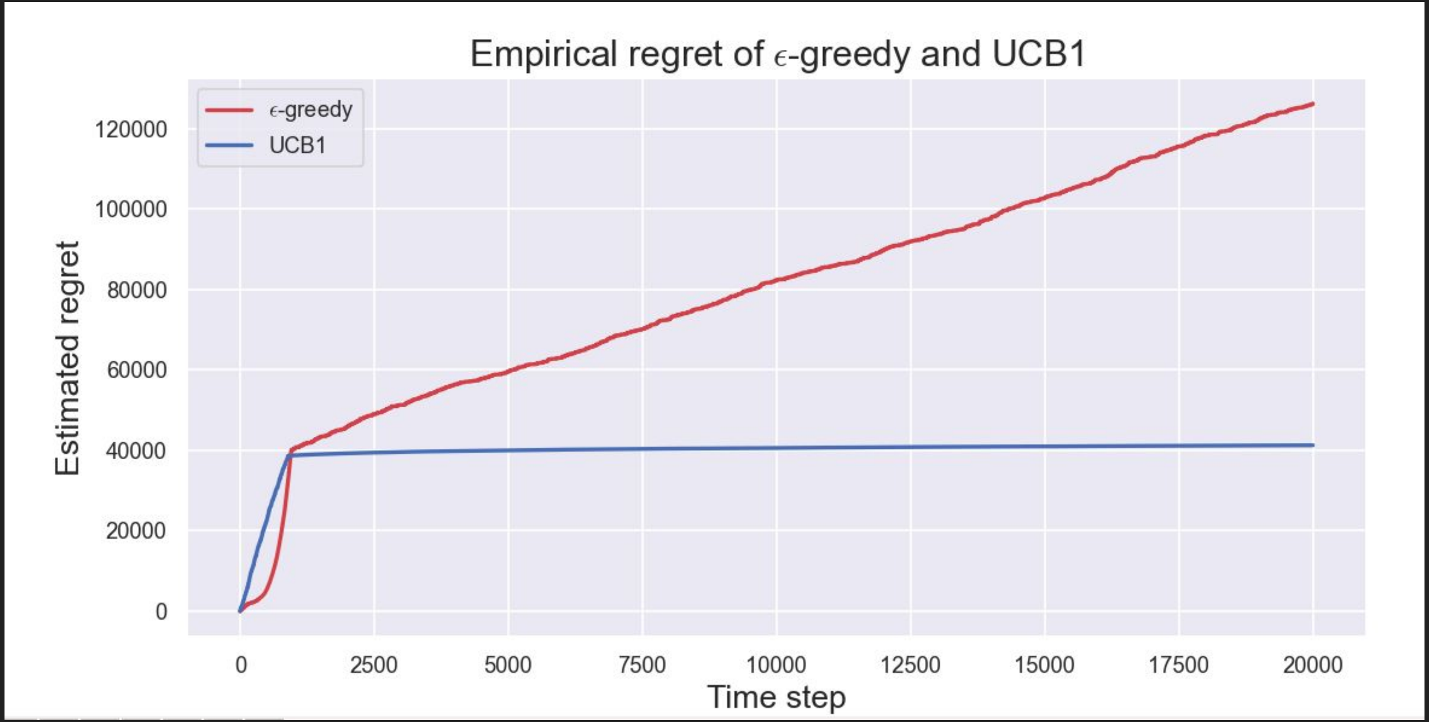
Agents: UCB1 and ϵ-greedy

Result: Gaussian random variables





Result: Gaussian random variables



$$\mu = 3.10, \ \ \sigma^2 = 2.82^2$$

Conclusions

- Present indirect inference as a stochastic bandit problem
- Demonstrated to work on simple example

Example: Bimodal Gaussian random variables

Data: $D \sim p \cdot N\left(\mu_1, \sigma_1{}^2\right) + (1 - p) \cdot N\left(\mu_2, \sigma_2{}^2\right)$

Models: $M(\theta) = \theta_5 \cdot N\left(\theta_1, \theta_2{}^2\right) + \left(1 - \theta_5\right) \cdot N\left(\theta_3, \theta_4{}^2\right),$

$\theta = \left(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5\right)^T$

Auxiliary parameters: k-quantiles

Agents: UCB1

Result: Bimodal Gaussian random variables