

File Edit View Insert Cell Kernel Widgets Help Trusted Python 3 (ipykernel) ○

Title: 4.2 Exercises
Author: Chad Wood
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Modified By: Chad Wood
Description: This program demonstrates the use of python to perform mathematical calculations and interpretations

Calculating Limits Numerically

Let $f(x) = 2x^3 - 4x + 1$ and let $g(x) = (e^x - 1)/x$

(a) Use Python code to evaluate the function f at appropriate values to calculate $\lim_{x \rightarrow 3} f(x)$. Hint: Evaluate f at values closer and closer to 3 and see what value the outputs get close to.

```
In [25]: # Builds a list that gets exponentially closer to limit
def h(lim):
    return [abs(lim + 0.1**i) for i in range(7)]
```

```
In [26]: # The provided mathematical function
def f(x):
    return (2*x)**3 - (4*x) + 1

# Evaluates the derivative using approximate values
lim = 3
for x in h(lim):
    der = f(x)
    print(der)

# Rounding whole happens to work here.
print(f'The derivative of f(x) at x={lim} is {round(der)}.')

8193.0
2956.2672000000007
2627.7331843199995
2596.4577283840317
2593.345617280385
2593.0345601728004
2593.003456001729
The derivative of f(x) at x=3 is 2593.
```

(b) Use Python code to evaluate the function g at appropriate values to calculate $\lim_{x \rightarrow 0} g(x)$.

```
In [27]: import math

# The provided mathematical function
def g(x):
    return (math.exp(x)-1) / x

# Evaluates the integral using approximate values
lim = 0
for x in h(lim):
    der = g(x)
    print(der)

print(f'The derivative of g(x) at x={lim} is {round(der)}.')

1.718281828459045
1.0517091807564771
1.0050167084167947
1.0005001667083844
1.0000500016671408
1.0000050000069647
1.0000004999621832
The derivative of g(x) at x=0 is 1.
```

(c) Could the limits in parts (a) and (b) be calculated by plugging in the value where the limit is taken into the function? Explain. What does this mean in terms of continuity?

With example (a), the limit can be calculated simply by plugging in the value where the limit is taken. This is because it is a polynomial and polynomials are continuous. It also means the left handed and right handed limits are equal.

Example (b), on the other hand, can not be calculated by plugging in the value where the limit is taken. Attempting to do so would result in an undefined solution due to division by 0.

Average Rate of Change

The formula for the average rate of change of a function f between two values a and b is given by:

Average Rate of Change = $\frac{f(b) - f(a)}{(b - a)}$

Create a Python function that takes in a mathematical function, e.g., $f(x) = 3x^2$ and two numbers a and b and returns the average rate of change of the function between a and b .

```
In [4]: # Function distributes variables a, b to f(x)
# using lambda with eval() interpreter
def AvgRateOfChng(fun: str, a, b):
    f = lambda x: eval(fun)
    return (f(b) - f(a)) / (b - a)

# Tests function
AvgRateOfChng('3*x**2', 2, 4)
```

Out[4]: 54.0

Average Rate of Change to Instantaneous Rate of Change

A baseball is dropped from a tall cliff. Neglecting air resistance, the distance traveled by the baseball in meters after t seconds is given by the function $f(t) = 4.9t^2$.

(a) Find the average speed (average rate of change of distance) of the baseball between 5 and 6 seconds.

(b) Find the average speed (average rate of change of distance) of the baseball between 5 and 5.5 seconds.

(c) Find the average speed (average rate of change of distance) of the baseball between 5 and 5.1 seconds.

(d) What is the instantaneous speed of the baseball at $t = 5$ seconds? Hint: You might want to do a few more calculations similar to those done in parts (a) - (c).

(e) Find the derivative of f . Hint: The derivative of a quadratic function $f(x) = ax^2 + bx + c$ is given by $f'(x) = 2ax + b$.

(f) Evaluate the derivative of f at $t = 5$. How does this value compare to the value found in part (d)? Explain what is happening.

```
In [5]: # Used in solution D
def f(t):
    return 4.9*(t**2)

# Used in solutions A-C
fun = '4.9*(t**2)'
```

```
In [6]: # Solutions A-C
sol_a = AvgRateOfChng(fun, 5, 6)
sol_b = AvgRateOfChng(fun, 5, 5.5)
sol_c = AvgRateOfChng(fun, 5, 5.1)

# Solution D
lim = 5
sol_d = [f(t) for t in h(lim)]

# Solution E
a, x, b = 4.9, 5, 0
sol_e = 2*x*x + b

# Solution F
sol_f = f(5)
```

```
In [7]: # Prints answers
from pprint import pprint

# w refers to written in answers
answers = {
    '(a)': sol_a,
    '(b)': sol_b,
    '(c)': sol_c,
    '(d)': sol_d,
    '(e)': sol_e,
    '(f)': sol_f,
    '(fw)': '''The solution of exercise (f) is the same as that found with exercise (d). This is because the type of evaluation used in (f) is compatible with the function due to the function being continuous.'''
}

pprint(answers)
```

{'(a)': 53.89999999999999,

'(b)': 51.4500000000002,

'(c)': 49.49000000000016,

'(d)': [78.4,

117.6490000000003,

122.01049000000002,

122.4510048999999,

122.4951000490003,

122.4999510000049,

122.4999951000004,

122.4999951000002,

122.4999951000001,

'(e)': 49.0,

'(f)': 122.5000000000001,

'(fw)': 'The solution of exercise (f) is the same as that found with exercise (d). This is because the type of evaluation used in (f) is compatible with the function due to the function being continuous.'

'(d).\\n'

' This is because the type of evaluation used in (f) is compatible '

'with the function \\n'

' due to the function being continuous.'

}
pprint(answers)

{'(a)': 44200,

'(b)': 2400*exp(2400*c - 1800*y + 1600),

'(bw)': ''The partial derivative of C is simply the slope of the tangent. Geometrically, S is the y-axis and C is the x-axis. We get 2400 because the remaining portion are constants and equal 0.'',

'(c)': 'The partial derivative of S is simply the slope of the tangent. Geometrically, S is the y-axis and C is the x-axis. We get -1800 because the remaining portion are constants and equal 0.'',

'(cw)': ''The partial derivative of Y is simply the slope of the tangent. Geometrically, S is the y-axis and Y is the x-axis. We get -1800 because the remaining portion are constants and equal 0.'',

'(d)': -1800*exp(2400*c - 1800*y + 1600),

'(e)': 'The partial derivative of Y is simply the slope of the tangent. Geometrically, S is the y-axis and Y is the x-axis. We get -1800 because the remaining portion are constants and equal 0.'',

'(f)': -1800*exp(2400*c - 1800*y + 1600),

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'(c)': 'The partial derivative of S is simply the slope of the tangent. Geometrically, S is the y-axis and C is the x-axis. We get -1800 because the remaining portion are constants and equal 0.'',

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