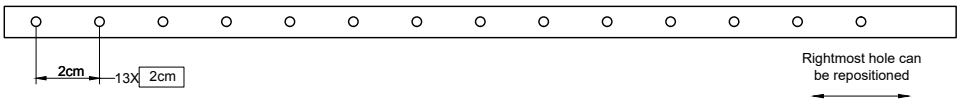
Synthesis Challenge 1

You are challenged to design a simple gear train! Your gear train will use the frame shown below. There are 14 holes positioned along the same straight line. The hole centers are spaced 2 cm apart. To specify the gear train you must decide how many rotating shafts you are going to use, which of the 14 holes they will go in, and the radius of the gear that is to be placed on each shaft.



Your gear train must transmit power from the input shaft (leftmost hole) to the output shaft (rightmost hole). For 2023, it must have an 8:1 speed reduction ratio, which means that 8 turns of the input shaft must result in 1 turn of the output shaft, in the same direction!

- You can specify gear shafts to be placed in any of the holes.
- You can specify up to 14 rotating shafts, and there must be exactly one gear fixed to each shaft.
- Imagine the gears to be circles–a pair of gears mesh properly when the circles just touch each other.
- The gears can be of any radii between 0.5 cm and 20 cm as long as all the radii can be written accurately as decimals (e.g. 1.14 cm is acceptable, $1\frac{1}{3}$ cm is not allowed).
- The rightmost hole (hence the output shaft) can be moved to the left or right of its preferred position, as far as needed to make the gears mesh.

Design objectives

More than one design could satisfy the requirements stated above! But some designs will be better than others. Your aim is to minimise (1) the number of shafts used and the total area of the circles representing the gears, which indicates cost, and (3) the distance the output shaft has to be repositioned from its preferred location, 26 cm from the input shaft.

Submission

You must submit your solution using the Canvas Quiz by the specified date and time. The quiz provides 14 text boxes to represent the 14 holes in the same sequence. If your design calls for a gear shaft to go in a particular hole, enter the radius of the gear into the corresponding text box. If your design does not call for a shaft in a particular hole, leave that text box blank.

For example, if your design calls for two gears, where the first hole has a gear of radius 10cm, and the 14th hole has a gear of radius 18 cm, enter 10 in the first text box and 18 in the last text box, leaving the others empty.

Assessment

This challenge is worth 2 points and you should allow a couple of hours to complete it. For 2023, 50% of the mark will be gained by submitting a design that satisfies the requirements. The remaining 50% is determined by how well your design performs in comparison with the rest of the class. So it will be worth spending time to iterate and improve on your first solution!The performance of your design, P, will be calculated as:

$$P = An + B\sum_{i=1}^{n} r_i^2 + Cd^3$$

- $A = 7.0$, $B = 0.3$, $C = 9.0$.
- n = the total number of gears used in your design
- r_i = radius of gear i, in cm
- d = distance the output gear must be repositioned from its preferred location which is 26 cm away from the input shaft, in cm.

It is essential to remember that a lower value of P is better.

We have provided a doodling template to help you develop your ideas. Hint: You might not find a solution using algebra. You will need to iterate between synthesis and analysis, proposing ideas, checking how good they are,

and iterating to improve them. Remember, this is the essence of design! It will help to be systematic and keep track of what you have tried.

A little bit of theory

You will need to know that the speed ratio of two meshing gears is governed by the following formula:

$$\frac{v_2}{v_1} = -\frac{r_1}{r_2} \tag{1}$$

Where r_1 and r_2 are the radii of the two gears, and v_1 and v_2 are their speeds. The minus symbol indicates that when two gears mesh, their directions of rotation will be opposite.

For a gear train with more than two meshing gears this equation can be expanded. For example, consider a train of 3 gears in which gear 1 meshes to gear 2, then gear 2 meshes to gear 3. For this case we would write:

$$\frac{v_3}{v_2}\frac{v_2}{v_1} = (-\frac{r_1}{r_2})(-\frac{r_2}{r_3}) \tag{2}$$

Simplifying,

$$\frac{v_3}{v_1} = \frac{r_1}{r_3} \tag{3}$$

Note how the middle gear (Gear 2) is eliminated from the calculation and does not affect the reduction ratio! But it does affect the direction of rotation.

Teamwork

This is an individual assignment. You can discuss the challenge with your friends but if you share a solution, or use someone else's solution, you will be CHEATING. There are an infinite number of possible solutions, so it is unlikely that we will see many identical ones.

David Wynn, March 2023.